

# A Digital Filter for Electrical Drive with Elastic Shaft

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**Abstract** In the paper analytically is constructed digital filter for DC-electrical drive with elastic junctions. It could be realized with two discrete integrators and five operational amplifiers. The filter would be insert sequentially on the output of the speed's regulator. As results transient processes similar to transients of dc-electric drive with rigid mechanical shaft are obtained.

**Keywords:** DC -thyristor electrical drive, elastic shaft, digital filter

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## 1. Introduction

On the power technological machines, including metallurgical rolling mills and paper machines with the purpose of the reliability mainly are used thyristor dc electric drives. Because mentioned technological machines have long mechanical shafts in their dynamics frequently arise strong elastic vibrations. In the continuous control systems in practice are widely applied soft differential feedbacks with R-C circuits to damping oscillations in the dynamics, as well as filters with R-L-C elements [1,2]. Such filters are not suitable for digitally controled elastic dc drives and we get naturally raised problem.

## 2. Analytical Design of a Digital Filter

To solve the problem we suggest digital filter based on integrators and operational amplifiers similar to the Kalman-Bucy filter [3,4]. To construct analitically such a filter first we write equations of movement of elastic electrical drive due to D'alambert's principle:

$$\begin{cases} M - M_e = J_1 \frac{d\omega_1}{dt}; \\ M_e - M_M = J_2 \frac{d\omega_2}{dt}; \\ M_e = c_{12} \int (\omega_1 - \omega_2) dt + b(\omega_1 - \omega_2), \end{cases} \quad (1)$$

where  $M, M_e$  and  $M_M$  are moment of rotation of the dc motor, elastic moment of mechanical transmission shaft and resistance moment of the mechanism reduced on the shaft of the motor, respectively;  $J_1$  and  $J_2$  are moments of inertia of the drive and mechanism;  $\omega_1$  and  $\omega_2$  are angular speeds of the motor and mechanism;  $c_{12}$  is coefficient of

elasticity of long mechanical shaft;  $b$  is damping coefficient of torsional vibrations of inner viscous friction of long shaft.

If we write the variables in the relative increments and use simple transformations in the (1) we get:

$$\begin{cases} \frac{d\Delta v_1}{dt} = \frac{1}{T_1} (\Delta\mu - \Delta\mu_e); \\ \frac{d\Delta v_2}{dt} = \frac{1}{T_2} (\Delta\mu_e - \Delta\mu_M); \\ \frac{d\Delta\mu_e}{dt} = \frac{1}{T_c} (\Delta v_1 - \Delta v_2) - \frac{T_d}{T_c} \cdot \left( \frac{1}{T_1} + \frac{1}{T_2} \right) \Delta\mu_e, \end{cases} \quad (2)$$

where  $T_1 = \frac{J_1 \omega_{ST}}{M_{ST}}$  and  $T_2 = \frac{J_2 \omega_{ST}}{M_{ST}}$  are mechanical time constants of the motor and mechanism, respectively;

$T_c = \frac{M_{ST}}{c_{12} \cdot \omega_{ST}}$  and  $T_d = \frac{b}{c_{12}}$  are time constants of long (elastic) shaft;  $M_{ST}$  and  $\omega_{ST}$  are base (static) values of the moment of resistance and angular speed of the drive of mechanism.

In the Cauchy form the system of equations (2) can be write as following:

In the Cauchy form the system of equations (2) can be write as following:

$$\begin{cases} \frac{dx}{dt} = A \cdot x + B \cdot u; \\ y = Cx, \end{cases} \quad (3)$$

where  $y$  is output signal of the object (angular speed of the motor);

$$A = \begin{bmatrix} 0 & \frac{1}{T_2} & 0 \\ -\frac{1}{T_c} & -\frac{T_d}{T_1 T_2 T_c} & \frac{1}{T_c} \\ 0 & -\frac{1}{T_1} & 0 \end{bmatrix};$$

$$B = \begin{bmatrix} 0 \\ \frac{T_d}{T_1 T_C} \\ \frac{1}{T_1} \end{bmatrix}; x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}; \quad (4)$$

$C = [0 \ 0 \ 1]$ ;  $x_1$ ,  $x_2$  and  $x_3$  - are state variables of the object, namely  $x_1$  - angular speed of the motor;  $x_2$  - elastic moment of the long shaft;  $x_3$  - angular speed of the mechanism;  $T_\Sigma = T_1 + T_2$  - total mechanical time constant of the drive.

To obtain transfer function of regulator first of we should define transfer function of the object via input signal  $u$  and output signal  $y = x_3$ . With this purpose let us write the system of equations given by (3) in the operational form ( $s = d/dt$ ) as following:

$$\begin{cases} sX(s) = A \cdot X(s) + B \cdot U(s); \\ Y(s) = C \cdot X(s), \end{cases} \quad (5)$$

From the first equation of (5) we can write:

$$X(s) = (s \cdot E - A)^{-1} B \cdot U(s), \quad (6)$$

Then from the (5) we get

$$Y(s) = C(s \cdot E - A)^{-1} \cdot B \cdot U(s). \quad (7)$$

Using inner current feedback and matrices from (4), an approximate transfer function of the object of control system could be given as following:

$$\theta_{OB}(s) = \frac{Y(s)}{U(s)} = \frac{X_3(s)}{U(s)} \approx \frac{1}{k_t \cdot T_\Sigma \cdot s} \cdot \frac{T_2'^2 \cdot s^2 + T_d \cdot s + 1}{T_1'^2 \cdot s^2 + T_d \cdot s + 1}, \quad (8)$$

where  $T_1' = (T_1 \cdot T_2 \cdot T_C / T_\Sigma)^{1/2}$ ;  $T_2' = (T_2 \cdot T_C)^{1/2}$ .

By substituting numerical values of the parameters  $T_1 = 1.5$  sec.;  $T_2 = 10$  sec;  $T_d = 0.002$  sec;  $T_c = 0.0004$  sec;  $T_1 = 1.5$  sec and  $k_t = 0.1$  for the electrical drive of press of paper machine we get:

$$\theta_{OB}(s) = \frac{0.004 \cdot s^2 + 0.002 \cdot s + 1}{s \cdot (0.0006 \cdot s^2 + 0.0025 \cdot s + 1.15)}. \quad (9)$$

Expressing transfer function of (9) in the discrete z-transform form we use well-known [5] approximation  $z = e^{T_0 s} \Rightarrow z \approx 1 + T_0 s \Rightarrow s = (z-1)/T_0$ , where sample time  $T_0 = 0.01$  sec. and then we have:

$$\theta_{OB}(z) = \frac{0.06367 \cdot z^2 - 0.1254 \cdot z + 0.06334}{z^3 - 2.774 \cdot z^2 + 2.734 \cdot z - 0.9592}. \quad (10)$$

On Figure 1,a are given Bode diagrams simulated corresponding to the transfer function (10).

In the expression (8) second multiplier denotes transfer function of elastic link of the object of dc drive. Compensation of its action could be realized via inversely given digital filter:

$$\begin{aligned} \theta_F(s) &= \frac{T_1'^2 \cdot s^2 + T_d \cdot s + 1}{T_2'^2 \cdot s^2 + T_d \cdot s + 1} \\ &= \frac{0.00053 \cdot s^2 + 0.002 \cdot s + 1}{0.004 \cdot s^2 + 0.002 \cdot s + 1}. \end{aligned} \quad (11)$$

z-transform of the (11) similarly to (10), has following view:

$$\theta_f(z) = \frac{0.1325 \cdot z^2 - 0.2459 \cdot z + 0.1383}{z^2 - 1.97 \cdot z + 0.995}. \quad (12)$$

On Figure 1,b are given Bode diagrams corresponding to transfer function (12). To purge resonant peak on the  $L_f$  -logarithmic amplitude frequency response, it is necessary second summand in denominator increase 20-times. Therefore, adjusted transfer function of digital filter could be write as following:

$$\theta_F(s) = \frac{0.00053 \cdot s^2 + 0.002 \cdot s + 1}{0.004 \cdot s^2 + 0.04 \cdot s + 1}. \quad (13)$$

Transfer function (13) in z-transform form looks as following:

$$\theta_F(z) = \frac{U_2(z)}{U_1(z)} = \frac{0.1325 \cdot z^2 - 0.2466 \cdot z + 0.1378}{z^2 - 1.881 \cdot z + 0.9048}, \quad (14)$$

where  $U_1(z)$  and  $U_2(z)$  are input and output signals of the filter.

On Figure 1,c Bode diagrams corresponding to (14) are presented, where resonant peak is suppressed.

To plot Bode diagrams (Figure 1,d) of the external open loop of speed we should use following transfer function:

$$\begin{aligned} \mathcal{G}_{OP} &= \theta_{OB}(z) \cdot \mathcal{G}_F(z) \cdot \mathcal{G}_{SR}(z) \\ &= \frac{\left( \begin{array}{l} 0.06 \cdot z^5 - 0.28 \cdot z^4 + 0.56 \cdot z^3 \\ -0.91 \cdot z^2 + 0.28 \cdot z - 0.06 \end{array} \right)}{\left( \begin{array}{l} z^6 - 5.68 \cdot z^5 + 13.56 \cdot z^4 - 17.48 \cdot z^3 \\ + 12.84 \cdot z^2 - 5.12 \cdot z + 0.86 \end{array} \right)}, \end{aligned} \quad (15)$$

where  $\mathcal{G}_{SR}(z) = \beta \cdot \frac{z-a}{z-1} = 5 \cdot \frac{z-0.975}{z-1}$  is PI-type digital transfer function of speed regulator which is obtained from  $\mathcal{G}_{SR}(s) = \beta \cdot \frac{\tau s + 1}{\tau s} = 5 \cdot \frac{0.4 \cdot s + 1}{0.4 \cdot s}$  via Matlab. On

Bode diagrams  $L_{OP}$  and  $\phi_{OP}$  are not resonant peaks that usually are features of the elasaticities.

To construct electrical scheme of the filter from (14) we should rewrite it as following:

$$U_2 = \frac{1}{z} \left[ \begin{array}{l} (0.1378 \cdot U_1 - 0.9048 \cdot U_2) \frac{1}{z} \\ + (1.881 \cdot U_2 - 0.2466 \cdot U_1) \end{array} \right] + 0.1325 \cdot U_1. \quad (16)$$

Digital block diagram with two digital integrators and five operational amplifiers are presented on the computer based block digram (Figure 2) as separate part, which is involved in the output of digital speed regulator (SR) of the electrical drive.

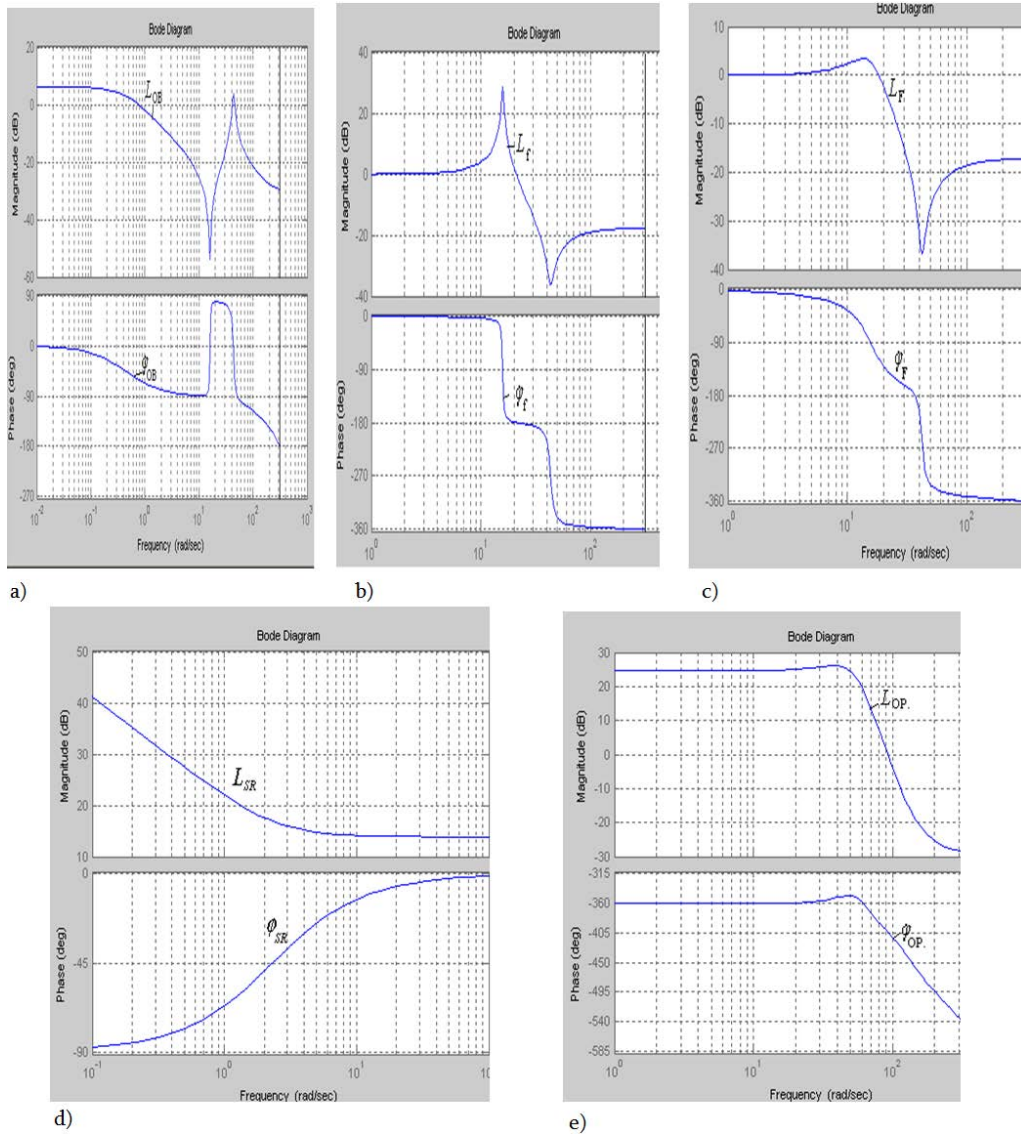


Figure 1. Bode diagrams: a) of the object of control system; b) of unadjusted filter according to (12); c) of adjusted filter according to (14); d) of the open loop of speed contour (together with regulator)

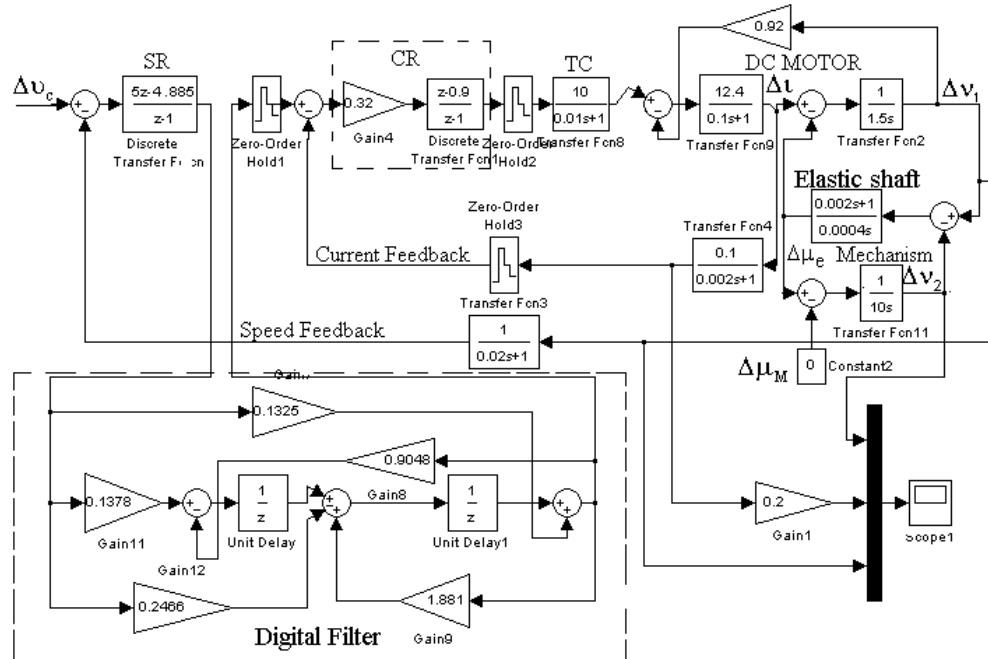
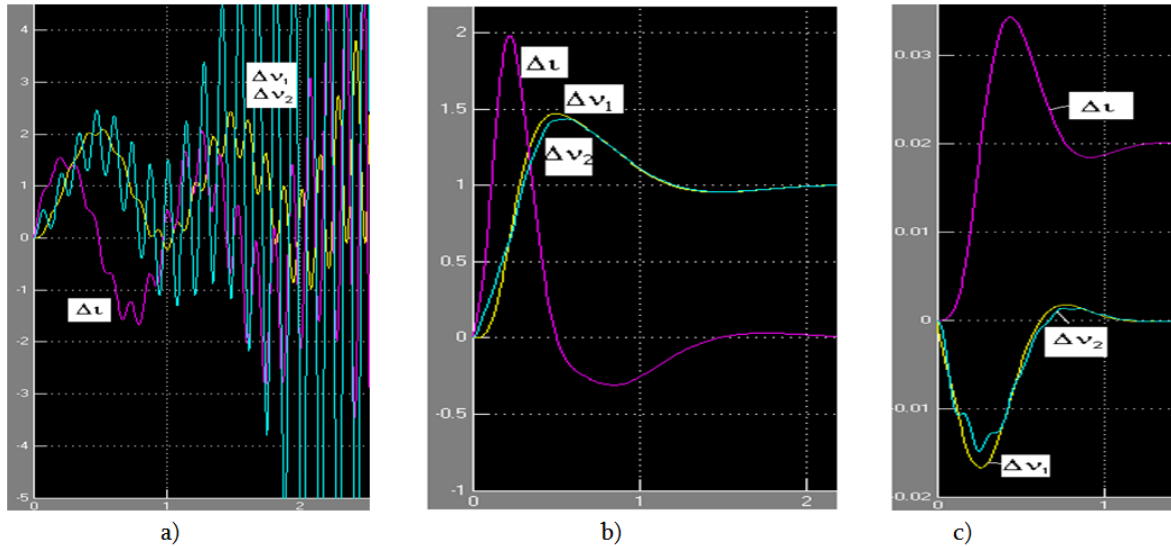


Figure 2. Numerical block diagram of elastic thyristor dc electric drive under digital control

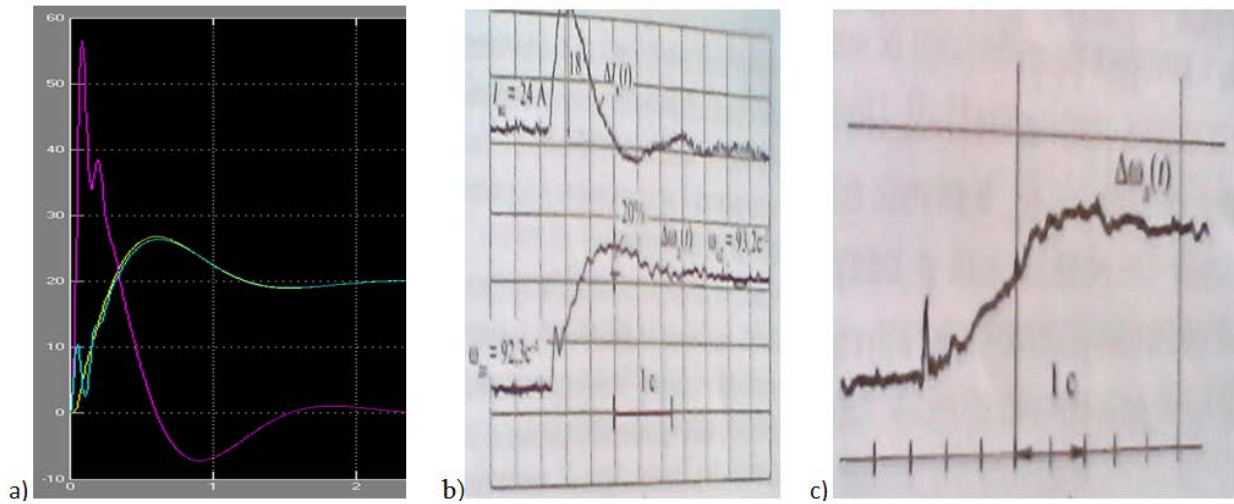
On Figure 2 following notations are used:  $\Delta v_c$  denotes relative increment of control (input) signal; SR and CR are digital regulators of speed and current; TC is thyristor converter;  $\Delta t$  denotes relative increment of motor anchor;  $\Delta \mu_M$  is moment of resistance of the drive from mechanism.

Investigations of dynamical modes using programm MatLab shows efficiency of suggested digital filter. Qualitative characteristics of transients of constructed digital filter ensures work of elastic drive system almost

similar to the drive system with short (hard) shaft:  $\sigma = 45\%$ ,  $t_T \approx 1\text{sec}$  and  $\Delta v_{dyn} = 0.015$ . It must noticed that constructed filter is better than feedbacks on the base R-C circuits. Such type of discrete filter completely cleans the power mechanical part of the drive systems from adverse impulsive strikes, for this reason we explore beginnings of the speed transient curves (Figure 4, a,b,c - [6]).



**Figure 3.** Transient curves of thyristor electrical drive with elastic shaft under digital control: a) without digital filter; With digital filter: b) under stepwise change of  $\Delta v_c$  control signal; c) under stepwise change of loading  $\Delta \mu_M$



**Figure 4.** Transient processes of dc elastic electrical drive with feedback using R-C circuits: a) obtained via Matlab simulations; b) and c) are experimental results on the acting paper machine

### 3. Conclusions

In the work we suggest new type digital filter based on discrete integrators to damping elastic vibrations of signals in dynamical modes of dc drives. For tuning of optimal parameters frequency characteristic method is used. Transient curves similar to non-elastic (with short, i.e. hard shaft) electrical drive are obtained  $\sigma = 45\%$ ,  $t_T \approx 1.0\text{sec}$ ,  $\Delta v_{dyn} = 0.015$ .

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