convex hull of a group of ellipsoids, instead of a single ellipsoid that would have resulted from the use of a single Lyapunov function. A numerical example was worked out to illustrate the results obtained in the technical note.

REFERENCES


Comments on “Disturbance Decoupling of Boolean Control Networks”

Juan Yao and Jun-e Feng

Abstract—A systematic method of disturbance decoupling for Boolean control networks has been developed in paper [1]. However, the purpose of this note is to show that the necessary and sufficient condition for the existence of disturbance decoupling controllers needs a mild modification, which is provided.

Index Terms—Boolean control network, disturbance decoupling, output-puty-subspace.

I. COUNTEREXAMPLE

Proposition V.1 of [1] says that

\[
F^2 (z(t), u(t), \xi(t)) = F^2 (z^2(t))
\]

iff \( Q^j (z^2(t), u(t), \xi(t)) = \text{constant} \) for every \( j = 1, 2, \ldots, k \).

This is true for the controller \( u(t) \) being a constant vector or a logical function of \( z(t) \). But Proposition V.1 is not necessary when the controller \( u(t) \) involves \( z^2(t) \), which will be shown by the following example. For the reader’s convenience, we use the symbols of paper [1].

Example 1: A disturbed 2-dimension Boolean control network with the following equation:

\[
\begin{align*}
z(t + 1) &= F_1 (z(t), z^2(t), u(t), \xi(t)), \\
\xi(t) &= z^2(t)
\end{align*}
\]

where

\[
F^2 (z(t), z^2(t), u(t), \xi(t)) = z^2(t) \land \left[ (z^2(t) \rightarrow \xi(t)) \lor u(t) \right]
\]

while

\[
F^1 (z^2(t), u(t), \xi(t)) = \text{any logical function of} \ z^2(t), \ z^2(t), \ u(t), \xi(t) \; \text{and all variables} \ z^2(t), \ z^2(t), \ u(t), \ y(t) \; \text{and} \ \xi(t) \; \text{belong to} \ \Delta_2.
\]

It is obvious that \( \mathcal{Z} \{ z^2(t) \} \) is an output-friendly subspace. Via computing, we get

\[
F^2 (z^2(t), z^2(t), u(t), \xi(t)) = \Xi_1 z^2(t) z^2(t) \xi(t) u(t) \tag{1}
\]

where \( \Xi_1 = \delta_2 [1, 1, 1, 1, 2, 2, 2, 2, 2, 2, 2, 2, 2] \).

Thus,

\[
F^2 (z^2(t), z^2(t), u(t), \xi(t)) = \left[ z^2(t) \land Q^1 (z^2(t), \xi(t), u(t)) \right] \lor \left[ z^2(t) \land Q^2 (z^2(t), \xi(t), u(t)) \right]
\]

where Boolean matrices \( \delta_2 [1, 1, 1, 2, 1, 1, 1, 2] \) and \( \delta_2 [2, 2, 2, 2, 2, 2, 2, 2, 2] \) are the structure matrices of \( Q^1 (z^2(t), \xi(t), u(t)) \) and \( Q^2 (z^2(t), \xi(t), u(t)) \), respectively.

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Taking \( u(t) \equiv 1 \), the condition of Proposition V.1 is satisfied. Hence it solves the problem. But it is easy to check that \( u(t) = z^2(t) \) is another solution, which does not satisfy the requirement of Proposition V.1.

Hence, Proposition V.1 is only a sufficient condition for 
\( F^2(z(t), u(t), \xi(t)) = F^2(z^2(t)) \). But when the controller \( u(t) \) is restricted to be a constant vector or a logical function of \( z^1(t) \), Proposition V.1 still holds as a necessary and sufficient condition.

Thus Theorem V.2 is also a sufficient condition for the solvability of disturbance decoupling problem (DDP) since Theorem V.2 is obtained based on Proposition V.1. In other words, when the disturbance decoupling controller \( u(t) \) is restricted to be a constant vector or a logical function of \( z^1(t) \), Theorem V.2 still holds as a necessary and sufficient condition.

II. MODIFICATION OF THEOREM V.2

Although, when the the disturbance decoupling controller \( u(t) \) is restricted to be a constant vector or a logical function of \( z^1(t) \), Proposition V.1 and Theorem V.2 all hold as a necessary and sufficient condition. But we can not get all disturbance decoupling controllers via them. In order to derive all disturbance decoupling controllers, we need to make a mild modification on them.

Using the same technique proposed in [1], we first give a modification to Proposition V.1. Consider the following system (which is also equations (34) in [1])

\[
\begin{cases}
  z^1(t + 1) = F^1(z(t), u(t), \xi(t)), \\
  z^2(t + 1) = F^2(z(t), u(t), \xi(t)), \\
  y(t) = G(z^2(t)).
\end{cases}
\]

Assume \( u(t) = z^2(t) \), it can be expressed as

\[
u = Kz^2z^1 = \varphi_{i_1i_2\ldots i_k}e_{i_1i_2\ldots i_k}(z^2)K_{i_1i_2\ldots i_k}z^1.\]

where

\[
e_{i_1i_2\ldots i_k}(z^2) = (z^2)_{i_k} \wedge \ldots \wedge (z^2)_{i_2} \wedge (z^2)_{i_1}
\]

and \( (z^2)_{i_l} = z^2 \) if \( i_l = 1 \) otherwise \( (z^2)_{i_l} = -z^2, i_l = 2; \ l = 1, 2, \ldots, k \). Furthermore, the relationship between \( K_{i_1i_2\ldots i_k} \) and \( K \) can be described as

\[
K = [K_1, K_2], K_{i_1} = [K_{i_11}, K_{i_12}], i_1 = 1, 2, K_{i_1i_2} = [K_{i_1i_21}, K_{i_1i_22}], i_1 = 1, 2; \ l = 1, 2, \\
\ldots \\
K_{i_1i_2\ldots i_k} = [K_{i_1i_2\ldots i_{k-1}1}, K_{i_1i_2\ldots i_{k-1}2}], i_1 = 1, 2; \ l = 1, 2, \ldots, (k - 1).
\]

Using controller (3), \( F^2(z(t), u(t), \xi(t)) \) can be expressed as

\[
F^2_j(z(t), u(t), \xi(t)) = \bigvee_{i_1 \ldots i_k} e_{i_1}(z^2(t)) \wedge Q_j^i(z^1(t), \xi(t)).
\]

Thus Proposition V.1 in [1] can be expressed as

**Proposition:** \( F^2(z(t), u(t), \xi(t)) = F^2(z^2(t)), \) iff there exist logical matrices

\[
K_{i_1i_2\ldots i_k}, i_1 = 1, 2; \ l = 1, \ldots, k
\]

with appropriate dimensions such that

\[
Q_j^i(z^1(t), \xi(t)) = \text{constant}
\]

where \( j = 1, \ldots, k; i = 1, \ldots, 2^k \).

The proof is exactly the same as that of Proposition V.1 in [1], so it is omitted here. Correspondingly, the second condition of Theorem V.2 in [1] can be described as

In (2) (which is also (34) of [1]) when \( F^2 \) is expressed as (4), there exist logical matrices

\[
K_{i_1i_2\ldots i_k}, \ i_1 = 1, 2; \ l = 1, \ldots, k
\]

with appropriate dimensions such that

\[
Q_j^i(z^1(t), \xi(t)) = \text{constant}
\]

where \( j = 1, \ldots, k; i = 1, \ldots, 2^k \).

Recall Example 1, assume that

\[
u = Kz^2z^1 = (z^2 \wedge K_1z^1) \vee (-z^2 \wedge K_2z^1).
\]

Substituting it into (4), we have that

\[
F^2(z^1(t), z^2(t), u(t), \xi(t)) = \Xi_2z^2(K_1z^1)(K_2z^1), z^1 \xi
\]

\[
= \left[ z^2 \wedge Q^1(1, K_1z^1, K_2z^1, \xi) \right] \vee \left[ -z^2 \wedge Q^2(z^1, K_1z^1, K_2z^1, \xi) \right]
\]

where

\[
\Xi_2 = \delta_2[1, 1, 1, 1, 1, 1, 1, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2],
\]

\[
Q^1(z^1, K_1z^1, K_2z^1, \xi) = \Xi_3(K_1z^1)(K_2z^1), z^1 \xi, Q^2(z^1, K_1z^1, K_2z^1, \xi) = \Xi_4(K_1z^1)(K_2z^1), z^1 \xi,
\]

\[
\Xi_3 = \delta_3[1, 1, 1, 1, 1, 1, 1, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2],
\]

\[
\Xi_4 = \delta_2[1, 1, 1, 1, 1, 1, 1, 2, 1, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 2, 2, 1, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2].
\]

It can be checked that for any \( K_2 \in \mathcal{L}_{2, n, 2}, Q^1(z^1, K_1z^1, K_2z^1, \xi) \) and \( Q^2(z^1, K_1z^1, K_2z^1, \xi) \) are all constants when \( K_1 = \delta_2[1, 1] \). That means we have four disturbance decoupling controllers.

1) When \( K_2 = \delta_2[1, 1] \), we have \( u(t) \equiv 1 \); 2) When \( K_2 = \delta_2[2, 2] \), we have \( u(t) = z^2(t) \); 3) When \( K_2 = \delta_2[1, 2] \), we have

\[
u(t) = z^2(t) \vee (-z^2(t) \wedge z^1(t))
\]

4) When \( K_2 = \delta_2[2, 1] \), we have

\[
u(t) = z^2(t) \vee (-z^2(t) \wedge z^1(t))
\]

Using the modified method we can derive all disturbance decoupling controllers. However, via the design method proposed by [1], we only can get the first one. Comparing with the method of [1], we get three more disturbance decoupling controllers.

**REFERENCES**