Output feedback control of a mechanical system using magnetic levitation

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This paper presents an application of a nonlinear magnetic levitation system to the problem of efficient active control of mass–spring–damper mechanical systems. An output feedback control scheme is proposed for reference position trajectory tracking tasks on the flexible mechanical system. The electromagnetically actuated system is shown to be a differentially flat nonlinear system. An extended state estimation approach is also proposed to obtain estimates of velocity, acceleration and disturbance signals. The differential flatness structural property of the system is then employed for the synthesis of the controller and the signal estimation approach presented in this work. Some experimental and simulation results are included to show the efficient performance of the control approach and the effective estimation of the unknown signals.

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1. Introduction

Magnetic levitation has shown its potential in many engineering fields. Practical applications of magnetic levitation can be found in vibration control systems, magnetic bearings, high-speed passenger trains as well as in other real engineering systems (see, e.g., [1–5] and references therein). Valuable nonlinear and linear control schemes based on sliding modes, feedback linearization, backstepping, classical control, neural networks, optimal control, adaptive control, and other control design approaches have been successfully applied to magnetic levitation systems. Thereby, control of magnetic levitation systems to robustly suspend a magnetic ball has been an active and challenging research topic in the few years (see, e.g., [5–16]). However, most of the contributions require measurements of position, velocity and electric current, and hence state observers should be synthesized to estimate the unavailable signals of the nonlinear dynamical system from the measurable output variable [17]. Moreover, it is well known that the nonlinear behavior of the magnetic levitation system hinders the design of state observers and output feedback tracking controllers.

Some nonlinear dynamical systems present the structural property known as differential flatness. A dynamical system is a differentially flat system if there is a set of independent outputs, called flat outputs and equal in number to control inputs, which completely parameterizes every state variable and control input [18,19]. The analysis and design of control schemes for nonlinear dynamical systems is greatly simplified by means of differential flatness. Trajectory planning and tracking constituted for a controlled nonlinear system are easily accomplished as well. Thus, nominal system trajectories can be completely described in terms of reference trajectories specified for flat output variables, without solving the system of differential equations. This allows establishing the desired behavior for the closed-loop system in the control design stage, considering possible technological constraints for the state and control variables, and energy efficiency criteria. Therefore, differential flatness qualifies as a suitable tool for proposing simple and efficient solutions to the problem of active control of vibrating mechanical systems using magnetic levitation.

Many vibrating mechanical systems can be characterized using several topological configurations of mass–spring–damper systems [20–22]. Classical dynamic vibration absorbers are modeled as mass–spring–damper secondary systems, which are coupled to the flexible mechanical system to be protected [23]. A wide variety of these vibration control devices can be found in bridges, civil structures, machine tools and other real engineering systems [21–23]. A Jeffcott rotor model can be used to properly describe
the first vibration modes in rotating machinery [24,25]. Here, the dynamics of the responses in the x and y directions of the geometric center coordinates of the unbalanced rotating mechanical systems are described by mass–spring–damper system models. The vibration problem of metal-cutting machine tools is analyzed using mass–spring–damper models for each motion axis direction [26,27]. Mechanical suspension systems for vibration isolation of high precision industrial machinery are also practical examples of mass–spring–damper systems [28]. Hence the efficient control of mass–spring–damper mechanical systems is a high relevance research topic in practical engineering systems.

There are three fundamental control methodologies for vibrating mechanical systems described as passive, semi-active and active control [21,22]. Passive control relies on the addition of stiffness and damping to the system to reduce the primary response, and serves for stable operating conditions. Semi-active control deals with adaptive spring or damper characteristics, which are tuned according to the operating conditions. Active control achieves better dynamic performance by adding controlling actuator forces depending on feedback information of the system obtained from sensors [28,29].

The use of force control devices based on magnetic levitation represents a growing trend in active control applications of flexible mechanical systems. These electromagnetic control devices present some better performance indicators than mechanical force actuators from perspectives of useful life, energy efficiency, equipment maintenance, fast control response and high operation velocities. The absence of mechanical contact between the electromagnetic actuator and machine parts reduces some relevant problems of wearing, material fatigue, friction, lubrication, poor finishes of manufactured products, loosening of fasteners, efficiency loss, and malfunction of instrumentation. Hence magnetic levitation offers areas of opportunity for design and implementation of efficient active control schemes for flexible mechanical systems (see, e.g., [2] and references therein).

This paper presents an application of magnetic levitation to the stabilization and tracking control problem of a single degree-of-freedom mass–spring–damper vibrating mechanical system. It is shown that electromagnetically controlled flexible mechanical systems exhibit the differential flatness property. The presented control approach can be extended to fully actuated or under-actuated, differentially flat, mass–spring–damper vibrating mechanical systems with multiple degrees of freedom. For instance, one could apply the presented control approach to the active vibration control problem using active–passive vibration absorbers [30–32]. In this case, the control force is supplied by the magnetic levitation device. In the same way, the outcomes of this study can be extended to the balancing problem of rotating machinery using active magnetic bearings involving several magnetic levitation devices [2,33]. Therefore, the developments presented in this study admit a wide variety of real applications in active control of vibration. Hence, we can say that our contribution is pertinent by taking traditional magnetic levitation to a concrete application case. In this paper, an output feedback control scheme is proposed for reference position trajectory tracking tasks on a mass–spring–damper vibrating mechanical system, including its stabilization at a desired equilibrium position. It is considered that only measurements of the position output variable are available. This because the actively controlled vibrating mechanical systems are commonly equipped with position sensors (e.g., encoders and proximity sensors) for implementation of control polices. However, acceleration and electric current sensors could also be used. Then, the reconstruction of the unavailable signals is easily carried out. Nevertheless, the problem of control is of major interest when output measurements of a nonlinear dynamic system are only allowed due to cost reduction reasons. A Luenberger-like extended state observer is also included in the control scheme to estimate velocity, acceleration and disturbance signals. The differential flatness property exhibited by the three degree-of-freedom nonlinear system and Taylor polynomial expansions is used for the synthesis of the observer-based control scheme proposed for vibrating mechanical systems. In addition, unlike other contributions, our control and observer design approach considers electromagnetic circuit dynamics in the synthesis of a control voltage algorithm to regulate the position of the mechanical system in accordance with specified motion planning. Moreover, the electric current signal is algebraically reconstructed through the estimated signals as a bonus, thanks to the differential flatness property.

The main differentiation and originality of this contribution with respect to the previous proposals on magnetic levitation systems (e.g., [5–16]) is then summarized through the following highlights of the presented control approach. (i) The application of a magnetic levitation system is extended to the efficient active control problem of vibrating mechanical systems. (ii) An input–output mathematical model of the differentially flat vibrating mechanical system is obtained for the synthesis of the controller and extended state observer proposed in this work. (iii) An output feedback controller for stabilization and desired reference position trajectory tracking tasks is proposed for mass–spring–damper flexible mechanical systems electromagnetically controlled by voltage using position output variable measurements only, and avoiding the employ of mechanical force actuators, which is quite common in active control of vibrating mechanical systems. (iv) An asymptotic estimation approach based on differential flatness, trajectory planning and tracking, and Taylor polynomial expansion of the disturbance signal affecting the transformed input–output system dynamics is proposed to estimate velocity, acceleration and disturbance signals. (v) The dynamics of the electromagnetic subsystem is considered in the analysis and synthesis of the controller and observer.

Experimental and simulation results spotlight the efficient performance of the control approach and the effective estimation of the unknown signals. Neglected dynamics and parametric uncertainty are considered in our performance assessment as disturbance signals to be estimated by the observer. Motion planning is initially specified to smoothly transfer the system from a rest position to another. Additionally, the robustness of the control and estimation scheme against actuator saturation and additive stochastic noise corrupting the measurement and control signals is verified for a closed-loop time-varying reference position trajectory tracking task showing satisfactory results.

2. An introductory case study: control of a mass–spring–damper system

2.1. Mathematical model

To illustrate the basic ideas of the proposed active control approach based on on-line estimation of signals, consider the n degree-of-freedom (DOF) mass–spring–damper mechanical system shown in Fig. 1. The generalized coordinates are the n positions of the mass carriers, x_i, i = 1, 2, ..., n. In addition, m_i, k_i and c_i denote mass, stiffness and viscous damping associated to the i-th DOF. u represents the force control input and y = x_1 is the output position variable to be controlled.

Fig. 1. Schematic diagram of a n DOF flexible mechanical system.
We then propose the following feedforward and feedback controller to asymptotically track some reference position trajectory \(y^*(t)\):

\[
u = m_1[v - \xi(t)]
\]
(1)

with \(v = \dot{y}^* - \alpha_1(y - \dot{y}^*) - \alpha_2(y - y^*) - \alpha_0 \int_0^t (y - y^*) \, dt
\)
(2)

where \(\xi(t)\) is a state-dependent disturbance signal due to neglected dynamics and, possibly, parametric uncertainty. Nevertheless, the controller (1) requires measurements of position, velocity and disturbances. Thus, in the next subsection, an asymptotic estimation scheme is proposed to estimate the unavailable signals. The estimation scheme is based on the so-called GPI observer design approach introduced in [34].

2.2. Disturbance estimation

For observer design purposes, disturbance signal \(\xi(t)\) is locally approximated by the Taylor polynomial expansion [35]:

\[
\xi(t) \approx \sum_{i=0}^{r-1} p_i t^i
\]
(3)

where all the coefficients \(p_i\) are completely unknown. In addition, we have assumed that \(\xi(t)\) and its time derivatives up to \(r\)-th order are uniformly absolutely bounded. This is a necessary and sufficient condition for global-in-time existence of solutions of the system [36].

An extended state space local model for the perturbed dynamics associated with the mass \(m_1\) is then given by

\[
\begin{align*}
\dot{\eta}_1 &= \eta_2 \\
\dot{\eta}_2 &= \frac{1}{m_1} u + \xi_1 \\
\dot{\xi}_1 &= \xi_2 \\
\dot{\xi}_2 &= \frac{1}{k_1} (\xi_1 - \zeta_1 - \eta_1) \\
&\vdots \\
\dot{\xi}_{r-1} &= \xi_r \\
\dot{\xi}_r &= 0
\end{align*}
\]
(4)

where \(\eta_1 = y, \eta_2 = \dot{y}, \xi_1 = \xi, \xi_2 = \dot{\xi}, \xi_3 = \ddot{\xi}, \ldots, \xi_r = \dddot{\xi} (t - 1)\).

Hence, we propose the Luenberger-like extended state observer

\[
\begin{align*}
\dot{\tilde{\eta}}_1 &= \tilde{\eta}_2 + \beta_{r+1}(\eta_1 - \tilde{\eta}_1) \\
\dot{\tilde{\eta}}_2 &= \frac{1}{m_1} u + \tilde{\xi}_1 + \beta_1(\eta_1 - \tilde{\eta}_1) \\
\dot{\tilde{\xi}}_1 &= \tilde{\xi}_2 + \beta_{r+1}(\eta_1 - \tilde{\eta}_1) \\
\dot{\tilde{\xi}}_2 &= \tilde{\xi}_3 + \beta_2(\eta_1 - \tilde{\eta}_1) \\
&\vdots \\
\dot{\tilde{\xi}}_{r-1} &= \tilde{\xi}_r + \beta_1(\eta_1 - \tilde{\eta}_1) \\
\dot{\tilde{\xi}}_r &= \beta_0(\eta_1 - \tilde{\eta}_1)
\end{align*}
\]
(5)

The estimation error dynamics is then obtained as

\[
\begin{align*}
\dot{e}_1 &= -\beta_{r+1} e_1 + e_2 \\
\dot{e}_2 &= -\beta_1 e_1 + e_3 \\
\dot{e}_p &= -\beta_{r-1} e_1 + e_p \quad (p > 0) \\
\dot{e}_p &= -\beta_{r-2} e_1 + e_p \quad (p > 0) \\
&\vdots \\
\dot{e}_{p-1} &= -\beta_1 e_1 + e_p \quad (p > 0) \\
\dot{e}_p &= -\beta_0 e_1
\end{align*}
\]
(6)

where \(e_1 = \eta_1 - \tilde{\eta}_1, e_2 = \eta_2 - \tilde{\eta}_2, \ldots, e_p = \xi_p - \tilde{\xi}_p, k = 1, 2, \ldots, r\).

The characteristic polynomial associated with (6) is given by

\[
p(s) = s^{r+1} + \beta_{r+1} s^{r} + \beta_{r-1} s^{r-1} + \ldots + \beta_0 s + \beta_0
\]
(7)

which is completely independent of any coefficients \(p_i\) of the Taylor polynomial expansion of the disturbance signal \(\xi(t)\). The design parameters \(\beta_i\) are then chosen so that the polynomial (7) is a Hurwitz polynomial.

2.3. Experimental results

Some real-time experiments were performed on a three degree-of-freedom mass–spring–damper system characterized by the set of system parameters: \(m_1 = 2.82\) kg, \(c_1 = 3.64\) N s/m, \(k_1 = 265\) N/m, \(m_2 = 2.59\) kg, \(c_2 = 1.75\) N s/m, \(k_2 = 700\) N/m, \(m_3 = 2.59\) kg, \(c_3 = 1.75\) N s/m, \(k_3 = 700\) N/m. The experimental setup used to test the proposed control and estimation approach is a rectilinear mechanical plant (Model 210a) provided by Educational Control Products.

The design parameters of the controller were selected to have the following third order characteristic polynomial for the closed-loop tracking error dynamics:

\[
p_c(s) = (s + p_1) (s^2 + 2\zeta_s c s + \omega_s^2)
\]
(8)

with \(\omega_s = 10\) rad/s, \(p_1 = 8\) rad/s and \(\zeta_s = 0.7071\).

The perturbation signal \(\xi(t)\) was modeled as a second order time polynomial. Therefore, the characteristic polynomial for the fifth order resulting observation error dynamics was set to be of the following form:

\[
p_o(s) = (s + p_0) (s^2 + 2\zeta_o c s + \omega_o^2)^2
\]
(9)

with \(\omega_o = 80\) rad/s, \(p_0 = 80\) rad/s and \(\zeta_o = 5\).

Fig. 2 illustrates the acceptable performance of the control (1) using signal estimation (4). The satisfactory tracking of the reference position trajectory \(y^*\) is verified. This profile was planned to transfer the mass \(m_1\) from the rest position to the nominal operation position of 0.01 cm in approximately 3 s. Therefore, in the next section the presented control and estimation approach is extended to flexible mechanical systems controlled actively by magnetic levitation.

3. Active mechanical suspension system

Consider the mass–spring–damper suspension system shown in Fig. 3, where \(m, c, k\) and \(k\) are its mass, viscous damping and stiffness constant of the helical spring, respectively. Here, an electromagnetic is used to induce an electromagnetic force \(f_{em}\) to control the position \(x\) of the mechanical system. The displacement \(x\) is measured from its static equilibrium position, in which the upward spring force exactly balances the downward gravitational force on the mass [20].

![Fig. 2. Position reference trajectory tracking planned for the controlled mass.](image-url)
The mathematical model describing the dynamics of the mechanical system with an electromagnet is then given by

\[
\begin{align*}
\ddot{x} & = -kx - cx + \frac{k_m m^2}{2l_0 x + a}^2 \\
I \frac{d}{dt} & = -Ri + u
\end{align*}
\tag{10}
\]

where \(l_0\) is the initial length between the core and the static equilibrium position \((u = 0)\) of the system, \(k_m\) is the electromagnetic force constant and \(a\) is a constant, which is commonly determined by experimentation. In addition, \(R\) is the winding resistance plus any additional series resistance in the control circuit, \(L\) is the coil inductance, and \(i\) denotes the control voltage. The inductance is assumed to be practically constant in the operation bandwidth of the system [8].

Defining the state variables as \(z_1 = x\), \(z_2 = \dot{x}\) and \(z_3 = i\), one obtains from (10) the state space description

\[
\begin{align*}
\dot{z}_1 &= z_2 \\
\dot{z}_2 &= -\frac{k}{m}z_1 - \frac{c}{m}z_2 + \frac{k_m}{2m(\delta - z_1)^2} \\
\dot{z}_3 &= -\frac{R}{L}z_3 + \frac{1}{L}u \\
y &= z_1
\end{align*}
\tag{11}
\]

where \(\delta = l_0 + a\)

The electromechanical system (11) is differentially flat, with flat output given by the position of the system \(y = z_1\). Then, all system variables can be differentially parameterized in terms of flat output and a finite number of its time derivatives [18,19]. For this, the time derivatives up to third order for \(y\) are obtained as

\[
\begin{align*}
y &= z_1 \\
\dot{y} &= z_2 \\
\ddot{y} &= -\frac{k}{m}\dot{y} - \frac{c}{m}y + \frac{k_m}{2m(\delta - y)^2} \\
\dddot{y} &= -\frac{k}{m}\ddot{y} - \frac{c}{m}\dot{y} + \frac{k_m}{m(\delta - y)^2}z_3 + \frac{k_m}{mL(\delta - y)^2}z_1 u
\end{align*}
\tag{12}
\]

Therefore the differential parameterization results in

\[
\begin{align*}
z_1 &= y \\
z_2 &= \dot{y} \\
z_3 &= \frac{2m(k + c\dot{y} + \dot{y})}{k_m} (\delta - y) \\
u &= \frac{L(\delta - y)}{2k_m} \left[ y^{(3)} + \frac{c}{m} \dot{y} + \frac{k}{m} \ddot{y} \\
&- 2 \left( \frac{y}{\delta - y} R \left( \frac{k}{m} \dot{y} + \frac{c}{m} \ddot{y} + \dot{y} \right) \right) \right]
\end{align*}
\tag{13}
\]

The flat output \(\delta\) then satisfies the following input–output differential equation:

\[
y^{(3)} = q + bu
\tag{14}
\]

with

\[
q = -\frac{k}{m} \dot{y} - \frac{c}{m} y + 2 \left( \frac{y}{\delta - y} R \left( \frac{k}{m} \dot{y} + \frac{c}{m} \ddot{y} + \dot{y} \right) \right)
\tag{15}
\]

In the next sections, the structural property of differential flatness is used to design a trajectory tracking controller based on asymptotic estimation of signals.

4. Output feedback tracking control

For control design purposes, consider the simplified mathematical model

\[
y^{(3)} = b^s(t)u + \xi(t)
\tag{16}
\]

with

\[
b^s(t) = \frac{1}{L(\delta - y)} \sqrt{2k_m \left( \frac{\dot{y} + \frac{c}{m} \ddot{y} + \frac{k}{m} \dddot{y}}{\delta - y} \right)}
\tag{17}
\]

where \(\xi(t)\) is considered as an unknown state-dependent disturbance input signal, which includes \(q(t)\) and deviations of \(b^s(t)\) with respect to the actual gain \(b(t)\). Note that \(\xi(t)\) could also include small perturbations due to parametric uncertainty, unknown external forces and possibly neglected dynamics.

From the perturbed nonlinear differential equation (16), we propose the following output feedback controller based on differential flatness for asymptotic tracking tasks of some desired reference position trajectory \(y^*(t)\):

\[
u = b\left(v - \xi\right)
\tag{18}
\]

with

\[
v = y^{(3)} - \alpha_2(y - y^*) - \alpha_1(y - y^*) - \alpha_0(y - y^*)
\tag{19}
\]

The use of this controller yields the closed loop dynamics for the tracking error, \(e = y - y^*\),

\[
e^{(3)i} + \alpha_2 \dot{e} + \alpha_1 \ddot{e} + \alpha_0 \dddot{e} = 0
\tag{19}
\]

Therefore, selecting the design parameters \(\alpha_i, i = 0, 1, 2\), such that the characteristic polynomial associated with (19) is Hurwitz, one can guarantee that the error dynamics is globally asymptotically stable. Hence the asymptotic tracking of the reference position trajectory is accomplished, i.e., \(y \rightarrow y^*\).

Note that by defining the reference position trajectory \(y^*\), from the differential parameterization (13), the nominal or reference trajectories of the state and control variables are described by \(z_1^* = y^*\)
where $e_1 = \eta_1 - \tilde{\eta}_1$, $e_2 = \eta_2 - \tilde{\eta}_2$, $e_3 = \eta_3 - \tilde{\eta}_3$, $e_k = \xi_k - \tilde{\xi}_k$, $k = 1, 2, \ldots, r$.

In fact, the estimation errors can be parameterized in terms of the output error $e_1$ and a finite number of its time derivatives as follows:

$$
e_2 = \dot{e}_1 + \beta_{r+1} e_1$$
$$e_3 = \dot{e}_1 + \beta_{r+1} e_1 + \beta_{r+2} e_1$$
$$e_{pi} = \dot{e}_1 + \beta_{r+1} e_1 + \beta_{r+2} e_1 + \beta_{r+3} e_1 + \beta_{r+4} e_1 + \beta_{r+5} e_1 + \beta_{r+6} e_1 + \beta_{r+7} e_1 + \beta_{r+8} e_1 + \beta_{r+9} e_1$$

The characteristic polynomial associated with (24) is given by

$$p(s) = s^r + \beta_{r+2} s^{r+1} + \beta_{r+3} s^r + \beta_{r+4} s^{r-1} + \beta_{r+5} s^{r-2} + \cdots + \beta_{r} s + \beta_{0}$$

which is completely independent of any coefficients $p_i$ of the Taylor polynomial expansion of the disturbance signal $\zeta(t)$. The design parameters $\beta_i$ are then chosen so that the polynomial (25) is a Hurwitz polynomial. Thus, estimates of the velocity, acceleration and disturbance signals are obtained by using the observer (23). Moreover, the electric current signal can be computed from (13) as

$$z_3 = \sqrt{\frac{2m \left( k C0 \right)}{k m \left( y + C0 \tilde{m} \tilde{y} \right)}} (\delta - y)$$

6. Simulations results

Some computer simulations were performed for an electromagnetically controlled mass–spring–damper mechanical system with an electromagnet characterized by the parameter set: $R=11.88$ $\Omega$, $L=0.8052$ H, $k_{m}=0.0015$ N m/A$^2$ and $d=0.008114$ N m A$^{-2}$. The mass of the machine was $0.54$ kg, $c=1$ N s/m, $k=100$ N/m and $l_b=0.02$ m, respectively.

The design parameters of the controller were selected to have the following third order characteristic polynomial for the closed-loop tracking error dynamics:

$$p_{c}(s) = (s + p_1)(s^2 + 2\zeta_0 s + \omega_0^2)$$

with $p_1 = a_0 = 20$ rad/s and $\zeta_0 = 0.7071$.

The perturbation signal $\zeta(t)$ was modeled as a second order time polynomial. Therefore, the characteristic polynomial for the sixth order resulting observation error dynamics was set to be of the following form:

$$p_{o}(s) = (s^2 + 2\zeta_0 a_0 s + a_0^2)$$

with $a_0 = 100$ rad/s and $\zeta_0 = 0.7071$.

The performance of the proposed output feedback controller was initially assessed for a reference trajectory tracking task to smoothly transfer the mechanical system from the rest position to the desired position of 0.01 m in approximately 2 s, with the purpose of avoiding problems of voltage and current peaks, and consequently actuator saturation. This takes advantage of the differential flatness property exhibited by the system, allowing to characterize the reference trajectories of the state and control variables in terms of the reference position trajectory $y^*(t)$ as it is
described by (20). Thus, the motion planning for the actively controlled mechanical system is given by the smooth function:

\[
y_n = \begin{cases} 
  y_1 & \text{for } 0 \leq t < T_1 \\
  y_1 + (y_2 - y_1) \psi(t, T_1, T_2) & \text{for } T_1 \leq t \leq T_2 \\
  y_2 & \text{for } t > T_2 \end{cases} 
\]

(29)

where \( y_1 = 0 \) m, \( y_2 = 0.01 \) m, \( T_1 = 2 \) s, \( T_2 = 4 \) s, and \( \psi(t, T_1, T_2) \) is a Bézier polynomial, with \( \psi(T_1, T_1, T_2) = 0 \) and \( \psi(T_2, T_1, T_2) = 1 \), given by

\[
\psi(t) = \left( \frac{T - T_1}{T_2 - T_1} \right)^5 \left[ r_1 - r_2 \left( \frac{T - T_1}{T_2 - T_1} \right) \right] \\
+ r_3 \left( \frac{T - T_1}{T_2 - T_1} \right)^2 - \cdots - r_6 \left( \frac{T - T_1}{T_2 - T_1} \right)^5
\]

with \( r_1 = 252, r_2 = 1050, r_3 = 1800, r_4 = 1575, r_5 = 700, r_6 = 126 \).

Fig. 4. Closed-loop tracking response of the reference position trajectory (29).

Fig. 5. Estimation of velocity, acceleration and disturbance signals for the trajectory tracking task (29).

Fig. 6. Control voltage and electric current for the trajectory tracking task (29).

Fig. 7. Tracking of the reference trajectory (31) under noise contamination and actuator saturations.

Fig. 4 depicts the efficient performance of the controller (18) using estimates of the velocity, acceleration and disturbance signals provided by the implementation of the observer (23). The satisfactory tracking of the reference position trajectory (29) can be clearly observed.

The acceptable estimation of the velocity, acceleration and disturbance signals is presented in Fig. 5. One can see that the estimated and actual responses are practically above each other. Therefore, the proposed Taylor polynomial family allows to locally reconstruct the unknown signals.

Fig. 6 describes the closed-loop signals of the control voltage and electric current required to perform the motion planning (29). One can observe the reduction of peaks of voltage and current thanks to the selected closed-loop position profile.

Moreover, the robust performance of the controller and observer have been tested when measurement and control signal are contaminated with reasonably small additive noise as follows:

\[
y_n = y + 0.1(\zeta_y - 0.5)|y| \\
u_n = u + 1(\zeta_u - 0.5)|u|
\]

(30)

where \( y_n \) and \( u_n \) are the noisy measurement and control signals used in the controller and observer implementation, \( \zeta_y \) and \( \zeta_u \) are stochastic processes consisting of computer-generated random variables with rectangular distribution in the interval \([0, 1]\).

For this case study, a closed-loop tracking task was planned for the time-varying reference position trajectory described by the first two terms of the Fourier series of a square wave with an
offset:

\[ y^* = \frac{4}{\pi} A \sum_{n=1,3} \frac{1}{n} \sin \left( \frac{n\pi}{T_s} t \right) + y \]  

(31)

with amplitude \( A = 0.002 \text{ m} \), period \( T_s = 2 \text{ s} \) and \( y = 0.01 \text{ m} \).

In order to consider possible actuator saturations, a maximum control input voltage of \( u_{\text{max}} = 20 \text{ V} \) was also imposed. The control voltage \( u \) can only take values into the closed interval \( [0, u_{\text{max}}] \). Fig. 7 displays the satisfactory tracking of the reference trajectory (31) under actuator saturations and noise contamination in measurements and control voltage (see Fig. 8). The reasonable estimation of the velocity, acceleration and disturbance signals can be verified in Fig. 9. Here, we have collocated a second order low pass filter of the form

\[ y_f = \frac{\omega_{nf}^2}{s^2 + 2\zeta_f\omega_{nf}s + \omega_{nf}^2} y_{\text{input}} \]  

(32)

with \( \omega_{nf} = 50 \text{ rad/s} \) and \( \zeta_f = 0.7071 \). Thus, \( y_f \) denotes the low pass filtered output signal.

Finally, we have accomplished a comparison of the proposed control approach with the traditional differential flatness control (18) without disturbance rejection \( (\xi = 0) \). Fig. 10 clearly describes the inefficient tracking performance of the reference trajectory (31). In addition, controller implementation requires measurements of position, velocity and acceleration signals. Nevertheless, the differential flatness control can be combined with signal estimation to improve its robustness property and reduce the number of sensors as described in our study.

7. Conclusions

In this work we have proposed an output feedback control scheme based on differential flatness for global stabilization and asymptotic tracking tasks of some desired reference position trajectory for a linear mass–spring–damper mechanical system controlled electromagnetically by voltage. The dynamics of the electromagnetic circuit was included for the synthesis of the control voltage algorithm to efficiently regulate the mechanical system toward the desired nominal operation reference system trajectories. The closed-loop behavior for the electromechanical system was established to avoid problems of voltage and current peaks, and, consequently, actuator saturation, without solving the nonlinear system of differential equations, taking advantage of the differential flatness property. In addition, a robust linear observation scheme was proposed to estimate in real-time the velocity, acceleration and disturbance signals in order to avoid the use of more than one sensor for control implementation. Differential
flatness, Taylor polynomial expansion, and trajectory planning and tracking were properly used for the synthesis of the robust extended state observer. Experimental and simulation results show the efficient and satisfactory performance of the tracking control scheme and the acceptable signal estimation. In addition, the robustness of the control and estimation scheme against neglected dynamics, parametric uncertainty and actuator saturation was verified for a closed-loop time-varying reference position trajectory tracking task showing reasonable results. Therefore, we can conclude that the proposed control scheme represents a very good choice for active control of vibrating mechanical systems using magnetic levitation and position measurements only. Future work will be oriented to verify experimentally the robustness of the controller and observer against endogenous and exogenous disturbances in electromagnetically controlled diverse practical engineering systems.

References