Multiband distributed sensing based on compressed measurements via two AICs implementations

J. Verlant-Chenet     F. Horlin
Université Libre de Bruxelles
OPERA department, Group WCG
jverlant@ulb.ac.be    fhorlin@ulb.ac.be

Abstract

We develop a distributed multiband spectrum sensing detector for cognitive radios based on compressed measurements that does not rely on signal reconstruction. A fusion centre collects the measurements from different sensing nodes and then makes a sensing decision based on a simplified maximum likelihood criterion which is valid for both analog to information implemented in the paper (MWC and NUP) and does not require prior signal information. Simulation results for probability of erroneous detection and ROC curves show that the performance of the proposed detector is good. Plus, it has a low computational complexity.

1 Introduction

It is well known that static frequency allocation has lead to the scarcity of spectral resources. Assigning fixed frequency bands to ever-evolving new applications is both expensive as well as unfeasible. However, this necessitates very large sampling rates (proportional to the spectrum bandwidth) which can heavily stress analog-to-digital converters (ADCs) in terms of power consumption. Recently, the theory of compressed sampling (CS) [3] has received considerable attention among research community as a means to reduce the sampling-rate constraints on the design of CR systems. In the context of CRs, CS is based on the fact that given the sparsity of the signal in the frequency domain, sampling rates can be made significantly lower than the Nyquist rate without losing much information. This may potentially facilitate simpler implementation of the ADCs and digital processors.

In the traditional CS framework, signal needs to be recovered from its compressed samples. A plethora of algorithms are available to provide reliable recovery of the sparse signal: matching pursuit (MP), orthogonal matching pursuit (OMP) [4], compressive sampling matching pursuit (CoSaMP) [5], basis pursuit (BP) [6], least absolute shrinkage and selection operator (LASSO) [7]. Note that most of these algorithms are quite complex and often consume a lot of computational resources. However, signal reconstruction may not be necessary in many signal processing applications as one may only be interested in solving an inference problem. Davenport et al. have demonstrated that it is possible to tackle the problem of detecting a known signal buried in noise, i.e., classification of the signals, directly in the compressive domain without first resorting to a complex signal reconstruction [8], [9]. Many other works focusing on solving a detection problem directly from the compressed samples are also available, e.g., [10], [11]. Continuing this direction of research, we focus on developing efficient detectors for CR systems, based on compressed measurements.

We have recently extended [9] to the optimal maximum likelihood (ML) detection of linearly modulated signals of unknown parameters occupying unknown frequency sub-channels [12] by using the compressed measurements only. We basically focused on the
functionality of an individual CR. However, the use of distributed spectrum sensing algorithms is recommended to cope with the fading phenomenon present in all wireless communications systems. In this paper, we basically combine our proposed approach in [12] with the distributed signal processing, where instead of focusing on a single CR, multiple CRs generate compressed measurements which are then transmitted to the fusion centre (FC). This results in saving sensing resources at individual CRs as well as reduces the capacity requirements of the control channels. This is in contrast with the existing contributions in literature which aim at reconstructing the wideband signal spectrum at the FC by defining a sparsity model common to all sensing nodes [16], [14]. We propose the detection of a primary signal directly in the compressive domain which does not rely on signal reconstruction from its compressed measurements. The proposed multiband signal detection procedure is optimized according to the ML detection criterion for a distributed CR scenario. A closed-form expression of the detection metric is obtained by carefully approximating the likelihood function. We demonstrate that the CS analog-to-information converter (AIC) can advantageously be implemented by a distributed network of sensing nodes. In this paper we consider two different realizations of the AIC. The first is a modulated wideband converter (MWC) [15], where the received signal is first multiplied with a high-rate binary spread code and then sampled at a low sampling rate after a low-pass filtering stage. The second is a non-uniformly periodic (NUP) sampling, where the received signal is delayed then sub-sampled. Each sensing node implements a branch of MWC or the NUP. The sensing nodes then transfer the low-rate sample sequence of the received signal to the FC for multiband detection. The performance of the proposed detector is exhibited by means of numerical simulations for probability of erroneous detection and receiver operating characteristic curves.

2 System model

The primary network consists of multiple mobile terminals communicating to a base station (uplink transmission). The overall bandwidth is divided into $M$ frequency bands that may be allocated to different terminals for their communication in a frequency division multiple access (FDMA) fashion. Each frequency band has a bandwidth $\frac{1}{T}$, where $T$ is the symbol duration for each user. We assume there are $K \leq M$ active terminals in the network. The secondary network are mobile platforms that embed a MWC or NUP AIC and send all the information to the fusion centre, where a maximum likelihood based detection algorithm is applied. Since both AIC are meant to be implemented on USRP2 platforms, the following system model comprises imperfections that might occur in a real-time demonstration: carrier frequency offset (CFO), sampling clock offset (SCO), channel effect, etc.

2.1 Transmitter architecture

Fig. 1 describes the transmitter architecture for primary user $k$. Symbols $I_k[n]$ are convoluted with a half-root Nyquist filter $g[n]$ after being upsampled by a factor $M$. The modulated symbols are then converted to an analog signal $s_k(t)$ at rate $\frac{1}{T}$. The result is

$$s_k(t) = \sum_{n=1}^{N} I_k[n] g(t - nT).$$

(1)

The modulated signal is then shifted to the allocated band of center frequency $\Delta f_k$, and then passed through a typical analog front-end. The baseband signal transmitted
by the primary user $k$ is expressed as $s_k(t) e^{j2\pi \Delta f_k t}$.

At any sensing node, the received signal is the sum of the signals for all contributing primary users, that is

$$s(t) = \sum_{k=1}^{K} s_k(t) e^{j2\pi \Delta f_k t}. \quad (2)$$

At the sensing node $q$, this signal is corrupted by a channel response $c_q(t)$ and additive white Gaussian noise $w_q(t)$. The received signal at node $q$ is then

$$x_q(t) = c_q(t) \ast s(t) + w_q(t). \quad (3)$$

Fig. 2 represents the spectrum of this signal, assuming that $K = 3$ and that the channel response is flat in the band of interest.

2.2 Receiver architecture

Fig. 3 describes the receiver architecture for secondary user $q$. Receiver analog front-end comprises a band-pass filter, a low noise amplifier, an automatic gain control, a
Figure 3: Secondary user receiver architecture

down-converter and a low-pass filter. The latter impulse response is denoted $h_{\text{LPF}}(t)$. Note that the down-converter center frequency might generally not match that of the transmitter, hence the CFO term $\Delta f_{c,q}$. The resulting received signal is

$$r_q(t) = x_q(t) e^{j2\pi \Delta f_{c,q} t} * h_{\text{LPF}}(t) = c_q(t) * s(t) e^{j2\pi \Delta f_{c,q} t} * h_{\text{LPF}}(t) + v_q(t),$$

(4)

where

$$v_q(t) = w_q(t) e^{j2\pi \Delta f_{c,q} t} * h_{\text{LPF}}(t)$$

(5)

is the noise restricted to the band of interest $\frac{M}{T}$. Since the sensing nodes focus their detection on all available sub-bands, we further only consider the power $\sigma_v^2$ of the noise in the overall bandwidth $\frac{M}{T}$. The power $\sigma_s^2$ of the signal $s(t)$ depends on the number of active primary users $K$. Since we want to compare our algorithms performance for different values of $K$, we need to define the SNR so that it is independent of the number of that parameter. Thus, we further only consider the average signal power per sub-band $\langle \sigma_{s_k}^2 \rangle = \frac{\sigma_s^2}{K}$. In our case, the SNR is defined as follows:

$$\text{SNR} = \frac{\langle \sigma_{s_k}^2 \rangle}{\sigma_v^2}.$$  

(6)

As shown in Fig. 3, the received signal is then passed through an AIC. In the following sections, we consider two different AIC implementations: NUP and MWC. We also show that the output signal $y_q[n]$ of both AIC can be expressed in the same model.

### 2.2.1 NUP AIC

Fig. 4 describes the AIC architecture for NUP. It simply waits a time $\tau_{\text{NUP},q}$ before sub-sampling at rate

$$T_s = \frac{c}{M} T + \Delta T_s$$

(7)

where $c$ is the sub-sampling factor, and $\Delta T_s$ is the sampling clock offset (SCO) between the transmitter and the receiver (assuming all transmitters clocks are frequency synchronized). Not that to achieve a non-uniformly periodic sampling, $\tau_{\text{NUP},q}$ has to be strictly different for each sensing node.

The AIC output is given by
\[
y_q[m] = r_q(t)
\]
\[
\bigg|_{t=mT_s-\tau_q} = \alpha_q s(mT_s - \tau_q) e^{j2\pi \Delta f_{c,q}(mT_s-\tau_q)} + \varphi_q + v_q(mT_s - \tau_q).
\]

where \( \alpha_q \) and \( \varphi_q \) represents the effect of the flat channel. In the expression of the delay

\[
\tau_q = \tau_{p,q} + \tau_{s,q} + \tau_{NUP,q},
\]

\( \tau_{p,q} \) is the propagation delay, \( \tau_{s,q} \) is the phase difference between transmitter and receiver sampling clocks (assuming all transmitters clocks are phase synchronized), and \( \tau_{NUP,q} \) is the artificial delay specific to the NUP AIC. Giving (2), the output (8) can be written in the general form

\[
y_q[m] = \sum_{k=1}^{K} c_{k,q}[m] s_{k,q}[m] + n_q[m]
\]

where

\[
c_{k,q}[m] \triangleq \alpha_q e^{j2\pi (\Delta f_k + \Delta f_{c,q})(mT_s-\tau_q)} + \varphi_q
\]

\[
s_{k,q}[m] \triangleq s_k(mT_s - \tau_q)
\]

\[
n_q[m] \triangleq v_q(mT_s - \tau_q)
\]

2.2.2 MWC AIC

Fig. 5 describes the AIC architecture for MWC. The received signal is first mixed with a chip sequence \( p_q(t) \). The sequence is \( T \)-periodic and can thus be expanded as a Fourier series, i.e.,

\[
p_q(t) = \sum_{m=-\infty}^{+\infty} c_q[m] e^{j2\pi m \frac{t}{T}}
\]

where \( c_q[m] \) are the Fourier coefficients of \( p_q(t) \) expansion. The mixing of (14) and (4) implies the product

\[
p_q(t)s(t) = \sum_{m=-\infty}^{+\infty} \sum_{k=1}^{K} c_q[m] e^{j2\pi m \frac{t}{T}} s_k(t) e^{j2\pi \Delta f_k t}.
\]

By defining \( m_k \) as

\[
\Delta f_k \triangleq \frac{m_k}{T},
\]

this product becomes

\[
p_q(t)s(t) = \sum_{m=-\infty}^{+\infty} \sum_{k=1}^{K} c_q[m] s_k(t) e^{j2\pi (m_k+m) \frac{t}{T}},
\]

\[
\text{Figure 4: NUP AIC architecture}
\]
which means that each occupied sub-band \( m_k \) is now shifted in every other part of the spectrum and summed with other PU contribution. In particular, after the low-pass filter, we only keep the contributions in (18) for every \( m = -m_k \). The product becomes

\[
p_q(t)s(t) = \sum_{k=1}^{K} c_q[-m_k] s_k(t).
\]

Thus, the output of the AIC is

\[
y_q[m] = r_q(t)p_q(t) * h_T(t) \big|_{t=mT_s-\tau_q}
\]

where \( h_T(t) \) is the impulse response of a low-pass filter of bandwidth \( \frac{1}{T_s} \), and

\[
T_s = T + \Delta T_s.
\]

Similarly to (8), and with (18), we find

\[
y_q[m] = \alpha_q \sum_{k=1}^{K} c_q[-m_k] s_k(mT_s - \tau_q) e^{j2\pi f_{c,q}(mT_s - \tau_q) + \phi_q} + n_q(mT_s - \tau_q),
\]

where \( n_q(mT_s - \tau_q) = v_q(t)p_q(t) * h_T(t) \big|_{t=mT_s-\tau_q} \) is still an AWGN. (21) can be expressed in the form of (10) if we define

\[
c_{k,q}[m] \triangleq \alpha_q c_q[-m_k] e^{j2\pi f_{c,q}(mT_s - \tau_q) + \phi_q}
\]

\[
s_{k,q}[m] \triangleq s_k(mT_s - \tau_q)
\]

\[
n_q[m] \triangleq n_q(mT_s - \tau_q)
\]

### 2.2.3 Common model

If we assume we only keep \( W \) samples from the AIC output \( y_q[m] \) of each sensing node \( q \), (10) can be expressed in the following matrix form:

\[
y_q = \sum_{k=1}^{K} C_{k,q} s_{k,q} + n_q
\]

Figure 5: MWC AIC architecture
where

\[
\begin{align*}
    y_q & \triangleq [y_q[0], \ldots, y_q[W - 1]]^T \\
    s_{k,q} & \triangleq [s_{k,q}[0], \ldots, s_{k,q}[W - 1]]^T \\
    n_q & \triangleq [n_q[0], \ldots, n_q[W - 1]]^T \\
    C_{k,q} & \triangleq \begin{bmatrix}
        c_{k,q}[0] & 0 \\
        \vdots & \ddots \\
        0 & c_{k,q}[W - 1]
    \end{bmatrix}
\end{align*}
\]

This model is valid for both AICs.

### 2.3 Information gathering at the FC

If we concatenate all AIC outputs from all \(Q\) sensing nodes at the FC, we obtain

\[
y = \sum_{k=1}^{K} C_k s_k + n
\]

where

\[
\begin{align*}
    y & \triangleq [y_1^T \cdots y_Q^T] \\
    s_k & \triangleq [s_{k,1}^T \cdots s_{k,Q}^T] \\
    n & \triangleq [n_1^T \cdots n_Q^T] \\
    C_k & \triangleq \begin{bmatrix}
        C_{k,1} & 0 \\
        \vdots & \ddots \\
        0 & C_{k,Q}
    \end{bmatrix}
\end{align*}
\]

### 3 Distributed Maximum Likelihood detector

If we apply the same method and assume the same hypotheses as in [12] with FC information (30), it is possible to demonstrate that the Maximum Likelihood approximate becomes

\[
\{ \hat{\Delta f}_k \} = \arg \max_{\{ \Delta f_k \}} \prod_{k=1}^{K} e^{-\rho_k} \left\| \sum_{q=1}^{Q} C_{k,q}^H y_q \right\|^2,
\]

in which

\[
\rho_k = W \frac{\sigma_s^2}{2K\sigma_n^2} \sum_{q=1}^{Q} \sum_{m=0}^{W-1} |c_{k,q}[m]|^2,
\]

where \(\frac{\sigma_s^2}{K}\) is the mean power of the signal in one single band. Note that for the NUP AIC, the definition (11) leads to

\[
\sum_{m=0}^{W-1} |c_{k,q}[m]|^2 = W |\alpha_q|^2
\]
which means that $\rho_k$ is independent of $k$ in that case. For that AIC, the ML criterion becomes

$$\left\{ \Delta \hat{f}_k \right\} = \arg \max_{\{\Delta f_k\}} \left\| \sum_{q=1}^{Q} C_{k,q}^H \mathbf{y}_q \right\|^2$$

(38)

4 Performance results

4.1 Simulation setup

The performance of our proposed detector is assessed numerically by computing probability of erroneous detection (PED) and receiver operating characteristic (ROC) curves. PED gives the average error rate. A detection is regarded as erroneous for even a single miss or false detection. For ROC curves, we also provide their theoretical limits. Further, we evaluate the performance of our proposed AIC MWC detector.

Assuming that the activity of each sub-band follows a Bernoulli distribution with probability $p$ and the overall bandwidth is sliced into $M$ uniform sub-bands, the number of enabled sub-bands $K$ follows a binomial distribution, i.e., $B(M, p)$. It is then possible to establish the extreme ROC curves for a perfect detection, i.e., for the noiseless case. In this case, a false alarm (FA) occurs when $\hat{K} > K$ while there is no misdetection (MD), whereas a MD is observed when $\hat{K} < K$ while there is no FA. Thus, the probability of false alarm $p_{FA}$ is given by the expectation of $\frac{K - \hat{K}}{K}$ over $K$ and the probability of misdetection $p_{MD}$ is found by computing the expectation of $\frac{K - \hat{K}}{K}$ over $K$. Both can be analytically computed for each value of $\hat{K}$ ranging from 1 to $M$.

We consider an overall bandwidth to be sensed as $\frac{M}{T} = 6.25$ MHz which is sliced into $M = 31$ uniform sub-bands of bandwidth 201.6 kHz each. When the number of licensed users $K$ is known and fixed, then $K = 6$. Otherwise, $K$ is a parameter. When evaluating ROC curves, the amount of used sub-bands follows a Bernoulli distribution with probability $p = 0.20$ and thus, the average usage of overall bandwidth is 20%. The total number of sensing node is fixed to $Q = 16$ except when the impact of this parameter on the performance is studied. Each simulation curve is generated by averaging over 1000 to 5000 realizations.

4.2 PED vs SNR

We obtain PED results against different values of SNR. Figure 6 and 7 show the PED simulation results for our proposed MLA based detector and the PSD based detector, respectively, for varying values of sensing nodes. In general, the performance improves with an increase in the number of sensing nodes $Q$ and with a decrease in the number of primary users $K$. Indeed, increasing the number of sensing nodes increases the average sampling rate and thus, more linear combinations of the enabled sub-bands are available which result in improved performance.

4.3 ROC curves

We generate the ROC curves for different values of $\hat{K}$, i.e., $\hat{K} \in [1; M]$. Figure 8 show the ROC curves for the proposed detector with MWC AIC for varying values of SNR. We also plot the curve for theoretical limit as a reference. Once again, the performance of both the detectors is comparable and also reaches the theoretical limit for very low values of SNR ($-9$ dB). However, our proposed detector has an edge in terms of low computational complexity.
5 Conclusion

In this paper, we have developed a distributed multiband spectrum sensing detector for cognitive radios which is based on compressed measurements and does not rely on signal reconstruction. The detector uses a simplified maximum likelihood metric which is valid for both MWC and NUP AIC and does not require prior signal information. Simulation results for probability of erroneous detection and ROC curves show that the performance of the proposed detector is good. Plus, it has a low computational complexity.

References


Figure 7: PED versus SNR for $Q = 16$


Figure 8: ROC curves for $Q = 16$


