Moses Schönfinkel and combinatory logic

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Based on a talk by Stephen Wolfram

I attended an online talk by Stephen Wolfram (2020d) celebrating the Russian logician Moses Schönfinkel (1888–1942). It was delivered exactly a century after a talk by Schönfinkel at the University of Göttingen in Germany, where he was a member of the group there headed by David Hilbert (1862–1943). Schönfinkel's talk in 1920 was entitled *Elemente der Logick* ("Elements of Logic"), where he summarised his foundational ideas on combinatory logic (Stanford, 2020), that he later published (Schönfinkel, 1924). Haskell Curry (1890–1982) also took up these ideas (Curry, 1927; 1930; Curry & Feys, 1958). This led to the term "currying", the technique of converting a function with multiple arguments into a sequence of functions each taking a single argument. This is important in lambda calculus (Cardone& Hindley, 2009), later mathematically modelled by Dana Scott in an early Programming Research Group Technical Monograph at Oxford (Scott, 1970). Subsequently, currying and lambda calculus gained importance in computer science, through functional programming.

Christopher Strachey (1916–1975) commented originally in lecture notes of 1967: "There is a device originated by Schönfinkel, for reducing operators with several operands to the successive application of single operand operators" (Strachey, 2000). Later, John Reynolds (1935-2013) commented: "In the last line we have used a trick called Currying (after the logician H. Curry) to solve the problem of introducing a binary operation into a language where all functions must accept a single argument." (Reynolds, 1998). The referee commented that although "Currying" is tastier, "Schönfinkeling" might be more accurate!

Stephen Wolfram's talk has an associated blog post on Moses Schönfinkel's contribution to the concept of combinators (Wolfram, 2020c) and other related blog posts (Wolfram, 2020a; 2020b). Wolfram and his team have undertaken

archival research on Schönfinkel's life. Sadly, not long after his contribution of combinatory logic, Schönfinkel later suffered from mental illness and spent the rest of his life in Moscow. On his death in poverty during 1942 at the time of World War II, his neighbours burnt his papers for heating. Despite his short period of research productivity, I believe that Moses Schönfinkel deserves to be better known for his contributions to computer science through combinatory logic. Stephen Wolfram's presentation was followed by an interesting online networking event, with participants able to roam around a virtual room and form parallel audio discussion groups (https://www.highfidelity.com), such are the rapid technological developments accelerated by the current pandemic.

Stephen Wolfram presenting on Zoom

Online networking afterwards

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