Characterization of Complex Permittivity Properties of Materials in Rectangular Waveguides Using a Hybrid Iterative Method

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Abstract—In many microwave applications, an accurate knowledge of the complex permittivity properties of materials is usually required. A new procedure for the accurate determination of these properties is presented, based on an optimization algorithm that makes use of measured scattering parameters and simulated results of a cylindrical rod of dielectric material passing completely through a rectangular waveguide. The simulation tool employed consists of a very accurate hybrid iterative method. Results for the permittivity properties of ethanol (high-loss liquid material) are presented and validated with results from the literature.

Index Terms—Dielectric materials, optimization methods, parameter estimation, rectangular waveguides, spectral domain analysis.

I. INTRODUCTION

THE precise knowledge of the permittivity properties of materials is of great interest for several microwave applications [1], [2]. The determination of these permittivity properties has received considerable attention in the technical literature. Cavity techniques usually give the most precise results, although they are limited to their typical restrictions (small size and low-loss samples), and they provide valid results only at one frequency [3]. In other extended techniques, based on transmission lines, the sample has to be completely embedded in the cross section of a waveguide or coaxial line [4]. Such a configuration is difficult to implement in certain cases due to the appearance of gaps between the sample and the transmission guide. Moreover, the bandwidth validity of these techniques can be reduced by the excitation of higher order modes, which depends on the permittivity properties of the sample.

To overcome the previous limitations, a new procedure for the accurate determination of the permittivity properties of materials is proposed in this paper. This method consists of placing a rod sample of dielectric material along the y-axis inside a rectangular waveguide, and then measuring the scattering parameters of such a structure with an automatic network analyzer (ANA). Next, an optimization algorithm based on these measurements and a precise electromagnetic multimode simulation tool is used to extract the permittivity properties of the material. The algorithm proposed also takes into account the presence of the sample holder in cases where the dielectric material is in liquid or granular state. To validate the procedure, the permittivity properties of a liquid sample of ethanol placed inside a Polytetrafluoroethylene (PTFE) pipe are determined and compared with previously reported results.

II. THEORY

For the simulation of the behavior of a multilayered dielectric rod placed inside a rectangular waveguide (see Fig. 1), a three-step analysis procedure is performed. First, the plates of the rectangular guide are split into smaller strips that are characterized individually (using a numerical method, i.e., the method of moments [5]). Next, the multilayered dielectric rod (the pipe filled with the material) is characterized using the analytic spectral method described in [6]. At this point, the scattering behavior of each element (the strips and the multilayered rod) is provided by an “individual scattering matrix” that only relates incident and scattered spectral waves to that object. Finally, the electromagnetic coupling among all elements previously characterized is solved by an iterative method, initially proposed in [7] for open-space problems, and which has been revisited in this paper for the accurate analysis of guided problems. Following this new iterative method, each scattering object is finally characterized by a “combined scattering matrix,” which relates incident and scattered spectra to that object but taking into account the presence of the other scatterers.

These combined scattering matrices give a full-wave solution of the structure in an open-space spectral domain. However, the scattering parameters of the guided structure are needed to compare with the measurements provided by the ANA. To obtain
the desired scattering parameters, a short-circuit is placed in the output reference plane (port 2 in Fig. 1). Then, the “combined scattering matrices” of the objects are used to compute the electric and magnetic fields in both reference planes, taking the fundamental mode in the input reference plane as the excitation of the structure. Finally, the admittance parameters $Y_{11}$ and $Y_{21}$ are easily computed as

$$Y_{11} = \frac{\int_0^a H_1 \cdot \vec{E}_1 \, dx}{\int_0^a \vec{E}_1 \cdot \vec{E}_1 \, dx} \bigg|_{\vec{B}_2 = 0}$$

and

$$Y_{21} = \frac{\int_0^a H_2 \cdot \vec{E}_1 \, dx}{\int_0^a \vec{E}_1 \cdot \vec{E}_1 \, dx} \bigg|_{\vec{B}_2 = 0}$$

where $\vec{E}_1$, $\vec{H}_1$, and $\vec{H}_2$ are the electric and magnetic fields in the input and output reference planes, $\vec{E}_1$ and $\vec{H}_1$ are the $H_{10}$ vector mode functions (see [8]), $x$ is a coordinate in the transversal direction, and $a$ is the width of the waveguide.

Once the admittance matrix has been obtained, the scattering matrix is directly computed from $S = (I - \vec{Y}) (I + \vec{Y})^{-1}$, where $\vec{Y}$ is the $\vec{Y}$-matrix normalized by the characteristic admittance of the fundamental mode ($H_{10}$).

### III. Experimental Procedure and Results

The measurement setup consists of a standard WR340 rectangular waveguide with a cylindrical pipe (of inner and outer radius $r_2$ and $r_1$) completely filled with the sample of material as shown in Fig. 1, where $\varepsilon_{r_1}$ and $\varepsilon_{r_2}$ are, respectively, the dielectric constants of pipe and material. To fix the sample of material in the center of the waveguide, a very small groove is recessed in the lower wall of the waveguide, where the pipe is firmly seated. Then, the pipe is completely filled with the material through a clearance hole in the upper wall of the waveguide. Finally, in order to avoid possible radiation leakage, the clearance hole is completely sealed with a metallic piston of the required dimensions.

The scattering measurements of the empty and full pipe have been performed in the $S$-band with an ANA HP-8720B calibrated following the standard full two-port procedure. Although the full wave analysis tool employed allows for good results at any frequency value, the monomode frequency range must be observed due to the practical limitation of the actual standard calibration techniques of network analyzers.

In order to estimate $\varepsilon_{r_1}$ and $\varepsilon_{r_2}$, two optimization processes must be performed, where the aim is to minimize a sort of difference (error function $U$) between simulated and measured scattering parameters. After testing several error functions, the best results versus accuracy and convergence rate were provided by the weighted amplitude and phase difference (norm 1), defined as follows:

$$U = 0.25 \left( |S_{11}^m| - |S_{11}^0| \right) + 0.75 \left( |S_{21}^m| - |S_{21}^0| \right) + 0.25 \left( |S_{12}^m| - |S_{12}^0| \right) + 0.75 \left( |S_{22}^m| - |S_{22}^0| \right)$$

where $S_{11}^m$ ($S_{21}^m$) and $S_{11}^0$ ($S_{21}^0$) are, respectively, the simulated and measured values of the reflection (transmission) coefficient.

The optimization method employed is based on a Nelder-Mead type simplex search algorithm [9]. To accelerate the optimization procedure, it is convenient to select a suitable starting point. In this case, the starting point has been obtained using the well-known perturbational theory described in [3]. Once the permittivity properties have been determined for each frequency, such values are further used as the starting point for the next frequency. Although the suitable choice of the error function and the starting points accelerated the convergence of the optimization procedure (about 70 function evaluations per frequency point), the method has proved to be very robust since it also converged with other error functions and nonoptimum starting points.

Using the analysis technique that has been described, a sample of ethanol inserted in a PTFE pipe ($r_2 = 16$ mm and $r_1 = 20$ mm) has been measured and characterized at ambient temperature in the range of 2–3 GHz.
The permittivity of the holder pipe is then introduced into a second optimization procedure to obtain the complex permittivity properties of ethanol, which are displayed in Fig. 2. As expected, the estimated values of $\varepsilon''_{pe}$ and $\varepsilon''_{pe}$ for the ethanol (solid line) are much more accurate than those obtained without modeling the PTFE holder (dotted line). From values depicted in Fig. 2, a good agreement with results from literature ([11], [12]) can be observed in the whole frequency range.

Finally, Fig. 3 shows the simulated and measured results of reflection and transmission coefficients of the structure. For the simulated results, the complex permittivity properties provided by the optimization procedure have been used. In this figure, a good match between both results is demonstrated, proving the good convergence of the optimization procedure.

IV. CONCLUSIONS

A new method for the accurate estimation of complex permittivity properties of materials following an optimization procedure is presented. Using this method, based on measurements and a very precise multimode simulating tool, a full-wave characterization of dielectric materials is possible, thus overcoming the main limitations of previous techniques. By characterizing multilayered dielectric rod samples, the method proposed can also determine the permittivity properties of liquid or granular materials in a very accurate way, since the distortion introduced by the sample holder is considered. Another advantage of this method is the simplicity of the cylindrical sample configuration employed for the measurements, although any other shape (square for instance) could be also considered, thus avoiding the possible presence of air gaps between the sample and the waveguide as happens in other techniques. Results for ethanol (high-loss liquid material) have been presented and validated with previously reported results.

REFERENCES


