Low Complexity Ranging Algorithm Based on TOA for IEEE 802.15.4a system

Sujin Kim, Na Young Kim, Joonhyuk Kang
Information and Communications University
Email:{kksj0613, nykim, jhkang}@icu.ac.kr

Youngok Kim
Kwangwoon University
Email:kimyoungok@kw.ac.kr

Kyung-Tae Nam, Sang-Moo Lee
KITECH
Email:{robotnam, lsmyce92}@kitech.re.kr

Abstract—A computationally efficient ranging scheme that estimates time of arrival (TOA) by a low complexity minimum mean square error (MMSE) and a matrix pencil techniques is proposed for IEEE 802.15.4a chirp spread spectrum system. In TOA estimation, it is known that accurate channel delay can be estimated by the MMSE technique. However, the major drawback of the MMSE is its high computational complexity, which grows with number of signal samples. In this paper, thus, a computational efficient MMSE technique based on the chirp signal characteristic is proposed for a low complexity ranging algorithm. The proposed TOA estimation method consists of two-step signal processing: proposed low complexity MMSE-based channel impulse response (CIR) estimation and the channel delay tracking by matrix pencil algorithm. The proposed TOA estimation method not only has low computational complexity, but also achieves small ranging error. The effectiveness of the proposed algorithm is demonstrated by simulation results.

I. INTRODUCTION

A location finding technique has been paid considerable attention as the location-based applications such as enhanced 911 (E911) services [1] and intrusion protection system become wide-spread. Usually these service and system require accurate location estimation especially in indoor environment. In order to obtain the accurate location, ranging information might be first needed. For instance, the location can be found by a trilateration method using three range information. Moreover 1 reference based location technique needs ranging information [2], [3]. Among several ranging estimation methods, i.e. TOA, RSSI (Received Signal Strength Indicator), TDOA (Time of Difference of Arrival) and so on, TOA is the most popular method for accurate positioning systems [4]. Hence, we focus on the ranging technique based on TOA estimation.

In a multipath environment, only the first arrived delay component is necessary for the TOA estimation. Ranging estimation algorithm from the TOA information has been researched with various techniques, such as multiple signal specification (MUSIC) [4], minimum-norm, matrix pencil [5], [6], and total least square estimation of signal parameter via rotational invariance techniques (TLS-ESPRIT) [7]. In addition, it is generally believed that the chirp spread spectrum (CSS) suits with high accuracy ranging application especially in indoor environment, where the multipath components are dense. An efficient ranging system using chirp signal, which employs a matrix pencil method to estimate time of arrival (TOA), is proposed in [8]. The proposed location system used a minimum mean square error (MMSE) criterion to estimate frequency channel impulse response for TOA information. However, MMSE criterion usually involves a high computational complexity because it requires the multiplication and inversion of known transmitted signal at each channel estimation. In this paper, the main contribution is to reduce the computational complexity in MMSE criterion using chirp signal characteristic. The proposed approach uses a low complexity CIR estimation and the matrix pencil method to extract the desired propagation delay. Simulation results demonstrate that the proposed scheme can achieve the small ranging error with less computational complexity, especially compared to conventional MMSE criterion.

The rest of the paper is organized as follows. In section II, system model is briefly described. In Section III, the proposed TOA estimation scheme including proposed low complexity MMSE is discussed. In section IV, the effectiveness of the proposed scheme is demonstrated with simulation results. The conclusion are given in Section V.

Notation: Throughout this paper, bold symbols denote matrices or vectors. $(\cdot)^T$, $(\cdot)^H$, $(\cdot)^+$ denote transpose, Hermitian transpose, pseudo inversion operation respectively.

II. SYSTEM DESCRIPTION

The proposed transceiver structure for ranging estimation based on the TOA is shown in Fig.1. As mentioned in the previous section, the CSS is a customized application of ranging under the requirement of high accuracy in indoor environment. Thus, we consider the system that employs the chirp signal based on the IEEE standard 802.15.4a [9]. The sub chirp sequence with its associated time gaps is generated.
from the following equation:

\[ g^n(t) = \sum_{n=0}^{\infty} \sum_{k=1}^{4} c_{n,k} e^{i(\omega_{n,k} + \frac{P}{2}) t} \cdot P_{RC}(t - T_{n,k,m}) \]

where \( m = 1, 2, 3, 4 \) defines which of the four different possible chirp symbols (sub-chirp sequences) is used, \( n = 0, 1, 2, \ldots \) is the sequence number of chirp symbols, \( P_{RC} \) is the raised cosine function and the other variables are expressed in IEEE standard 802.15.4a [9]. The generated chirp signal waveform is illustrated in Fig. 2.

After generating the signal, the transmitter produces a preamble by repeating the signal eight times since the preamble for 1Mb/s consists of 8 chirp symbols with all one preamble by repeating the signal eight times since the preamble for 1Mb/s consists of 8 chirp symbols with all one symbol. On the other hand, the pseudo inversion of the channel matrix in (5) with inversion of the signal shape matrix can be easily obtained by multiplying both sides of (5) with inversion of the signal shape matrix \( S^+ \).

\[
\hat{R} = \tilde{H}^T + \tilde{W}^T + \tilde{W}.
\]

where \( \tilde{S} \) is transmitted chirp signal matrix transformed into frequency domain. The channel matrix \( \tilde{H} \) is divided into delay matrix \( a \) and discrete sampled channel component \( \tilde{V} \).

\[
\hat{R} = [R(0)\ R(1)\ \cdots\ R(M-1)]^T,
\]

\[
a = [\alpha_0^T\ \alpha_1^T\ \cdots\ \alpha_{L_p-1}^T],
\]

\[
\tilde{V} = [\tilde{V}(0)\ \tilde{V}(1)\ \cdots\ \tilde{V}(\tau_{L_p-1})],
\]

where the noisy received signal is first averaged. The averaging preamble make the noise be diminished by getting a sort of time diversity since the noise is assumed as an AWGN. After that, the CIR is estimated by employing the proposed low complexity MMSE criterion. Then, it is possible to compute the distance between two devices using matrix pencil.

### III. Proposed TOA Estimation

In this section, we introduce the proposed TOA estimation method with two-step signal processing: low complexity MMSE-based channel estimation and the shortest channel delay tracking by matrix pencil algorithm.

#### A. Low complexity MMSE CIR estimation

Typically, the transmitted signals can be recovered by applying the inversion or pseudo inversion of the channel matrix in signal detection area. On the other hand, the pseudo inversion of the known signal matrix is needed here since the considered problem is to find the channel impulse response information.

1) **Conventional MMSE CIR estimation:**

The channel impulse response can be easily obtained by multiplying both side of (5) with inversion of the signal shape matrix \( S^+ \). That is,

\[
S^+R = S^+SH + S^+W.
\]

or

\[
\hat{R} = \tilde{H}^T + \tilde{W}^T.
\]

Based on the MMSE criterion, \( S^+ \) is expressed as

\[
S^+ = S^H(S^HS^H + (N_0/2)\cdot I)^{-1},
\]
where \( N_0 \) is noise variance and \( \mathbf{I} \) represents an identity matrix. Channel matrix \( \mathbf{H} \) to apply the TOA estimation is estimated by MMSE method.

2) Proposed Low complexity MMSE CIR estimation: From Eq.(8), MMSE based estimation has relatively high computational complexity since it requires the inversion and multiplication operation of transmit signal matrix \( \mathbf{S} \). As the complexity of MMSE estimation grows exponentially with the observation samples, a more refined strategy can reduce the number of signal samples cut off within bandwidth \( f_{\text{BW}} \). Therefore, MMSE CIR estimation based on the reduced signal matrix has extremely less computational complexity than conventional MMSE one. The Eq.(8) can be rewritten with the reduced received signal matrix \( \tilde{\mathbf{S}} \) as follows

\[
\tilde{\mathbf{S}}^+ = \mathbf{S}^H (\mathbf{S} \cdot \mathbf{S}^H + (N_0/2) \cdot \mathbf{I})^{-1},
\]

where

\[
\tilde{\mathbf{S}} = \left[ s(f_c - f_{\text{BW}}/2), \ldots, s(f_c + f_{\text{BW}}/2) \right].
\]

To estimate the channel impulse response using the proposed reduced signal matrix, the received signal also cut within same boundary \([f_c - f_{\text{BW}}/2, f_c + f_{\text{BW}}/2]\). To apply the proposed MMSE, finding the adequate cutoff bandwidth \( f_{\text{BW}} \) is crucial. It is observed that the difference between high frequency and low frequency is a good approximation for the cut-off bandwidth \( f_{\text{BW}} \) of the chirp pulse in Fig.2(c). Here, high and low frequency means the max and min frequency value of instantaneous frequency varies with time. The instantaneous frequency for chirp signal is defined as the slop of the phase, thus the range of frequency change can be derived from time varying phase of chirp signal. From Eq.(1), the phase function \( \psi(t) \) is represented

\[
\psi(t) = (\tilde{\omega}_{k,m} + \frac{\mu}{2} \xi_{k,m}(t - T_{n,k,m})) (t - T_{n,k,m})
\]

and the instantaneous frequency \( f_i(t) \) can be obtained by

\[
f_i(t) = \frac{1}{2\pi} \frac{d}{dt} \psi(t)
\]

where, the units of \( \psi(t) \) are rad/sec. So we divide by 2\( \pi \) to obtain Hz [11]. Then it is possible to compute difference between high and low frequency by Eq.(11), which is shown in Fig.2(b).

B. TOA Estimation using Matrix Pencil

The frequency response of the estimated noisy CIR from the Eq. (4) can be written as

\[
H(j2\pi f \Delta f) = \sum_{k=0}^{L_p-1} \alpha_k z_k^l + W_k,
\]

where \( z_k = e^{-j2\pi \Delta f \tau_k} \) with \( \Delta f = 1/L \Delta t \). Note that once \( z_k \) is estimated, the delay component, \( \tau_k \), and TOA also can be estimated.

Now the well-known matrix pencil algorithm to estimate the desired information \( \tau_k \) is briefly introduced [12]. We begin by defining estimated CIR matrix from Eq. (7) and (12) as a matrix pencil input \( \mathbf{X} \) which has \((L - P) \times (P + 1)\) dimension representing the \( l \)-th frequency sample at \( l \Delta f \).

\[
\mathbf{X} = \begin{bmatrix}
H(0) & \cdots & H(P) \\
\vdots & \ddots & \vdots \\
H(L - P - 1) & \cdots & H(L - 1)
\end{bmatrix}, \tag{13}
\]

where \( P \) represents the pencil parameter. Let us define two \((L - P) \times P\) matrices \( \mathbf{X}_0 \) and \( \mathbf{X}_1 \), each of which consists of the first \( P \) and last \( P \) columns of \( \mathbf{X} \). These two matrices can be expressed as \( \mathbf{X}_0 = \mathbf{Z}_1 \mathbf{A} \mathbf{Z}_2 \) and \( \mathbf{X}_1 = \mathbf{Z}_1 \mathbf{A} \mathbf{Z}_0 \mathbf{Z}_2 \), where \( \mathbf{Z}_0 \) and \( \mathbf{A} \) are the diagonal matrices with diagonal elements \( z_1 \sim z_{L_p} \) and \( \alpha_1 \sim \alpha_{L_p} \), respectively. The desired parameter can be obtained from the matrix pencil, \( \mathbf{X}_1 - \lambda \mathbf{X}_0 = \mathbf{Z}_1 \mathbf{A} [\mathbf{Z}_0 - \lambda \mathbf{I}] \mathbf{Z}_2 \), if we choose \( \lambda = z_k \). Thus, we can find the \( z_k \) as the generalized eigenvalues of the matrix pair \([\mathbf{X}_1, \mathbf{X}_0]\). The delay component \( \tau_k \) can be evaluated by using

\[
\tau_k = -\Im \left[ \frac{\ln(z_k)}{2\pi \Delta f} \right], \quad k = 0, 1, \ldots, L_p - 1. \tag{16}
\]

where \( \Im \) is the imaginary part operator.

A similar TLS matrix pencil can be applied to matrix pencil. This scheme use singular value decomposition (SVD) of \( \mathbf{X} \). A SVD of data matrix is \( \mathbf{X} = U' \sum V' \), where \( U' \) is the first \( L_p \) left singular vectors, \( V' \) is the first \( L_p \) right singular vectors like \( \mathbf{X}_0, \mathbf{X}_1 \) and \( \sum' \) is the first \( L_p \) singular values. The remaining process of TLS matrix pencil is similar with that of matrix pencil.

Finally, we can assess the range with multiplying \( \tau_k \) by the propagation speed of an electric wave.

IV. Simulation Results

In this section, the effectiveness of the proposed TOA estimation scheme is demonstrated with both the complexity and performance analysis. Moreover, the proposed low complexity MMSE scheme with matrix pencil ranging estimation technique is compared with the conventional MMSE criterion.

The chirp signal described in Eq. (1) is generated by the raised cosine window pulse shaping. The preamble for ranging consists of eight chirp symbols with all ones sequence. The IEEE 802.15.4a channel model 1, which was measured within coverage from 7m to 20m in residential line of sight (LOS) environment [13], is employed.
A. Complexity Analysis

The computational complexity between the conventional MMSE criterion and proposed scheme is investigated. Recalling that \( N \) is the total number of data samples, \( L \) is the number of samples in frequency domain, \( P \) is matrix pencil parameter, and now we define \( N_{re} \) is a reduced number of samples in frequency domain by applying the low complexity MMSE. Table I summarizes the basic operations for matrix pencil algorithm based on MMSE estimator, where GE stands for generalized eigenvalue decomposition. The overall flow of the proposed algorithm proceeds as follows:

- **FFT**: Operate \( N_{fft} \)-point FFT, where \( N_{fft} \) is an integer power of 2 and is larger than \( N \)
- **Preamble averaging**: Average the preamble (8 preamble used based on IEEE 802.15.4a)
- **MMSE estimator**: Consist of matrices multiplication and matrix inversion from Eq.(8), (9)
- **Matrix Pencil**: Find the pencil matrix and perform generalized eigenvalue decomposition
- **Delay finding**: Compute the delay component by Eq.(16)

With these operations, the complexity of proposed algorithm can be investigated with the number of real operation needed as shown in Table II. In fact, Matrix Pencil uses Hessenberg-Triangulart reduction to compute the Q and Z matrix and performs QZ algorithm to obtain the generalized eigenvalue, which is equivalent to one eigenvalue decomposition \((L-P) \times (L-P)\) matrix followed by an inversion of a matrix of same dimension [14], [15]. MMSE criterion is very computationally intensive operation since the ratio of MMSE to total operations is 80\%. Therefore, the computational complexity can be significantly reduced by the proposed scheme. Low complexity MMSE algorithm uses less data samples \( N_{re} \) which are within \( f_{BW} \). Thus Matrix dimension of MMSE has \((M_{re} \times L)\) instead of \((M \times L)\), where reduced multi-snapshot \( M_{re} \) can be acquired by \( M_{re} = N_{re} - L + 1 \). Note that, the reduced the number of frequency domain samples \( N_{re} \) is much smaller than the number of total signal samples \( N \), thus the size of frequency domain sinal matrix \( M_{re} \times L \) can be reduced remarkably. If we define, the relative ratio of computational complexity is

\[
\text{Ratio}(f_{BW}) = \frac{\text{Total number of operation in } f_{BW}}{\text{Total number of operation in full bandwidth}},
\]

the relative complexity ratio when we use 40 MHz is

\[
\text{Ratio (40 MHz)} = \frac{1.286e+006}{3.088e+007} = 41.66\%.
\]

This complexity ratio is calculated over the same conditions with \( N = 476, L = 50, M = 427, P = L_p = 21, N_{re} = 190, M_{re} = 141 \). From this result, it is worth noting that the proposed algorithm has many less MMSE operations than conventional MMSE method.

### Table I

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Complexity</th>
<th>Operations</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N_{m} ) point FFT</td>
<td>( N_{m} ) point complex multiplication and addition</td>
<td>10 \cdot N_{m} \log_2 N_{m}</td>
</tr>
<tr>
<td>Preamble Averaging</td>
<td>( N \times ) preamble complex addition and ( N ) complex division with real divisors</td>
<td>( 2N(\text{ preamble} + 1) )</td>
</tr>
<tr>
<td>MMSE</td>
<td>( 1_{(M \times L)} ) Matrix Pencil</td>
<td>( 15ML^2 + 10ML + 5L^3/3 + 9L^2/2 + 83L/6(+M) )</td>
</tr>
<tr>
<td>Matrix Pencil</td>
<td>Hessenberg-Triangular Reduction with ((L-P) \times (L-P)) matrix</td>
<td>( (L-P) - 2 \sum_{k=1}^{(L-P-k+1)} {228(N-P) - 76(k-1) + 256} )</td>
</tr>
<tr>
<td>QZ method for GE with ((L-P) \times (L-P)) matrix</td>
<td>((L_p) ) complex division with complex divisors</td>
<td>( 58(L-P)^3 - 66(L-P)^2 + 264(L-P) - 326 )</td>
</tr>
<tr>
<td>Delay Finding</td>
<td>( L_p ) complex division with real divisors</td>
<td>( 13L_p )</td>
</tr>
</tbody>
</table>

### Table II

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Complexity</th>
<th>Operations</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N_{m} ) point FFT</td>
<td>( N_{m} ) point complex multiplication and addition</td>
<td>10 \cdot N_{m} \log_2 N_{m}</td>
</tr>
<tr>
<td>Preamble Averaging</td>
<td>( N \times ) preamble complex addition and ( N ) complex division with real divisors</td>
<td>( 2N(\text{ preamble} + 1) )</td>
</tr>
<tr>
<td>MMSE</td>
<td>( 1_{(M \times L)} ) Matrix Pencil</td>
<td>( 15ML^2 + 10ML + 5L^3/3 + 9L^2/2 + 83L/6(+M) )</td>
</tr>
<tr>
<td>Matrix Pencil</td>
<td>Hessenberg-Triangular Reduction with ((L-P) \times (L-P)) matrix</td>
<td>( (L-P) - 2 \sum_{k=1}^{(L-P-k+1)} {228(N-P) - 76(k-1) + 256} )</td>
</tr>
<tr>
<td>QZ method for GE with ((L-P) \times (L-P)) matrix</td>
<td>((L_p) ) complex division with complex divisors</td>
<td>( 58(L-P)^3 - 66(L-P)^2 + 264(L-P) - 326 )</td>
</tr>
<tr>
<td>Delay Finding</td>
<td>( L_p ) complex division with real divisors</td>
<td>( 13L_p )</td>
</tr>
</tbody>
</table>
Fig. 3. Average ranging errors with different bandwidth

Fig. 4. Average ranging error and standard deviation according to the SNR

The performance of TOA estimation algorithms are compared in Fig. 4 with the mean error and the corresponding standard deviation of ranging estimation. The performance of conventional MUSIC algorithm is degraded in dense multipath environment [16]. Especially standard deviation of MUSIC much larger than that of MP. Since MUSIC finds the delay component by extracting the peak point for each delay component using pseudo-spectrum function, the error could vibrate as long as channel delay duration. The simulation results show that the ranging estimation base on the MP is suitable method in indoor environment. Furthermore, the proposed low complexity MMSE algorithm has almost same mean error and standard deviation compare with conventional one by using only less than half of the operations. It is observed that the average ranging error of low complexity MMSE scheme has less 10ns error over 15dB SNR, and the standard deviation exists within 20ns at the overall SNR.

V. CONCLUSION

We proposed a computationally efficient ranging scheme that estimates TOA by a low complexity MMSE and a matrix pencil techniques. The proposed scheme not only has low computational complexity, but also achieves small ranging error. The effectiveness of the proposed scheme for indoor environment is demonstrated by simulation results.

ACKNOWLEDGMENT

This work was supported in part by KITECH (Korea Institute of Industrial Technology) through Division of Applied Robot Technology, Korea.

REFERENCES

[9] P802.15.4a/D7, Approved Draft Amendment to IEEE Standard for Information technology-Telecommunications and information exchange between systems-PART 15.4:Wireless Medium Access Control (MAC) and Physical Layer (PHY) Specifications for Low-Rate Wireless Personal Area Networks (LR-WPANs): Amendment to add alternate PHY (Amendment of IEEE Std 802.15.4a), Jan. 2007