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Josef Stefan: His life and legacy in the thermal sciences

John Crepeau *

Department of Mechanical Engineering, University of Idaho, 1776 Science Center Drive, Idaho Falls, ID 83402, USA

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Abstract

This paper discusses the life of Josef Stefan, namesake of both the Stefan–Boltzmann constant, used in radiation heat transfer, and the Stefan number, the dimensionless variable used in solid–liquid phase change processes. Stefan was also the first to accurately measure the thermal conductivity of gases. He was a beloved teacher and a mentor to Ludwig Boltzmann. Although Stefan made broad and seminal contributions to the thermal sciences, he is not a widely known figure.

Keywords: Josef Stefan; Thermal conductivity; Radiation heat transfer; Solid–liquid phase change

1. Introduction

It has been said that, "Eponymity, not anonymity, is the standard of recognition in science" [1]. Reynolds and Planck are cases in point. However, one who is multiply eponymous, yet virtually anonymous, is the 19th century physicist, Josef Stefan (Fig. 1). He empirically determined the T^4 radiation law, and along with his student, Ludwig Boltzmann, who independently determined the law from first principles, lent their names to the Stefan–Boltzmann constant. Stefan also studied the moving boundary problem, specifically water freezing on the polar ice caps, and from that work his name is given to the ratio of sensible heat to the latent heat of fusion, the Stefan number. Despite such honors, Stefan remains almost unknown in the scientific community.

2. Early life and education

Josef Stefan was born March 24, 1835 in the small village of St. Peter, just outside the town of Klagenfurt, in what is now Austria. Although his childhood home has been destroyed, there is a plaque commemorating his birth on a home rebuilt on the same location at 88 Ebentalerstrasse (Fig. 2). Klagenfurt is home to a large ethnic Slovenian population, and his parents descended from these peoples. His family was poor growing up; his father Aleš worked as a miller and baker, and his mother Marija Startinik was a maidservant. Early on he showed great academic talent and wished to study at the local gymnasium, but as an illegitimate child was unable to attend, so his parents married when Josef was eleven years old. In 1848, the March Revolution heightened awareness of the many ethnicities within the Austro-Hungarian empire, and the teenage Stefan began to publish poetry and other musings in Slovenian. His poems touched on romantic, patriotic and scientific themes. However, the writings were not well-received by some within the Slovenian literary community, and by his early twenties he abandoned altogether his Slovenian poetry and prose [2,3].

Stefan was an energetic and focused boy, and after completing his studies at the gymnasium, he chose mathematics and physics over the Benedictines and enrolled at the University of Vienna in 1853. While at the University, he worked with Karl Ludwig, a professor of physiology, and began to hone his experimental expertise by studying the flow of water in tubes. After graduating in 1857, he taught physics for pharmacy students. During his studies, Stefan

* Tel.: +1 208 282 7955; fax: +1 208 282 7950.
E-mail address: crepeau@uidaho.edu

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Nomenclature

a	empirical constant	Q	total heat transfer, W
A	proportionality constant, W/m^2 K^4	t	time, s
A_c	cross-sectional area, m^2	T	temperature, K
B	integration constant	x	distance, m
c_p	specific heat at constant temperature, kJ/kg K		
c_v	specific heat at constant volume, kJ/kg K	*Greek*	
E	radiative power, W/m^2	β	integration constant
$f(t)$	general temperature variation, K	θ	temperature difference, K
h	ice thickness, m	λ	latent heat of fusion, J/kg
k	thermal conductivity, W/mK	μ	material constant
m	mass, kg	ρ	density, kg/m^3
p	pressure, N/m^2	σ	Stefan–Boltzmann constant, W/m^2 K^4

Fig. 1. Portrait of Josef Stefan, taken around 1880. (Courtesy of the Austrian Academy of Sciences).

Fig. 2. Plaque commemorating the birthplace of Josef Stefan. It reads, "In this house, the physicist Josef Stefan, the discoverer and namesake of the radiation law, was born on 24 March, 1835." The plaque is located on 88 Ebentalerstrasse in the District of St. Peter, Klagenfurt, Austria. Photo by the author.

realized he had an aptitude for research and began to publish his work in the scientific literature. He accepted a position with Ludwig at the Physiology Institute at the University of Vienna, where he improved and strengthened his experimental skills. Just a year after graduating, he passed his doctoral examination and became a *Privatdozent* (Instructor) at the University of Vienna. Although Stefan found the position at the Physiological Institute rewarding, especially given his impoverished childhood and years of study, he desired a more physics-related position where he could apply his formal education. Ludwig and a colleague, Ernst Brücke, lobbied hard to appoint Stefan a corresponding member of the Austrian Academy of Sciences and he was granted this status in 1860. Still, his goal of performing physics research seemed distant until events turned in his favor. He was offered a full professorship in mathematics and physics at the University of Vienna in 1863, becoming the youngest to hold that rank in Austria. Then due to an untimely death of one of its researchers, a position opened at the University's Institute of Physics, which Stefan immediately accepted. Two years later, in 1865, Andreas von Ettinghausen retired as Director of the Institute and Stefan was offered the post. Within the space of a few years, Stefan went from being in a fortunate but not wholly satisfactory position in physiology to the head of what would become a renowned center for scientific research. For the rest of his life, Stefan took advantage of this great opportunity. He began publishing in diverse subjects including acoustics, electrodynamics and optics. His peers in the scientific community began to recognize his work when he received the inaugural Ignaz L. Lieben Prize in 1865, which was awarded every three years to young citizens of the Austro-Hungarian empire for the best scientific paper. The Lieben Prize, which has its own fascinating history [4], was discontinued in 1938 because of economic and social unrest in Europe, but was revived again in 2004, available for young scientists from all of the nations that comprised the former Hapsburg empire.

During his lifetime, Stefan ably served the University of Vienna in various administrative positions, including Dean of the Philosophical Faculty from 1869 to 1870, and Rector from 1876 to 1877. He was also the Secretary and Vice President of the Austrian Academy of Sciences. In addition to his scientific and administrative talents, Stefan was a warm and beloved teacher. He gave very energetic, animated lectures and was said to be exhausted upon their completion [5]. His students not only felt comfortable around him but were motivated to do high-level scientific research. One of his students later remarked on the collegial atmosphere that Stefan maintained at the cramped, underfunded Institute on Erdbergstrasse, far from the central campus buildings of the University, "Nothing diminishes the excellence of his character, the magic [Stefan] worked on the young academics. That magic could only be experienced personally...Erdberg stayed with me my whole life as a symbol of serious, inspired experimental activity" [6]. That student, who walked into the Institute to complete his doctoral studies under Stefan just as he became its Director, was none other than Ludwig Boltzmann.

3. Stefan and Boltzmann

Stefan and his star pupil Boltzmann were a study in contrasts. Although he was the Director of the Institute, Stefan was only nine years older than Boltzmann. Boltzmann grew up in a solid middle-class family, one which valued education; Stefan was born into a peasant family with illiterate parents. Stefan was an outstanding experimentalist and an able theoretician; Boltzmann was a talented experimentalist whose genius and fame came from his analytical work. Boltzmann was a peripatetic traveler, especially for his time, and an active promoter of his ideas, while Stefan preferred sleeping in his lab, sometimes not leaving the Institute building for days at a time, and publishing in the local scientific journal. Stefan spent his entire academic career at the University of Vienna; Boltzmann used his academic success to extract favorable employment terms from universities throughout Europe. Boltzmann married soon after receiving his doctorate and had five children; Stefan remained single until almost the end of his life and had no children. He was personable and outgoing, a well-liked teacher and administrator, but very focused on his work, so much so that he had few friends and almost no social life. Boltzmann, on the other hand, enjoyed social settings but lacked interpersonal skills and suffered from severe mood swings [7]. Despite these differences in personalities, both were dedicated and hard-working physicists.

In the collection of his writings, *Populäre Schriften*, Boltzmann described his mentor Stefan: "He used the tools of advanced mathematics and understood how to present the most difficult developments in the clearest and most lucid form without ever having to resort to mathematical formalism...[he] never tried to flaunt [his] mental superiority. [His] uplifting humor, which turned the most difficult discussion into an entertaining game for the student, made such a deep impression on me" [6].

4. The diathermometer and thermal conductivity measurements of gases

Upon receiving his doctorate, Stefan embarked on a research program that covered many fields. He was instrumental in bringing Maxwell's electrodynamics and kinetic theory to continental Europe and encouraged Boltzmann to improve his English by studying Maxwell's works. One of Stefan's early, major contributions to the field of heat transfer was the first accurate measurement of the thermal conductivity of gases.

To put this in context, it must be understood that a great debate was roiling about whether or not gases could even conduct heat. In 1780, Priestley performed experiments wherein he believed he was measuring a gas' "power to conduct heat." However, this turned out to be the specific heat, which at the time was a relatively new scientific concept. Later, in 1786, Rumsford examined "the conducting power of the artificial airs or gases," and while doing so stumbled on a completely new mode of heat transfer, convection. This forced him to conclude in 1799 that a gas was unable to conduct heat, despite his own experimental evidence and physical intuition to the contrary. Unfortunately, because of Rumsford's reputation within the scientific community, his conclusions were not seriously challenged for many years. It was not until 1861 that Magnus used an electrically heated platinum wire surrounded by different gases to show conclusively that gases do conduct heat. For an early history of thermal conductivity and experimental measurements, see Burr [8].

At about the same time that Magnus published his results, two greats weighed in on the controversy, albeit from the theoretical perspective. Maxwell published in 1860 his groundbreaking paper on the dynamical theory of gases and in it calculated a theoretical value of the thermal conductivity of a gas and showed its dependence on the temperature and pressure. In it he noted, "It would be almost impossible to establish the value of the conductivity of a gas by direct experiment, as the heat radiated from the sides of the vessel would be far greater than the heat conducted through the air, even if currents could be entirely prevented."[9] Two years later, Clausius both admired and admonished Maxwell. Clausius recognized the work for its "elegance of mathematical developments," but pointed out that Maxwell "has treated the conduction of heat too incompletely" [10]. Clausius then showed that the thermal conductivity increased with temperature and was independent of pressure when the gas was ideal, and using his own methods calculated the thermal conductivity of air to be $k = 0.0115$ W/mK. Subsequently, Maxwell revised his work [11] and rederived relations for the thermal conductivity of a gas and calculated its value for air and showed that oxygen, nitrogen and carbon monoxide had about the same thermal conductivity, carbon dioxide had a much lower value than the diatomic gases, and that of hydrogen to be higher by about a factor of six. Using Maxwell's equation for the thermal conductivity of a gas, the value for air can be calculated to give $k = 0.0218$ W/mK.

Not one to back down from the challenge, Stefan brought to bear his enormous experimental capabilities and insight to defy Maxwell's words. He knew of Clausius' conclusions on the behavior of thermal conductivities and was well aware of the convincing experimental work of Magnus. Understanding that convection would skew the results, Stefan built an apparatus which heated the air at the top and cooled it at the bottom. In fulfillment of Maxwell's words, he abandoned this design because he was unable to control the heat loss through the walls. He then constructed a glass bulb to contain the air and submerged it in an ice bath. However, for small bulbs, the temperature changes were too small to measure, and for large bulbs convection affected the results. Stefan realized that measuring the thermal conductivity of a stationary gas would entail too large a risk of introducing convection effects, so he formulated a way to incorporate time-dependency into his measurements. He envisioned placing a gas in the gap between two surfaces and equating the total heat transfer by Fourier's law with the change of energy absorbed by the gas [12]

$$-kA_c\frac{\theta}{\Delta x}\,\mathrm{d}t = \mathrm{d}Q = mc_v\,\mathrm{d}\theta \tag{1}$$

After rearranging and integrating, he showed

$$\frac{\theta}{\theta_0} = \exp\left(-\frac{kA_c}{mc_v\Delta x}t\right) \tag{2}$$

where θ_0 was the initial temperature difference between the two surfaces. For an ideal gas at constant volume, the relative change in the temperature is equal to the relative change in pressure, so

$$\frac{\Delta p}{\Delta p_0} = \exp\left(-\frac{kA_c}{mc_v\Delta x}t\right) \tag{3}$$

Stefan then devised an apparatus which he called a diathermometer (Fig. 3), made of concentric copper cylinders containing the candidate gas in between. A manometer was connected to the inside portion of the inner cylinder and the outer cylinder was immersed in a constant temperature bath. He measured the change in pressure over a given time interval and from Eq. (3) calculated the thermal conductivity of the gas.

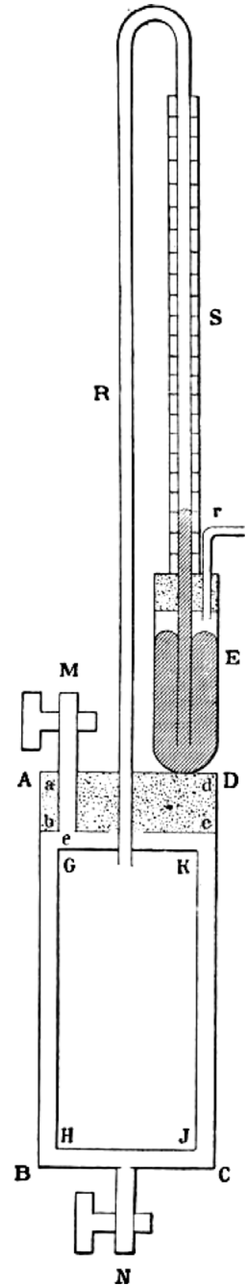

Fig. 3. This is a reproduction of the original schematic of the diathermometer, used by Josef Stefan to make the first accurate measurements of the thermal conductivity of gases. The candidate gas is introduced through valve M into gap e, between the concentric cylinders ABCD and GHJK. Keeping the gap small minimizes convection effects. Manometer E measures the pressure within cylinder GHJK. After ensuring that the system is in thermal equilibrium, the concentric cylinders are placed in a constant temperature bath, and the change in pressure versus time is measured. Using this data and Eq. (3), Stefan calculated the thermal conductivity of the gas between the cylinders. (Courtesy of the Austrian Academy of Sciences).

Using his diathermometer, Stefan calculated the thermal conductivity of air to be $k = 0.0234$ W/mK [2], which is

11% off today's accepted value of $k = 0.0263$ W/mK (at 300 K) [13]. Almost as importantly, his data also conclusively showed that the thermal conductivity did not vary with the pressure, verifying the predictions of Maxwell and Clausius. Remarkably, Stefan's value also compared very well (7%) to the theoretical value calculated using Maxwell's model. Later, Stefan [14] used his diathermometer to measure the thermal conductivities of hydrogen, nitrous oxide, methane, carbon monoxide and carbon dioxide with similar success. The diathermometer which is described by Stefan in his first thermal conductivity paper [12], while the diagram appeared in his second [14], is one of the very few figures that Stefan published in his 88 papers. Boltzmann described the diathermometer as "fantastically simple," and that it not only served to measure the thermal conductivity "with an exactness not previously thought possible," but "it proved to lend glowing support to every other prediction of kinetic theory' '[6].

5. The T^4 radiation law

In one of the wonderfully coincidental scientific events, where seemingly disparate threads are woven together in one cloth, Stefan's thermal conductivity measurement work helped him determine the law for which he is most recognized. Dulong and Petit [15] published in 1817 experimental results from what they considered to be purely radiation heat transfer between a spherical bulb and a spherical chamber. Both bare and silvered bulbs were tested and heated only up to about 573 K, while the chamber temperature was kept around 273 K. Various gases filled the gap between the two, and they measured the rate of change of temperature of the bulb over a range of pressures. Their model for the radiative power was,

$$E(T) = \mu a^T \tag{4}$$

where μ was a constant dependent on the material and size of the body, a was an empirical constant for all materials, $a = 1.0077$, and the temperature T, was given in degrees centigrade. By extrapolating the data to zero pressure, they thought that the effects due to conduction and convection had been eliminated. However, Stefan understood from his previous work that the thermal conductivity of gases was not a function of pressure. By lowering the pressure between the two spheres, Dulong and Petit managed to eliminate convection effects, but not those of conduction.

Stefan suspected the accuracy of their relation (Eq. (4)) and began to reformulate a model to better describe the data. He found that the difference of the temperatures raised to the fourth power matched the trends of Dulong and Petit's experimental values and gave good agreement [16]. His equations for the radiative power of the two bodies were,

$$E_1(T) = AT_1^4, \quad E_2(T) = AT_2^4 \tag{5}$$

where "A depends on the size and the surface the body," and importantly the temperature was given in absolute values.

By including a simple conduction term to his radiative model and making some straightforward assumptions based on the Dulong and Petit apparatus, Stefan estimated that 10–15% of the cooling for the bare bulb and up to 50% of the cooling of the silvered bulb was due to conduction. Using his T^4 model, Stefan found that he could get slightly better agreement with the experimental data than the Dulong and Petit model. The question was, how would it perform at higher temperatures?

It was widely known at the time that the cooling rate was much higher at higher temperatures, and Stefan was eager to test his model in that range. He came across the results from Tyndall [17] who reported heat transfer data for a platinum wire over a wide temperature range. As stated by Stefan [16], "From weak red heat (about 525 °C) to complete white heat (about 1200 °C) the intensity of radiation increases from 10.4 to 122, thus nearly 12-fold (more precisely 11.7). The ratio of the absolute temperature 273 + 1200 and 273 + 525 raised to the fourth power gives 11.6." (translation from Strnad [18]) This observation gave Stefan additional confidence in his T^4 model, especially at high temperatures. He then applied his T^4 model to the experimental results of Provostaye and Desains [19], Draper [20], and Ericsson [21] and found much better agreement using his model rather than the Dulong–Petit model, especially at higher temperatures.

After distilling the data from all of the sources, he concluded that for a body at 373 K and another at 273 K, the radiative power was 697.8 W/m^2, although he was not terribly confident in the result. He noted that this analysis had a "hypothetical nature and reasoned support for [it] was impossible, so long as measurements are not made of radiation to surroundings at absolute zero, or at least a very low temperature" (translation from Dougal [22]). Although Stefan himself never computed a value for the proportionality between the radiative power and the differences in the temperature to the fourth power, based on his deduced heat flux between the two bodies mentioned above, it can easily be determined to be 5.056×10^{-8} W/m^2 K^4.

In the last portion of his paper, Stefan used his model to determine the temperature of the Sun. He first reviewed the results of the experimenter Pouillet, who used the Dulong and Petit model to calculate the temperature of the Sun to be between 1734 and 2034 K, which was recognized to be low. Using the Pouillet data and his T^4 model, Stefan estimated the Sun's temperature to fall between 5859 and 10,420 K. Stefan then examined the experimental data of Soret, who measured the radiant energy of the Sun and estimated its temperature to be between 2446 and 2546 K. Stefan reevaluated Soret's data using his T^4 model and estimated a temperature range of 5580–5838 K. Both of his estimates, using his model and the experimental data of others gave the first accurate calculation of the temperature of the Sun and is in line with the currently accepted value of about 5770 K.

Despite the success Stefan had in modeling heat transfer due to radiation, there were errors which fortunately balanced out so that his model gave reasonable agreement with the data. First, there were mistakes in the experimental data reported by Tyndall, over which, of course, Stefan had no control. It has since been shown that for platinum, the ratio of temperatures for "radiation increases from 10.4 to 122" is 18.6, and not the 11.6 as given by Tyndall. In addition, platinum is not a perfect radiator, and the T^4 law only holds for a blackbody; for real materials, the emissivity must be considered. Regardless, Stefan was able to get good agreement using his T^4 model with a variety of independent data sets.

In his defense, Stefan provided some important insights. From his thermal conductivity measurements and studies of kinetic theory, he understood the importance of using absolute temperature units and not those based on an arbitrary scale to measure the radiation heat transfer. This was not well understood in his day. Without his intuition and experience of thermal behavior, the T^4 law could not have been deduced from experimental data. Also, Stefan was able to present a formula which was accurate over a wide temperature range, and was valid especially at high temperatures, where the models of the day had trouble getting good agreement.

Why a man of Stefan's technical ability did not simply replicate the experiments and check the data himself is not known. Although the Institute produced outstanding scientific results, it was not well-supported either financially or politically by the University, so there may not have been sufficient funds to carry out the work independently. Boltzmann complained about the poor quality of students in Vienna, indicating that perhaps there were not enough talented or motivated students to do the experiments.

Despite the success Stefan had modeling radiation heat transfer data, his results were not well accepted until five years later when Boltzmann, while a professor in Graz, derived independently an equation with the same temperature dependence using the radiation pressure of light [23,24],

$$E(T) = \sigma T^4 \tag{6}$$

The proportionality constant between the emissive power for a blackbody and T^4, familiar to us now as the Stefan–Boltzmann constant, is $5.67 \times 10^{-8}\ \mathrm{W/m^2\,K^4}$, about 11% higher than that estimated by Stefan, and Eq. (6) is known as the Stefan–Boltzmann law.

6. The moving boundary problem: freezing on the polar ice caps

After publishing his model of the T^4 radiation law, Stefan turned his attention to heat conduction and diffusion in fluids, as well as phase change problems, including evaporation. It is not known precisely why Stefan chose to study this class of problems, but certain events occurred that permit an educated guess. From 1872 to 1874, Karl Weyprecht helped lead the Austro-Hungarian Polar Expedition, and took many meteorological measurements. While their ships were stuck in ice floes during the winter, crew members regularly recorded the ice growth rate and air temperature. In 1876, Weyprecht and co-leader Julius von Payer reported their results to the Austrian Academy of Sciences. Since Stefan was a member of the Academy he surely was aware of their data [25].

A colleague of Stefan's at the University of Vienna, Julius von Hann, the director of the Institute of Meteorology and Geodynamics, alerted Stefan to ice growth and air temperature data taken by British and German explorers during earlier expeditions. These trips were financed by their respective governments, presumably to find the Northwest Passage, allowing easier access to the riches of the Far East [26]. These data provided an interesting challenge to Stefan. Instead of calculating the heat transfer across a fixed boundary, as he did in the measurement of the thermal conductivity of gases, modeling the growth rate of ice had to include a time-dependent or moving boundary.

Unbeknownst to Stefan, some work on the moving boundary problem had already been done. In 1762, Joseph Black, a professor of medicine at the University of Glasgow in Scotland, studied the ice–water phase change problem and identified the phenomenon of latent heat [27], while Franz Neumann presented solutions to the moving boundary problem in a series of lectures given around 1860. However, his work was not published until 1901 by Weber [28].

Stefan began his paper [29] by acknowledging the data taken during the British and German expeditions, then proceeded to outline his first solution to the moving boundary problem. He applied a simple conservation of energy model at the solid–liquid interface. As the liquid became solid, it liberated heat per unit area in the amount, $\lambda\rho\,\mathrm{d}h$. Assuming a linear temperature profile in the ice, Stefan used Fourier's law to calculate the amount of heat conducted away from the freezing front per unit area, $k\frac{\Delta T}{h(t)}\,\mathrm{d}t$, where ΔT was the temperature difference within the ice between the water–ice interface, which is at the fusion temperature, and the air–ice interface. In the first part of his analysis the air–ice interface temperature remained constant. Stefan's experience measuring the thermal conductivity using time-dependent methods served him well in formulating this model. The explorers reported the temperature data of the air, so ΔT was readily available. By equating the liberation of the latent heat with the amount conducted away then integrating, Stefan showed that the square of the ice thickness was a linear function of time,

$$h^2(t) = \frac{2}{\rho}\frac{k\Delta T}{\lambda}t \tag{7}$$

Stefan realized that this was an oversimplification of the problem, but nevertheless compared his model with the experimental data at hand and gave some error estimates, showing rough agreement between the ice thickness and the square root of time. One of the problems in achieving agreement between the theory and experimental results was obtaining an accurate value for the thermal conductivity of ice. Stefan used an average of three quoted values, which turned out to be about 20% smaller than what is currently accepted. Also, since Stefan himself did not take the measurements, it was impossible to control the experimental parameters or guarantee the accuracy of the results. Therefore he used the data at hand and made appropriate assumptions.

Knowing that a linear temperature profile in the solid phase gave insufficient agreement, Stefan then modeled the air temperature above the air–solid interface as a function of time and assumed this as the interface temperature. When solving for the ice thickness using this time varying temperature, Stefan found,

$$h^2(t) = \frac{2k}{\rho\lambda}\int_0^t \Delta T\,\mathrm{d}t \tag{8}$$

calling the integral term the *Kältesumme*, or the cold sum. Using this model, he found much better agreement with the data taken from both the British and German expeditions [26]. Although it is strongly presumed that Stefan was aware of the results of the Austro-Hungarian Polar Expedition, for unknown reasons he never mentioned their data in his paper.

Stefan then outlined the mathematical model of the moving boundary problem, now commonly referred to as the Stefan problem, by giving the heat diffusion equation,

$$\frac{\partial T(x,t)}{\partial t} = \frac{k}{\rho c_p}\frac{\partial^2 T(x,t)}{\partial x^2} \tag{9}$$

He defined the location of the ice–air interface as $x = 0$, and $x = h(t)$ the location of the ice–liquid interface. The temperature at the ice–liquid interface he denoted as zero and at the ice–air interface, $T = f(t)$, where f was a general temperature distribution. He then wrote the heat balance at the ice–liquid interface,

$$\rho\lambda\frac{\mathrm{d}h}{\mathrm{d}t} = -k\left.\frac{\partial T}{\partial x}\right|_{x=h} \tag{10}$$

Then, by expressing the temperature, which is a function of both space and time as a total differential,

$$\mathrm{d}T = \frac{\partial T}{\partial t} + \left.\frac{\partial T}{\partial x}\right|_{x=h}\frac{\mathrm{d}h}{\mathrm{d}t} = 0 \tag{11}$$

he was then able to eliminate the $\mathrm{d}h/\mathrm{d}t$ term between Eqs. (10) and (11) to get,

$$\frac{\partial T}{\partial t} = \frac{k}{\rho\lambda}\left(\left.\frac{\partial T}{\partial x}\right|_{x=h}\right)^2 \tag{12}$$

which yielded the solution,

$$T(x,t) = B\int_{\frac{x}{2\sqrt{\frac{k}{\rho c_p}t}}}^{\beta} \mathrm{e}^{-z^2}\,\mathrm{d}z \tag{13}$$

where B and β were integration constants.

By applying the boundary condition, $T(x = 0,t) = f(t) = \Delta T$, where ΔT was the constant temperature difference between the ice–liquid/ice–air interface, he obtained,

$$\Delta T = B\int_0^{\beta} \mathrm{e}^{-z^2}\,\mathrm{d}z \tag{14}$$

At the ice–liquid interface, $T(x = h(t),t) = 0$, he found from Eq. (13),

$$h(t) = 2\beta\sqrt{\frac{k}{\rho c_p}t} \tag{15}$$

To eliminate the constant B, Stefan then calculated the space and time derivatives of the temperature using Eq. (13),

$$\begin{aligned}\frac{\partial T}{\partial t} &= \frac{Bx}{4t\sqrt{(k/\rho c_p)t}}\exp\left[-\frac{x^2}{4(k/\rho c_p)t}\right]\\ \frac{\partial T}{\partial x} &= \frac{-B}{2\sqrt{(k/\rho c_p)t}}\exp\left[-\frac{x^2}{4(k/\rho c_p)t}\right]\end{aligned} \tag{16}$$

and by inserting these back into Eq. (12), obtained the transcendental equation,

$$\beta e^{\beta^2}\int_0^{\beta} \mathrm{e}^{-z^2}\,\mathrm{d}z = \frac{c_p\Delta T}{2\lambda} \tag{17}$$

In a first approximation to the solution of Eq. (17), he found,

$$h^2(t) = \frac{2k\Delta T}{\rho\lambda}t \tag{18}$$

which is exactly Eq. (7), calculated earlier using the linear temperature profile.

Finally, Stefan attempted to solve Eqs. (9) and (10) for general, time-dependent temperatures on the ice–air interface, $T = f(t)$, using series expansion techniques and obtained some approximate solutions, but no closed form, exact solution.

Because Stefan's journal of choice, the *Sitzungberichte der Kaiserlichen Akademie der Wissenschaften* of Vienna was not widely distributed and his results were considered important, his entire paper was reprinted in the *Annalen der Physik und Chemie* in 1891 [30], which had a higher circulation. For this reason dual references to this same work exist. This was one of the last papers that Stefan published, and it was not until the mid-1900s that the moving boundary problem began to be seriously studied by scientists, mathematicians and engineers.

7. Epilogue

In 1891, at the age of 56, Stefan ended his lifelong bachelorhood and married the widow Marija Neumann. About a year later, he suffered a stroke, and after decades of service to the University of Vienna and the scientific community, he died on January 7, 1893. He was buried in Vienna's famed *Zentralfriedhof* and although the location is known, there is no marker, in contrast to the famous headstone of his student Boltzmann. The University of Vienna recognized Stefan's outstanding contributions by erecting a memorial to him in its courtyard arcade (Fig. 4), alongside

Fig. 4. Memorial to Josef Stefan, located in the courtyard arcade of the University of Vienna (Courtesy of B. Jezovnik).

such luminaries as Doppler and Schrödinger [31]. Boltzmann delivered the dedicatory address, which is printed as a chapter in his *Populäre Schriften* [6]. Very little is known about Stefan's life, and most of what we do know comes from a small booklet [32] published just months after his death by one of his students, Albert von Obermayer, who rose to the rank of colonel in the Austrian army. Obermayer was also the driving force behind the creation of the arcade memorial. Recently, a doctoral dissertation about Stefan [33], appropriately enough submitted to the University of Vienna, has been written which describes his broad scientific contributions.

Upon Stefan's death, Boltzmann became the Director of the Institute of Physics. In addition to his theoretical solution of the radiation law, he helped develop kinetic theory and spent a good portion of his career and life defending it and its basis, the existence of atoms, against Ernst Mach and his followers who argued that atoms did not exist since no one had ever seen them [7]. Boltzmann committed suicide on September 5, 1906, never receiving full credit during his lifetime for the seminal contributions he made to science.

The thermal conductivity measurements of gases made by Stefan were steadily improved upon as experimental techniques became more refined. His experimental insight and ability to separate convection and radiation effects from the physical mechanism of conduction, as predicted by kinetic theory, provided the first accurate benchmark data. Today there are whole scientific fields devoted to measuring thermophysical properties. Reproductions of the diathermometer that Stefan invented can be used today as an excellent classroom or laboratory demonstration device [34].

Max Planck incorporated the radiation law developed by Stefan and Boltzmann to help begin quantum mechanics. It was most likely Planck who named the proportionality between the radiant energy and temperature to the fourth power the Stefan–Boltzmann constant. Dougal [22] provides an excellent technical analysis and summary along with excerpted translations of Stefan's landmark paper.

The moving boundary problem lay dormant for decades. Brillouin [35] in 1931 discussed the solution methods of both Neumann and Stefan, then expanded their work for various geometries and conditions. In the late 1940s, Soviet scientists Dacev and Rubenstein [36] directed serious efforts into solving the moving boundary problem, and it appears that they were the first to call this, "the Problem of Stefan." Later in the west, mathematicians began to delve into its rich and complex behavior, exploring existence and uniqueness solutions [37]. Carslaw and Jaeger [38] widely disseminated the Stefan problem and its solutions to a broader audience in their treatise on heat conduction. Since then, the Stefan problem has been a broad field of study. As late as 1966, Catchpole and Fulford [39] stated that the Stefan number was the ratio of radiation and conduction heat transfer, $\frac{\sigma L T^3}{k}$, although this usage has virtually disappeared. The commonly used definition, $Ste = \frac{c_p \Delta T}{\lambda}$, which is the sensible heat divided by the latent heat was coined by Lock in 1969 [40]. Vuik [26] gives an excellent in-depth analysis of Stefan's paper on the freezing of water on the polar ice caps. Unfortunately, it is the inverse problem that Stefan studied, melting of the polar ice caps, which garners popular attention today.

Both Austria and Slovenia claim Josef Stefan as their own. Austria issued a stamp on the 150th anniversary of his birth in 1985, and Slovenia did the same to commemorate the 100th anniversary of his death in 1993 [41]. Slovenians, justifiably proud of their famous scientific son, named their premier research institute in Ljubljana after Jožef Stefan. This humble, hard-working scientist made a permanent impact on the field of heat transfer. It is remarkable that a single figure, about whom so little is known, could make such important contributions to conduction, convection and radiation heat transfer.

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