A dual adaptive control theory inspired by Hebbian associative learning

<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>As Published</td>
<td><a href="http://dx.doi.org/10.1109/CDC.2009.5400831">http://dx.doi.org/10.1109/CDC.2009.5400831</a></td>
</tr>
<tr>
<td>Publisher</td>
<td>Institute of Electrical and Electronics Engineers</td>
</tr>
<tr>
<td>Version</td>
<td>Final published version</td>
</tr>
<tr>
<td>Citable Link</td>
<td><a href="http://hdl.handle.net/1721.1/59424">http://hdl.handle.net/1721.1/59424</a></td>
</tr>
<tr>
<td>Terms of Use</td>
<td>Article is made available in accordance with the publisher’s policy and may be subject to US copyright law. Please refer to the publisher’s site for terms of use.</td>
</tr>
<tr>
<td>Detailed Terms</td>
<td></td>
</tr>
</tbody>
</table>
A dual adaptive control theory inspired by Hebbian associative learning

Jun-e Feng, Chung Tin and Chi-Sang Poon IEEE Fellow

Abstract—Hebbian associative learning is a common form of neuronal adaptation in the brain and is important for many physiological functions such as motor learning, classical conditioning and operant conditioning. Here we show that a Hebbian associative learning synapse is an ideal neuronal substrate for the simultaneous implementation of high-gain adaptive control (HGAC) and model-reference adaptive control (MRAC), two classical adaptive control paradigms. The resultant dual adaptive control (DAC) scheme is shown to achieve superior tracking performance compared to both HGAC and MRAC, with increased convergence speed and improved robustness against disturbances and adaptation instability. The relationships between convergence rate and adaptation gain/error feedback gain are demonstrated via numerical simulations. According to these relationships, a tradeoff between the convergence rate and overshoot exists with respect to the choice of adaptation gain and error feedback gain.

I. INTRODUCTION

The remarkable ability of the nervous system to discern unknown or underdetermined changes in the environment has attracted much attention in the control systems and computer science literature. One reason of its rising popularity is that it inspires new insights for tackling some difficult issues such as robustness and computation efficiency, which are important for control systems in real-life problems. After all, the brain is an excellent example of an intelligent controller in many aspects.

In this regard, a biologically plausible adaptive control strategy will be of extreme interest to the parties of both control theory and their biological counterpart. Questions of biological realism are important not only for the sake of physiological validity, but also because they may reveal important design principles that are unique to biological neural networks. Such biologically based principles may lend improved control performance more compatible brain-like behavior that is otherwise not possible. Biological plausibility is also essential consideration in any hardware (e.g. VLSI) implementation of the model for massively parallel computation.

On the other hand, the theory of adaptive control has been extensively studied in the last several decades, and has inspired many applications in the areas of robotics, aircraft and rocket control, chemical processes, power system and ship steering, etc. These theories may lend new concepts to the characterization of biologically-based neural control paradigms. In particular, high-gain adaptive control (HGAC) and model reference adaptive control (MRAC) are two basic adaptive control strategies that have found wide applications in these areas [7], [8], [12], [21]. However, there have been few studies on the dependence of convergence rate on various control parameters. Paper [20] establishes an adaptive observer for a class of nonlinear systems with arbitrary exponential rate of convergence. Tracking errors and prediction errors are combined in [29] in order to further improve the performance of the adaptive controller. In [30], the adaptive controller is constructed using an estimate of prediction error and the proportional-integral or integral-relay adjustment law. These previous results suggest that suitable combinations of different adaptation strategies may be advantageous in improving the overall performance of the resultant adaptive system.

In this paper, we propose a biologically plausible neural control paradigm based on the theory of classical conditioning and operant conditioning [1] and show that they are analogous to a combination of HGAC and MRAC. Figure 1 shows a hypothetical neuronal mechanism of reinforcement learning based on operant conditioning. A strong stimulus (unconditioned stimulus, $z_{US}(t)$) arriving at a synaptic site may induce significant changes in post-synaptic potential which, when paired with a weak input (conditioned stimulus, $z_{CS}(t)$), may strengthen the synapse in the conditioned stimulus pathway through some associative form of Hebbian long-term potentiation [2], [3]. In operant conditioning, the reinforcement signal $z_{US}(t)$ may derive from the behavioral response $y(t)$ in a closed loop. In particular, if $z_{US}(t)$ is a feedback error signal (e.g. tracking error in a movement task) the resultant learning rule is same as the delta rule commonly used in feedforward neural networks [4], [5], [6] and MRAC [29], [8].

It turns out that implementing MRAC with Hebbian learning rules concurrently brings about HGAC, since $z_{US}(t)$ itself is a powerful input that may exert profound influence on system response. The resultant dual control model is interesting in two respects. From an engineering standpoint, dual adaptive control proves to afford improved system performance that is not possible with either mode alone. From a biological standpoint, dual adaptive control by Hebbian learning suggests a possible mechanism whereby intelligent control may be achieved by biological systems.

This paper is organized as follows. We begin in the follow-
Section with a brief overview of HGAC and MRAC and a necessary lemma to be used in the subsequent analysis. We also examine the stability and convergence rate of HGAC and MRAC and their relations to the corresponding adaptation gains. In section III, we will formulate the dual adaptive control (DAC) strategy. Its performance is tested against several types of uncertainties via numerical simulation. Finally, section IV concludes the whole paper.

![Diagram](image)

Fig. 1. Feedback error learning based on operant conditioning. A conditioned stimulus $z_{CS}(t)$ with weak connectivity may become sensitized after repetitive pairing with a strong conditioned stimulus, $z_{US}(t)$. In this case, $z_{US}(t)$ is the error feedback sign and is itself a powerful input that may exert profound influence on system response. The $z_{US}(t)$ or $e(t)$ pathway also undergoes homosynaptic Hebbian learning so that its connectivity is strengthened by the continued presence of the feedback signal.

II. PRELIMINARIES AND BACKGROUND

In this section we give a brief outline for two stable adaptive control strategies: high-gain adaptive control (HGAC) and model reference adaptive control (MRAC). Both strategies have been extensively studied in traditional control literature. In what follows we consider only linear, strictly minimum-phase SISO systems unless otherwise stated.

A. High-gain adaptive control

Let $y_m, y_p$ be the reference target and output of the closed-loop system, respectively, and $h_p(s) \equiv y_p \cdot \alpha_p(s)/\beta_p(s)$ be the process to be controlled, where $\alpha_p$ and $\beta_p$ are monic, coprime polynomials and $y_p$ is a non-zero constant. Assume that:

(p1) The sign of $y_p$ is known.

(p2) The relative degree (pole excess) $\Delta_p \equiv \text{deg}[\beta_p(s)] - \text{deg}[\alpha_p(s)]$ is 0 or 1.

Suppose the target is zero origin, i.e., $y_m \equiv 0$ for all $t \geq 0$. Then, from classical control theory the control $u = -\text{sgn}(y_p)k_0e$, where $e = y_p - y_m$ is the tracking error, will stabilize the system for some gain $k_0 > 0$ that is sufficiently large. For any fixed $k_0$, the system structure represents a proportional feedback controller.

In practice, the value of $k_0$ needed to stabilize the process may not be known a priori because $\alpha_p$ and $\beta_p$ are not completely specified. In this event the feedback gain may be adaptively tuned, starting from an arbitrary initial value, by using an adaptation law that causes $k_0$ to increase whenever $e \neq 0$. A commonly used law is

$$k_0(t) = \eta e^2$$

where the adaptation rate $\eta > 0$ is an arbitrary constant. It is well known that $e$ decays to 0 asymptotically as $k_0(t)$ converges to some finite limiting value. This adaptive stabilization scheme using high-gain negative feedback may also be generalized to other situations where the sign of $y_p$ is unknown [26, 32]; the pole excess is greater than 1 (see [22, 23]); the process is nonlinear [18]; and plants are with multiple inputs and multiple outputs [13].

On the other hand, the high-gain adaptive controller (1) suffers from a lack of robustness with respect to bounded disturbances. Sigma modification achieves boundedness of all solutions [14, 17], but it does not maintain stability in the noise-free case [27]. In [28], two data-driven stability indicators have been introduced in order to improve robustness. However, if $y_m$ is non-zero and/or time-varying, then $e$ does not generally go to zero for any finite $k_0$. Thus HGAC is not applicable to problems involving regulation about nonzero targets or tracking of time-varying targets.

B. Model reference adaptive control

Suppose the process as defined above is to be controlled to track a nonzero and time-varying target $y_m(t)$ derived from a $n$-th order reference model with given transfer function $h_m(s) \equiv g_m \cdot \alpha_m(s)/\beta_m(s)$, where $\alpha_m$ and $\beta_m$ are monic, coprime polynomials. Assume that $h_m(s)$ is strictly proper and, further, is strictly positive real (SPR) [8], [24], [29], which implies that $\Delta_m = 1$, where $\Delta_m \equiv \text{deg}[\beta_m(s)] - \text{deg}[\alpha_m(s)]$ is the pole excess of $h_m(s)$. Systems satisfying the above conditions are said to be dissipative. For simplicity and without loss of generality we assume that $g_m > 0$ and $y_p > 0$. In addition, we assume that the order of the process is related to the model order as follows:

(p3) $\text{deg}[\beta_p(s)] \leq n$.

(p4) $0 \leq \Delta_p \leq \Delta_m$.

Compared to HGAC, more assumptions about the process dynamics are needed here. If $h_p(s)$ is completely given, one may readily design appropriate feedback and feedforward compensators that would balance any differences between process and model dynamics.

If $h_p(s)$ is not completely known, the above compensation gains must be adaptively tuned to achieve perfect tracking. A common controller

$$u(t) = k^T(t) \cdot z(t)$$

and the corresponding adaptive law is the delta rule [8], [24], [29]:

$$\dot{k}(t) = -\text{sgn}(g_p)\eta \Lambda z(t)e(t)$$

where $k = \begin{bmatrix} k_1 & k_2 & \kappa_1^T & \kappa_2^T \end{bmatrix}^T$, $z = \begin{bmatrix} r & y_p & v_1^T & v_2^T \end{bmatrix}^T$, $v_1$ and $v_2$ are the outputs of the compensation filters, $\text{sgn}(g_p) = 1$ (by assumption), and $\Lambda = \text{diag} [\lambda_1, \lambda_2, \ldots, \lambda_{2n}]$ with $\lambda_i > 0$, $i = 1, 2, \ldots, 2n$.

Such tuning may not always be feasible because the adaptive system may not converge. However, the SPR condition ensures global stability for the above adaptive system, such that $e$ goes to 0 asymptotically. Under this condition, the
gain $k$ will converge to the desired values provided $z$ is persistently exciting [7], [8], [21].

In the more general case where $\Delta_m > 1$, the SPR condition no longer holds but stability may be achieved by using an "augmented error" for gain adaptation [8], [24], [29]. In this paper, however, we will only consider reference models that satisfy SPR condition unless stated otherwise.

Finally we introduce one lemma which is called Meyer-Kalman-Yacubovich Positive Real Lemma or Popov-Kalman-Yacubovich Positive Real Lemma [7], [8], [21].

**Lemma 1:** Let $H(s)$ be a SPR rational transfer function with $H(\infty) = 0$. Then $H(s)$ has a minimal state variable realization \( \{ A, b, c \} \) with $H(s) = c(sl - A)^{-1}b$, where

$$
A + A^T = -QQ^T - \gamma^2I, \quad b = c^T
$$

(4)

for some non-zero matrix $Q$ and non-zero constant $\gamma$.

III. CONVERGENCE RULES FOR HGAC AND MRAC

A. Convergence of HGAC

From (1) it can be shown that convergence speed of HGAC is increased with increasing $\eta$. The maximum convergence rate that can be achieved by increasing $\eta$ is dependent on $\Delta_p$. For $\Delta_p = 1$, convergence of the adaptive system can be made arbitrarily fast by increasing $\eta$ toward infinity [19]. In the limit, the response of the closed-loop system is dictated by the high-gain feedback. For $\Delta_p = 2$, convergence rate is limited by system dynamics which is characterized by a pair of complex conjugate poles whose real part is independent of the loop gain (as long as it exceeds a critical value) [22]. For $\Delta_p > 2$, the adaptive system may become unstable for larger $\eta$ since some complex pole pairs of the closed-loop system may move toward the right hand-plane as the loop gain is increased [12], [23].

B. Convergence of MRAC

Although MRAC is known to be stable, factors governing its rate of convergence are poorly understood. Generally, one cannot increase the convergence rate of MRAC by simply increasing the adaptation gain. Transient behaviors of adaptive control have not been well considered in the literature until [34]. Afterwards, there has been increasing interest in this subject (see [11], [25], [31]). A MRAC algorithm which utilizes multiple models is proposed in [11] to improve the transient performance under large parameter variations for a class of SISO systems. Reference [25] develops a strategy for improving the transient response by switching between multiple models of the plant. In [31], the estimation error generated by the identification scheme is used as a control signal to counteract errors resulting from the certainty equivalence design for improving the transient performance of adaptive systems. However, none of these references deals with the relationship between the convergence rate and the adaptation gain(feedback gain).

For MRAC (2) and (3), the error equation is

$$
e(t) = y_p(t) - y_m(t) = \frac{g_p}{g_m}h_m(s)[z^T(t)\delta k(t)]$$

(5)

where $\delta k(t) = k(t) - k^*$ and $k^*$ is the set of optimal parameter values for the controller. From (3), we know that $\delta k(t)$ satisfies the following differential equation:

$$
\delta k(t) = -sgn(g_p)\eta Az(t)e(t) = -\eta Az(t)e(t).
$$

(6)

Now we present the general result on the convergence of MRAC for SPR models. The proof of exponential stability for the adaptive system (5) and (6) can be obtained from Lemma B.2.3 in reference [21] via some system transformations and a coordinate transformation.

**Lemma 2:** (Exponential stability for MRAC) For any positive $\eta > 0$, the adaptive system (5) and (6) is exponentially stable, if

1. $h_m(s)$ is SPR;
2. $||z(t)||$ and $||\dot{z}(t)||$ are uniformly bounded;
3. $z(t)$ is persistently exciting.

**Proof.** Let $\delta x = [x_1 \quad x_2 \ldots \quad x_n]^T$ be the model state and $\{A_m, b_m, c_m \}$ be a minimal realization satisfying (4) in Lemma 1, and $c(t) = c_m \delta x(t)$, which means $h_m(s) = c_m(sI - A_m)^{-1}b_m$. Then

$$
A_m + A_m^T = -QQ^T - \gamma^2I, \quad b_m = c_m.
$$

Then the error equation can be written as

$$
\begin{bmatrix}
\delta \dot{x}(t) \\
\delta \dot{k}(t)
\end{bmatrix} =
\begin{bmatrix}
A_m & \frac{g_p}{g_m}b_mz^T(t) \\
-\eta Az(t)c_m & 0
\end{bmatrix}
\begin{bmatrix}
\delta x(t) \\
\delta k(t)
\end{bmatrix}.
$$

(7)

Taking $\bar{b}_m = \frac{g_p}{g_m}b_m$ and $\bar{\eta} = \frac{\eta g_p}{g_m}$, we get the equivalent form of (7):

$$
\begin{bmatrix}
\delta \dot{x}(t) \\
\delta \dot{k}(t)
\end{bmatrix} =
\begin{bmatrix}
A_m & \bar{b}_mz^T(t) \\
-\bar{\eta}Az(t)b_m^T & 0
\end{bmatrix}
\begin{bmatrix}
\delta x(t) \\
\delta k(t)
\end{bmatrix}.
$$

(8)

Without loss of generality we assume $\Lambda = I$. Indeed, if $\Lambda \neq I$, consider the change of coordinates

$$
\delta x = \delta x, \quad \delta k = (L^T)^{-1}\delta k, \quad \Lambda = L^TL, \quad L \in \mathbb{R}^{2n \times 2n}.
$$

In the new coordinates, define $\tilde{z}(t) = Lz(t)$, then (8) can be equivalently rewritten as

$$
\begin{bmatrix}
\delta \dot{x}(t) \\
\delta \dot{k}(t)
\end{bmatrix} =
\begin{bmatrix}
A_m & \bar{b}_m\tilde{z}^T(t) \\
-\bar{\eta}\tilde{z}(t)b_m^T & 0
\end{bmatrix}
\begin{bmatrix}
\delta x(t) \\
\delta k(t)
\end{bmatrix}.
$$

(9)

Let $\bar{\eta}^2 \delta k = \delta k$, then we can describe (9) equivalently as

$$
\begin{bmatrix}
\delta \dot{x}(t) \\
\delta \dot{k}(t)
\end{bmatrix} =
\begin{bmatrix}
A_m & \bar{\eta}^2\bar{b}_m\tilde{z}^T(t) \\
-\bar{\eta}^2\tilde{z}(t)b_m^T & 0
\end{bmatrix}
\begin{bmatrix}
\delta x(t) \\
\delta k(t)
\end{bmatrix}.
$$

(10)

Furthermore, it is easy to check that $\tilde{z}(t)$ is persistently exciting and that $||\tilde{z}(t)||$ and $||\dot{\tilde{z}}(t)||$ are uniformly bounded, since $z(t) = Lz(t)$ and $L$ is nonsingular. Hence we have verified that system (10) satisfies all conditions in Theorem 2.3 of reference [7]. Therefore, system (10) is exponentially stable.

C. Relationship between convergence rate $\beta$ and adaptation gain $\eta$ for MRAC

In this subsection, we shall discuss the relationship between the convergence rate $\beta$ and adaptation gain $\eta$ for MRAC via numerical example, which will show that, when adaptation gain exceeds a certain threshold, the system
converges more slowly as the adaptation gain increases. Therefore, there is a limit for improving the convergence rate by increasing the adaptation gain \( \eta \) alone.

**Example 1:** Consider the second-order system with

\[
A_p = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad B_p = \begin{bmatrix} 2 \\ 0 \end{bmatrix}, \quad C_p = \begin{bmatrix} 1 & 2 \end{bmatrix}.
\]

Take the model reference system with

\[
A_m = \begin{bmatrix} -4 & 1 \\ 1 & -4 \end{bmatrix}, \quad B_m = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad C_m = \begin{bmatrix} 1 & 0 \end{bmatrix},
\]

\( r(t) = \sin(t) + \sin(10t) \), and the initial values \( x_p = x_m = \begin{bmatrix} 0 & 0 \end{bmatrix}^T \), \( z(0) = \begin{bmatrix} 1 & 1 & 1 & 0 \end{bmatrix}^T \), \( e(0) = 0 \), \( \delta k(0) = \begin{bmatrix} 0.6 & 2.5 & 7.5 & -3.5 \end{bmatrix}^T \) and compensation filters initial values \( v(0) = \begin{bmatrix} 1 & 1 \end{bmatrix}^T \).

The simulation results are shown in Figure 2. Figures 2(a), 2(b) and 2(c) show the tracking error for \( \eta = 500 \), \( \eta = 1000 \) and \( \eta = 1500 \), respectively, over time. These figures show that the error converges the fastest with \( \eta = 1000 \), compared with \( \eta = 500 \) or 1500. Hence, by further increasing the adaptation gain from 1000 to 1500, the system performance actually get worse. A more sophisticated tweak to the control strategy will be necessary to further improve the system performance. The following section will introduce more efficient controller, DAC.

IV. DUAL ADAPTIVE CONTROL

A. Establishment of Dual adaptive controller

We now derive the dual adaptive control (DAC) scheme that draws together MRAC and HGAC, and discuss the relationship between convergence rate and error feedback gain. First introduce the error feedback into controller (2), i.e.,

\[
u(t) = k^T(t) \cdot z(t) - k_0 e(t)
\]

where \( k(t) \) still satisfies (3). We call the corresponding control scheme dual adaptive control (DAC), see Fig.3. In this case, the equivalent error equation for DAC system is

\[
e(t) = \frac{g_p}{g_m} h_m(s) [z^T(t) \delta k(t) - k_0 e(t)].
\]

The corresponding error equation becomes

\[
\begin{bmatrix}
\frac{\Delta_m}{\eta} & \frac{g_p}{g_m} b_m z^T(t) \\
-\eta \Lambda z(t) c_m & 0
\end{bmatrix}
\begin{bmatrix}
\delta x(t) \\
\delta k(t)
\end{bmatrix}.
\]

(13)

where \( \Delta_m = A_m - k_0 \frac{g_p}{g_m} b_m c_m \). Since \( A_m + A_m^T = -QQ^T - \gamma^2 I \), we have

\[
A_m - k_0 \frac{g_p}{g_m} b_m c_m + (A_m - k_0 \frac{g_p}{g_m} b_m c_m)^T = -(QQ^T + \gamma^2 I + 2k_0 \frac{g_p}{g_m} b_m c_m)
\]

\[
= -(D + 2k_0 \frac{g_p}{g_m} b_m c_m)
\]

\[
= -(D + 2k_0 \frac{g_p}{g_m} b_m b_m^T)
\]

\[\triangleq -\bar{D}.
\]

Noting that error equations (13) and (7) have essentially the same structure, we obtain the following convergence rule for DAC, by direct applying of Lemma 2.

**Theorem 1:** *(Convergence rule for DAC with given \( k_0 \))

For any positive \( \eta > 0 \) and \( k_0 > 0 \), if the DAC system (13) satisfies all conditions in Lemma 2, then (13) is exponentially stable.

The advantages of introducing of error feedback into MRAC will be shown via the following example.

**Example 2:** Consider the system in Example 1. In order to improve the speed of tracking, we introduce error feedback. Figure 4 shows the tracking errors for error feedback gain \( k_0 = 100 \) (pink line) and \( k_0 = 0 \) (black line) under adaptation gain \( \eta = 1000 \). It can be seen that the tracking error converges much faster with error feedback than without.
Remark 1: Advantages of introducing error feedback into MRAC: (1) It can improve the convergence rate, which cannot be achieved by simply increasing adaptation gain \( \eta \); (2) It can reduce overshoot, as shown in Example 2.

B. Robustness analysis

Adaptive systems are known to be sensitive to unknown disturbances or uncertainties in the system structure. Such non-robust behavior may be exacerbated by over-adaptation with excessive adaptation gains. Therefore, robustness of adaptive control has received much attention in recent years (see [9], [10], [15], [16], [33] and the references therein). In this subsection, we shall examine the robustness of DAC, which will be accessed by subjecting the system to unknown disturbance input, unknown system dynamics and high-frequency reference input.

1) Unknown disturbance input: For simplicity, we first study first-order plants with disturbance input:

\[
y_p = -a_p y_p - d + b_p u
\]

where \( d \) is an unknown nonzero constant, which can be regarded as a constant disturbance input. Furthermore, the corresponding model reference is

\[
y_m = -a_m y_m + b_m r(t)
\]

where \( r(t) \) is reference input, \( a_m \) and \( b_m \) are known constant parameters, and \( a_m \) is required to be strictly positive. Without loss of generality \( b_m \) can be chosen strictly positive. In such case, the dual adaptive controller becomes

\[
u = \hat{a}_r(t) r + \hat{a}_y(t) y_p + \hat{a}_d - k_0 e
\]

(16)

where \( \hat{a}_r, \hat{a}_y \) and \( \hat{a}_d \) are variable feedback gains. The adaptive law is

\[
\begin{align*}
\dot{\hat{a}}_r &= -sgn(b_p) \eta er \\
\dot{\hat{a}}_y &= -sgn(b_p) \eta ey_p \\
\dot{\hat{a}}_d &= -sgn(b_p) \eta e.
\end{align*}
\]

(Note when \( k_0 = 0 \), controller (16) with the above adaptive law is equivalent to MRAC [29]). It is obvious that the controller above is robust for any constant disturbance input \( d \).

For high-order systems, \( y_p = h_p(s) u + h_f(s) d \), where \( d \) is unknown constant and \( h_p(s) \) satisfying same conditions as before. Then the corresponding control law still has the form of (11), but there \( k = \begin{bmatrix} k_1 & k_2 \hat{a}_d \kappa^T_1 \kappa^T_2 \end{bmatrix}^T \), \( z = \begin{bmatrix} r & y_p & 1 & u_1^T & u_2^T \end{bmatrix}^T \). Furthermore, \( k \) and \( z \) still satisfy (3), where \( \Lambda = \text{diag} \left[ \lambda_1 \lambda_2 \ldots \lambda_{2n+1} \right] \) with \( \lambda_i > 0, i = 1, 2, \ldots, 2n + 1 \).

2) Fast-adaptation instability: For convenience of discussion, we will confine our analysis to the following example of first-order plant with unknown dynamics:

\[
\begin{align*}
\dot{x} &= -x + bw + u \\
\mu \dot{w} &= -w + 2u \\
y &= x
\end{align*}
\]

(17)

where \( b > 1/2 \) is a constant parameter, and \( \mu \) is a small positive number which corresponds to the time constant of an un-modeled state \( w \). The control objective is to track the output \( x_m \) of a reference model

\[
\dot{x}_m = -x_m + r.
\]

(18)

For \( \mu = 0 \), the MRAC law

\[
u = kr, \quad \dot{k} = -\eta e, \quad e = x - x_m
\]

(19)

guarantees that \( e \rightarrow 0 \) as \( t \rightarrow \infty \) and all signals are bounded for any bounded input \( r \). For \( 1 \gg \mu > 0 \), the closed-loop plant is:

\[
\begin{align*}
\dot{e} &= -e + bw + (k - 1) r \\
\mu \dot{w} &= -w + 2kr \\
\dot{k} &= -\eta e.
\end{align*}
\]

(20)

For large \( \eta \) the above MRAC system may become unstable. In particular, for any \( r = constant \) the system is linear and time invariant. Hence, it can be shown by using the Routh-Hurwitz criterion that (20) is stable iff the adaptation gain \( \eta \) satisfies

\[
\eta^2 < \frac{1}{\mu} \frac{1 + \mu}{2b - \mu} \approx \frac{1}{2b \mu}
\]

(21)

where the approximation in the above inequality is from \( \mu \ll 1 \).
Now suppose that in addition to the above MRAC law there is a negative error feedback with a gain $k_0 >> 1$. The first two equations in (20) becomes
\[
\dot{e} = -(1 + k_0) e + bu - (k + 1)r
\]
\[
\mu \dot{w} = -w + 2kr - 2k_0 e.
\] (22)

Using the Routh-Hurwitz criterion as before it can be shown that the dual adaptive control system is stable if
\[
\eta r^2 < \frac{1}{\mu} \left( \frac{1 + k_0 + 2b\mu}{2b} \right) \frac{1 + (1 + k_0)\mu}{\mu}.
\] (23)

For $k_0 = 0$ the stability condition is identical to (21). For $\frac{1}{2\mu} << k_0 < -1 + 2b/\mu$, however, the right-hand side of (23) tends to $k_0/\mu >> 1/2b\mu$. This result shows that the dual adaptive control system is considerably more robust than MRAC.

V. CONCLUSION

Combining HGAC and MRAC, we have formulated DAC which outperforms either of the controller implemented alone. DAC scheme has proved superior in terms of convergence rate and robustness properties, which are demonstrated with several simulation results in this paper. Furthermore, DAC also shows to be promising in dealing with a class of nonlinear systems, which is omitted since the space limited.

REFERENCES