A Methodology for Fault Detection, Isolation, and Identification for Nonlinear Processes with Parametric Uncertainties

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This paper presents a novel methodology for systematically designing a fault detection, isolation, and identification algorithm for nonlinear systems with known model structure but uncertainty in parameters. The proposed fault diagnosis methodology does not require historical operational data and/or a priori fault information in order to achieve accurate fault identification. This is achieved by a two-step procedure consisting of a nonlinear observer, which includes a parameter estimator and a fault isolation and identification filter. Parameter estimation within the observer is performed by using the unknown parameters as augmented states of the system, and robustness is ensured by application of a variation of Kharitonov's theorem to the observer design. The filter design for fault reconstruction is based upon a linearization, which has to be repeatedly computed at each step where a fault is to be identified. However, this repeated linearization does not pose a severe drawback because linearization of a model can be automated and is computationally not very demanding for models used for fault detection. It is not possible to simultaneously perform parameter estimation and fault reconstruction because faults and the parametric uncertainty influence one another. Therefore, these two tasks are performed at different time scales, where the fault identification takes place at a higher frequency than the parameter estimation. It is shown that the fault can be reconstructed under some realistic assumptions and the performance of the proposed methodology is evaluated on a simulated chemical process exhibiting nonlinear dynamic behavior.

1. Introduction

Early and accurate fault detection and diagnosis is an essential component of operating modern chemical plants in order to reduce down time, increase safety and product quality, minimize the impact on the environment, and reduce manufacturing costs.1,2 As the level of instrumentation in chemical plants increases, it is essential to be able to monitor the variables and interpret their variations. While some of these variations are due to changing operating conditions, others can be directly linked to faults. Extracting essential information about the state of a system and processing the data for detecting, isolating, and identifying abnormal readings are important tasks of a fault diagnosis system,3 where the individual goals are defined as follows:

(i) Fault detection: a Boolean decision about the existence of faults in a system.

(ii) Fault isolation: determination of the location of a fault, e.g., which sensor or actuator is not operating within normal limits.

(iii) Fault identification: estimation of the size and type of a fault.

There exist numerous techniques for performing fault diagnosis.4 The majority of these approaches are based upon data from past operations in which statistical measures are used to compare the current operating data to earlier conditions of the process where the state of the process was known. While these techniques are often easy to implement, they do have the drawback that it is not possible to perform fault identification, that is, a large amount of past data are required, that the method may not be able to detect a fault if operating conditions have changed significantly, or that processes exhibiting highly nonlinear behavior may be difficult to diagnose.5 To address these points, this paper presents an approach for fault diagnosis based upon nonlinear first-principles models, which can include parametric uncertainty. Incorporating fundamental models into the procedure allows for accurate diagnosis even if operating conditions have changed, while the online estimation of model parameters takes care of plant–model mismatch. The parameter estimation is performed using an augmented nonlinear observer, where a concept from Kharitonov's theory about stability under the influence of parametric uncertainty is utilized in order to ensure a certain level of robustness for the designed observer. The fault diagnosis itself uses the computation of residuals (i.e., the mismatch between the measured output and estimated output using the model) for fault detection,3 and appropriate filters are designed to achieve fault isolation and identification as well. Because it is not possible to simultaneously perform parameter estimation and fault detection, as a result of the interactions of these two tasks, an approach where these computations are taking place at different time scales is implemented. It is shown that fault detection, isolation, and identification for nonlinear systems containing uncertain parameters can be per-
formed under realistic assumptions with the presented approach.

2. Previous Work

Extensive research on fault detection has been undertaken over the last few decades. The majority of methods are based on statistical techniques, however, a significant body of literature also exists for fault detection and identification of measurement bias based upon first-principles models.

Work on model-based fault detection has included unknown input observers (UIOs), which are based upon the idea that the state-estimation error can be decoupled from unknown inputs acting as disturbances and thereby decoupling residuals from uncertainties. This concept was generalized in a subsequent work for detecting and isolating both sensor and actuator faults by considering the case when unknown inputs also appear in the output equation. A different approach, but using a similar concept of decoupling the disturbances from the residuals for fault detection, made use of the eigenstructure assignment of the observer. In this technique, the observer is designed to decouple the residual from unknown inputs rather than from the state-estimation error. The above two approaches work satisfactorily for linear time-invariant (LTI) systems; however, they can result in poor performances for nonlinear systems because these systems are not always affine in the unknown inputs. To cope with nonlinearities of the process, it has been proposed to develop nonlinear UIOs for residual generation, which requires a suitable nonlinear state transformation. However, the conditions under which these transformations exist are very restrictive in nature. Moreover, it is assumed that the unknown disturbances affect the system in a piecewise linear affine fashion. The restrictive conditions required to develop UIOs and attain eigenstructure assignment have led to frequency domain and optimization methods for robust fault diagnosis. The main emphasis is on the formulation of robust fault diagnosis and isolation (FDI) problems using frequency performance criteria. Limitations of using an optimization method for robust fault diagnosis are due to the assumption that no modeling errors are present or that the modeling errors can be viewed as disturbances.

A major advantage of observer-based fault diagnosis techniques as compared to data-driven techniques is that the residual’s sensitivity to faults of a specific frequency range can be tailored. Uncertainties in the model can then be taken into account by considering them to be slowly time-varying faults. However, this involves the risk that fault signals of low frequency may not be detected because the enhancement of robustness is associated with an accompanying decrease of the ability to detect slowly time-varying fault signals. To overcome this difficulty, it was proposed to use adaptive observers where certain effects of nonlinearities and model uncertainties may be handled as unknown parameters that can be decoupled from the residuals. The formulation of the above problem is based on the assumption that the slowly time-varying unknown parameters appear as an affine unknown input to the system. However, most chemical processes are nonlinear and exhibit an exponential dependence of unknown parameters in the process model, e.g., the activation energy. Moreover, because industrial processes operate in a closed loop with appropriate output feedback to attain certain performance objectives, it is important not just to detect and isolate instrument faults but to reconstruct them at the same time in order to implement fault-tolerant control.

3. Preliminaries

In section 3.1, observer-based fault diagnosis for LTI systems is reviewed. Required background information about the concept of stability of an interval family of polynomials is presented in section 3.2. This serves as a foundation for the presentation of the new technique in section 4.

3.1. Fault Diagnosis for LTI Systems. Consider a LTI system with no input

\[ \dot{x} = Ax + Bu \]

where \( x \in \mathbb{R}^n \) is a vector of state variables and \( y \in \mathbb{R}^m \) is a vector of output variables, \( n \) is the number of states, and \( m \) refers to the number of output variables. \( A \) and \( C \) are matrices of appropriate dimensions, and \( f_s \) is the sensor fault of unknown nature with the same dimensions as the output. Assuming that the above system is observable, a Luenberger observer for the system can be designed

\[ \dot{x} = Ax + L(y - y) \]

\[ y = Cx \]

where \( L \) is chosen to make the closed-loop observer stable and achieve a desired observer dynamics. Further, define a residual

\[ r(t) = \int_0^t Q(t - \tau) (y(t) - \hat{y}(\tau)) d\tau \]

which represents the difference between the observer output and the actual output passed through a filter \( Q(t) \). Taking a Laplace transform of eqs 1–3 results in

\[ r(s) = Q(s) (I - C(sl - (A - LC))^{-1}L)f_s(s) \]

where \( Q(s) \) is chosen such that \( Q(s) \) is a RH\(_{\infty}\) matrix. It can be shown that \( Q(s) \) depends on \( f_s(t) \) and \( f_t(t) \) if \( f_s(t) \neq 0 \) and \( f_t(t) = 0 \), indicating that the value of \( r(t) \) predicts the existence of a fault in the system.

In addition, if one uses the dedicated observer scheme as shown for a system with two outputs in Figure 1, then the fault detection system can also determine the location of the fault: \( r_i(t) = 0 \) if \( f_i(t) = 0 \), \( r_i(t) \neq 0 \) if \( f_i(t) \neq 0 \), \( i = 1, 2, 3, \ldots, m \). To meet the above conditions, the following restrictions on the choice of \( Q(s) \) are imposed: (a) \( Q(s) \neq 0, \forall s \in \mathbb{C} \); (b) \( Q(s) = (1 - C(sl - (A - LC))^{-1}L)^{-1} = (C(sl - A))^{-1}L + 1 \).

Linear, observer-based fault detection, isolation, and identification schemes work well if an accurate model exists for the process over the whole operating region and if appropriate choices are made for \( L \) and \( Q \).
Consider a set $\delta(s)$ of real polynomials of degree $n$ of the form

$$\delta(s) = \delta_0 + \delta_1 s + \delta_2 s^2 + \delta_3 s^3 + \delta_4 s^4 + \ldots + \delta_n s^n$$

where the coefficients lie within given ranges

$$\delta_0 \in [\delta_0^-, \delta_0^+], \delta_1 \in [\delta_1^-, \delta_1^+], \ldots, \delta_n \in [\delta_n^-, \delta_n^+]$$

Denote that $\delta = [\delta_0, \delta_1, \ldots, \delta_n]$ and define a polynomial $\delta(s)$ by its coefficient vector $\delta$. Furthermore, define the hyperrectangle of coefficients.

$$\Omega := \{ \delta : \delta \in \mathbb{R}^{n+1}, \delta_1^- \leq \delta_i \leq \delta_1^+, i = 0, 1, 2, \ldots, n \}$$

Assume that the degree remains invariant over the family, so that $0 \in \{ \delta_0^-, \delta_0^+ \}$. A set of polynomials with the above properties is called an interval polynomial family. Kharitonov’s theorem provides a necessary and sufficient condition for the Hurwitz stability of all members contained in this family.

**Theorem 1 (Kharitonov’s Theorem).** Every polynomial in the family $\delta(s)$ is Hurwitz if and only if the following four extreme polynomials are Hurwitz:

- $\delta^-(s) = \delta_0^- + \delta_2^- s^2 + \delta_3^- s^3 + \delta_4^- s^4 + \delta_5^- s^5 + \ldots + \delta_n^- s^n$
- $\delta^+(s) = \delta_0^+ + \delta_2^+ s^2 + \delta_3^+ s^3 + \delta_4^+ s^4 + \delta_5^+ s^5 + \ldots + \delta_n^+ s^n$
- $\delta^{--}(s) = \delta_0^- + \delta_2^- s^2 + \delta_3^- s^3 + \delta_4^- s^4 + \delta_5^- s^5 + \ldots + \delta_n^- s^n$
- $\delta^{+-}(s) = \delta_0^+ + \delta_2^+ s^2 + \delta_3^+ s^3 + \delta_4^+ s^4 + \delta_5^+ s^5 + \ldots + \delta_n^+ s^n$
- $\delta^{--}(s) = \delta_0^- + \delta_2^- s^2 + \delta_3^- s^3 + \delta_4^- s^4 + \delta_5^- s^5 + \ldots + \delta_n^- s^n$
- $\delta^{+-}(s) = \delta_0^+ + \delta_2^+ s^2 + \delta_3^+ s^3 + \delta_4^+ s^4 + \delta_5^+ s^5 + \ldots + \delta_n^+ s^n$

While this theorem has been used extensively in parametric approaches to robust control, it will be utilized for developing observers that handle parametric uncertainties and nonlinearities in the model.

4. Robust Fault Detection, Isolation, and Identification

4.1. Problem Formulation. Consider a nonlinear system with possibly multiple outputs of the following form:

$$\dot{x} = f(x, \theta)$$
$$y = h(x, \theta) + f_s$$

where $x \in \mathbb{R}^n$ is a vector of state variables and $y \in \mathbb{R}^m$ is a vector of output variables. It is assumed that $f(x, \theta)$ is a smooth analytical vector field in $\mathbb{R}^n$ and $h(x, \theta)$ is a smooth analytical vector field in $\mathbb{R}^m$. Let $\theta \in \mathbb{R}^k$ be a parameter vector assumed to be constant with time but a priori uncertain, and $f_s$ is the sensor fault of unknown nature with the same dimensions as the output. The goal of this paper is to estimate the state vector with limited information about the parameters describing the process model and under the influence of output disturbances such that $\lim_{t \to \infty} (x - \hat{x}) = 0$, where $\hat{x}$ is the estimate of the state vector, $x$, and to the design of a set of filters $Q(t)$ so that the residuals, given by the expression $r(t) = \int_0^t Q(t-r) [y(t) - \hat{y}(t)] \, dr$, have all of the five properties discussed in section 3.1.

One of the main challenges in this research is that both faults and plant–model mismatch will have an effect on the fault identification. To perform accurate state and parameter estimation, it is desired to have reliable measurements, while at the same time, an accurate model of the process is required to identify the fault. This will be taken into account by performing the parameter estimation and the fault detection at different time scales. Each time the parameters are estimated, it is assumed that the fault is not changing at that instance, while the values of the parameters are not adjusted during each individual fault detection. A variety of different techniques exist for designing nonlinear observers. However, because the class of problems under investigation includes parametric uncertainty, it would be natural to address these issues through a parametric approach instead of the often-used extended Kalman filter or extended Luenberger observer. The procedure for designing nonlinear observers under the influence of parametric uncertainty is outlined in the next subsections, which is followed by a description of the fault detection, isolation, and identification algorithm.

4.2. Estimator Design: A Parametric Approach. A nonlinear system of the form given by eq 6 can be rewritten by viewing the parameters as augmented states of the system

$$\dot{x} = \begin{bmatrix} f(x, \theta) \\ 0 \end{bmatrix}$$
$$y = h(x, \theta) + f_s$$

and with a change of notation

$$x = \begin{bmatrix} x \\ \theta \end{bmatrix}, \quad \dot{x}(x, \theta) = \begin{bmatrix} f(x, \theta) \\ 0 \end{bmatrix}$$

where $x \in \mathbb{R}^n$ is a vector of state variables, $\theta \in \mathbb{R}^k$ is a vector of parameter variables, and $f_s \in \mathbb{R}^m$ is a vector of sensor faults.

Figure 1. Schematic of a dedicated observer scheme for a system with two measurements.
this results in the following system:

\[ \dot{x} = f(x) \]
\[ y = h(x) + f_s \]  

(9)

For the state and parameter estimation step, it is assumed that the sensor faults are known because they are identified at certain “sampling times”, and the assumption is made that they remain constant over the time interval between two “sampling points”. Furthermore, assume that each component \( \theta_i \) of the parameter vector

\[ \theta := [\theta_0, \theta_1, ..., \theta_{k-1}] \]  

(10)
can vary independently of the other components and each \( \theta_i \) lies within an interval where the upper and lower bounds are known

\[ \Pi := \{ \theta: \theta_i^- \leq \theta_i \leq \theta_i^+ \}, i = 0, 1, 2, ..., k - 1 \]  

(11)

Also, let \( \theta = \theta_{ss} \in \Pi \) be a vector of constant a priori unknown parameters and \((x_{ss}, \theta_{ss})\) be an equilibrium point of eq 7. The augmented system needs to be observable in order to design an observer, which can also estimate the values of the parameters. A sufficient condition for local observability of a nonlinear system is if the observability matrix

\[ W_{ss}(x) = \begin{bmatrix} \frac{\partial h(x)}{\partial x} & \frac{\partial^2 h(x)}{\partial x^2} & ... & \frac{\partial^{n+k-1} h(x)}{\partial x^{n+k-1}} \end{bmatrix} \]  

(12)

has rank \( n + k \) for \((x, \theta) = (x_{ss}, \theta_{ss})\). Because the equilibrium points of the system depend on the values of the parameters that are not known a priori, it is required that the rank of \( W_{ss}(x) \) is checked for all \( \theta = \theta_{ss} \in \Pi \) and the resulting equilibrium points \((x_{ss}, \theta_{ss})\).

To proceed, it is assumed that the augmented system is observable over the entire hyperrectangle-like set \( \Pi \) and the equilibrium points corresponding to these parameter values. It is then possible to design an observer for the augmented system

\[ \begin{pmatrix} \dot{x} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} f(x, \theta) \\ \tilde{L}(x, \theta) (y - y) \end{pmatrix} + \begin{pmatrix} 0 \\ h(x, \theta) + f_s \end{pmatrix} \]  

(13)

where \( \tilde{L}(x, \theta) \) is the estimated observable of the system and \( \theta \) is the estimated state of the observer. It is a suitably chosen nonlinear observer gain. Also, note that the observer makes use of the assumption that the measurement fault is known from an earlier identification of the fault. When the observer is computed for the first time, it has no knowledge about possible sensor faults and assumes that no sensor fault was initially present.

4.2.1 Determining the Family of Polynomials for Observer Design. In this section, the result about Hurwitz stability of an interval family of polynomials from section 3.2 is utilized to determine a methodology for computing the gain \( L(x, \theta) \) of the nonlinear observer given by eq 13. Consider the linearized model of the augmented process model around an equilibrium point \((x_{ss}, \theta_{ss})\)

\[ \begin{pmatrix} \dot{x} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} \tilde{A}(x_{ss}, \theta_{ss}) \end{pmatrix} x \\ \begin{pmatrix} 0 \end{pmatrix} = \begin{pmatrix} \tilde{C}(x_{ss}, \theta_{ss}) \end{pmatrix} x + f_s \]  

(14)

where \( \tilde{A}(x_{ss}, \theta_{ss}) \) is the jacobian of \( f(x, \theta) \) at the point \((x_{ss}, \theta_{ss})\) and \( \tilde{C}(x_{ss}, \theta_{ss}) = [i \partial h(x, \theta)/\partial x(x_{ss}, \theta_{ss}) i \partial h(x, \theta)/\partial \theta(x_{ss}, \theta_{ss})] \). The characteristic polynomial of the system, which determines its stability, is given by

\[ \delta(s) = \text{det}[sI - \tilde{A}(x_{ss}, \theta_{ss})] = \delta_0(x_{ss}, \theta_{ss}) + \delta_1(s_{ss}, \theta_{ss}) s + \delta_2(s_{ss}, \theta_{ss}) s^2 + ... + s^n \]  

(15)

It can be seen that the coefficients of the characteristic polynomial are nonlinear functions of the parameter vector \( \theta_{ss} \) and \( x_{ss} \). Assuming that \( f(x_{ss}, \theta_{ss}) \) satisfies the conditions of the implicit function theorem, i.e., \( i \partial f(x, \theta)/\partial x(x_{ss}, \theta_{ss}) \neq 0 \), then \( x_{ss} \) can be solved for a given \( \theta_{ss} \); i.e., \( x_{ss} = \phi(\theta_{ss}) \), where \( \phi: R^k \rightarrow R^n \). The characteristic polynomial in \( s \) given by eq 15 can then be rewritten as

\[ \delta(s) = \delta_0(\phi(\theta_{ss}), \theta_{ss}) + \delta_1(\phi(\theta_{ss}), \theta_{ss}) s + \delta_2(\phi(\theta_{ss}), \theta_{ss}) s^2 + ... + s^n \]  

(16)

where \( \delta_i(\phi(\theta_{ss}), \theta_{ss}) = \bar{\delta}_i(\theta_{ss}), i = 0, 1, 2, 3, ..., n - 1 \). While it is generally not possible to derive an analytical expression of the coefficients \( \delta_i(\theta_{ss}) \) as a function of \( \theta_{ss} \), \( x_{ss} \) can be evaluated by numerically solving the equation \( f(x_{ss}, \theta_{ss}) = 0 \) for \( \theta_{ss} \in \Pi \). Because \( f(x, \theta) \) is assumed to be a smooth vector function, the coefficients of the characteristic polynomial are continuous functions of \((x_{ss}, \theta_{ss})\). Therefore, by discretization of the set \( \Pi \) and evaluating the maximum and minimum values for each coefficient \( \delta_i(\theta_{ss}) \) over all the points in the set \( \Pi \), the hyperrectangle of coefficients \( \Omega \) as described in section 3.2 can be obtained. In the case of a multidimensional parameter \( \theta \), discretizing the set \( \Pi \) can be computationally expensive; however, advanced nonlinear programming algorithms exist that facilitate the calculation of the required bounds on the coefficients. For the case where \( \theta \) is a scalar, the range within which the coefficients vary can be determined by plotting \( \delta_i(\theta_{ss}) \) against \( \theta_{ss} \) for \( i = 0, 1, 2, 3, ..., n - 1 \). Figure 2 shows a typical plot of the coefficient versus the one-dimensional parameter \( \theta \).

To enforce that the estimation error decays asymptotically for the linearized system, the observer gains are chosen to satisfy the condition

\[ \lambda[\tilde{A}(x_{ss}, \theta_{ss}) - \tilde{L}(x_{ss}, \theta_{ss}) \tilde{C}(x_{ss}, \theta_{ss})] \in C^- \quad \forall \theta_{ss} \in \Pi \]  

(17)

where \( \lambda(\cdot) \) refers to the eigenvalues of the matrix. The following section focuses on computing appropriate gains \( L \) by making use of Kharitonov’s theorem.

4.2.2 Observer Gain Computation. Because it is assumed that the augmented system given by eq 7 is observable over the entire hyperrectangle-like set \( \Pi \) and the equilibrium points corresponding to these parameter values, it is possible to find an invertible transformation \( T(x_{ss}, \theta_{ss}) \), s.t. \( \forall \theta_{ss} \in \Pi \); the LTI system given by eq 14
Consider the set of \( A \) and \( C \) in observable canonical form and the coefficients of the characteristic polynomial of \( A \) are subject to bounded perturbations. The following theorem guarantees that there exists a free parameter vector \( I \) such that the pair \( (A, C) \) can always be stabilized.

**Theorem 2.** For any set of nominal parameters, \( \{0, 1, \ldots, n-1\} \), and for any set of positive numbers \( \Delta_0, \Delta_1, \ldots, \Delta_{n-1} \), it is possible to find a vector \( I \) such that the entire family \( (s) \) is stable.

By theorem 2, there always exists a \( I \) such that \( [A(x_{ss}, \theta_{ss}) - IC] \in C \), \( \forall \theta_{ss} \in II \), which can be systematically computed (appendix B). The result is an observer given by eq 13, which locally estimates the states and parameters of the system given by eq 6 and with an observer gain \( L(x, \theta) = T^{-1}(x, \theta)I \). The proposed approach is considerably less computationally demanding than alternative state and parameter estimation techniques such as extended Luenberger observers.

Additionally, the presented method yields an analytical expression for observer gains irrespective of the dimension of the systems but guarantees convergence of the error dynamics locally around the operating point.

**4.3 Fault Detection.** The purpose of fault detection is to determine whether a fault has occurred in the system. It can be seen that \( \lim_{t \to \infty} (x - \hat{x}) = 0 \) in the presence of sensor faults. To extract the information about faults from the system, a residual needs to be defined as \( r(t) = j_{\theta}(x(t), y(t)) \), where \( Q(t) \) is any stable filter. It can be verified that \( (i) \lim_{t \to \infty} r(t) = 0 \) if \( f_x(t) = 0 \) and \( (ii) \lim_{t \to \infty} r(t) = f \) if \( f_x(t) \neq 0 \). Additional restrictions on the class of stable filters \( Q(t) \) will be imposed in the following sections in order to satisfy desired objectives.

**4.4 Fault Isolation.** Fault isolation is synonymous with determining the location of a fault, and its computation imposes additional restrictions on the choice of the filter \( Q(t) \). To perform fault isolation, the augmented system given by eq 7 is assumed to be separately locally observable through each of the outputs \( y \) \( \forall \theta_{ss} \in II \). It should be noted that this requirement is mandatory for the existence of a fault isolation filter and hence does not pose a stringent condition for using the presented approach.

To achieve fault detection as well as isolation, the proposed approach uses a series of dedicated nonlinear observers as shown in Figure 1. In this method, as many residuals are generated as the number of measurable outputs. It can be verified that \( (i) \lim_{t \to \infty} r_i(t) = 0 \) if \( f_{s_i}(t) \neq 0 \) and \( (ii) \lim_{t \to \infty} r_i(t) = 0 \) if \( f_{s_i}(t) = 0 \), \( i = 1, 2, 3, \ldots, m \) for an appropriately chosen filter \( Q(t) \).
4.5. Fault Identification. To estimate the shape and size of the fault, the residuals have to meet the following objective:

\[
\lim_{t \to \infty} [r_i(t) - f_i(t)] = 0 \quad i = 1, 2, ..., m
\]

Because a dedicated nonlinear observer scheme is utilized in the proposed approach, it remains to choose a suitable stable filter \( Q(t) \) to meet all of the conditions for fault detection, isolation, and identification. It was shown in section 3.1 that an appropriate choice of \( Q(t) \) for a LTI system described by eq 1 is given by

\[
Q(s) = \begin{bmatrix} C(sI - A)^{-1}L + 1 \end{bmatrix}
\]

where \( Q(s) \) is the Laplace transform of the filter \( Q(t) \). Similarly, for the nonlinear system given by eq 9, a linear filter

\[
Q(s) = \{ \mathbf{C}(x_{ss}, \theta_{ss}) [sI - \mathbf{A}(x_{ss}, \theta_{ss})]^{-1} \mathbf{L}(x_{ss}, \theta_{ss}) + 1 \} - I
\]

is locally applicable. Because the equilibrium point \( (x_{ss}, \theta_{ss}) \) is a priori unknown, the fault identification filter is modified:

\[
Q(s) = \{ \mathbf{C}(\hat{x}, \hat{\theta}) [sI - \hat{\mathbf{A}}(\hat{x}, \hat{\theta})]^{-1} \mathbf{L}(\hat{x}, \hat{\theta}) + 1 \}
\]

where \( Q(s) \) is the Laplace transform of the filter at any point \( (\hat{x}, \hat{\theta}) \) in the state space. However, because at least as many eigenvalues of \( \mathbf{A}(\hat{x}, \hat{\theta}) \) are identical to zero as there are parameters of the original system, the above \( Q(t) \) is not stable. To overcome the problem of choosing a stable filter for fault reconstruction, a lower dimensional observer that does not perform the parameter estimation but only estimates the states needs to be considered

\[
\dot{\hat{x}} = \mathbf{f}(\hat{x}, \hat{\theta}) + \mathbf{L}(\hat{x}, \hat{\theta})(\mathbf{y} - \hat{\mathbf{y}})
\]

\[
\dot{\hat{y}} = \mathbf{h}(\hat{x}, \hat{\theta}) + \mathbf{f}_s
\]

(20)

where \( \hat{x}(t) \) is the estimate of \( x(t) \) and

\[
\mathbf{L}(x_{ss}, \theta_{ss}) \text{ s.t. } \lambda [\mathbf{A}(x_{ss}, \theta_{ss}) - \mathbf{L}(x_{ss}, \theta_{ss}) \mathbf{C}(x_{ss}, \theta_{ss})] \in \mathbb{C}^-, \quad \forall \theta_{ss} \in \Pi
\]

(21)

where \( \mathbf{A}(x_{ss}, \theta_{ss}) \) is the \( J \) acobian of \( \mathbf{f}(x, \theta) \) at the point \( (x_{ss}, \theta_{ss}) \) and \( \mathbf{C}(x_{ss}, \theta_{ss}) = \partial \mathbf{h}(x, \theta) / \partial x \)

**Lemma 1.** The nonlinear system described by eq 20 in conjunction with the observer of the augmented system (13) is a locally asymptotic observer to the system given by eq 6 if \( \mathbf{f}_s \) is known.

**Proof.** \( \mathbf{L}(x_{ss}, \theta_{ss}) \) is chosen such that the condition in eq 17 is met, \( \lim_{t \to \infty} (\theta - \hat{\theta}) = 0 \). Linearizing the system given by eq 20 around the equilibrium point \( (x_{ss}, \theta_{ss}) \):

\[
\dot{x} = \mathbf{A}(x_{ss}, \theta_{ss}) \hat{x} + \mathbf{L}(x_{ss}, \theta_{ss})(\mathbf{y} - \hat{\mathbf{y}})
\]

\[
\dot{\hat{y}} = \mathbf{C}(x_{ss}, \theta_{ss}) \hat{x} + \mathbf{f}_s
\]

(22)

Similarly, linearizing the system given by eq 6 around the equilibrium point \( (x_{ss}, \theta_{ss}) \) results in

\[
x = \mathbf{A}(x_{ss}, \theta_{ss}) x
\]

\[
y = \mathbf{C}(x_{ss}, \theta_{ss}) x + \mathbf{f}_s
\]

(23)

The error of the state estimates, \( e = x - \hat{x} \), is then given by the following equation:

\[
e = [\mathbf{A}(x_{ss}, \theta_{ss}) - \mathbf{L}(x_{ss}, \theta_{ss}) \mathbf{C}(x_{ss}, \theta_{ss})] e
\]

(24)

Because \( \mathbf{L}(x_{ss}, \theta_{ss}) \) is chosen to satisfy the condition in eq 21, the estimation error in eq 24 converges asymptotically to zero.

Note that the gains for the observers given by eq 20 can be computed using the technique presented in section 4.2.2. Observability conditions similar to those in section 4.2 can be derived for the existence of gains that guarantee stability of the closed-loop observers in the neighborhood of the operating point.

For practical purposes, the original system given by eq 6 in the absence of faults is considered to be locally stable around the operating point as the parameters vary in the hyperrectangle as defined by eq 11. In other words, it is assumed that the \( J \) acobian \( \mathbf{A}(x_{ss}, \theta_{ss}), \forall \theta_{ss} \in \Pi \), is Hurwitz stable.

Using the above assumption, a stable linear fault identification filter \( Q(t) \) subject to the residual \( r(t) = \mathbf{y}(t) - \hat{\mathbf{y}}(t) \), having the property that \( \lim_{t \to \infty} r(t) = \mathbf{f}_s \), has the following state-space representation:

\[
\dot{\xi} = \mathbf{A}(\hat{x}, \hat{\theta}) \xi + \mathbf{L}(\hat{x}, \hat{\theta})(\mathbf{y} - \hat{\mathbf{y}})
\]

\[
r = \mathbf{C} \xi + 1(\mathbf{y} - \hat{\mathbf{y}})
\]

(25)

where \( \hat{\mathbf{y}} \) and \( \hat{x} \) are the output and state estimates obtained via the observer given by eq 20 and \( \xi \in \mathbb{R}^n \) is a state with initial condition \( \xi(0) = 0 \).

Putting all of these pieces together, the fault detection, isolation, and identification filter consists of the observers given by eqs 13 and 20 and is computed in parallel with eq 25 in order to generate residuals. The filter is recomputed at each time step by linearizing the model at the estimate of the location in state space of the augmented system.

In the presence of unknown sensor faults, the estimate \( \hat{\theta} \) for some \( \theta_{ss} \in \Pi \) may diverge from the actual value, and therefore the stability of the overall fault diagnosis system cannot be guaranteed. To overcome this problem, parameter estimation and fault reconstruction are performed at different time scales and it is assumed that the algorithm is initialized when no sensor fault occurs until a time \( t_0 \) such that some \( \epsilon > 0 \), \( \| \mathbf{y} - \hat{\mathbf{y}} \|_2 \leq \epsilon, \quad \forall t_0 \geq 0 \). The sensor fault is of the following form:

\[
\mathbf{f}_s(t) = f(t) S(t-t_0), \quad S(t-t_0) = \begin{cases} 1: & t \geq t_0 \\ 0: & t < t_0 \end{cases}
\]

The above assumption ensures that the parameter estimate from eq 17 converges to its actual value with a desired accuracy

\[
\| \theta_{ss} - \hat{\theta} \|_2 \leq \eta, \quad \eta(\epsilon) > 0
\]

(26)

before the onset of faults in the original process. Additionally, the parameters are adapted periodically by the augmented observer (13) in order to take process drifts into account.
In summary, the presented fault diagnosis system performs parameter estimation and fault reconstruction at different time scales, where the fault identification takes place at a higher frequency than the parameter estimation. The values of the parameters are assumed to stay constant during the fault identification, while the faults are assumed to be constant during the parameter estimation. Figure 3 illustrates this two-time scale behavior, where stages 2 and 3 are repeated alternatively throughout the operation and the time between the start of each stage is decided by the nature of the process. However, in general the parameter estimation is only performed sporadically and requires only short periods of time, so that the fault can be identified the vast majority of the time.

5. Case Study

5.1. Fault Diagnosis of a Reactor with Uncertain Parameters. To illustrate the main aspects of the investigated observer-based fault diagnosis scheme, a nonisothermal continuous stirred tank reactor (CSTR) is considered with coolant jacket dynamics, where the following exothermic irreversible reaction between sodium thiosulfate and hydrogen peroxide is taking place:

\[
2\text{Na}_2\text{S}_2\text{O}_3 + 4\text{H}_2\text{O}_2 \rightarrow \text{Na}_2\text{S}_3\text{O}_6 + \text{Na}_2\text{S}_2\text{O}_4 + 4\text{H}_2\text{O}
\]  

(27)

The capital letters A–E are used to denote the chemical compounds Na$_2$S$_2$O$_3$, H$_2$O$_2$, Na$_2$S$_3$O$_6$, Na$_2$S$_2$O$_4$, and H$_2$O. The reaction kinetic law is reported in the literature to be

\[
-r_A = k(T) C_A C_B = (k_0 + \Delta k_0) \exp \left( \frac{-E + \Delta E}{RT} \right) C_A C_B
\]

where \(\Delta k_0\) and \(\Delta E\) represent parametric uncertainties in the model. A mole balance for species A and energy balances for the reactor and the cooling jacket result in the following nonlinear process model:

\[
\frac{dC_A}{dt} = \frac{F}{V} (C_{A\text{in}} - C_A) - 2k(T) C_A^2
\]

\[
\frac{dT}{dt} = \frac{F}{V} (T_{\text{in}} - T) + 2 \frac{(-\Delta H)_R + \Delta(-\Delta H)_R}{\rho C_p} k(T) C_A^2 - \frac{UA + \Delta UA}{V \rho C_p} (T - T_j)
\]

\[
\frac{dT_j}{dt} = \frac{F}{V} (T_{\text{in}} - T_j) + \frac{UA + \Delta UA}{V w \rho w C_{pw}} (T - T_j)
\]

(28)

where \(F\) is the feed flow rate, \(V\) is the volume of the reactor, \(C_{A\text{in}}\) is the inlet feed concentration, \(T_{\text{in}}\) is the inlet feed temperature, \(V_w\) is the volume of the cooling jacket, \(T_{\text{in}}\) is the inlet coolant temperature, \(F_w\) is the inlet coolant flow rate, \(C_p\) is the heat capacity of the reacting mixture, \(C_{pw}\) is the heat capacity of the coolant, \(\rho\) is the density of the reacting mixture, \(U\) is the overall heat-transfer coefficient, and \(A\) is the area over which the heat is transferred. The process parameter values are listed in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(F)</td>
<td>1000 g/L</td>
</tr>
<tr>
<td>(C_{A\text{in}})</td>
<td>1 mol/L</td>
</tr>
<tr>
<td>(V)</td>
<td>100 L</td>
</tr>
<tr>
<td>(T_{\text{in}})</td>
<td>275 K</td>
</tr>
<tr>
<td>((-\Delta H)_R)</td>
<td>76 534.704 J/mol</td>
</tr>
<tr>
<td>((\Delta E)/R)</td>
<td>76 534.704 J/mol</td>
</tr>
</tbody>
</table>

The capital letters A–E are used to denote the chemical compounds Na$_2$S$_2$O$_3$, H$_2$O$_2$, Na$_2$S$_3$O$_6$, Na$_2$S$_2$O$_4$, and H$_2$O. The reaction kinetic law is reported in the literature to be

\[
2\text{Na}_2\text{S}_2\text{O}_3 + 4\text{H}_2\text{O}_2 \rightarrow \text{Na}_2\text{S}_3\text{O}_6 + \text{Na}_2\text{S}_2\text{O}_4 + 4\text{H}_2\text{O}
\]

(27)

The capital letters A–E are used to denote the chemical compounds Na$_2$S$_2$O$_3$, H$_2$O$_2$, Na$_2$S$_3$O$_6$, Na$_2$S$_2$O$_4$, and H$_2$O. The reaction kinetic law is reported in the literature to be

\[
2\text{Na}_2\text{S}_2\text{O}_3 + 4\text{H}_2\text{O}_2 \rightarrow \text{Na}_2\text{S}_3\text{O}_6 + \text{Na}_2\text{S}_2\text{O}_4 + 4\text{H}_2\text{O}
\]

(27)

The capital letters A–E are used to denote the chemical compounds Na$_2$S$_2$O$_3$, H$_2$O$_2$, Na$_2$S$_3$O$_6$, Na$_2$S$_2$O$_4$, and H$_2$O. The reaction kinetic law is reported in the literature to be

\[
2\text{Na}_2\text{S}_2\text{O}_3 + 4\text{H}_2\text{O}_2 \rightarrow \text{Na}_2\text{S}_3\text{O}_6 + \text{Na}_2\text{S}_2\text{O}_4 + 4\text{H}_2\text{O}
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\[
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\]

(27)
384.005 K; \( T_{jss} = 371.272 \) K) is stable and chosen as the point of operation. Because the activation energy appears exponentially in the state-space description of the process, the effect of uncertainty on the behavior of the system is significantly higher than that of the other parameters listed above. This observation has also been confirmed in simulations.

To validate the performance of the presented approach, it is in a first step compared to the results derived from a fault detection scheme based upon a Luenberger observer for the process under consideration. For now, the process parameters are assumed to be known and given the values in Table 1. The system matrices obtained by linearizing the process model (28) around the chosen steady state are

\[
A = \begin{bmatrix}
-123.749 & 972.4 & -0.073 & 473 & 630 & 19 & 0 \\
17 & 408.486 & 19 & 6.379 & 943 & 743 & 2.857 & 142 & 857 \\
0 & 28.571 & 428 & 57 & -31.571 & 428 & 57
\end{bmatrix}
\]

(29)

\[
C_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}^T, \quad C_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}^T
\]

with \( \lambda(A) = \{-112.94, -1.37, -34.63\} \). For performing fault isolation and identification, it is required to design observers for each of the two measurements as shown in Figure 1, and the eigenvalues of the closed-loop observers are placed at \( \{-6.85, -6.86, -6.87\} \). The observer gain calculated for a measurement of the reaction temperature is

\[
L_1 = \begin{bmatrix}
-53.912 \\
1.55E + 3 \\
5.79E + 5
\end{bmatrix}
\]

and the gain corresponding to the coolant temperature is found to be

\[
L_2 = \begin{bmatrix}
1.7E + 2 \\
2.7E + 4 \\
1.55E + 3
\end{bmatrix}
\]

Both reaction temperature and coolant temperature sensors are induced with an additive fault signal and random noise with normal distribution whose shape and size are shown in Figure 4.

Residuals generated by the technique based upon a Luenberger observer with uncertainty in the initial conditions are shown in Figure 5. When Figures 4 and 5 are compared, it is concluded that the Luenberger observer-based fault diagnosis scheme is able to isolate and identify the approximate nature of the fault in each sensor. Similar simulations have been carried out, where the process model includes uncertainties \( \Delta k_0 = 5\% k_0, \Delta E = 6\% E, \Delta(\Delta H) = 5\% \Delta H, \) and \( \Delta UA = 5\% UA \). Figure 6 shows the residual generated for the fault signal shown in Figure 4 for one specific case of parametric uncertainty. From Figure 6, it is evident that while the shape of the fault is reproduced almost perfectly, the bias in the residuals results from modeling uncertainties and can be misinterpreted as a response to a step fault in the sensor. To illustrate this point, simulations of the fault diagnosis scheme based upon the Luenberger observer are performed for a sufficiently large number of scenarios (10 000), which include a random occurrence of faults in either or both of the sensors as well as randomly chosen parametric uncertainty within the given intervals in order to determine the overall percentage of successfully identifying one or all of the scenarios. The scenarios denoted by 00, 01, 10, and 11 in Tables 2 and 3 stand for no faults in both sensors, no fault in the reaction temperature sensor and fault in the coolant temperature sensor, fault in the reaction temperature sensor and no fault in the coolant temperature sensor, and fault in both sensors, respectively. Step faults starting at time \( t = 0 \) and of magnitude 5 K were added to the sensors. Various thresholds are selected to determine whether a fault occurred in the sensors, and the fault isolation scheme (based upon a Luenberger observer) is tested in Monte Carlo simulations where the parametric uncertainty is chosen at random within the given intervals. As an example of this scheme, the scenario identifies the condition where no faults occur in both sensors for a chosen threshold \( \alpha \) if the following condition is satisfied: (a) if the time average of \( r_c(t) \) < \( \alpha \), where \( r_c(t) \) denotes the coolant temperature residual, and (b) if the
time average of $|r_r(t)| < \alpha$, where $r_r(t)$ denotes the reactor temperature residual.

The criteria used for the other (01, 10, and 11) scenarios are chosen accordingly. Table 2 summarizes the results of how efficiently the fault isolation scheme was able to predict the correct fault locations for random uncertainties in all of the parameters within the range described above. These results show that the parametric uncertainty can have a strong effect on robustness properties of a fault diagnosis scheme and hence requires techniques that can cope with model uncertainty. Because of these limitations, the nonlinear fault detection scheme presented in this work is applied to the same scenario. Because the effect of uncertainty in the process parameters other than the activation energy has been determined to be of lesser importance for fault isolation, only uncertainty in the activation energy $E := \{E := 0.94E_{ss} \leq E \leq 1.06E_{ss}\}$ is considered $E_{ss} = 76534.704$ J/mol. However, while the design is solely performed based upon uncertainty in this one parameter, the evaluation of the fault diagnosis scheme will consider uncertainty in all of the parameters to compare it to the Luenberger observer scheme. The interval polynomial computed by a step-by-step procedure as discussed in section 4.2.2 is as follows:

$$\delta(s) = \delta_0 + \delta_1s + \delta_2s^2 + s^3$$  \hspace{1cm} (30)
Figure 8. Reactor and coolant temperature fault signals.

where

\[ \delta_0 \in [2143, 11840], \quad \delta_1 \in [1648, 9090], \quad \delta_2 \in [79, 289] \]

It can be verified by theorem 1 that the interval polynomial family given by eq 30 is Hurwitz stable, thereby verifying that the nonlinear system given by eq 28 is locally stable around the operating points as \( \theta \) varies in the set \( \Theta := \{ \theta := 0.94 e_{ss} \leq \theta \leq 1.06 e_{ss} \} \), where \( e_{ss} = 76534.704 \) J/mol.

The detailed derivation of the observer gain computation is not presented here because of space constraints, but the procedure has been provided in section 4.2.2. Similarly the observer gains for the simultaneous state and parameter estimator from the reactor temperature is

\[
L(\hat{x}, \hat{\theta}) = T^{-1}(\hat{x}, \hat{\theta}) \begin{bmatrix} -5929 \\ -12970 \\ -11347.5 \\ -6113 \end{bmatrix}
\]

where \( T^{-1}(\hat{x}, \hat{\theta}) \) is the locally invertible transformation as shown in section 4.2.2. Similarly the observer gains for the state estimator of the form eq 20 to be used for fault isolation are computed to be

\[
L_1(\hat{x}, \hat{\theta}) = T_1^{-1}(\hat{x}, \hat{\theta}) \begin{bmatrix} -5929 \\ -4143 \\ 878.5 \end{bmatrix}
\]

\[
L_2(\hat{x}, \hat{\theta}) = T_2^{-1}(\hat{x}, \hat{\theta}) \begin{bmatrix} -5929 \\ -4143 \\ 878.5 \end{bmatrix}
\]

Using the presented technique and applying it to a system with uncertainty in all of the model parameters, it is found that the estimate of the activation energy converges to its true value after 7 min in the absence of sensor faults. The condition that there is no initial sensor fault is a reasonable assumption because one would like to have a certain level of confidence in the measurements before a fault diagnosis procedure is invoked. Figure 8 shows the fault signal \( f_i(t) \) that is affecting the sensors. The corresponding coolant and reactor temperature residuals generated by the Kharitonov theorem-based fault identification techniques are presented in Figure 9. It is apparent that the residuals converge to the values of the faults even when uncertainty exists in the model parameters. Additionally, the location, shape, and magnitude of the faults are correctly reconstructed, and sensor noise is filtered.

Because the performed simulation has only used uncertainty in the activation energy, Monte Carlo simulations have a 100% success rate for the scenarios considered in Table 2. However, because this is not a very realistic assumption and in order to compare the presented fault detection scheme to the Luenberger observer-based one, Monte Carlo simulations are performed by taking the uncertainty in all of the parameters (\( \Delta k_0 = 5\% k_0, \Delta E = 6\% E, \Delta H(\Delta H) = 5\% H, \) and \( \Delta UA = 5\% UA \)) into account. The results are summarized in Table 3 and clearly show that the fault detection, isolation, and identification scheme performs very well even under the influence of uncertainty in all of the model parameters. It should also be noted that the assumption that only the activation energy has a major impact on the fault diagnosis was a good one because the fault identification was only designed for uncertainty in this parameter; nevertheless, reliable fault diagnosis is possible even under the influence of uncertainty in several other parameters. Additionally, it can be concluded that it is an important task to choose an appropriate threshold for determining a fault.

5.2. Fault Diagnosis of a Reactor with Uncertain and Time-Varying Parameters. In this section, the performance of the proposed fault diagnosis scheme is evaluated for the nonisothermal CSTR problem as introduced in section 5.1 but with model parameters varying with time. Because the activation energy affects the behavior of the system significantly more strongly than any other parameter, it is assumed that only the activation energy varies with time possibly because of catalyst deactivation or coking. Figure 10 shows the plot of the activation energy and its estimate over the simulated time span, and Figure 11 presents the fault signal \( f_i(t) \) that is affecting the sensors. The corresponding coolant and reactor temperature residuals generated by the Kharitonov theorem-based fault identification technique are shown in Figure 12. The time period during which the parameter is identified within accept-
which fault detection and identification is invoked ranges from 10 to 200 min. The parameter is adapted from 200 to 210 min. This is followed by another fault detection period ranging from $t = 210$ to 400 min. It can be concluded from Figure 12 that the fault identification scheme is effective even in the presence of time-varying uncertain parameters. It should be noted that the system would not work as well if the parameters were not periodically reidentified, as can be seen from Figure 12 during the time period just before 200 min.

6. Conclusions

A new observer-based fault diagnosis scheme for nonlinear dynamic systems with parametric uncertainty was presented. This approach is centered around two main components: the design of a nonlinear observer, which includes uncertain parameters as augmented states, and the choice of an appropriate fault isolation and identification filter for reconstructing the location and nature of the fault. The observer design was performed based upon Kharitonov's theorem but takes into account the effect that changes in the parameters have on the steady state of the system. This resulted in a nonlinear, augmented observer, which has the property that it is locally stable for parametric uncertainty within a specified range. The fault isolation and identification filter was designed based upon a linearization of the nonlinear model at each time step. Repeatedly computing the linearization of the model does not pose a problem in practice because it is computationally inexpensive.

Because it is not possible to simultaneously perform parameter estimation and fault detection, these two tasks were implemented at different time scales. The parameters were estimated at periodic intervals where the fault was either assumed to be zero or known and constant, whereas the fault detection scheme was invoked at all times with the exception of the short periods used for the parameter estimation.

The performance of the proposed fault diagnosis method was evaluated using a numerical example of an exothermic CSTR and by performing Monte Carlo simulations on a bounded set of parametric uncertainties for a series of faults in both of the available measurements. The faults were reconstructed correctly even in the presence of severe uncertainties in the model parameters and measurement noise.

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Appendix A. Observer Form State Transformation

The state-space representation of a LTI system

$$\dot{x} = Ax$$

$$y = Cx$$

is given where $x \in \mathbb{R}^n$ and $y \in \mathbb{R}^1$. The characteristic polynomial of the matrix $A$ results in

$$\delta(s) = \delta_0 + \delta_1 s + \delta_2 s^2 + \ldots + \delta_{n-1} s^{n-1} + s^n$$

The aim is to find a coordinate transformation matrix
Appendix B. Observer Gain Computation

Consider a polynomial
\[ \delta(s) = \delta_0 + \delta_1 s + \delta_2 s^2 + \delta_3 s^3 + \ldots + \delta_{n-1} s^{n-1} + s^n \]
whose coefficients can vary independently within a given uncertainty range as follows:
\[ \left\{ \delta: \frac{\Delta \delta_i}{2} \leq \delta_i \leq \delta_i^0 + \frac{\Delta \delta_i}{2}, i = 0, 1, 2, \ldots, n - 1 \right\}, \]
where
\[ \Delta \delta_i = \delta_i^+ - \delta_i^- \]
and \( i = 0, 1, \ldots, n - 1 \).

The aim is to find a constant vector \( \mathbf{l} = (l_0, l_1, l_2, \ldots, l_{n-1}) \) to transform the interval polynomial family \( \delta(s) \) into another interval polynomial family described by
\[ \gamma(s) = (\delta_0 + l_0) + (\delta_1 + l_1)s + (\delta_2 + l_2)s^2 + \ldots + (\delta_{n-1} + l_{n-1})s^{n-1} + s^n \]
such that the entire family \( \gamma(s) \) remains Hurwitz.

1. Consider any stable polynomial \( R(s) \) of degree \( n - 1 \). Let \( \rho[R(s)] \) be the radius of the largest stability hypersphere around \( R(s) \). It can be shown that, for any positive real number \( \alpha \), \( \alpha \rho[R(s)] = \alpha \rho[R(s)]^6 \).

2. Thus, it is possible to find a polynomial \( \alpha R(s) \) such that
\[ \alpha = \sqrt{\frac{\sum_{i=0}^{n-1} (\Delta \delta_i)^2}{\rho[R(s)]}} \]

3. When \( R(s) = r_0 + r_1 s + r_2 s^2 + \ldots + r_{n-1} s^{n-1} + s^n \), the constant vector \( \mathbf{l} \) is calculated as follows:
\[ \{ \mathbf{l}: l_i = \alpha r_i - \delta_i^0, i = 0, 1, 2, \ldots, n - 1 \} \]

It can be seen from the above calculations that, for a given interval family \( \delta(s) \) with associated uncertainty ranges, there is an infinite number of possibilities for the constant gain vector \( \mathbf{l} \) that transform the given interval family \( \delta(s) \) into \( \gamma(s) \) such that \( \gamma(s) \) is Hurwitz.

Notation

- \( A, C \) = system matrices
- \( \hat{A}, \hat{C} \) = system matrices for the augmented system
- \( A, \hat{C} \) = canonically transformed matrices
- \( f \) = vector of faults
- \( h(x), \hat{h}(x) \) = vector fields in a state-space description of a continuous time nonlinear system
- \( l \) = constant observer gain vector
- \( L \) = constant observer gain matrix
- \( L(x, \hat{x}) \) = nonlinear observer gain for the augmented system
- \( L(x, \hat{x}) \) = nonlinear observer gain for the original nonlinear system
- \( \hat{L} \) = Lie derivative of \( h(x) \) with respect to \( f(x) \)
- \( q(t) \) = fault reconstruction filter
- \( r(t) \) = difference between the actual and estimated outputs
- \( t \) = time
- \( T, \hat{T} \) = invertible transformation matrices for the augmented and original systems, respectively
- \( W(x) \) = observability matrix
- \( x \) = vector of state variables
- \( \hat{x} \) = estimate of state variables of the original nonlinear system
- \( \hat{x} \) = estimate of state variables of the augmented system
- \( x \) = augmented state variables
- \( y \) = vector of output variables
- \( \hat{y} \) = estimate of output variables
- \( z \) = transformed state vector
Greek Letters

\( \delta(s) \) = open-loop interval polynomial family

\( \gamma(s) \) = closed-loop interval polynomial family

\( \Omega \) = hyperrectangle of coefficients of an interval polynomial family

\( \theta \) = uncertain parameter vector

\( \theta_0 \) = nominal parameter value

\( \Pi \) = hyperrectangle within which the uncertain parameter varies

\( f(x) \) = nonlinear map

\( \lambda(A) \) = eigenvalues of the matrix \( A \)

\( \xi \) = vector of state variables

\( \eta, \epsilon, \alpha \) = positive scalars

\( S(\cdot) \) = unit step function

Other Symbols

\( ||| \cdot ||| \) = Euclidean norm

\( R^n \) = n-dimensional Euclidean space

\( C \) = complex plane

\( C^- \) = left half complex plane

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