Network-based real option models

Joseph Y.J. Chow a,*, Amelia C. Regan b,1

a Institute of Transportation Studies, University of California, Irvine, 4000 Anteater Instruction and Research Bldg, Irvine, CA 92697-3600, United States
b University of California, Irvine, 4068 Bren Hall, Irvine, CA 92697, United States

ABSTRACT

Building on earlier work to incorporate real option methodologies into network modeling, two models are proposed. The first is the network option design problem, which maximizes the expanded net present value of a network investment as a function of network design variables with the option to defer the committed design investment. The problem is shown to be a generalized version of the network design problem and the multi-period network design problem. A heuristic based on radial basis functions is used to solve the problem for continuous link expansion with congestion effects. The second model is a link investment deferral option set, which decomposes the network investment deferral option into individual, interacting link or project investments. This model is a project selection problem under uncertainty, where each link or project can be deferred such that the expanded net present value is maximized. The option is defined in such a way that a lower bound can be solved using an exact method based on multi-option least squares Monte Carlo simulation. Numerical tests are conducted with the classical Sioux Falls network and compared to earlier published results.

1. Introduction

There is a compelling need to account for time-dependent uncertainty within network designs, such as in the transportation industry where high capital costs and individual traveler route choice behavior can lead to irreversible investments, subject to unaccountable factors that impact the outcome of design and investment decisions. This need has been further emphasized in recent years by more complex objectives considered by decision-makers managing a network (Yang and Bell, 1998), such as equity (Yang and Zhang, 2002; Chen and Yang, 2004; Duthie and Waller, 2008), robustness requirements (Karoonsoontawong and Waller, 2007; Mudchanatongsuk et al., 2008; Yin et al., 2009), as well as greater environmental, economic, and political uncertainties affecting such networks. These latter uncertainties may be reflected in volatility in fuel prices (transport costs), recurrent incidents such as accidents or floods (link capacities and travel demand), or shifts in economic and land use behavior such as the relocation of distribution centers affecting freight demand.

The need to account for time-dependent uncertainty is reflected in the literature. Time dependency of network designs have been dealt with since 1974 (Steenbrink) with the use of multiple periods and optimal staging of a design over time via dynamic programming. Despite more recent interest (Wei and Schonfeld, 1993; Kim et al., 2008; Lo and Szeto, 2009) in solving these problems for larger networks, the treatment of multiple periods has been restricted to deterministic settings for origin–destination (OD) demand and link capacities.

On the other hand, stochastic elements such as demand, transport fuel costs, construction costs, and capacity have generally been dealt with using stationary, static distributions with scenario planning approaches. Stochastic programming...
efforts have been applied to bi-level network design problems where congestion effects occur (Waller and Ziliaskopoulos, 2001; Chen and Yang, 2004), as well as in facility location and inventory management problems (Shu et al., 2005; Snyder et al., 2007). More complex objectives have been studied such as robust optimization with higher moments of the probability distribution or with reliable network designs (Sumalee et al., 2006; Li et al., 2007). Nonetheless, these studies have not considered multi-period, time-dependent design decisions with stochastic variables. Among the transportation-related papers involving uncertainty, Zhao and Kockelman (2002) examine the propagation of uncertainty over time and determined that uncertainties tend to have non-stationary variance.

The recent work by Damnjanovic et al. (2008) quantifies the value of flexibility in having recourse in a single period network design problem. Ukkusuri and Patil (2009) propose a multi-period stochastic network design problem formulation that accounts for elastic demand. They formulate the model of allocating design variables to a number of links as a mathematical program with equilibrium constraints (MPEC) with stochastic demand over multiple time periods. However, this formulation would not explicitly treat future period investments as options as adapted processes (that depend on the realization of all the stochastic elements up to that point). Furthermore, the inclusion of elastic demand allows for more realistic consideration of induced demand but does not address the interaction of infrastructure investments and land use allocations that is addressed with integrated land use models. A truly flexible network design approach with multiple periods, non-stationary stochastic variables, and adaptation in decision-making requires modeling and solution methodologies from real option theory.

This paper builds on earlier work (Chow and Regan, in press) that only quantified the option value of a network design investment as a bundle. Two models are proposed that better integrate the design elements with time-dependent uncertainty: the first one is a network design problem that maximizes option value, and the second one is an option evaluation model for a network design that treats each link or project investment as a separate, interacting option. Numerical tests with the classical Sioux Falls network are conducted to compare the two models with the network investment deferral option (NIDO) model developed by Chow and Regan (in press). These models provide new insight to designing networks with adaptive decision-making under time-dependent uncertainty.

2. Real options

While real option theory is an investment evaluation method derived from financial options for corporate finance, the underlying concept is simply to enable a decision to be made at any time within some time horizon. By doing so, the decision can be adapted to the latest data to maximize its value. In option theory, this argument considers an investment as a right that can expire as opposed to a static obligation. Real option methods have been shown to be most effective in high capital cost industries with high volatilities in the profit values.

To quantify the value of an option, Trigeorgis (1996) presents an expanded net present value (NPV) framework that consists of an additive expansion of the traditional static NPV with various option premiums:

$$\text{Expanded NPV (Option value)} = \text{NPV} + \text{Option Premium}. \quad (1)$$

A thorough introduction of real options and their applicability to transportation industry is provided in Chow (2010) and to a lesser extent, in Chow and Regan (in press). By framing a decision as a dynamic programming problem with non-stationary stochastic variables, it is possible to quantify the value of treating an investment decision as an option with flexibility. The value of this flexibility is known to increase as the volatility of the underlying stochastic processes increases; in other words, the greater the uncertainty, the more value there is to having the flexibility to make a decision.

A real option can be considered an optimal stopping problem that is solved with dynamic programming methods. For a simple real option problem with a stochastic variable defined as a basic Ito process as shown in general form in Eq. (2), it is possible to analytically solve the problem.

$$dx = a(x, t)dt + b(x, t)dW, \quad (2)$$

where \(x\) is a random variable, \(a\) and \(b\) are deterministic functions of \(x\) and time \(t\), and \(dW\) is an increment in a Wiener process. The Wiener process is a Brownian motion; the distribution is Normal with mean of zero and variance that is linearly proportional to time. When the volatility represented by \(b(x, t)\) is sufficiently high, the value of an option increases. For a network modeling application, consider one possible form for modeling OD demand as a multi-dimensional geometric Brownian motion:

$$dq_t = (\sigma dW_t)^2 q_t + (\sigma^2 dt)^2 q_t, \quad (3)$$

where \(I\) is the identity matrix, \(q_t\) is the OD demand at time period \(t\), \(x \in \mathbb{R}^{|N| \times |M|}\) is a diagonal drift matrix and \(\sigma \in \mathbb{R}^{|N| \times |M|\times|N|\times|M|}\) is a constant diffusion matrix. \(N\) is the set of nodes and \(M\) is the set of commodities in the network. If OD pairs are independent of each other, then the diffusion matrix reduces to a volatility vector of size \(|M|\times|M|\) times the identity matrix. Note that for large networks the estimation of the covariance matrix can become an issue, and that the test network examined in this research addresses independent OD pairs.

While the OD demand \(q_t\) is assumed to be exogenous and inelastic to reflect a base approach, it can easily be related to the equilibration of the network using the variable/elastic demand formulations from Sheffi (1985) and demonstrated by
Ukkusuri and Patil (2009). Such an extension would not address the issue of activity-based demand interaction with supply changes that require integrated land use models to address.

A review of real option studies in transportation shows that earlier real option methods introduced for transportation projects tend to ignore network interactions (Garvin and Cheah, 2004; Zhao et al., 2004; Pichayapan et al., 2003; Saphores and Boarnet, 2006; Galera and Soliño, 2010). Others (Friesz et al., 2008) have considered route choice as a European call option, which is a somewhat different problem than the network design problem. The potential complexity of evaluating network equilibrium conditions to compute payoff values when considering network interactions and congestion effects means that numerical methods are needed.

2.1. Solution methodologies

Trigeorgis (1996) describes three general numerical methods for determining the real option value: finite difference (Brennan and Schwartz, 1977), binomial lattice (Cox et al., 1979; Trigeorgis, 1991), and Monte Carlo simulation (Boyle, 1977). Chow and Regan (in press) argue that none of the three methods directly handle network-based option values very well because of the potential non-differentiability of a subset of the network design problems, the dimensionality of OD demand, and computational cost of evaluating network equilibrium. Therefore, a type of Monte Carlo method, the least squares Monte Carlo simulation (LSM) method (Longstaff and Schwartz, 2001), is proposed as a suitable method for solving the NIDO model, defined in Section 2.2.

Compared to the earlier binomial lattice methods and Monte Carlo simulation, LSM is a very cost-effective method of obtaining option values for high-dimensional variables. The method has been proven to converge toward the actual option value as the number of paths, time steps, and number of polynomials for regression approach infinity. However, the method is known to have a downward bias for small samples. Another disadvantage of the method is the inability to handle multiple interacting options.

To overcome this disadvantage, the LSM approach has been further extended by Gamba (2002) to a multi-option LSM approach that can handle three idealized cases: (1) the trivial case where the options are independent of each other, resulting in a summation; (2) purely compound options where one option depends on whether the prior option is exercised; and (3) mutually exclusive options where selecting one would eliminate the alternative options.

2.2. Network investment deferral option (NIDO)

Chow and Regan (in press) develop a model and solution method for computing the deferral option value of a generalized network design under multi-dimensional stochastic demand and multi-period time horizon. Such an option comes embedded with a premium to re-design the network, which can be interpreted as an opportunity cost to committing to preferred alternatives in transportation planning. This is shown in Eqs. (4) and (5), where the option premium $F$ is equal to the sum of the basic deferral premium $F_D$ and the network re-design premium $F_N$. $\Phi$ in Eq. (4) is the option value.

$$\Phi = NPV + F,$$

where

$$F = F_D + F_N.$$  

To solve the problem numerically, a Bellman equation is constructed to determine the optimal decision at any time state $t_n$ based on a backward recursion from the final time state. The Bellman equation for the NIDO model is shown in Eq. (6).

$$\phi(t_n,q_{tn}) = \max \left\{ \pi_{tn} \left( \phi(q_{tn},y_{tn}), (1 + \rho)^{t_n}E \left[ \phi(t_{n+1},q_{tn+1}) \right] \right) \right\},$$

where

$\phi(t_n,q_{tn})$ is the option value of an investment with underlying stochastic variable $q_{tn}$ at time step $t_n$;

$\pi_{tn}$ is the net present value of an immediate investment made at time step $t_n$;

$\phi$ is a network performance measure, such as the total travel time savings with a stochastic variable $q_{tn}$, and a design vector $y_{tn}$;

$y_{tn} \in \mathbb{R}^{|A|}$ is a vector of link design improvements of up to the size of the number of links $A$ in a network $G(N,A)$;

$\rho$ is a discount rate.

The design vector $y_{tn}$ represents a set of link improvements, such as a capacity expansion. However, this design vector is not a decision variable in this model. It is exogenously obtained from a network design problem (NDP) solution. The decision at each time state is to maximize the outcome by either stopping the process (investing immediately) or to defer the decision to the next time step. The immediate investment payoff is evaluated by solving a network design problem given current realized demand at time step $t_n$, so a simulation with $T$ time steps and $P$ simulation paths would require solving the NDP $T \times P$ times.

One assumption in the real option model is that the budget remains available over the time horizon and can be applied to the investment at a later time. In transportation planning, this often translates to funding needs, transportation
improvement programs (TIPs), and earmarks. Under these funding programs, local governments allocate funding towards a project or set of projects for a certain time, such as a 5- or 10-year time frame, as they try to secure funding from state and federal governments for approving the project. However, as discussed in Chow and Regan (in press) this often results in the commitment of preferred alternatives that may not be applicable over long term under highly volatile settings.

The network re-design premium can be computed by simultaneously obtaining the deferral option value where the initial design is committed – in each time state, the NDP is not solved again, and instead the network equilibrium is evaluated based on the initial design solution \( y_{t_n} \). The network re-design premium is then the difference between the NIDO value and the committed deferral option value. The Bellman equation for the committed design is shown in Eq. (7).

\[
\Phi_{t_n}(q_{t_n}, y_{t_n}) = \max \left\{ \pi_{t_n} \left( \phi(q_{t_n}, y_{t_n}) \right), (1 + \rho)^{-\Delta t} E \left[ \Phi_{t_{n+1}} (q_{t_{n+1}}) \right] \right\},
\]

where \( y_{t_n} \) is the initial period design vector. Although Eqs. (6) and (7) appear similar, they behave quite differently because Eq. (7) does not allow the design to change in future time periods. In Eq. (7) the decision variable is the timing of the complete bundled investment. Hence, there is no need for a budget constraint in Eqs. (6) and (7) since both of these represent only the timing of the investment. Chow and Regan demonstrate the model and LSM method for the classical Sioux Falls network. Given a five year time horizon, 6% discount rate, and OD pairs that evolve independently as geometric Brownian motions with zero drift and homogeneous volatility, the volatility threshold to defer the initial continuous network design is approximately 0.30. Whereas a network design solution with static NPV evaluation would suggest implementing the design immediately, the NIDO solution with volatilities at 0.35 shows there is greater value to defer the investment and to maintain the option to re-design the network.

This modeling framework is designed to allow for a number of different stochastic variables, OD demand only being one of them. In a real project implementation the modeler/planner should consider the variables that are most volatile to them and develop the option model based on that variable or set of variables as the primary concern. Future case studies (assuming the time series data is available) could not only examine supply/construction costs, but also fuel costs or any number of variables that can impact the network equilibration portion or the dynamic investment portion of the option model.

Despite these contributions, the NIDO model does not actually optimize the option value as a function of the design variables, nor can it be decomposed into a set of interacting link or project options. By forgoing the network re-design premium, however, it becomes possible to formulate and solve the following two proposed models.

3. Proposed model: network option design problem (NODP)

3.1. Model formulation

Suppose there is a graph \( G \) of a set of nodes \( N \), a set of links \( A \), and a set of commodities \( M \) subject to a set of constraints \( S \). Each origin – destination pair and commodity, \( q \in \mathbb{R}^{N \times M} \), evolves stochastically as discussed in the previous section. \(|N|\) and \(|M|\) are the number of nodes and number of commodities, respectively.

Other variables that need to be defined for this model are the initial budget \( B \), a non-negative discount interest rate of \( \rho \) for all projects, and a planning horizon \( T \) after which a managing agent’s option to invest in a project would expire. A time horizon \( T \) is divided into \( n_{\text{max}} \) discrete intervals such that \( t_{n} \) is the \( n \)th time step out of \( n_{\text{max}} \) steps. For realistic network design problems, there may be a subset of links \( A \subseteq A \) that can be invested upon.

The objective of the network option design problem is defined as the design decision to allocate a budget to a set of links or components such that the option value of the investment is maximized. The dynamic programming formulation for quantifying the deferral option value needs to be modified so that the design variables are endogenous to the problem. However, the NIDO model itself with the option to re-design the network in the future is intractable in terms of design variables because each time-state would generate a new set of design variables that need to be solved.

The lower bound basic deferral problem (letting \( F = F_{0} \)) is considered instead, which only has the initial time design variables and is thus solvable. Eq. (8) represents the proposed Bellman equation for the NODP with deferral option

\[
\Phi_{t_n}(q_{t_n}, y_{t_n}) = \max_{y_t} \left\{ \pi_{t_n} \left( \phi(q_{t_n}, y_{t_n}) \right), (1 + \rho)^{-\Delta t} E \left[ \Phi_{t_{n+1}} (q_{t_{n+1}}) \right] \right\},
\]

Subject to

\[
y_{t_n}^T d \leq B.
\]
\[
y_{t_n} \geq 0.
\]

where

- \( d \) is a vector of construction costs relative to each link improvement;
- \( B \) is the initial budget.

The difference between Eqs. (8) and (7) is that the decision variables are both the timing (as represented by the Bellman equation) and the design vector. Because of this change, a budget constraint is added but it applies only to the base period because the design is not allowed to change in the future. The equilibrium constraint that is typically the lower level problem
in the Bellman Equation. At any time step the design is always the initial \( y_{t0} \), and the objective value is a function of both the stochastic demand and the initial design variables: \( \Phi_n (q_{tn}, y_{t0}) \). The differences between the NIDO model and the proposed models are shown in Fig. 1.

3.2. Equivalency to network design problem

When the problem is represented by one time step and deterministic OD demand, the Bellman equation in Eq. (8) loses the second term in the maximization. The term: \( \pi_{t0} (\phi(q_{tn}, y_{t0})) \), is evaluated as the difference in total system travel time between a baseline network equilibrium and network equilibrium with a design solution, multiplied by a constant rate to obtain annual savings.

Essentially the problem collapses into a basic NDP. Therefore, the NODP can be considered a more generalized form of the NDP that takes into account stochastic elements such as demand, multiple time periods, and endogenous decision-making.

3.3. A solution method

Since NDP’s can be non-convex and non-differentiable especially when accounting for congestion effects, the more generalized NODP would also require global heuristics to solve. The multi-start local metric stochastic radial basis function (MSRBF) algorithm used by Chow et al. (in press) for the continuous network design problem (CNDP) is considered for the NODP with continuous link expansion design variables with congestion effects. In this type of problem, the decision-maker has to simultaneously make two sets of decisions: (1) allocate budget to increase link capacities assuming some monotonically increasing link cost function with capacity parameter such as the Bureau of Public Roads (BPR) function; and (2) decide whether to invest in the design immediately or to defer with no option to re-design in the future.

The multi-start local MSRBF algorithm was developed by Regis and Shoemaker (2007) as a faster converging global stochastic search algorithm based on response surfaces and interpolation of sample solutions using radial basis functions. The algorithm has been shown by Chow et al. to be much more effective than some more popular meta-heuristics such as genetic algorithm for a continuous network design problem for a network with up to 416 nodes, 914 links, and 31 link investments (Anaheim, CA). The study provides details on the absolute and relative computational costs of the method compared to other...
An initial set of designs are sampled with a Latin Hypercube Sampling method. A set of $P$ independent paths of realization are also simulated for the stochastic OD demand over the time horizon $T$. For each $i$th sample, the deferral option value and deferral decision is determined using the LSM algorithm and the same results from the stochastic OD demand simulation. After all sample designs are evaluated, they are used to fit an RBF interpolation function, which is then used to evaluate candidate points generated randomly in the neighborhood of the best design solution in the set. The candidate point that has the best weighted interpolation fit and distance criteria is selected for evaluation. If convergence criteria are met, the algorithm would reset and start again with a new LHS sample (multi-start local approach). The algorithm ends when the maximum number of iterations $N_{\text{max}}$ is reached.

4. Proposed model: link investment deferral option set (LIDOS)

4.1. Modeling issues

The objective of the link investment deferral option set (LIDOS) model is to treat the link components of a network design as individual options, so that deferral and investment decisions can be made piecemeal instead of as a bundle. From here on, the term project can refer to a single link investment or a set of link investments. The major contribution of this model is that projects in a design solution can be staged in a stochastic setting. For example, the application of this model to high-speed rail investment would enable policy-makers to make link investment staging plans based on time-dependent stochastic demand for high-speed rail service. It would be the first time a decision-maker can make link designs over multiple time periods under a non-stationary stochastic demand setting.

There are several challenges in breaking up the NIDO model into individual, interacting projects that are allowed to be deferred and redesigned. First, by definition the projects need to be considered as discrete components since each is an investment option in its own right. For a network design problem with continuous expansion variables, this would mean giving up the option to re-design from a continuous context. Second, and more importantly, the model requires backward recursion but features a forward constraint: the current investment decision at any time step depends on what has been invested earlier. In other words, decoupling a design into a set of interacting options would mean that the budget constraint needs enforcing in each time step, where the budget amount depends on investments made beforehand. However, the dynamic programming methodology requires backward time recursion. This feature makes the model intractable as defined.

4.2. Lower bound model formulation

We propose a lower bound model instead, by defining the model in such a way that a lower bound would exist, one that can be solved using existing numerical methods without having to maintain a budget constraint in each time step. Let’s recognize that at any time step under exogenous demand, the following completed investments are equivalent:

$$\{x_1, x_2, x_3\} \equiv \{x_2, x_1, x_3\} \equiv \{x_3, x_1, x_2\}$$
At any time step, if the payoff value were obtained for each of the three different permutations shown, the option value of the set itself would correspond to the highest payoff value in the set.

This insight leads to the following model representation: the total option value of all the projects is the sum of two components. The first component is the maximum of all the options, where each option treats the project investments as purely compound options where one project option depends on whether the prior project is exercised. Given a set of 3 links A, B, and C, the value is max(A-B-C, A-C-B, B-A-C, C-A-B), where A-B-C is the total option value of the three link investments treated in that particular sequence and B cannot be invested unless A is invested already, and C cannot be invested unless both A and B are invested.

The second component is the option to re-order the projects in the future. This is equivalent to the discrete form of the network re-design premium. In other words, the LIDOS model can be defined using the following expanded NPV framework:

$$\Phi = \sum_{l} NPV_l + \sum_{l} (F_{DJ} + F_{JI}) + F_{IS}$$

(9)

Where the total option value of the link investments $$\Phi$$ is equal to the sum of the individual project NPV’s, the sum of the deferral options $$F_{DJ}$$ with the option value of investing in the subsequent projects $$F_{IS}$$ plus the option to re-order the project investments in the future $$F_{LS}$$. The terms $$\sum_{l} NPV_l + \sum_{l} (F_{DJ} + F_{JI})$$ represent the option value of a committed set of ordered projects.

Analogous to the NODP, the LIDOS model can forgo the option to re-order the projects in the future so that the initial permutation chosen would be committed. Let us call this the ordered link investment deferral option set (OLIDOS). Since the option premiums are non-negative, the OLIDOS model in Eq. (10) provides a lower bound on the solution of the LIDOS model.

$$\Phi = \sum_{l} NPV_l + \sum_{l} (F_{DJ} + F_{I})$$

(10)

The advantage of defining this lower bound OLIDOS model is that it can be solved using a combination of Gamba’s (2002) multi-option LSM algorithm (this is the second stylized problem presented in that paper) for each possible permutation $$h \in L!$$ along with a selection of the sequence that offers the highest initial project (since that would capture the option value of all subsequent projects in the sequence).

Gamba’s treatment of an ordered set of compound options can be modeled with the following Bellman equation.

$$\Phi_{h, t_0}(q_{t_0}) = \max_{z_{h, t_0}} \left\{ \psi_{h, t_0}(\phi(q_{t_0}, z_{h, t_0})) + \Phi_{h, 1: t_0}(q_{t_0}, (1 + \rho)^{-\Delta t}[\Phi_{h, t_{0+1}}(q_{t_{0+1}}))] \right\},$$

(11)

where

- $$h$$ is the project among the set of L projects in the ordered $$h$$th set, $$h \in H$$, $$|H| = |L|!$$ are different permutations of projects;
- $$\Phi_{h, t_0}$$ is the option value of the $$h$$th project;
- $$\pi_{h, t_0}$$ is the net present worth of an immediate investment in the $$j$$th project at time step $$t_0$$;
- $$z_{h, t_0}$$ is the design vector of the $$j$$th project at time step $$t_0$$.

Compared to Eq. (8), the immediate payoff value in Eq. (11) is equal to the payoff from immediate investment of the $$j$$th project plus the opportunity of investing in the $$(j+1)$$th project. This adds a backward option recursion to the backward time recursion. The Solution Method section provides a clearer picture of how this would flow.

$$z_{h, t_0}$$ is the set of first $$j$$ projects at time state $$t_0$$ for ordered set $$h$$ equal to 1. For 5 interacting projects, there are 120 different permutations. Ten of these 120 permutations are shown in Table 1. To clarify the notation, if the optimal decision for $$h = 1$$ is to invest in the first project immediately and defer the rest, then $$z_{h, t_0} = 1$$ and $$z_{h, t_0} = 0$$ for $$j = 2, 3, 4, 5$$. For $$h = 2$$, $$h_2 =$$ project 5. After Eq. (11) is used to solve each permutation, the maximization needs to be taken over every permutation to find the permutation that offers the highest first option in the sequence:

$$\Phi_{t_0}(q_{t_0}, z_{t_0}) = \max_{h} \left\{ \sum_{l} \Phi_{h, t_0}(q_{t_0}) : h \in \arg\max_{h} \left\{ \Phi_{h, t_0}(q_{t_0}) \right\} \right\}$$

(12)

Eq. (12) supersedes the one first proposed in Chow (2010).

<table>
<thead>
<tr>
<th>Table 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ten of the 120 possible ordered sets for five projects.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Order set, $$h$$</th>
<th>$$h_1$$</th>
<th>$$h_2$$</th>
<th>$$h_3$$</th>
<th>$$h_4$$</th>
<th>$$h_5$$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>4</td>
<td>2</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>4</td>
<td>2</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>4</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>6</td>
<td>5</td>
<td>4</td>
<td>1</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>7</td>
<td>5</td>
<td>3</td>
<td>4</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>5</td>
<td>3</td>
<td>4</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>9</td>
<td>5</td>
<td>3</td>
<td>2</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>10</td>
<td>5</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>4</td>
</tr>
</tbody>
</table>
The assumption that demand is exogenous may not be appropriate in the context of transportation planning. As discussed in Magnanti and Wong (1984), there is a large range of problems that involve NDP’s for which roadway infrastructure investment is just one example. The approach itself described here should be applicable to a wide range of network design models including service delivery, vehicle fleet planning, and capital investment decisions involving airports, fleet acquisitions, mass transit, etc. It should also include decisions made at strategic, tactical, and operational levels reflecting a broad range of time horizons. These problems will have differing degrees of demand interactions with the investments. While demand uncertainty interaction is certainly an issue, it is also a persistent issue that has existed in travel demand forecasting that requires the use of integrated land use modeling such as the Production, Exchange, and Consumption Allocation System (PECAS) model (Abraham and Hunt, 2003) to resolve. We intend to pursue that further in future research, first with a real OD time series practical case study to investigate the effectiveness of this model with real data for transportation planning; and then to identify ways to integrate the real option methodologies with network models under an integrated demand interaction setting.

4.3. Equivalencies

In the project staging problem, a discrete set of projects are ordered and timed under a deterministic demand setting. The OLIDOS model collapses to a committed project staging problem when the stochastic elements are deterministic. Therefore, the OLIDOS model is a generalized project staging problem with exogenous stochastic elements such as demand.

If the cost of all the projects exceeds the budget constraint, then the problem becomes a project selection problem. This can be solved exactly by considering only the first j feasible options for each permutation h, and solving Eq. (11) for them. The only difference from project staging would be the additional number of permutations to consider minus the redundant permutations because of the budget constraint.

For example, consider three equally costly links A, B, and C as described earlier, with two additional links D and E. Consider a budget that can only finance three links. In the earlier example, there were six permutations to consider: max(A-B-C, A-C-B, B-A-C, B-C-A, C-A-B, C-B-A). Now there would be \( P_{3,5} = 5!/2! = 60 \) different ways to order the five links with a budget of three links. The rest of the model remains the same.

When demand is deterministic over a multiple period time horizon, the problem simplifies to the multi-period dynamic programming problem studied by Steenbrink (1974) and Wei and Schonfeld (1993).

When demand is deterministic over a single time period, the problem simplifies further into a basic discrete network design problem, where a budget is allocated to choose discrete components to add to a network. The OLIDOS is a generalized form of the discrete network design problem with multiple time steps (the project selection problem) and stochastic elements.

4.4. Solution method

The approach to solving the multi-option LSM from Gamba (2002) is to repeat the LSM from Longstaff and Schwartz (2001) from the last option in the ordered set h to the first option in the ordered set for each time state and simulation path. Each ordered set is enumerated to obtain the exact multi-option LSM solution value.

OLIDOS solution algorithm.

1. Obtain \( y_j^n \in \mathbb{R}^J \) using a discrete network design solution, or by creating a set of discrete projects \( z_a^n \in \mathbb{R}^{|J|} \) from a continuous network design solution, where \( z_a^n = \{z_{a1}, \ldots, z_{a|J|}\} \) and \( z_{a0} = -\sum_{a \in J} y_{0a} \) where \( J \) is a predefined set of projects among |J| projects and \( \{\cup a \in J\} = \{\cup_{|J|}, \cup \} \).
2. Determine the number of different permutations for ordering them. Let \( H = |J|! \).
3. For each sequence \( h \in H \), solve Eq. (11) using Gamba’s multi-option LSM method:
   a. Evaluate the cumulative payoff for each additional project at each every time step \( t_n \) and simulation path \( \omega \).
   b. The payoff value of the \( j \)-th project is the difference \( \pi_{h,t_j}(\phi(q_{1-1}, z_{h,t_{j-1}})) - \sum_{j=1}^{j-1} \pi_{h,t_j} \).
   c. If \( \pi_{h,t_j}(\phi(q_{1-1}, z_{h,t_{j-1}})) < 0 \) let
   \[
   \pi_{h,t_j}(\phi(q_{1-1}, z_{h,t_{j-1}})) = \pi_{h,t_{j-1}}(\phi(q_{1-1}, z_{h,t_{j-2}}))(1 + \rho)^{-1}
   \]
   d. If \( \pi_{h,t_j} = \phi(q_{1-1}, z_{h,t_{j-1}}) + \pi_{h,t_{j-1}}(q_{1-1}) < (1 - \rho h)^{-1}E[\pi_{h,t_{j-1}}(q_{1-1})] \), the current option and all subsequent options in the sequence need to be deferred, \( \theta_h(\omega, t)_{j-1, j} = 0 \). Otherwise let \( \theta_h(\omega, t)_{j-1, j} = 1 \).
   e. When the initial year is reached, discount back the option value for each j-th option and take the average:
   \[
   E[\pi_{h,t_j}(\omega, t_j)] = \sum_{j=1}^J \pi_{h,t_j}(\omega, t_j)(1 + \rho)^{-n}.
   \]
   f. Going backward from the final budget-feasible option, let:
   \[
   \Phi_{h,t_j} = \max(\pi_{h,t_j}, E[\pi_{h,t_j}(\omega, t_j)])/[\pi_{h,t_j}(\omega, t_j) - 1] \].

4. The option value is the maximum of the sum of option values obtained from the ordered set $h$ belonging to the set with the maximum first project, the solution to Eq. (12).

The algorithm is shown graphically as a flow diagram in Fig. 3. When the number of $J$ options is reduced to one, the algorithm collapses into the basic LSM algorithm for solving the fixed design deferral model from Eq. (7).

Since the algorithm is just an extension of LSM, the same convergence criteria apply. However, the multi-option LSM needs to be evaluated for each option in an ordered set and the option value needs to be evaluated for each sequence in $H$. Although $|H| = |L|^J$, some of the combinations are repeated so that the cumulative payoff of the $j$th option of $h = 1$ at time $t_n$ can be re-used for an option in another sequence $h = 2$. This effectively makes the computational cost of this exact enumeration method on the order of $O(P_j L^j C_0 C^n P_j)$. For five sets of links and 5 time periods with 300 sample paths, this is equivalent to $O((5 + 10 + 5 + 1) \times 300 + 5 \times UE(N)) = O(46,500 + UE(N))$. As a combinatorial problem there is still a challenge in solving this problem for a large number of link sets, and future research should address this need for larger network design considerations. For example, if all ten links in the typical Sioux Falls example are treated as separate options, this could lead to a computational time of $O((5 + 10 + 5 + 1) \times 300 + 5 \times UE(N))$, which is 33 times the computational cost from doubling the number of links from five to ten.

A brief discussion of the validity of the OLIDOS model under sub-optimal NDP solutions needs to be made. As a real option valuation model, the OLIDOS model seeks to optimize the timing of a set of investments. Whether or not the details of those investments are determined optimally does not influence the validity of the real option valuation model, which would still provide additional benefit by adding the time dimension to the decision-making process. In other words, it’s akin to providing extra decision-support tools to a planner who cannot optimally determine the preferred alternative. In reality, most NDP’s in transportation planning would be bi-level in nature and would be non-convex and non-differentiable (Yang and Bell, 1998), and the resulting design solutions from which the OLIDOS model would stage would likely not be global optima.

5. Numerical tests

The two models are conducted on the same Sioux Falls network parameters used to test the NIDO model in Chow and Regan (in press), which are partially obtained from Chen and Yang (2004) and Suwansirikul et al. (1987). In that test, each of the 552 OD pairs evolves independently as a geometric Brownian motion. For simplicity, the drift parameter $\mu_{rs} = 0$ for all OD pairs $rs$ with continuous link expansion design variables. The solution to the standard CNDP with the baseline demand flows is obtained using an iterative optimization algorithm (IOA), resulting in a total system travel time (TSTT) = 75.942. This is in comparison to a no investment user-optimal cost of 101.171. A conversion rate $\lambda = 56.604$ is chosen to translate $5.5\ M$ budget to 5.5 vehicle-hours traveled (VHT) cost with a 6% discount rate. The expected net present value of an immediate investment is $NPV = \$19.73\ M$. Fig. 4 shows the Sioux Falls network.
Under a deterministic setting, the positive NPV suggests that the decision-maker should invest immediately for a net payoff value of $19.73 M.

Under the NIDO model with $\sigma_{rs} = 0.35$ for all OD pairs $rs$ and a 5-year time horizon and 300 simulation paths, the option value is $22.6 M$ with the option to re-design the network in the future. Without that option, the basic deferral option of the IOA network design solution is $21.4 M$. This deferral option is composed of a static NPV of $19.7 M$ and a deferral premium of $1.7 M$.

5.1. NODP Solution

In this test, the NODP solution demonstrates that the design variables can be optimized with respect to the option value instead of the travel time objective. As shown in Fig. 5, the NODP solution algorithm is run for two local starts resulting in 266 option evaluations for the Sioux Falls network with the same 0.35 volatility, $T = 5$ years, $P = 300$, $II = 6$, and using the same random seed as the NIDO model run with same parameters.

The maximum option value of $21.71 M$ is obtained in the 255th iteration, which is the 135th iteration of the second local start. Contrary to the IOA solution value of TSTT = 75.942, the link designs for this solution value result in a TSTT = 76.062.
This value converts to a static NPV = $19.61 M, which is less than the static NPV of $19.73 M from the NIDO model. However, by deferring the fixed design the option value of $21.71 M is higher than the $21.50 M from the fixed design option with the IOA network design. The design variables are summarized in Table 4 in the Comparison of Model Results section below.

5.2. OLIDOS solution

The discrete links in this example are the initial design values from the NODP solution: each of the link capacity increases determined from the NODP are assumed to be fixed projects. In this context, the OLIDOS serves as a project staging problem under uncertainty, determining which link to defer and which to invest in immediately.

Instead of treating each of the 10 link investments as a separate option, the pairs of opposite direction links are combined into a set of $|L| = 5$ discrete projects to reduce computational cost: link 16 plus link 19, link 17 plus link 20, link 25 plus link 26, link 29 plus link 48, and link 39 plus link 74.

With five projects, there are 120 ordered sets to consider. For each one, the proposed solution algorithm is used to solve the option value. The same number of basis functions $II = 6$ are considered. The option value of the first project in each project sequence are shown in Fig. 6.

The optimal sequence is the 22nd set: $\{5, 1, 2, 4, 3\}$, with a total option value of $50.58 M and an initial option value of $21.56 M. On the contrary, the lowest initial option value belonging to an option set is $20.10 M, with a sequence of $\{4, 1, 2, 3, 5\}$ and an optimal decision to defer all the projects. The bundled investment is decoupled into a subset for investment and the remainder to be deferred. The results of the optimal solution are summarized in Table 2.
The volatility is allowed to vary to examine how the optimal solution set would change. Table 3 maps the optimal solution set decisions with total option value to the homogeneous volatility using the same random seeds from the NIDO model volatility test in Chow and Regan (in press). The presence of multiple projects results in a graduated threshold that shifts the decisions from immediate investment of all projects to partial investment of some projects, and finally to deferring all projects. The first option’s value increases as the volatility increases.

5.3. Comparison of model results

The three models are summarized along with baseline link design values to compare the performance under the 0.35 volatility for a five year planning horizon. The IOA column refers to the solution to the CNDP using IOA approach for Sioux Falls.
The NIDO column computes the option value of the IOA design solution, assuming at each time state the IOA is applied to find the optimal design at that time. The NIDO option value $22.68$ M includes the flexibility to defer and re-design the network. The fixed design option column includes only the flexibility to defer and results in a slightly lower option value of $21.50$ M.

The models are run on Matlab R2009b on a 64-bit Windows 7 Intel Core 2 Quad, 2.33 GHz processor with 4 GB RAM. If a large network were to be examined, these run times could be shortened significantly by re-coding portions or all of the models in C or C++.

The fixed design option is the same as NIDO, but constrains the design to be the initial design at all time states. NODP is an approximate optimal design solution using an RBF heuristic to maximize the option value as a function of both the design variables and the deferral decision. This option value of $21.71$ M is the maximum that can be obtained when the decision-maker can only defer a design under demand uncertainty.

Lastly, the OLIDOS value including all the link project options together is $50.58$ M. The sum of the deferral premiums is $1.95$, which is slightly less than the deferral premium from the NODP result. This is because the least squares estimation for OLIDOS is conducted for each individual link project as opposed to the overall design, so it can lead to slight differences in value. The ordered staging premium is the benefit of treating the link projects individually as options, which gives the design much more flexibility in adapting to volatile conditions.

6. Discussion

By using real option methodologies, it is possible to expand a network design problem to include the flexibility to defer, flexibility to re-design the solution, or flexibility to invest in each subset of links separately considering uncertainty. It is also possible to optimize the design with a deferral decision with regards to the value of the investment opportunity. Furthermore, the models proposed represent generalized forms of basic network design, staging, and project selection problems that can take into account multiple time periods and stochastic elements.

There are many applications with these two models. The NODP provides a new objective for designing a network that takes into account non-stationary stochastic elements, whether it is a transportation network with congestion or any other type of network design. Unlike prior treatments of stochastic elements, the NODP provides a solution that not only optimizes the design variable but also optimizes the timing of the decision.

The OLIDOS model allows decision-makers to compare priorities between competing projects that may interact with each other in the same network setting. Although the model is developed for a network setting, the same approach can be applied to any setting with interacting options. As the results demonstrate, the order of projects can make a significant difference in the value of the option under high volatility. Clear applications of this model can be observed in many project financing and program management problems, such as the staging of alternative fuel infrastructure investments in a network setting or the investment of multiple high-speed rail links in the US network today with a limited budget and high uncertainty in ridership demand.

Future research in this direction will look at development of heuristics for the OLIDOS solution to handle the computational cost of the combinatorial problem. As discussed under the solution method for OLIDOS, doubling the number of projects from 5 to 10 would increase the computational time by 33 times. Heuristics need to be able to select a near maximum option value and also have a consistent set of decisions regarding the project investments. Note however that there are likely many problems in which the networks considered are large (not a limiting factor in our model) but the number of competing projects (the limiting factor in our model) is fairly small.

A case study using OD demand time series data for an actual network would illustrate the added value of the real option strategies to the managing agent of that network to consider staging investments, committing to a preferred set of alternatives, or deferring the set of alternatives altogether.

A shortcoming of the proposed models (and perhaps to transportation planning models in general) is that OD demand is assumed to be exogenous. While examples exist in which this is a valid assumption, in some transportation planning applications this is clearly not accurate. For example, using the alternative fuel infrastructure example just discussed, the current models cannot be applied directly because they assume that demand for alternative fuel vehicles would not change with the investment of new fuel stations at a particular location. Future research should address this shortcoming, keeping in mind that the current formulation of the LIDOS model depends on having exogenous demand that does not respond to investments made to the network.

Acknowledgements

This work was conducted as part of a dissertation by Joseph Chow, who was supported during his studies by a Federal Highway Administration Dwight D. Eisenhower Transportation Graduate Fellowship. The research was also partially supported by a grant from the University of California Transportation Center. We gratefully acknowledge that support. We are also grateful to the two anonymous reviewers who provided helpful comments to us. Naturally, the findings and any errors found are those of the authors alone.
References


Chow, J.Y.J., Regan, A.C., in press. Real option pricing of network design investments. Transportation Science.


