EXPLORING GENERATIVE MOMENTS OF INTERACTION BETWEEN MATHEMATICS TEACHERS ON SOCIAL MEDIA

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Stimulating sustainable mathematics teacher collaboration can be challenging in many commonly found professional development contexts. Despite this, an unprompted, unfunded, unmandated, and largely unstudied mathematics teacher community has emerged where mathematics teachers use social media to communicate about the teaching and learning of mathematics. This paper presents an analysis of one episode where teachers engage in a prolonged exchange about responding to a common mathematical error. Analytical tools drawn from complexity theory are used to explain moments of productivity. Results indicate that enough redundancy and diversity among members is necessary to make conversations productive. Identified sources of redundancy indicate the 'taken-as-shared' values of this group.

INTRODUCTION

Teacher professional development is essential for enhancing the quality of teaching and learning in schools (Borko, 2004). As such, various approaches to professional development, such as lesson study (Stigler & Hiebert, 1999) and communities of practice (Wenger, 1998), have been explored. What is known from this research, is that the robustness of a professional development initiative is dependent on ensuring both teachers and facilitators adopt a stance of inquiry, activities reflect and are driven by teacher needs and interests, and community building and networking are at the core (Lerman & Zehetmeier, 2008). This means that ongoing teacher collaboration is indispensable. However, due to constraints around time, funding, and facilitation, teacher professional development initiatives are commonly limited to sparse one-time professional development workshops held in face-to-face synchronous settings. Such workshops, due to their temporal nature, are generally unconducive to building communities that engender ongoing professional growth.

In contrast to these centrally organized, and sometimes compulsory, professional development initiatives, teachers from across North America are participating in decentralized, virtual, and autonomous professional communities. One such community involves mathematics teachers who regularly use Twitter and blog pages to asynchronously communicate their musings and practices, and have come to be identified as the Math Twitter Blogosphere (MTBoS) (Larsen, 2016). This unprompted, unfunded, and unevaluated teacher community is a rich phenomenon of interest that is largely unstudied and deserving of attention. As such, the study presented in this paper is driven by the overarching question – what can participation in the MTBoS occasion for mathematics teachers?

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THEORETICAL FRAMEWORK

With an aim to understand the autonomous organism of the MTBoS, this study is guided by complexity theory (Davis & Simmt, 2003; Davis & Sumara, 2006). Complexity theory provides the tools to describe a system of individual agents who seem to generate emergent macro-behaviours. Complex systems don't merely exist, they also learn. In complexity theory, learning is expanding the space of the possible and is primarily concerned with "ensuring conditions for the emergence of the as-yet unimagined" (Davis & Sumara, 2006, p. 135). The goal of complexity theory is not to identify interpersonal collectivity, as do other social theories of learning, but rather to understand 'collective-knowing', where knowledge is not attributed to any one member, but sits atop of the social network.

To this end, Davis and Simmt (2003) identify five interdependent conditions necessary for complex emergence, that is, for a complex system to learn. These conditions include *internal diversity*, *redundancy*, *neighbour interactions*, *decentralized control*, and *organized randomness*. Davis and Sumara (2006) further theorize these conditions into complementary pairs: specialization (tension between *diversity* and *redundancy*), trans-level learning (*neighbour interactions*[1] enabled through *decentralized control*), and enabling constraints (balancing *randomness* and *coherence*). These conditions form the basis of the theoretical framework that informs the overall study. For purposes of brevity, only the first pair of conditions, *diversity* and *redundancy*, will be used in the analysis presented in this paper.

The interplay between *diversity* and *redundancy*, also referred to as the 'zone of creative adaptability', is a key contributor to the ability of a system to adapt to changing conditions. *Diversity* allows for novel actions and possibilities because it refers to the diversity among the agents, while *redundancy* allows for stability and coherence because it refers to the common ground among agents. Without *redundancy*, agents may not be able to communicate, but without *diversity*, agents may never have anything to communicate about. Therefore, both are necessary for a system to be productive. Further, because of *decentralized control*, no agent is ever in a position of final authority, and knowledge is always tentative. Holding authority within a complex system means to have the capacity to use a prevailing discourse, or to act within the consensual domain of the system, with the overall aim of occasioning 'collective-knowing' (Davis & Sumara, 2006).

As such, this study takes interest in the possibility of 'collective-knowing' in the MTBoS, and pursues the question of how *diversity* and *redundancy* can contribute to the complex emergence of 'collective-knowing' in the MTBoS.

METHODS

Given that the MTBoS began developing as early as 2007 when mathematics teacher bloggers began to incorporate the use of Twitter into their blogging practice, and that there are over 500 self-identified MTBoS members, many of whom post multiple times a day, the sheer mass of data that has accumulated over the past few years makes the phenomenon too large to study within the confines of this paper. As such, a very specific subset is chosen as the data set for this paper. This subset contains all responses to a given Twitter post made by one particularly well-followed member. This conversation reflects the breadth and depth of MTBoS because it includes both very brief responses that do not continue conversation, and responses that initiate further conversation, both of which are generally encountered within the MTBoS.

Since Twitter is an ultra-personalized environment where users only see posts made by members they subscribe to as 'followers', we have taken an ethnographic approach as participant observers by immersing ourselves in the MTBoS community and subscribing to over 500 mathematics teachers who engage in the MTBoS. Without such an immersion, noticing and identifying the data set would be near to impossible. In addition, Twitter offers a feature which gives updates on the most relevant and most replied-to tweets one has missed. This feature enabled us to identify one particular post that generated a significant number of replies from mathematics teachers around the world. This post was made by Michael Fenton, who has over 4000 followers, and asked about how users would respond to a student's mathematical error (see fig. 1).

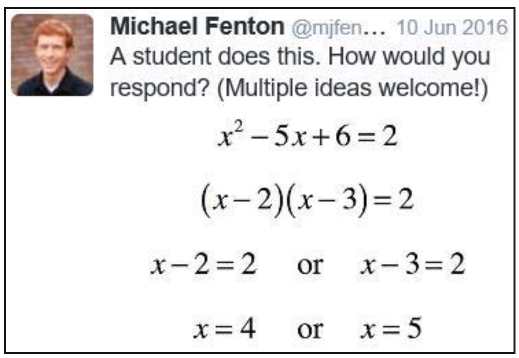

Figure 1: Fenton's initial math mistake query

Fenton's post elicited 254 replies from a total of 87 users, 52 of whom identify themselves as mathematics teachers. Replies included explaining the error, explaining why the error could have been made, describing a teaching approach to help the student come to a deeper understanding about the nature of the error, and generating activities to use with students to help mitigate this error. With an effort to maintain the reply structure as well as the chronological order of the posts, the data was organized into threads. Some of these threads were considered as *non-continuing* replies because they were made by one user and spawned little to no discussion. Other threads were considered as *continuing* because they included conversation between at least two users and elicited more than four subsequent replies. Out of the total 254 tweets, 84 were identified as non-continuing, 155 were identified as continuing, and 15 were irrelevant. The 155 continuing tweets were reconstructed into ten threads based on both chronology and logical conversation order.

With an aim of understanding the substantial variance in engagement in this conversation and to illuminate the complex emergence and 'collective-knowing' of the MTBoS, we analysed the data using the five conditions for complex emergence, as outlined by Davis and Simmt (2003) and further elaborated by Davis and Sumara (2006). As mentioned, in this paper, we discuss only the aspects of *diversity* and *redundancy* within the continuing threads in pursuit of the more specific question – what are possible sources of internal *diversity* and *redundancy* within a series of self-organized neighbour interactions in the MTBoS, and what complex emergence do they contribute to?

RESULTS AND ANALYSIS

In what follows, we exemplify continuing interchanges through the presentation of three interchanges along with an analysis of each with respect to *diversity* and *redundancy*, and draw out key conclusions.

Example 1: Check your answers

Kathy Howe (@kdhowe1) responds to Michael Fenton's (@mjfenton) math mistake query by explaining that she would get students to check their answers.

> That's a popular error. I focus on "lots of things multiply to 2, so there are lots of answers to that factored equation" … also, "Great! Do those answers check in the original equation? Oh, they don't? Why not?" (@kdhowe1, June 10, 2016, 7:15 AM)

Fenton provokes her by responding with a sample student response to her approach.

> "But Mr. Fenton, I checked the first one, and it worked. I figured the second one would work too." (@mjfenton, June 10, 2016, 7:23AM)

Howe then notes that she explains to her students what counts as a valid response.

> I explain to them early on that "right for the wrong reason" is still not a correct solution. (@kdhowe1, June 10, 2016, 7:54AM)

Howe's last comment is ignored, and the conversation does not continue further.

In example 1, some *redundancy* is evident in that Howe and Fenton seem to both have familiarity with the student error and with the mathematics. They have a 'taken-as-shared' understanding of a general classroom context where a teacher explains to students what to do. They both can envision a prototypical student. This enables them to communicate. However, there is *diversity* in approaches. Howe focuses on explaining to students that they need to check their answers and that they should know what's 'right' and what's 'wrong'. Fenton offers a potential student response to Howe's request for checking answers. Fenton is not only challenging the request for 'checking answers', but is also illuminating that he chose to design the mistake so that one factor works and the other doesn't. There is an opportunity to continue discussing the design of the task here that is not recognized by Howe. Fenton's responses elsewhere in the data indicate that he is interested in more than a typical response. The *diversity* in intentions seems to halt the conversation, and Howe's last comment is ignored. This *diversity* can also be attributed to the different levels of membership in the MTBoS between Howe and Fenton. Howe is a newer member, with less than 200 followers, while Fenton joined early on and has over 4000 followers. In this example, there seems to be too much *diversity* between Fenton and Howe in terms of how they approach interpreting each other's posts, their pedagogical approaches, and their membership in the MTBoS to continue conversation.

Example 2: Looking for patterns

Avery Pickford (@woutgeo) responds to Michael Fenton's (@mjfenton) query by expressing he loves the mistake and offers a string of equations from which he'd have students notice patterns.

> <3 this mistake. I'd probably try to subtly slip them (x-2)(x-3)=0, (x-2)(x-3)=9, & (x-2)(x-3) = 13 & ask them 2 look for patterns. (@woutgeo, June 10, 2016, 4:00PM)

Pickford further notes that he thinks discussion around this mistake can lead to new approaches to finding roots.

> what i love about this mistake is that i can see it naturally leading to a new method for finding roots involving factor pairs. (@woutgeo, June 10, 2016, 4:04PM)

A few hours later, Max Ray-Riek (@maxmathforum) asks him to predict patterns that could be noticed. He also asks if these patterns could "get kids thinking about the new method of factoring [she] mentioned" (@maxmathforum, June 10, 2016, 6:27PM). Pickford responds with a few options.

> idk. maybe 1) not the same answers (hmm) 2) 1st is easy, 2nd is medium (should have made it =20, not 9), 3rd is hard (@woutgeo, June 10, 2016, 6:17PM)

Ray-Riek agrees with Pickford that this is a useful mistake to entertain and claims that "it stretched [his] math brain" (@maxmathforum, June 10, 2016, 6:14PM). The conversation does not continue further.

In example 2, Pickford and Ray-Riek seem to have a fair amount of *redundancy* in their pedagogical approaches, which both involve asking learners to notice patterns among several examples chosen specifically to illuminate properties *without telling*. In fact, Pickford invokes a 'problem string' structure, known as a practice where "students answer related questions, the teacher models student thinking, [and] students construct relationships and connections" (Harris, n.d., para 3). This structure shows up elsewhere in the data, and is used by members who are relatively active in the MTBoS. It is referred to as an *instructional routine,* and acts as a source of *shared language*. Pickford and Ray-Riek are both familiar with this approach, and both agree that using a 'string' helps students notice mathematical properties without direct instruction. They also both entertain the idea of finding some sort of new mathematical approach given this student scenario. Since Ray-Riek offers similar examples as Pickford elsewhere in the data, the only source of *diversity* is in the specific examples they provide, and the choices they make in ordering and selecting numerical values with aims of illuminating various features. Pickford emphasizes the increasing difficulty in the examples, while Ray-Riek focuses on merely changing the product in different ways. In this example, there seems to be too much *redundancy* between Pickford and Ray-Riek to generate any further conversation because they both agree on their approach to interpreting each other's posts, and their pedagogical approaches. They are both also relatively well-connected with the MTBoS and its overarching values.

Example 3: Generating strings

Ray-Riek responded to Fenton's post earlier that day, responding to himself several times in a journal-like fashion.

(x-3)(x-2) = 2 still only has 2 answers ... there is only one set of factors of 2 that make this true. Why those? Hmm ... (@maxmathforum, June 10, 2016, 7:50AM)
I think the direction I'd go is to look at solving a bunch of quadratics that = 2. They all have different factors. Compare to = 0 (@maxmathforum, June 10, 2016, 7:56AM)
I think I'd look at (x+8)(x+4) = 12, (x-1)(x-2) = 12, and (x-6)(x-10) = 12. Analytically we could come up w/ different sol'ns … (@maxmathforum, June 10, 2016, 8:22AM)

About five days later, Michael Pershan (@mpershan) replies to Ray Riek's musings with examples of 'equation strings'.

How does the approach this equation string aims at compare to what you'd be aiming for? $(x-2)(x-4)=15$ $(A-3)(A-5)=15$ $(A-3)(A-5)=35$ $(Y-3)(Y-10)=0$ (@mpershan, June 15, 2016, 5:19PM)

Ray Riek responds by saying that he's "not thinking of it as eqn string . . . [but that] each has solns at different factors of 2" (@maxmathforum, June 15, 2016, 7:30PM). However, he then notes that he can see the 'string' Pershan is referring to.

oh now I see the string you are talking about. Is the idea here that 1) is easy and 2) is not b/c hard to get 7*5? (@maxmathforum, June 15, 2016, 7:30PM)
oh! Now I see the whole string. X=7, A=0, A=10, Y=3 or 10 ... No, I don't think your string gets at the same idea I had. (@maxmathforum, June 15, 2016, 7:33PM)

Pershan replies that he thought it was referring to the same idea because he's emphasizing multiplication in his example. However, Ray-Riek notes that although it's related, he wants "three problems that all = 12 but in different ways" (@maxmathforum, June 15, 2016, 3:35AM). They continue discussing their intentions, and both offer additional examples. Ray-Riek explains he expects that students ignore negative and non-obvious solutions, and wants to emphasize this through different ways of factoring. Ray-Riek then offers an alternative option that may further elicit the type of student noticing they both expect.

@mpershan @mjfenton I wonder about a #wodb with
$A: (x-2)(x-1)=12,$
$B: (x-2)(x-1)=0,$
$C: (x-5)(x+2)=0$ What might kids notice? (@maxmathforum, June 15, 2016, 5:55AM)

Ray-Riek's 'which one doesn't belong' example attracts another member to engage in thinking through the options and entertaining what students may notice. This conversation includes a total of 29 tweets, and prompts Pershan to post further about it in other threads.

In example 3, similarly as in example 2, both Ray-Riek and Pershan are active members of the MTBoS, and exhibit *redundancy* around the way they interpret each other's posts through inquiry and their general pedagogical approach of *teaching without telling* by asking students to observe patterns within a series of examples, guiding them towards *mathematical generalization*. They are both familiar with the *instructional routines* of 'problem strings' and 'which one doesn't belong', both common approaches to teaching discussed throughout the MTBoS, and they are able to communicate their intentions through examples of these. However, there is a slight amount of *diversity* in their approaches to and representations of the mathematics and to the *instructional routines*. They seem to use the *redundancy* to explore sources of *diversity* in a productive manner that leads them to generating several examples for use in teaching mathematics.

Overall, members who are connected to the MTBoS exhibit patterns of interaction such as *thinking like a learner*, *generating examples*, *invoking shared language*, and *using instructional routines*. They also indicate 'taken-as-shared' pedagogical approaches of *teaching without telling* that involve a teacher helping students arrive at a generalization through carefully chosen examples that will be discussed, which follows the 'notice and wonder' approach commonly exhibited in MTBoS discussions. These are all sources of *redundancy* in the MTBoS that allow users to communicate meaningfully. When this *redundancy* is not available, as in example 1, the conversation cannot become generative. When this *redundancy* is not paired with enough *diversity*, as in example 2, the conversation ends with agreeance. However, when this *redundancy* is paired with enough *diversity*, which is exposed through communication, there is possibility for the system to generate new as-yet unimagined tasks and approaches.

CONCLUSIONS

Engaging in the MTBoS with authority means to act within the consensual domain, which is to share sources of *redundancy* unique to the MTBoS. This investigation shows that the consensual domain of the MTBoS includes patterns of interaction such as *thinking like a learner*, *generating examples*, *invoking shared language*, and *using instructional routines*, as well as being guided by pedagogical values related to *teaching without telling* and guiding students towards *mathematical generalization*. Without these sources of *redundancy*, it is difficult to communicate productively.

However, it is also essential for there to be *diversity* around approaches and representations of mathematical ideas to allow for emergence of novel ideas for teaching and learning mathematics. Those with authority over the consensual domain of the MTBoS have greater capacity to push new meanings, and in turn, contribute to the complex emergence of the MTBoS.

This study indicates the potential for complex emergence in the MTBoS, and points to sources of *redundancy* and *diversity* that can contribute to the MTBoS as an autonomous asynchronous complex system that occasions space for generating an ideational network of mathematical tasks, pedagogy, and beliefs about mathematical teaching and learning. Further study should explore other cases where productivity occurs within the MTBoS to identify conditions that contribute to this productivity. The products of the MTBoS have great potential implications for teaching that need to be explored given that they are quickly unfolding and are developing at every moment.

Note

[1]Neighbour interactions refer to ideational interaction rather than social interaction. However, a physical component such as oral or written expression through various representations is often used for ideas to interact.

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