# LOW FREQUENCY SECOND ORDER WAVE EXCITING FORCES ON FLOATING STRUCTURES

PROEFSCHRIFT

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*Aan Martha*
*en de kinderen*

Dit proefschrift is goedgekeurd
door de promotoren

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## I. INTRODUCTION

Stationary vessels floating or submerged in irregular waves are subjected to large, so-called first order, wave forces and moments which are linearly proportional to the wave height and contain the same frequencies as the waves. They are also subjected to small, so-called second order, mean and low frequency wave forces and moments which are proportional to the square of the wave height. The frequencies of the second order low frequency components are associated with the frequencies of wave groups occurring in irregular waves.

The first order wave forces and moments are the cause of the well known first order motions with wave frequencies. Due to the importance of the first order wave forces and motions they have been subject to investigation for several decades. As a result of these investigations, methods have evolved by means of which these may be predicted with a reasonable degree of accuracy for many different vessel shapes.

This study deals with the mean and low frequency second order wave forces acting on stationary vessels in regular and irregular waves in general and, in particular, with a method to predict these forces on basis of computations. Knowledge concerning the nature and magnitude of these forces is of importance due to the effect they have been shown to have on the general behaviour of stationary structures in irregular waves.

The components of mean and low frequency second order wave forces can affect different structures in different ways and although of the same origin have even been called by different names. The horizontal components of the mean and low frequency second order wave forces are also known as wave drift forces since, under the influence of these forces, a floating vessel will carry out a steady slow drift motion in the general direction of wave propagation if it is not restrained.

The importance of the mean and low frequency wave drift forces from the point of view of motion behaviour and mooring loads on vessels moored at sea has been recognized only within the last few years. Verhagen and Van Sluijs [I-1], Hsu and Blenkarn [I-2] and Remery and Hermans [I-3] showed that the low frequency components of the wave drift forces in irregular waves could, even though relatively small in magnitude, excite large amplitude low frequency horizontal motions in moored vessels. It was shown that in irregular waves the drift forces contain components with frequencies coinciding with the natural frequencies of the horizontal motions of moored vessels. Combined with the fact that the damping of low frequency horizontal motions of moored structures is generally very low, this leads to large amplitude resonant behaviour of the motions. See Figure I-1.

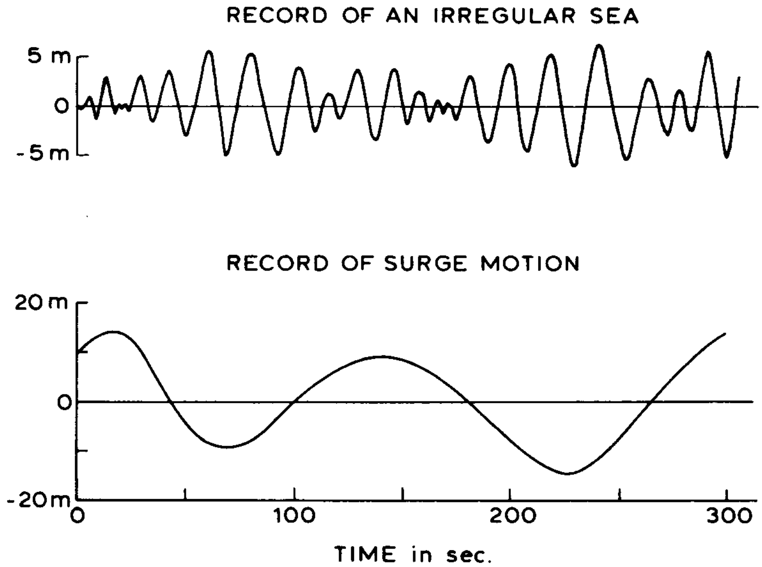



Fig. I-1 Low frequency surge motions of a moored LNG carrier in irregular head seas.

Remery and Hermans [I-3] established that the low frequency components in the drift forces are associated with the frequencies of groups of waves present in an irregular wave train. See Figure I-2.

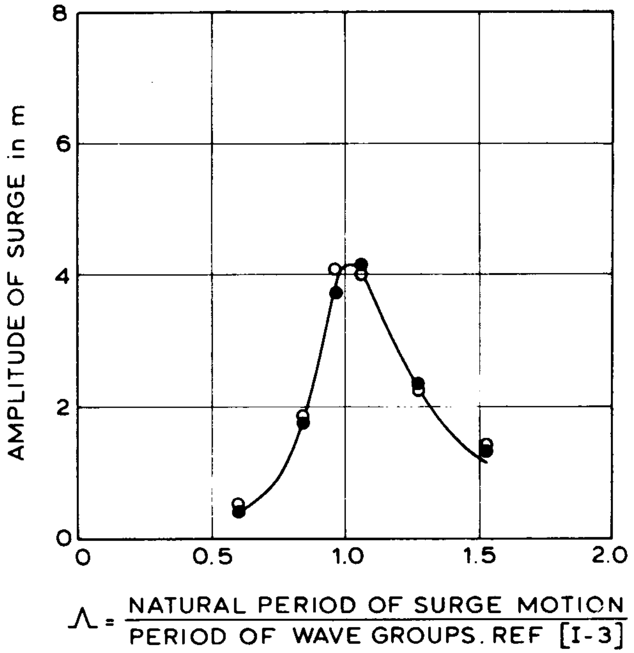



Fig. I-2 Surge motions of a moored barge in regular head wave groups. Ref. [I-3].

Dynamically positioned vessels such as drill ships which remain in a prescribed position in the horizontal plane through the controlled use of thrust generated by propulsion units are also influenced by mean and low frequency wave drift forces. The power to be installed in these vessels is dependent on the magnitude of these forces. The frequency response characteristics of the control systems must be chosen so that little or no power is expended to compensate the large oscillatory motions with wave frequencies, while the mean and low frequency horizontal motions caused by the mean and low frequency drift forces should be reduced to values commensurate with the task of the vessel. This has led to the development of sophisticated control systems. See Figure I-3.

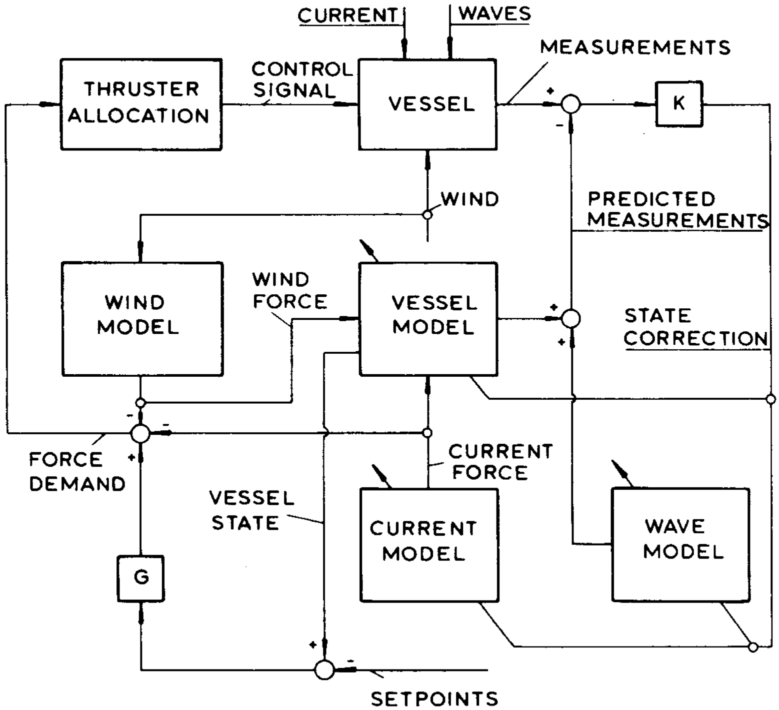



Fig. I-3 Block diagram for typical dynamic positioning system.

The vertical components of the second order forces are sometimes known as suction forces. This term is generally applied in connection with the mean wave induced vertical force and pitching moment acting on submarine vehicles when hovering or travelling near the free surface. It is shown by Bhattacharyya [I-4] that in extreme cases the upward acting suction force due to waves can cause a submarine vehicle to rise and broach the surface, thus posing a problem concerning the control of the vehicle in the vertical plane. See Figure I-4.

The vertical components of the second order wave forces have also been connected with the phenomena of the steady tilt of semi-submersibles with low initial static stability as indicated by Kuo et al [I-5]. Depending on the frequency of the waves it has been found that the difference in the suction forces on the floaters of a semi-submersible can result in a tilting moment, which can cause the platform to tilt towards or away from the oncoming waves. Such effects are of importance in judging the minimum static stability requirements for such platforms. From observations in reality and from the results of model tests it has been found that large, deep floating storage vessels can carry out low frequency heave motions in irregular waves which are of the same magnitude as the heave motions with wave frequencies.

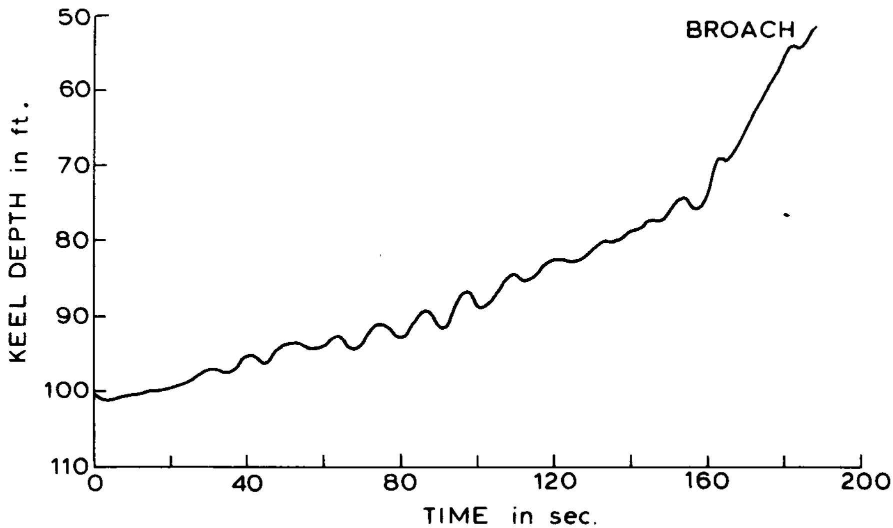



Fig. I-4 Depth record showing effect of suction force on submarine under waves. Quartering sea, wave height = 18 ft., vessel speed = 0 knot. Ref. [I-4].

From the foregoing it can be seen that, depending on the kind of structure or vessel considered, one or more of the six components of the mean and low frequency second order wave forces in irregular waves can be of importance. In order to be able to evaluate the influence of such forces on the performance or behaviour of a structure the most reliable method available, which can take into account in a relatively straightforward way those factors which are deemed of importance for the behaviour of a system, is by means of model tests. In many practical cases sufficient insight in the complex behaviour of, for instance, a large tanker moored to a single point mooring system is still lacking for reliable prediction of the motion behaviour and forces in the mooring system to be made by means other than physical model testing.

Simulation techniques based on numerical computations are becoming of increasing importance in the design phase of many floating structures however. For instance, in order to evaluate the effectiveness of control systems for dynamically positioned vessels, time domain simulations, which take into account the equations of motion of the vessel and the behaviour of external loads such as the mean and low frequency wave drift forces, are carried out. In such cases, due to the complexity of the control system and the objectives of the study, it is more practical to make use of simplified equations describing the environmental forces and the reaction of the structure or vessel to external forces than to simulate the characteristics of the control systems during a model test. See for instance Sjouke and Lagers [I-6], Sugiura et al [I-7] and Tamehiro et al [I-8]. For such simulation studies accurate numerical data on the behaviour of the mean and low frequency wave forces are desirable, so that meaningful results can be given regarding the systems under investigation. See for instance Van Oortmerssen [I-9] and Arai et al [I-10]. In order to produce numerical results, however, a theory must be available on which calculations can be based. In this study such a theory is developed based on potential theory. The final expressions are valid for all six degrees of freedom and are obtained through direct integration of the fluid pressures acting on the instantaneous wetted surface of the body. The final expressions are evaluated using an existing computer program based on three-dimensional linear potential theory. Numerical results are compared with analytical results obtained for a simple shaped body using a different theory. Experimental results for different, more practical shapes of vessels and structures are compared with results of computations. It is shown that the expressions obtained for the mean and low frequency second order wave forces can be used to gain more insight in the mechanism by which waves and structure interact to produce the forces. It is also shown that the insight gained using the method of direct integration can be used to enhance the positioning accuracy of dynamically positioned vessels in irregular waves. This is effected through the use of a wave-feed-forward control signal based on the instantaneous relative wave height measured around the vessel.

# II. PAST DEVELOPMENTS CONCERNING THE COMPUTATION OF MEAN AND LOW FREQUENCY WAVE FORCES

## II.1. Introduction

In this section, in which a review is given of developments in the past concerning theories which may be used to predict the second order wave forces, theories concerning the prediction of the added resistance of ships travelling in waves will also be taken into account, since the physical aspects are the same in both cases. In fact the added resistance is simply the longitudinal component of the mean second order wave forces for the case of non-zero forward speed. Indeed, initially emphasis was placed on obtaining good estimates of the added resistance in waves of vessels with forward speed. Only in recent years, due to the enormous increase in the number of vessels being moored at sea, have theories been developed which did not have to take into account the effect of forward speed which is of great importance for the added resistance. Most of the work carried out in the past has been concerned solely with the mean second order wave forces on a vessel or structure travelling or stationary in regular waves.

Maruo [II-1] and Gerritsma [II-2] show that on basis of this information the mean component of the second order wave force can be determined in irregular waves. As shown by Dalzell [II-3] the low frequency component of the second order wave forces on bodies in irregular waves can, strictly, only be determined from knowledge of the low frequency excitation in regular wave groups consisting of combinations of two regular waves with different frequencies. The low frequency wave force will then have the frequency corresponding to the difference frequency of the component regular waves. As will be seen in this section only in recent times have attempts been made to determine these components of the second order forces.

## II.2. Historical review

The existence of non-zero mean components in the total wave force acting on a floating vessel was first noted by Suyehiro

[II-4] who, from experiments, found that a vessel rolling in regular beam waves was subjected to a mean sway force. Suyehiro contributed this force to the capability of the vessel to reflect part of the incoming wave.

Watanabe [II-5] gave an expression for the mean sway force in regular waves based on the product of the first order roll motion and the Froude-Kryloff component of the roll moment, which indicated that the phenomenon involved was of second order. Results of Watanabe's calculations accounted for about half of the mean forces measured by Suyehiro.

Havelock [II-6] gave a similar second order expression for the mean longitudinal component of the second order wave force or added resistance on vessels in head seas involving the Froude-Kryloff parts of the heave force and pitching moment and the heave and pitch motions. This expression was used to estimate the increase in resistance experienced by a vessel travelling into head waves. The results obtained using Havelock's expression generally overestimate the added resistance at pitch resonance and underestimate the added resistance in the range of short wave lengths, where diffraction effects become more important. Watanabe's and Havelock's expressions for the mean second order wave forces in regular waves neglected diffraction effects.

Maruo [II-7] presented expressions for the longitudinal and transverse components of the mean horizontal second order wave force on stationary vessels in regular waves. The theory is valid for two and three-dimensions and is exact to second order within potential theory. It is based on the application of the laws of conservation of momentum and energy to the body of fluid surrounding the vessel. The final expressions derived are evaluated based on knowledge of the behaviour of the potential describing the fluid motions at great distance from the body. Numerical results given by Maruo are, however, limited and do not give satisfactory verification of the applicability of the theory since no correlation is given with experimental results.

Kudou [II-8] has given analytical results on the mean horizontal wave force on a floating sphere in regular waves using Maruo's [II-7] theory and shows reasonable correlation between computed and measured data.

Newman [II-9] rederived Maruo's three-dimensional expressions for the horizontal force components and extended the theory by including an expression for the mean yaw moment. The expressions were evaluated using slender body assumptions and results of computations compared with experimental results given by Spens and Lalangas [II-10]. Through lack of sufficient experimental data no final conclusions could be drawn regarding the validity of the theory.

Faltinsen and Michelsen [II-11] modified Newman's expression and evaluated their result by using a computer program based on three-dimensional potential theory using a distribution of singularities over the surface of the body. Results of computations compared with experimental results of the mean horizontal force on a box shaped barge in regular waves showed good agreement.

Recently Molin [II-12] modified Maruo's expression for the horizontal force and evaluated it using a numerical fluid finite elements method of computing the potential describing the fluid motion. The modification to the original formulation lies in the change of the surface of integration. Molin used the mean surface of the vessel while Maruo applied asymptotic expansions valid at great distance from the vessel. Molin's results compare well with experimental results on the mean longitudinal and transverse force and yawing moment on a stationary tanker in head, beam and bow quartering regular waves.

Kim and Chou [II-13] have made use of Maruo's [II-7] expression for the two-dimensional case of a vessel in beam seas to derive the mean sway force on stationary vessels in oblique waves. Comparisons made by Faltinsen and Løken [II-14] with results obtained by other methods and from experimental results with the method of Kim and Chou indicate that the method can show large deviations.

Joosen [II-15] has determined, by application of slender body theory, the added resistance of ships using Maruo's [II-7] expression. The final result is similar to that found by Havelock [II-6]. In Joosen's case the added resistance is independent of speed.

Lee and Newman [II-16] have given expressions to determine the mean vertical force and pitching moment acting on deeply submerged slender cylinders. The method is based on momentum considerations. No computed results are given.

Karppinen [II-17] has developed a method to determine the mean second order wave force and moment on semi-submersible structures based on three-dimensional potential theory. Karppinen assumes that the structure may be subdivided into slender elements which do not interact. The total mean forces and moments are found by summation of the contributions of the elements. The mean force on each element is determined from momentum considerations in a manner similar to that given by Lee and Newman [II-16]. Karppinen gives computed results for a semi-submersible. No comparisons are made with experimental results. Mean forces on simple elements are compared with results obtained by others.

Lin and Reed [II-18] have presented a method, based on momentum consideration and through the use of an asymptotic form of the Green's function valid at a large distance, for the mean horizontal second order force and yaw moment on ships travelling at a constant speed in oblique regular waves. No results of computations are given.

An approximative theory for the added resistance in regular waves is given by Gerritsma and Beukelman [II-19]. In this method the mean force is derived by equating the energy radiated by the oscillating vessel to work done by the incoming waves. The expression obtained has been applied to the case of ships travelling in head seas and the correlation between the computed and measured added resistance is good. Strip theory methods are used to evaluate the final expression. No experimental data are given for the case of a stationary vessel.

Kaplan and Sargent [II-20] have proposed to use the approach of Gerritsma and Beukelman to the case of oblique seas. No comparisons with experimental data are given by these authors.

Ogilvie [II-21] developed expressions based on two-dimensional potential theory for the mean second order vertical and horizontal wave forces on submerged circular cylinders fixed, free floating or with forced motions in regular beam waves. The problem is solved analytically and the results are exact within potential theory. No assumptions are made regarding the slenderness of the cylinders. No comparisons are given with experimental results.

Goodman [II-22] has determined, by direct integration of pressure acting on the hull, the mean vertical force acting on a submerged cylinder in regular beam and head waves for wave lengths in the order of the diameter of the cylinder. No comparisons are given between computed and experimental results.

Salvesen [II-23] has derived expressions for the total mean and low frequency second order wave force and moment on floating structures which is three-dimensional and exact to second order within potential theory. The expressions were derived through integration of pressure over the hull surface. The final results, however, make use of the asymptotic behaviour of the velocity potentials at great distance from the body. The theory was applied to the case of stationary vessels and to vessels with forward speed in regular waves. In order to finally evaluate the expressions slender body assumptions were applied. Comparisons made by Faltinsen and Løken [II-14] with other theories show that the slender body assumptions can scarcely be applied in many practical cases.

Dalzell and Kim [II-24] have computed the mean and low frequency components of the second order forces on a vessel using Salvesen's [II-23] equation for the mean force in regular waves. Comparisons are given between computed and measured data which show reasonable qualitative agreement.

Ankudinov [II-25], [II-26] gives expressions for the mean second order force and moment on stationary ships and the added resistance of ships travelling in regular waves in deep or shallow water. The theory is exact and based on integration of pressure on the body's wetted surface. The expressions of Havelock, Maruo and Newman are derived as particular cases of Ankudinov's final expressions. No numerical results are given on the mean force on stationary vessels. The added resistance of ships travelling in waves is computed using strip theory methods and compared with experimental data for deep and shallow water. The results compare reasonably well.

Based on direct integration of pressure, Boese [II-27] approximated the added resistance of ships in regular waves from the relative wave height and the product of heave force and pitch motion. The final expressions were evaluated by strip theory methods. Results of computations agree reasonably with experimental data.

Pinkster [II-28] gave an expression based on direct integration of pressure for the mean and low frequency second order horizontal wave force on a vessel in irregular waves. This expression included the components used by Boese [II-27]. Using strip theory methods, only the same components could be evaluated. Results of computations of the mean and low frequency surge motions of a vessel moored in irregular head seas were compared with experimental results and showed reasonable agreement.

Pinkster and Van Oortmerssen [II-29] presented results of computations of the mean longitudinal and transverse force and yaw moment on a stationary free floating rectangular barge in regular waves based on the method of direct integration of pressures. Evaluation of the complete expressions given, which are exact within potential theory, requires accurate and detailed knowledge of the flow around the hull. This was determined using a numerical three-dimensional sink and source technique utilizing Green's functions. See Boreel [II-30]. The results of computations were compared with experimental results and good correlation was found. Computed results of the mean vertical force are also shown. No comparisons are given with experimental data for this component.

Faltinsen and Løken [II-31] presented a two-dimensional method based on potential theory to compute the mean and low frequency components of the second order transverse force on cylinders floating in beam seas. The method takes into account the force contribution arising from the second order non-linear velocity potential as well as the usual components arising from products of the first order quantities. The expressions obtained are exact within potential theory and results of computation of the mean and low frequency transverse force on a number of cylinders with different forms and breadth to draft ratios are presented by Faltinsen and Løken in [II-32]. No comparisons were given with experimental results.

Pinkster and Hooft [II-33] and Pinkster [II-34], [II-35] extended the method of direct integration to include the low frequency components of the second order wave forces on stationary free floating bodies in regular wave groups. The contribution arising from the second order, non-linear, potential is included using an approximation based on the transformation of a first order wave exciting force. The approximation for this component is compared with two-dimensional exact results given by Faltinsen and Løken for the case of a floating cylinder in beam waves. Pinkster [II-34] compared results of computations of the mean longitudinal wave force in regular head waves on a semi-submersible with experimental results. The comparison indicates that potential effects rather than viscous effects dominate in the second order force on semi-submersibles.

Pinkster [II-35] computed by the method of direct integration the low frequency component of the second order longitudinal force on a semi-submersible in head waves and compared the results with experimental results obtained from tests in irregular head waves using cross-bi-spectral analysis techniques as developed by Dalzell [II-3]. The agreement was reasonable.

Bourianoff and Penumalli [II-36] determine the total hydrodynamic force including the first order force and the second order mean and low frequency forces by means of time domain solution of the Euler hydrodynamic equation coupled with the rigid body equation of motion for the ship. The method allows non-linear treatment of ship-wave interaction and arbitrary two-dimensional geometry. Furthermore ship motions are calculated in regular or irregular waves and the effect of arbitrary mooring forces can be included. Results of computations are compared with experimental results regarding the low frequency motions of a vessel in irregular beam waves. The correlation is reasonable but computation time exceeds real time by a factor of about four.

Pijfers and Brink [II-37] developed expressions by means of which the mean horizontal wave force on semi-submersible structures consisting of slender elements could be determined. The method is based on the use of Morison's equation and the relative motion concept to determine the wave loads on the structural elements. Results of computations indicate that the viscous drag plays an important role in the mean force. In regular waves the mean force as determined by Pijfers and Brink is not a quadratic function of the wave height. No comparisons with results of experiments are given. Previously Wahab [II-38] presented a similar method to that of Pijfers and Brink. The results of computations were compared with limited data from experiments. However, no general conclusions could be drawn.

Huse [II-39] has given an expression for the mean horizontal force on semi-submersibles from which a qualitative indication is drawn regarding the influence of viscous effects. Comparisons are made with experimental results for two semi-submersibles. For the computation of the mean force the restriction of long waves relative to the platform dimensions is imposed.

## II.3. Conclusions

Resuming the foregoing it can be seen that the theories, developed in the past, may be grouped in four main categories:

1. Potential theories which deduce the mean second order forces based on momentum and energy considerations applied to the body of fluid surrounding the vessel. The change in momentum (or moment of momentum) of the fluid is equated to the mean force (or moment) acting on the vessel. These theories generally make use of knowledge of the far-field behaviour of the potentials describing the fluid motions. Theories in this category are due to:

   - Maruo [II-7]
   - Newman [II-9]
   - Faltinsen and Michelsen [II-11]
   - Molin [II-12]
   - Kim and Chou [II-13]
   - Joosen [II-15]
   - Lee and Newman [II-16]
   - Karppinen [II-17]
   - Lin and Reed [II-18]

   The theory of Maruo, Newman, Faltinsen and Michelsen and Molin are three-dimensional and exact to second order within potential theory. Their basic expressions do not impose restrictions on the hull form. Other methods in this category make use of slender body assumption. The theory of Lin and Reed includes the effect of forward speed.

2. Potential theories which deduce the mean and in some cases also the low frequency second order forces and moments through direct integration of the fluid pressure acting on the wetted part of the hull. In a number of these cases the final expressions are, by application of Gauss's theorem, transformed to equivalent expressions which have to be evaluated on a fictitious boundary at great distance from the vessel, thus making use of the asymptotic or far-field behaviour of the potential describing the flow. Theories in this category are due to:

   - Watanabe [II-5]
   - Havelock [II-6]
   - Ogilvie [II-21]
   - Goodman [II-22]
   - Salvesen [II-23]

- Dalzell and Kim [II-24]
- Ankudinov [II-25], [II-26]
- Boese [II-27]
- Pinkster [II-28], [II-34], [II-35]
- Faltinsen and Løken [II-31]
- Bourianoff and Penumalli [II-36]

Of the theories for the mean second order forces, those due to Ogilvie and Faltinsen and Løken are two-dimensional and exact to second order. The theories of Salvesen, Ankudinov and Pinkster are three-dimensional and exact to second order. The theories which have been used to determine the low frequency part of the second order forces are those due to Dalzell and Kim, Faltinsen and Løken, Pinkster and Bourianoff and Penumalli. The theory of Dalzell and Kim makes use of slender body assumption and is approximative. The theory of Faltinsen and Løken is two-dimensional and exact to second order in basic formulation and in the results obtained. The theory of Pinkster is three-dimensional and exact to second order in basic formulation and, for the greater part, in the results obtained. The theory due to Bourianoff appears to be fully non-linear in basic formulation and in the results obtained. This theory is, of all theories discussed here, the only one solved in time domain.

3. Potential theories which deduce the mean second order forces by equating the damping energy radiated by the oscillating vessel to work done by the incoming waves. These theories are approximative and in all cases make use of slender body assumption. Theories in this category are due to:

   - Gerritsma and Beukelman [II-19]
   - Kaplan and Sargent [II-20]

4. Approximative theories which make use of Morison's equation and the relative motion concept. These methods apply typically to semi-submersible structures which are assumed to consist of slender elements. These theories are due to:

   - Wahab [II-38]
   - Pijfers and Brink [II-37]
   - Huse [II-39]

In the following chapter the hydrodynamic theory for the general three-dimensional case of a body floating in arbitrary wave conditions will be treated. Expressions will be derived for the mean and low frequency second order wave forces for six degrees of freedom based on the method of direct integration of pressure over the wetted hull.

From the review of work already published in this field it would appear that similar derivations may have been given by other authors. This is, however, not the case. With respect to the method of direct integration of pressure, partial results of the same nature as given in chapter III have been given by Ogilvie [II-21] and Boese [II-27]. In neither case has the general hydrodynamic theory been discussed or have the complete and general expressions for the mean and low frequency second order forces been derived.

# III. HYDRODYNAMIC THEORY

## III.1. Introduction

In this section the hydrodynamic theory which forms the basis for computations of the mean and low frequency second order wave drift forces on floating or submerged objects will be treated. The theory is developed based on the assumption that the fluid surrounding the body is inviscid, irrotational, homogeneous and incompressible. The fluid motions may then be described by a velocity potential $\Phi$ from which the velocity field can be derived by taking the gradient:

$$\overline{V} = \overline{\nabla}.\Phi \quad \ldots\ldots \quad \text{(III-1)}$$

with: $$\Phi = \Phi(\overline{X},t) \quad \ldots\ldots \quad \text{(III-2)}$$

in which $\overline{X}$, t are respectively the position vector relative to a fixed system of rectangular co-ordinate axes and time.

For an arbitrary case the motions of the body and the potential $\Phi$ are unknown quantities which have to be determined taking into account certain boundary conditions applicable to the flow and the equations of motion of the body. In accordance with classical hydrodynamic theory - see for instance Stoker [III-1] - it will be assumed that the velocity potential $\Phi$ of the flow and all quantities derivable from the flow, such as the fluid velocity, wave height, pressure, hydrodynamic forces and the motions of the object, may be expanded in a convergent power series with respect to a small parameter $\varepsilon$, for instance:

- the potential:

$$\Phi = \varepsilon\Phi^{(1)} + \varepsilon^2\Phi^{(2)} + O(\varepsilon^3) \text{ ----- } \varepsilon<<1 \quad \ldots\ldots \quad \text{(III-3)}$$

- the wave elevation:

$$\zeta = \zeta^{(0)} + \varepsilon\zeta^{(1)} + \varepsilon^2\zeta^{(2)} + O(\varepsilon^3) \quad \ldots\ldots \quad \text{(III-4)}$$

- the motion of the object:

$$\overline{X} = \overline{X}^{(0)} + \varepsilon\overline{X}^{(1)} + \varepsilon^2\overline{X}^{(2)} + O(\varepsilon^3) \quad \text{(III-5)}$$

where the affix $^{(0)}$ denotes the static value, $^{(1)}$ indicates first order variations and $^{(2)}$ the second order variations, etc.

In waves the first order quantities are oscillatory quantities with wave frequencies. In the most general case second order quantities, besides containing low frequency components, also contain high frequency components with a frequency in the order of twice the wave frequencies. For some problems, for instance hull vibrations, the high frequency components of the second order wave forces may be of interest. In that case the exciting forces can be obtained by taking the high frequency components of the second order forces. Force and motion components of this type are, however, of no consequence for the problem at hand and will therefore be left out of consideration in this study.

It will be understood hereafter that first order quantities are oscillatory with wave frequencies, while second order quantities are restricted to low frequencies with frequencies lower than the wave frequencies.

In the following quantities are of second order if preceded by $\varepsilon^2$. If, as in many cases, the $\varepsilon$ or $\varepsilon^2$ are discarded this will be due to the fact that the expression involved will contain only first or second order quantities. In such instances first order quantities will be recognizable by the affix $^{(1)}$ and second order quantities by the affix $^{(2)}$ or by the fact that a component is the product of first order quantities with affix $^{(1)}$. For instance, the pressure component:

$$-\tfrac{1}{2}\rho|\overline{\nabla}\Phi^{(1)}|^2 \quad \text{(III-6)}$$

is recognized as a second order quantity.

For the derivation of the second order wave forces on an object in waves it is sufficient that the expansions in a power series with respect to a small parameter $\varepsilon$ only be carried out up to and including the second order. In order to develop a consistent theory it is assumed that the low frequency motions induced by the low frequency second order forces are of second order and small in relation to the first order motions. From measurements of low frequency motions of moored vessels it is known that in the range of practical wave heights this assumption is in some cases strictly speaking incorrect. This is due to the large dynamic magnification of the motions resulting from small damping for low frequency motions. The theory, however, is based on the assumption of infinitesimal wave height ($\varepsilon \rightarrow 0$) in which case, in spite of low system damping, the low frequency motions induced by the low frequency second order wave drift forces are always small in relation to the first order motions.

In this chapter the hydrodynamic boundary problem for the potential $\Phi$ will be formulated to first and second order. If the potential $\Phi$ is known the pressure in a point in the fluid may be determined using Bernoulli's equation:

$$p = p_0 - \rho g X_3 - \rho \Phi_t - \tfrac{1}{2}\rho|\overline{\nabla}\Phi|^2 + C(t) \quad \ldots\ldots \quad \text{(III-7)}$$

where:

$p_0$ = atmospheric pressure

$X_3$ = vertical distance of the point below the mean water surface

$C(t)$ = a function independent of the co-ordinates

$t$ = time

$\rho$ = mass density of the fluid

$g$ = gravity constant.

The fluid forces acting on the body are determined by the method of direct integration using the following basic equation for the forces:

$$\overline{F} = -\iint_S p.\overline{N}.dS \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad \text{(III-8)}$$

and for the moments:

$$\overline{M} = -\iint_S p.(\overline{X} \times \overline{N}).dS \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad \text{(III-9)}$$

in which:

p = fluid pressure

S = total wetted surface of the body

dS = a surface element

$\overline{N}$ = outward pointing normal vector of dS

$\overline{X}$ = co-ordinates of dS

The numerical method used to finally evaluate the fluid forces and moments will be discussed in chapter IV.

## III.2. Co-ordinate systems

Use is made of three systems of co-ordinate axes (see Figure III-1). The first is a right-handed system of $G-x_1-x_2-x_3$ body axes with as origin the centre of gravity G and with positive $G-x_3$ axis vertically upwards in the mean position of the oscillating vessel. The surface of the hull is uniquely defined relative to this system of axes. A point on the surface has as position the vector $\overline{x}$. The orientation of a surface element in this system of axes is defined by the outward pointing normal vector $\overline{n}$.

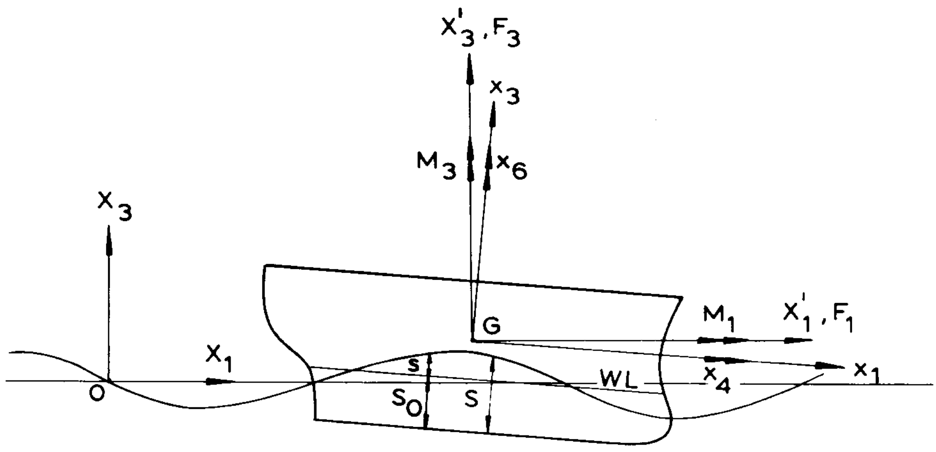



Fig. III-1 Systems of co-ordinates.

The second system of co-ordinate axes is a fixed $O\text{-}X_1\text{-}X_2\text{-}X_3$ system with axes parallel to the $G\text{-}x_1\text{-}x_2\text{-}x_3$ system of axes with the body in the mean position and origin O in the mean free surface.

The third system of co-ordinate axes is a $G\text{-}X_1'\text{-}X_2'\text{-}X_3'$ system of axes with origin in the centre of gravity G of the body and axes which are at all times parallel to the axes of the fixed $O\text{-}X_1\text{-}X_2\text{-}X_3$ system.

The angular motions of the body about the body axes are denoted by the Eulerian angles $x_4$, $x_5$ and $x_6$.

III.3. Motion and velocity of a point on the hull of the body

If the body is carrying out small amplitude motions in six degrees of freedom under the influence of oscillatory first order and low frequency second order wave forces the position vector of a point on the hull of the body relative to the fixed system of $O\text{-}X_1\text{-}X_2\text{-}X_3$ axes is:

$$\bar{X} = \bar{X}^{(0)} + \varepsilon\bar{X}^{(1)} + \varepsilon^2\bar{X}^{(2)} \quad \text{(III-10)}$$

where $\bar{X}^{(0)}$ denotes the mean position vector with:

$$\bar{X}^{(0)} = \bar{X}_g^{(0)} + \bar{x} \quad \text{(III-11)}$$

and $\bar{X}^{(1)}$ denotes the first order oscillatory motion with:

$$\bar{X}^{(1)} = \bar{X}_g^{(1)} + \bar{\alpha}^{(1)} \times \bar{x} \quad \text{(III-12)}$$

where $\bar{\alpha}^{(1)}$ is the oscillatory first order angular motion vector with components $x_4^{(1)}$, $x_5^{(1)}$ and $x_6^{(1)}$ respectively and $\bar{X}_g^{(1)}$ is the oscillatory first order motion vector of the centre of gravity of the body. Similarly the second order low frequency motion is:

$$\bar{X}^{(2)} = \bar{X}_g^{(2)} + \bar{\alpha}^{(2)} \times \bar{x} \quad \text{(III-13)}$$

where $\bar{\alpha}^{(2)}$ is the low frequency second order angular motion vector with components $x_4^{(2)}$, $x_5^{(2)}$ and $x_6^{(2)}$ respectively and $\bar{X}_g^{(2)}$ is the low frequency second order motion vector of the centre of gravity. The velocity $\bar{V}$ is:

$$\bar{V} = \dot{\bar{X}} = \varepsilon\dot{\bar{X}}^{(1)} + \varepsilon^2\dot{\bar{X}}^{(2)} \quad \text{(III-14)}$$

where: $\dot{\bar{X}}^{(1)} = \bar{V}^{(1)} = \dot{\bar{X}}_g^{(1)} + \dot{\bar{\alpha}}^{(1)} \times \bar{x}$ ........... (III-15)

and : $\dot{\bar{X}}^{(2)} = \bar{V}^{(2)} = \dot{\bar{X}}_g^{(2)} + \dot{\bar{\alpha}}^{(2)} \times \bar{x}$ ........... (III-16)

in which the components of the angular velocity vectors $\dot{\bar{\alpha}}^{(1)}$ and $\dot{\bar{\alpha}}^{(2)}$ are $\dot{x}_4^{(1)}$, $\dot{x}_5^{(1)}$, $\dot{x}_6^{(1)}$ and $\dot{x}_4^{(2)}$, $\dot{x}_5^{(2)}$ and $\dot{x}_6^{(2)}$ respectively.

The orientation of surface elements of the hull of the body relative to the body axes $G-x_1-x_2-x_3$ are denoted by the outward pointing normal vector $\bar{n}$. Relative to the fixed system of co-ordinate axes $O-X_1-X_2-X_3$ and the $G-X_1'-X_2'-X_3'$ axes the normal vector of a surface element becomes:

$$\bar{N} = \bar{N}^{(0)} + \varepsilon\bar{N}^{(1)} + \varepsilon^2\bar{N}^{(2)} \quad \text{(III-17)}$$

where it is found that:

$$\bar{N}^{(0)} = \bar{n} \quad \text{(III-18)}$$

$$\bar{N}^{(1)} = \bar{\alpha}^{(1)} \times \bar{n} \quad \text{(III-19)}$$

$$\bar{N}^{(2)} = \bar{\alpha}^{(2)} \times \bar{n} \quad \text{(III-20)}$$

## III.4. Fluid motions and boundary conditions

### III.4.1. Boundary conditions within the fluid, at the free surface and on the sea floor

The fluid domain is bounded by the free surface, the surface of the body and the sea floor. Assuming that the fluid is inviscid, irrotational, homogeneous and incompressible the fluid motion may be described by means of the velocity potential $\Phi$:

$$\Phi = \varepsilon\Phi^{(1)} + \varepsilon^2\Phi^{(2)} \quad \ldots\ldots\ldots\ldots \quad \text{(III-21)}$$

The potentials are defined relative to the fixed system of O-$X_1$-$X_2$-$X_3$ axes with:

$$\Phi = \Phi(\overline{X},t) \quad \ldots\ldots\ldots\ldots \quad \text{(III-22)}$$

where t denotes time and $\overline{X}$ the position vector of the point under consideration.

The potential $\Phi$ must comply with the following boundary conditions:

- Everywhere within the fluid domain the equation of continuity must be satisfied or:

  $$\nabla^2\Phi = 0 \quad \ldots\ldots\ldots\ldots \quad \text{(III-23)}$$

  In order to satisfy this requirement to first and second order it follows that:

  $$\nabla^2\Phi^{(1)} = 0 \quad \ldots\ldots\ldots\ldots \quad \text{(III-24)}$$

  $$\nabla^2\Phi^{(2)} = 0 \quad \ldots\ldots\ldots\ldots \quad \text{(III-25)}$$

- The boundary conditions at the free surface. The (unknown) free surface is a surface of constant pressure and the velocity component of the fluid normal to the free surface is equal to the velocity of the surface in the same direction. The latter statement implies that no fluid particles pass through the free surface. The boundary conditions on the moving free surface may be expressed as boundary conditions, which must be satisfied on the mean, fixed free surface. According to Stoker [III-1] the boundary condition is satisfied to first order if:

  $$g\Phi^{(1)}_{X_3} + \Phi^{(1)}_{tt} = 0 \qquad \text{on } X_3 = 0 \quad \ldots\ldots\ldots \quad \text{(III-26)}$$

  The boundary condition is satisfied to second order if:

$$g\Phi^{(2)}_{X_3} + \Phi^{(2)}_{tt} = -2\overline{\nabla}\Phi^{(1)}.\overline{\nabla}\Phi^{(1)}_{t} + \Phi^{(1)}_{t}(\Phi^{(1)}_{X_3X_3} + \frac{1}{g}\Phi^{(1)}_{ttX_3})$$

$$\text{on } X_3 = 0 \quad \ldots\ldots\ldots\ldots \quad \text{(III-27)}$$

- The boundary condition at the sea floor, which states that to first and second order no fluid particles shall pass through this boundary or:

$$\overline{\nabla}\Phi^{(1)}.\overline{n}_b = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad \text{(III-28)}$$

$$\overline{\nabla}\Phi^{(2)}.\overline{n}_b = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad \text{(III-29)}$$

where $\overline{n}_b$ is the normal vector of a point on the surface of the sea floor.

## III.4.2. Boundary conditions on the body

In general the boundary condition on the body states that the relative velocity between the fluid and the body in the direction of the normal to the body be zero. This means that no fluid passes through the hull. This boundary condition has to be satisfied at the instantaneous position of the body surface and is as follows:

$$\overline{\nabla}\Phi.\overline{N} = \overline{V}.\overline{N} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad \text{(III-30)}$$

Taking into account equations (III-3) and (III-14) through (III-29) and grouping powers of $\varepsilon$ results in the first and second order body boundary conditions.

The boundary condition for the first order potential $\Phi^{(1)}$ on the body, which states that, to first order, there is no relative motion between the fluid and the body surface in the direction of the outward pointing normal vector $\overline{N}$, is as follows:

$$\overline{\nabla}\Phi^{(1)}.\overline{n} = \overline{V}^{(1)}.\overline{n} \quad \ldots\ldots\ldots\ldots\ldots \quad \text{(III-31)}$$

The boundary condition for the second order potential $\Phi^{(2)}$ states that, to second order, the relative velocity in the direction of the outward pointing normal $\overline{N}$ be zero or:

$$\overline{\nabla}\Phi^{(2)}.\overline{n} = (\overline{V}^{(1)} - \overline{\nabla}\Phi^{(1)}).\overline{N}^{(1)} + \overline{V}^{(2)}.\overline{n} \quad \text{. . . . . (III-32)}$$

Equations (III-31) and (III-32) have to be satisfied at the instantaneous position of the surface of the body. Assuming that the motions are small and applying a Taylor expansion similar conditions may be posed on the potentials at the mean position of the surface. The first order boundary condition becomes:

$$\overline{\nabla}\Phi^{(1)}.\overline{n} = \overline{V}^{(1)}.\overline{n} \quad \text{. . . . . . . . . . . . . . . . (III-33)}$$

The second order boundary condition becomes:

$$\overline{\nabla}\Phi^{(2)}.\overline{n} = -(\overline{X}^{(1)}.\overline{\nabla}).\overline{\nabla}\Phi^{(1)}.\overline{n} + (\overline{V}^{(1)} - \overline{\nabla}\Phi^{(1)}).\overline{N}^{(1)} +$$

$$+ \overline{V}^{(2)}.\overline{n} \quad \text{. . . . . . . . . . . . . . . (III-34)}$$

where the additional term in equation (III-34) arises from the second order correction to equation (III-31) when applying the Taylor expansion to the velocity $\overline{\nabla}\Phi^{(1)}$.

In equation (III-33) and (III-34) the potentials and their derivatives have to be taken at the mean position of the body. We may decompose $\Phi^{(1)}$ in the following way:

$$\Phi^{(1)} = \Phi_w^{(1)} + \Phi_d^{(1)} + \Phi_b^{(1)} \quad \text{. . . . . . . . . . . . (III-35)}$$

in which $\Phi_w^{(1)}$ is the first order potential associated with the undisturbed incoming waves. Substitution of equation (III-35) in boundary condition (III-33) gives the following:

$$(\overline{\nabla}\Phi_w^{(1)} + \overline{\nabla}\Phi_d^{(1)} + \overline{\nabla}\Phi_b^{(1)}).\overline{n} = \overline{V}^{(1)}.\overline{n} \quad \text{. . . . . . . (III-36)}$$

Since the expression is linear $\Phi^{(1)}$ we may decompose this equation into two parts:

$$\overline{\nabla}\Phi_d^{(1)}.\overline{n} = -\overline{\nabla}\Phi_w^{(1)}.\overline{n} \quad \text{. . . . . . . . . . . . . . (III-37)}$$

and: $\overline{\nabla}\Phi_b^{(1)}.\overline{n} = \overline{V}^{(1)}.\overline{n}$ . . . . . . . . . . . . . . . . (III-38)

Equation (III-37) defines the diffraction potential $\Phi_d^{(1)}$ which compensates the normal velocity components due to the undisturbed incoming waves at the motionless body surface. Equation (III-38) defines the body motion potential $\Phi_b^{(1)}$ which must be introduced in order to satisfy the first order boundary condition on the body oscillating in still water. From the solution of the diffraction potential $\Phi_d^{(1)}$ combined with the potential of the undisturbed incoming waves $\Phi_w^{(1)}$ the so-called first order wave exciting forces are found. The body motion potential $\Phi_b^{(1)}$ is used to determine the hydrodynamic reaction forces known as added mass and damping for unit amplitude acceleration and velocity of the body motions. From the first order wave exciting forces the added mass and damping coefficients and the equations of motion of the body, the unknown first order motions and hence the total first order potential $\Phi^{(1)}$ can be determined; see for instance Van Oortmerssen [III-2]. Substitution of the first order potential $\Phi^{(1)}$ of equation (III-35) in the non-homogeneous second order free surface boundary condition of equation (III-27) shows that the second order potential has, in general, the following components:

$$\Phi^{(2)} = \Phi_{ww}^{(2)} + \Phi_{dd}^{(2)} + \Phi_{bb}^{(2)} + \Phi_{wd}^{(2)} + \Phi_{wb}^{(2)} + \Phi_{db}^{(2)} + \\ + \Phi_{dw}^{(2)} + \Phi_{bw}^{(2)} + \Phi_{bd}^{(2)} + \Phi_{0}^{(2)} \quad \text{. . . . . . . . . (III-39)}$$

where the first nine components on the right-hand side are potentials which are particular solutions to the following type of boundary condition at the mean free surface, e.g.:

$$g\Phi_{ww_{X_3}}^{(2)} + \Phi_{ww_{tt}}^{(2)} = -2\overline{\nabla}\Phi_w^{(1)}.\overline{\nabla}\Phi_{w_t}^{(1)} + \Phi_{w_t}^{(1)}(\Phi_{w_{X_3X_3}}^{(1)} + \frac{1}{g}\Phi_{w_{ttX_3}}^{(1)}) \quad \text{. . . . . . . . . . . . . . . . . (III-40)}$$

The last potential $\Phi_0^{(2)}$ is a potential which satisfies the homogeneous boundary condition:

$$g\Phi_{0_{X_3}}^{(2)} + \Phi_{0_{tt}}^{(2)} = 0 \quad \text{. . . . . . . . . . . . . . . . . (III-41)}$$

$\Phi_0^{(2)}$ is therefore an "ordinary" potential which satisfies the linearized free surface condition. We will simplify equation (III-39) by putting:

$$\Phi^{(2)} = \Phi_w^{(2)} + \Phi_0^{(2)} \quad \ldots\ldots\ldots\ldots\ldots \quad \text{(III-42)}$$

in which $\Phi_w^{(2)}$ represents the sum of the first nine components on the right-hand side of equation (III-39). $\Phi_w^{(2)}$ may be regarded as the second order equivalent of the first order undisturbed incoming wave potential $\Phi_w^{(1)}$. We will decompose $\Phi_0^{(2)}$ as follows:

$$\Phi_0^{(2)} = \Phi_d^{(2)} + \Phi_b^{(2)} \quad \ldots\ldots\ldots\ldots\ldots \quad \text{(III-43)}$$

Both these potentials satisfy the linearized free surface condition of equation (III-41). Substitution in equation (III-42) gives:

$$\Phi^{(2)} = \Phi_w^{(2)} + \Phi_d^{(2)} + \Phi_b^{(2)} \quad \ldots\ldots\ldots\ldots\ldots \quad \text{(III-44)}$$

Substitution of equation (III-44) in the second order boundary condition (III-34) gives:

$$(\overline{\nabla}\Phi_w^{(2)} + \overline{\nabla}\Phi_d^{(2)} + \overline{\nabla}\Phi_b^{(2)}).\overline{n} = -(\overline{X}^{(1)}.\overline{\nabla}).\overline{\nabla}\Phi^{(1)}.\overline{n} + (\overline{V}^{(1)} - \overline{\nabla}\Phi^{(1)}).\overline{N}^{(1)} + \overline{V}^{(2)}.\overline{n} \quad \ldots\ldots\ldots\ldots\ldots \quad \text{(III-45)}$$

which may be decomposed in:

$$\overline{\nabla}\Phi_d^{(2)}.\overline{n} = \{-\overline{\nabla}\Phi_w^{(2)} - (\overline{X}^{(1)}.\overline{\nabla}).\overline{\nabla}\Phi^{(1)}\}.\overline{n} + (\overline{V}^{(1)} - \overline{\nabla}\Phi^{(1)}).\overline{N}^{(1)} \quad \ldots\ldots\ldots\ldots \quad \text{(III-46)}$$

$$\overline{\nabla}\Phi_b^{(2)}.\overline{n} = \overline{V}^{(2)}.\overline{n} \quad \ldots\ldots\ldots\ldots\ldots \quad \text{(III-47)}$$

Equation (III-46) defines the second order diffraction potential $\Phi_d^{(2)}$ which firstly compensates the second order velocity components of $\Phi_w^{(2)}$ and the second order correction to the first order velocity $\overline{\nabla}\Phi^{(1)}$, which results from the first order motion $\overline{X}^{(1)}$ in a direction along the normal n to the surface and secondly the second order velocity component of the difference between the first order velocity $\overline{V}^{(1)}$ of the body surface and the first order fluid velocity $\overline{\nabla}\Phi^{(1)}$ in a direction along the first order normal $\overline{N}^{(1)}$. From the solution of the second order diffraction potential $\Phi_d^{(2)}$ combined with the undisturbed second order potential $\Phi_w^{(2)}$ the low frequency second order wave exciting forces are found. Equation (III-47) defines the second order body motion potential $\Phi_b^{(2)}$ which must be introduced in order to satisfy the boundary condition on the body carrying out low frequency second order motions in still water. This potential satisfies the same boundary condition as the first order body motion potential $\Phi_b^{(1)}$. The only difference is that the motions are low frequency and of the second order in magnitude. The same techniques may therefore be employed in solving $\Phi_b^{(2)}$ as used in solving $\Phi_b^{(1)}$. This means that $\Phi_b^{(2)}$ may be expressed in terms of hydrodynamic reaction forces for unit amplitude of motion velocity and acceleration of the body, better known as added mass and damping.

### III.4.3. Boundary conditions at infinity

For the potentials $\Phi_d^{(1)}$, $\Phi_b^{(1)}$ and $\Phi_d^{(2)}$, $\Phi_b^{(2)}$ a radiation condition, which states that at a great distance from the body the waves associated with these potentials move outwards, must be satisfied. This restriction imposes a uniqueness which would not otherwise be present. Since the components of $\Phi_w^{(2)}$ are particular solutions to the free surface boundary condition (III-27), which is defined over the complete free surface, a radiation condition need not be imposed.

## III.5. Pressure in a point within the fluid

If the velocity potential $\Phi$ is known the fluid pressure in a point is determined by Bernoulli's equation:

$$p = p_0 - \rho g X_3 - \rho \Phi_t - \tfrac{1}{2}\rho|\overline{\nabla}\Phi|^2 + C(t) \quad \ldots\ldots \quad \text{(III-48)}$$

where:

$p_0$ = atmospheric pressure
$X_3$ = vertical distance below the mean free surface
$\Phi$ = velocity potential
C(t) = a function independent of the co-ordinates
t = time
$\rho$ = mass density of the fluid.

In Bernoulli's equation $p_0$ and C(t) may be taken equal to zero without loss of generality, see ref. [III-1].

Assuming that the point is carrying out first order wave frequency motions $\overline{X}^{(1)}$ and low frequency second order motions $\overline{X}^{(2)}$ about a mean position $\overline{X}^{(0)}$ and applying a Taylor expansion to the pressure in the mean position the following expression is found:

$$p = p^{(0)} + \varepsilon p^{(1)} + \varepsilon^2 p^{(2)} \quad \text{(III-49)}$$

where:

- hydrostatic pressure:

$$p^{(0)} = -\rho g X_3^{(0)} \quad \text{(III-50)}$$

- first order pressure:

$$p^{(1)} = -\rho g X_3^{(1)} - \rho \Phi_t^{(1)} \quad \text{(III-51)}$$

- second order pressure:

$$p^{(2)} = -\tfrac{1}{2}\rho |\overline{\nabla}\Phi^{(1)}|^2 - \rho \Phi_t^{(2)} - \rho(\overline{X}^{(1)} . \overline{\nabla}\Phi_t^{(1)}) - \rho g X_3^{(2)} \quad \text{(III-52)}$$

In the above the derivatives of the potentials have to be taken in the mean position of the point.

We have assumed that the point is moving within the fluid domain. The same expression will be used to determine the pressure on a point on the hull of the body. This means that derivatives of the potentials are taken at the mean position of the hull which is alternately within and outside the actual fluid domain. This appears to be permissible if the potential functions are sufficiently "smooth" at the boundaries, see ref. [III-3]. This is assumed to be satisfied in this case.

## III.6. Second order wave force and moment

### III.6.1. Second order wave force

In determining the second order wave force consideration must first be given to the choice of the system of axes to which will be referred. Since in general we are concerned with the slow wave drifting force induced motions of bodies in the horizontal plane we have chosen to determine the wave drifting force along the axes of the $G-X_1'-X_2'-X_3'$ system of co-ordinates. See Figure III-1.

The fluid force exerted on the body relative to the $G-X_1'-X_2'-X_3'$ system of axes, which is the system with axes parallel to the axes of the fixed system $O-X_1-X_2-X_3$ but with origin in the centre of gravity G of the body, follows from:

$$\bar{F} = -\iint_S p.\bar{N}.dS \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad \text{(III-53)}$$

where S is the instantaneous wetted surface and $\bar{N}$ is the instantaneous normal vector to the surface element dS relative to the $G-X_1'-X_2'-X_3'$ system of axes. $\bar{N}$ is given by equation (III-17) and p by equation (III-49).

The instantaneous wetted surface S is split into two parts, viz.: a constant part $S_0$ up to the static waterline on the hull and an oscillating part s between the static waterline on the hull and the wave profile along the body. See Figure III-1.

Substitution of the pressure p as given by equation (III-49) and the normal vector $\bar{N}$ as given by equation (III-17) gives:

$$\overline{F} = -\iint_{S_0} (p^{(0)} + \varepsilon p^{(1)} + \varepsilon^2 p^{(2)})(\overline{n} + \varepsilon\overline{N}^{(1)} + \varepsilon^2\overline{N}^{(2)})dS +$$

$$-\iint_{s} (p^{(0)} + \varepsilon p^{(1)} + \varepsilon^2 p^{(2)})(\overline{n} + \varepsilon\overline{N}^{(1)} + \varepsilon^2\overline{N}^{(2)})dS \quad \ldots\ldots\ldots\ldots\ldots \text{(III-54)}$$

$$= \overline{F}^{(0)} + \varepsilon\overline{F}^{(1)} + \varepsilon^2\overline{F}^{(2)} + O(\varepsilon^3) \quad \ldots\ldots\ldots \text{(III-55)}$$

The hydrostatic force $\overline{F}^{(0)}$ follows from integration of the hydrostatic pressure $p^{(0)}$ over the mean wetted surface $S_0$:

$$\overline{F}^{(0)} = \rho g \iint_{S_0} x_3^{(0)}.\overline{n}.dS = (0,0,\rho g V) \quad \ldots\ldots\ldots \text{(III-56)}$$

The total first order oscillatory fluid force $\overline{F}^{(1)}$ follows from:

$$\overline{F}^{(1)} = -\iint_{S_0} (p^{(1)}.\overline{n} + p^{(0)}.\overline{N}^{(1)})dS \quad \ldots\ldots\ldots \text{(III-57)}$$

$$= -\iint_{S_0} p^{(1)}.\overline{n}.dS + \overline{\alpha}^{(1)} \times (0,0,\rho g V) \quad \ldots\ldots \text{(III-58)}$$

The first part of this expression is the total first order fluid force relative to the body axes $G\text{-}x_1\text{-}x_2\text{-}x_3$. The second order force is found by integration of all products of pressure p and normal vector $\overline{N}$ which give second order force contributions over the constant part $S_0$ of the wetted surface and by integration of first order pressures over the oscillating surface s:

$$\overline{F}^{(2)} = -\iint_{S_0} (p^{(1)}.\overline{N}^{(1)} + p^{(2)}.\overline{n} + p^{(0)}.\overline{N}^{(2)})dS +$$

$$-\iint_{s} p^{(1)}.\overline{n}.dS \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(III-59)}$$

Taking into account that:

$$\overline{N}^{(1)} = \overline{\alpha}^{(1)} \times \overline{n} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(III-60)}$$

Since angular displacements are the same for all surface elements dS, the first part of the first integral becomes:

$$-\iint_{S_0} p^{(1)}.\overline{N}^{(1)}.dS = \overline{\alpha}^{(1)} \times -\iint_{S_0} p^{(1)}.\overline{n}.dS \quad \ldots\ldots \text{(III-61)}$$

The integral in this expression corresponds with the first term in equation (III-58) which is the total first order fluid force relative to the body axes $G-x_1-x_2-x_3$. Equation (III-61) indicates that a second order force contribution relative to the $G-X_1'-X_2'-X_3'$ system of axes arises from rotation of the first order fluid force relative to the body axes. In the same way the gravity force acting on the body relative to the body axes must be accounted for in the second order force. This force relative to the body axes is:

$$-\bar{\alpha}^{(1)} \times (0,0,-mg) = \bar{\alpha}^{(1)} \times (0,0,\rho gV) \quad \ldots\ldots \quad \text{(III-62)}$$

Adding this component to equation (III-61) gives:

$$\bar{\alpha}^{(1)} \times \{-\iint_{S_0} p^{(1)}.\bar{n}.dS + \bar{\alpha}^{(1)} \times (0,0,\rho gV)\} = \bar{\alpha}^{(1)} \times \bar{F}^{(1)}$$

$$\ldots\ldots\ldots\ldots\ldots \quad \text{(III-63)}$$

where $\bar{F}^{(1)}$ is the total first order fluid force including the hydrostatic restoring force, the wave exciting force and the hydrodynamic reaction force. See equation (III-58). Consequently, according to Newton's law, we may put:

$$\bar{F}^{(1)} = M.\ddot{\bar{X}}_g^{(1)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad \text{(III-64)}$$

from which it follows that:

$$\bar{\alpha}^{(1)} \times \bar{F}^{(1)} = \bar{\alpha}^{(1)} \times (M.\ddot{\bar{X}}_g^{(1)}) \quad \ldots\ldots\ldots\ldots \quad \text{(III-65)}$$

The second part of the first integral in equation (III-59) involves straightforward integration of the pressure $p^{(2)}$ as given in equation (III-52). The third part of the first integral is a second order hydrostatic component:

$$-\iint_{S_0} p^{(0)}.\bar{N}^{(2)}.dS = \bar{\alpha}^{(2)} \times -\iint_{S_0} p^{(0)}.\bar{n}.dS$$

$$= \bar{\alpha}^{(2)} \times (0,0,\rho gV) \quad \ldots\ldots\ldots \quad \text{(III-66)}$$

The second integral in equation (III-59) over the oscillating surface is solved by substituting $p^{(1)}$ from equation (III-51) and writing the surface element dS as:

$$dS = dX_3.d\ell \quad \ldots\ldots \quad (III\text{-}67)$$

Also taking into account that at the waterline:

$$-\rho\Phi_t^{(1)} = \rho g\zeta^{(1)} \quad \ldots\ldots \quad (III\text{-}68)$$

this integral becomes:

$$-\int_{WL}\int_{X_{3WL}^{(1)}}^{\zeta^{(1)}} (-\rho g X_3 + \rho g\zeta^{(1)})\bar{n}.dX_3.d\ell \quad \ldots\ldots \quad (III\text{-}69)$$

which results in:

$$-\int_{WL} \tfrac{1}{2}\rho g\zeta_r^{(1)^2}.\bar{n}.d\ell \quad \ldots\ldots \quad (III\text{-}70)$$

in which $\zeta_r^{(1)}$ is the relative wave elevation defined by:

$$\zeta_r^{(1)} = \zeta^{(1)} - X_{3WL}^{(1)} \quad \ldots\ldots \quad (III\text{-}71)$$

The final expression for the total second order force thus becomes:

$$\begin{aligned}\bar{F}^{(2)} = &-\int_{WL} \tfrac{1}{2}\rho g\zeta_r^{(1)^2}.\bar{n}.d\ell + \bar{\alpha}^{(1)} \times (M.\ddot{\bar{X}}_g^{(1)}) + \\ &-\iint_{S_0} \{-\tfrac{1}{2}\rho|\bar{\nabla}\Phi^{(1)}|^2 - \rho\Phi_t^{(2)} - \rho(\bar{X}^{(1)}.\bar{\nabla}\Phi_t^{(1)})\}\bar{n}.dS + \\ &-\iint_{S_0} -\rho g X_3^{(2)}.\bar{n}.dS + \bar{\alpha}^{(2)} \times (0,0,\rho g V)\end{aligned}$$

$$\ldots\ldots \quad (III\text{-}72)$$

III.6.2. Second order wave moment

The moment about the axes of the $G\text{-}X_1'\text{-}X_2'\text{-}X_3'$ system of coordinates follows from:

$$\bar{M} = -\iint_S p.(\bar{X}' \times \bar{N}).dS \quad \ldots\ldots \quad (III\text{-}73)$$

The derivation is analogous to that followed for the force. The final expression for the second order wave moment is:

$$\overline{M}^{(2)} = -\int_{WL} \tfrac{1}{2}\rho g {\zeta_r^{(1)}}^2 . (\overline{x} \times \overline{n}) . d\ell + \overline{\alpha}^{(1)} \times (I . \ddot{\overline{\alpha}}^{(1)}) +$$
$$-\iint_{S_0} \{-\tfrac{1}{2}\rho |\overline{\nabla}\Phi^{(1)}|^2 - \rho\Phi_t^{(2)} - \rho(\overline{X}^{(1)} . \overline{\nabla}\Phi_t^{(1)})\} .$$
$$. (\overline{x} \times \overline{n}) . dS - \iint_{S_0} -\rho g X_3^{(2)} . (\overline{x} \times \overline{n}) . dS$$

. . . . . . . . . . . . . . (III-74)

Equations (III-72) and (III-74) give the total second order forces acting on a vessel, thus including the wave exciting force and the hydrodynamic and hydrostatic reaction forces. In most cases prime interest is focussed on the second order wave exciting forces and moments. It will be clear from the aforegoing that the second order hydrodynamic reaction forces are contained in the contributions due to the total second order potential $\Phi^{(2)}$. The hydrostatic reaction forces are contained in the last parts of equations (III-72) and (III-74). Taking into account equation (III-44) the second order wave exciting force and moment become:

$$\overline{F}^{(2)} = -\int_{WL} \tfrac{1}{2}\rho g {\zeta_r^{(1)}}^2 . \overline{n} . d\ell + \overline{\alpha}^{(1)} \times (M . \ddot{\overline{X}}_g^{(1)}) +$$
$$-\iint_{S_0} \{-\tfrac{1}{2}\rho |\overline{\nabla}\Phi^{(1)}|^2 - \rho(\Phi_{w_t}^{(2)} + \Phi_{d_t}^{(2)}) +$$
$$- \rho(\overline{X}^{(1)} . \overline{\nabla}\Phi_t^{(1)})\}\overline{n} . dS$$

. . . . . . . . . . . . . . (III-75)

$$\overline{M}^{(2)} = -\int_{WL} \tfrac{1}{2}\rho g {\zeta_r^{(1)}}^2 . (\overline{x} \times \overline{n}) . d\ell + \overline{\alpha}^{(1)} \times (I . \ddot{\overline{\alpha}}^{(1)}) +$$
$$-\iint_{S_0} \{-\tfrac{1}{2}\rho |\overline{\nabla}\Phi^{(1)}|^2 - \rho(\Phi_{w_t}^{(2)} + \Phi_{d_t}^{(2)}) +$$
$$- \rho(\overline{X}^{(1)} . \overline{\nabla}\Phi_t^{(1)})\} . (\overline{x} \times \overline{n}) . dS$$

. . . . . . . . . . . . . . (III-76)

The hydrodynamic reaction forces due to motions induced by second order forces may be expressed in terms of added mass and damping forces as has been shown in the aforegoing.

## III.7. Conclusions

In this section it was shown that within potential theory, as is also the case with first order forces and motions, the total second order problem may be split into two parts, namely: determination of the second order wave exciting forces in the absence of motions induced by these forces and determination of the hydrodynamic reaction forces (added mass and damping).

From the expressions (III-75) and (III-76) it is seen that the wave exciting forces can be obtained only after the first order solution, the solution to the second order "undisturbed" wave potential and the second order diffraction potential have been found. In the following sections it will be shown that in many practical cases the contributions arising from components dependent on first order quantities, which can be evaluated using existing techniques, tend to be dominant. Finding the solution to the second order potential $\Phi_w^{(2)}$ becomes difficult due to the complexity of the free surface boundary conditions given in equation (III-27). The second order diffraction potential $\Phi_d^{(2)}$ on the other hand has to satisfy the homogeneous free surface boundary condition of equation (III-26). This means that in principle this potential can again be solved by existing first order methods. In this case, however, the boundary condition on the mean wetted surface of the hull of equation (III-46) contains the unknown second order "undisturbed" wave potential. In the following sections it will be shown that in practice a simple approximation of these second order potential contributions may be used.

# IV. EVALUATION OF THE SECOND ORDER WAVE EXCITING FORCES

## IV.1. Introduction

In the previous section general expressions for the second order wave exciting forces and moments have been obtained based on the method of direct integration of pressure acting on the wetted surface of a body. The expressions obtained are, however, not in a form which is easily used for practical applications. In this section it will be shown that the second order forces may be expressed more conveniently in terms of time independent quadratic transfer functions by means of which it is possible to express the second order wave exciting forces in the frequency domain in terms of force spectra or in the time domain as time histories of second order forces.

The components of the transfer functions for the second order forces which depend on first order quantities can be evaluated using an existing method of computation based on three-dimensional linear potential theory of which a brief account will be given. The contributions due to second order potential effects will be determined by an approximation using results of computations based on the same method. Comparisons of this approximation with some exact results will be given. Only the low frequency second order forces are treated here. The same procedure applies to the low frequency second order moments.

## IV.2. The quadratic transfer function

### IV.2.1. General

In this study the total quadratic transfer function is split up in contributions arising from the following components of equation (III-75):

I : First order relative wave elevation

$$-\tfrac{1}{2}\rho g \int_{WL} \zeta_r^{(1)^2} . \bar{n} . d\ell \quad . . . . . . . . . . . . . . . . (IV-1)$$

II : Pressure drop due to first order velocity

$$-\iint_{S_0} -\tfrac{1}{2}\rho|\overline{\nabla}\Phi^{(1)}|^2.\overline{n}.dS \quad \ldots\ldots\ldots\ldots\ldots \quad \text{(IV-2)}$$

III : Pressure due to product of gradient of first order pressure and first order motion

$$-\iint_{S_0} -\rho(\overline{X}^{(1)}.\overline{\nabla}\Phi_t^{(1)}).\overline{n}.dS \quad \ldots\ldots\ldots\ldots\ldots \quad \text{(IV-3)}$$

IV : Contribution due to products of first order angular motions and inertia forces

$$\overline{\alpha}^{(1)} \times (M.\ddot{\overline{X}}_g^{(1)}) \quad \ldots\ldots\ldots\ldots\ldots \quad \text{(IV-4)}$$

V : Contribution due to second order potentials

$$-\iint_{S_0} -\rho(\Phi_{w_t}^{(2)} + \Phi_{d_t}^{(2)}).\overline{n}.dS \quad \ldots\ldots\ldots\ldots\ldots \quad \text{(IV-5)}$$

The procedure to obtain the quadratic transfer functions of the forces dependent on first order quantities (I, II, III and IV) will be illustrated by taking the low frequency part of the longitudinal component of the force contribution due to the relative wave elevation:

$$F_1^{(2)} \doteq F_1^{(2)}(t) = -\int_{WL} \tfrac{1}{2}\rho g\zeta_r^{(1)^2}(t,\ell).n_1(\ell).d\ell$$

$$\ldots\ldots\ldots\ldots\ldots \quad \text{(IV-6)}$$

in which:

$\zeta_r^{(1)}(t,\ell)$ = time dependent relative wave elevation in a point $\ell$ along the waterline

$n_1(\ell)$ = direction cosine of a length element $d\ell$ in longitudinal direction.

In irregular long-crested waves the elevation, to first order, of the incoming undisturbed waves - referred to the mean position of the centre of gravity of the floating body - may be written as:

$$\zeta^{(1)}(t) = \sum_{i=1}^{N} \zeta_i^{(1)}.\cos(\omega_i t + \underline{\varepsilon}_i) \quad \ldots\ldots\ldots \quad \text{(IV-7)}$$

The first order relative wave elevation at a point $\ell$ on the waterline of the body may be written as follows:

$$\zeta_r^{(1)}(t,\ell) = \sum_{i=1}^{N} \zeta_i^{(1)} . \zeta_{r_i}^{(1)'}(\ell) . \cos\{\omega_i t + \underline{\varepsilon}_i + \varepsilon_{r_i}(\ell)\}$$

. . . . . . . . . . . . . . (IV-8)

in which:

$\zeta_i^{(1)}$ = amplitude of i-th regular wave component

$\underline{\varepsilon}_i$ = random phase uniformly distributed over 0 - 2π

$\omega_i$ = frequency of i-th component

$\zeta_{r_i}^{(1)'}(\ell)$ = transfer function of the amplitude of the first order relative wave elevation at point $\ell$ in the waterline

$\varepsilon_{r_i}(\ell)$ = phase angle of the relative wave elevation at point $\ell$ related to the undisturbed wave crest passing the centre of gravity.

Substitution of (IV-8) in equation (IV-6) leads to:

$$F_1^{(2)}(t) = \sum_{i=1}^{N} \sum_{j=1}^{N} \zeta_i^{(1)} . \zeta_j^{(1)} . P_{ij} . \cos\{(\omega_i - \omega_j)t + (\underline{\varepsilon}_i - \underline{\varepsilon}_j)\} +$$

$$+ \sum_{i=1}^{N} \sum_{j=1}^{N} \zeta_i^{(1)} . \zeta_j^{(1)} . Q_{ij} . \sin\{(\omega_i - \omega_j)t + (\underline{\varepsilon}_i - \underline{\varepsilon}_j)\} +$$

$$+ \text{ high frequency terms}$$

. . . . . . . . . . . . . . (IV-9)

where $P_{ij}$ and $Q_{ij}$ are the in-phase and out-of-phase components of the time independent transfer function with:

$$P_{ij} = P(\omega_i, \omega_j) = \int_{WL} \tfrac{1}{4}\rho g \zeta'_{r_i}(\ell) . \zeta'_{r_j}(\ell) . \cos\{\varepsilon_{r_i}(\ell) + - \varepsilon_{r_j}(\ell)\} n_1(\ell) . d\ell$$

. . . . . . . . . . . . . . (IV-10)

$$Q_{ij} = Q(\omega_i, \omega_j) = - \int_{WL} \tfrac{1}{4}\rho g \zeta'_{r_i}(\ell) . \zeta'_{r_j}(\ell) . \sin\{\varepsilon_{r_i}(\ell) + - \varepsilon_{r_j}(\ell)\} n_1(\ell) . d\ell$$

. . . . . . . . . . . . . . (IV-11)

Taking the low frequency part of the square of the wave elevation given by equation (IV-7) results in:

$$\zeta^{(1)^2}(t)_\ell = \sum_{i=1}^{N} \sum_{j=1}^{N} \tfrac{1}{2}\zeta_i^{(1)} . \zeta_j^{(1)} . \cos\{(\omega_i - \omega_j)t + (\underline{\varepsilon}_i - \underline{\varepsilon}_j)\}$$

$$\text{. . . . . . . . . . . . . . (IV-12)}$$

Comparison with equation (IV-9) shows that $P_{ij}$ and $Q_{ij}$ are transfer functions which give that part of the wave drifting force which is in-phase and out-of-phase respectively with the low frequency part of the square of the incident wave elevation.

It will be clear that similar developments can be made for other contributions to the wave drifting forces which depend only on first order quantities. The total in-phase and out-of-phase transfer functions are found by simple summation of the contributions from the five components. The wave drifting forces may thus be presented as transfer functions which, as can be seen from the aforegoing, are a function of two frequencies. In general the quadratic transfer functions will also be functions of the direction of the waves.

### IV.2.2. Evaluation of the components dependent on first order quantities

Evaluation of these components of the quadratic transfer functions of the low frequency wave drifting forces requires detailed knowledge of the first order vessel motions and fluid motions. As can be seen from equations (IV-10) and (IV-11) knowledge of the first order amplitude and phase transfer functions as a function of the wave frequency are sufficient to evaluate these components of the quadratic transfer functions.

A numerical method by means of which such detailed information may be obtained (using a distribution of sources over the mean wetted surface of the body) has been developed by Boreel [IV-1] and Van Oortmerssen [IV-2]. A brief description of this method is given in Appendix A. The computer program DIFFRAC based on the theory given in this appendix has been used to evaluate these components of the quadratic transfer functions.

## IV.2.3. Contribution of the second order potential

### IV.2.3.1. General

When a body is floating in a regular wave group consisting of two regular waves with frequencies $\omega_i$ and $\omega_j$ part of the second order wave exciting forces are due to second order velocity potentials $\Phi_w^{(2)}$ and $\Phi_d^{(2)}$ as was already indicated in chapter III. The second order potential $\Phi_w^{(2)}$ has to be determined taking into account the following boundary conditions at the mean free surface:

$$g\Phi^{(2)}_{w_{X_3}} + \Phi^{(2)}_{w_{tt}} = -2.\overline{\nabla}\Phi^{(1)}.\overline{\nabla}\Phi^{(1)}_t + \Phi^{(1)}_t.(\Phi^{(1)}_{X_3X_3} + \frac{1}{g}\,\Phi^{(1)}_{ttX_3}) \qquad \text{(IV-13)}$$

in which $\Phi^{(1)}$ represents the total first order potential containing the contributions from the undisturbed incoming waves, diffracted waves and waves due to first order body motions. The corresponding boundary condition for the second order diffraction potential $\Phi_d^{(2)}$ is as follows:

$$g\Phi^{(2)}_{d_{X_3}} + \Phi^{(2)}_{d_{tt}} = 0 \qquad \text{(IV-14)}$$

The second order diffraction potential $\Phi_d^{(2)}$ also has to satisfy the following boundary condition at the surface of the body in the mean position:

$$\overline{\nabla}\Phi_d^{(2)}.\overline{n} = -\overline{\nabla}\Phi_w^{(2)}.\overline{n} - \{(\overline{X}^{(1)}.\overline{\nabla}).\overline{\nabla}\Phi^{(1)}\}\overline{n} + (\overline{V}^{(1)} - \overline{\nabla}\Phi^{(1)})\overline{N}^{(1)} \qquad \text{(IV-15)}$$

Besides these conditions other boundary conditions discussed in chapter III have to be satisfied. However, for the discussion in this section only the above conditions are relevant.

As can be seen from equation (IV-14) the second order diffraction potential $\Phi_d^{(2)}$ satisfies the free surface boundary condition which is applicable to normal first order potentials. This means that in principle the same numerical techniques as indicated in Appendix A can be used to determine this potential for arbitrary body shapes.

The boundary conditions at the mean position of the body given in equation (IV-15) contains first order contributions which are known on the basis of the first order solution and an unknown second order contribution due to the "undisturbed" second order potential $\Phi_w^{(2)}$ which has first to be determined taking into account the non-homogeneous free surface boundary condition (IV-13). This boundary condition prescribes the behaviour of $\Phi_w^{(2)}$ over the complete mean free surface. No elementary solutions for this potential are known for the general three-dimensional case. This is due to the complexity of the right-hand side of the free surface boundary condition equation (IV-13).

In the three-dimensional case of a vessel floating in a wave field consisting of two regular waves with frequencies $\omega_1$ and $\omega_2$ approaching from the same direction the total first order potential $\Phi^{(1)}$ will contain contributions of the two long-crested incoming regular waves and a complex pattern of cylindrical outgoing waves due to diffraction and body motion effects. The right-hand side of the non-homogeneous free surface condition of equation (IV-13) contains products of potentials associated with these long-crested incoming and outgoing cylindrical waves. As shown in chapter III, the potential $\Phi_w^{(2)}$ can be split into the following parts:

$$\Phi_w^{(2)} = \Phi_{ww}^{(2)} + \Phi_{dd}^{(2)} + \Phi_{bb}^{(2)} + \Phi_{wd}^{(2)} + \Phi_{wb}^{(2)} + \\ + \Phi_{dw}^{(2)} + \Phi_{db}^{(2)} + \Phi_{bw}^{(2)} + \Phi_{bd}^{(2)} \quad \ldots\ldots\ldots \quad \text{(IV-16)}$$

in which:

$\Phi_{ww}^{(2)}$ = second order potential associated with the undisturbed incoming first order wave potential

$\Phi_{dd}^{(2)}$, $\Phi_{bb}^{(2)}$ = second order potentials associated with the outgoing diffraction waves and waves due to body motions

$\Phi_{wd}^{(2)}$---$\Phi_{bd}^{(2)}$ = second order potentials associated with interactions between incoming waves and outgoing waves.

Of these potentials an analytical solution is only known for $\Phi_{ww}^{(2)}$. The other potentials may be written as complicated two-dimensional Fourier integrals. Evaluation of such integrals, however, presents a considerable computational problem. Therefore we prefer a description in terms of a source distribution. In order to solve the second order potential $\Phi_{w}^{(2)}$ a considerable increase in the number of sources is necessary. The general procedure using this numerical technique will be described here briefly. Firstly, a source distribution is defined over the mean wetted surface of the vessel and over the mean free surface in the vicinity of the vessel. In the numerical method described in Appendix A the Green's function chosen for the elementary sources which are distributed over the mean surface of the vessel satisfies the homogeneous free surface condition equation (IV-14). The source distribution over the free surface cannot make use of the same formulation for the elementary source since in that case the non-homogeneous free surface condition equation (IV-13) cannot be satisfied. Instead we may choose the elementary source function of the $\frac{1}{r}$-type, which corresponds to the first two terms of equation (A-7) given in Appendix A. This type of source satisfies only the equation of continuity equation (III-23) and the kinematic condition equation (III-29) at the sea floor. The extent of the distribution over the free surface will be the result of a compromise between the magnitude of the error in the results due to the truncation some distance away from the vessel and the increase in computation times. Secondly, the first order solutions obtained for two regular waves with frequency $\omega_1$ and $\omega_2$ are used to compute the values of the second order free surface conditions of equation (IV-13) at the centre of the sources distributed over the free surface. Since the source distribution over the hull of the vessel satisfies the homogeneous free surface condition equation (IV-14) these do not contribute to the source strengths in the free surface. The source strength in the free surface may therefore be solved without consideration of the source distribution over the hull surface. The non-homogeneous free surface condition equation (IV-13) leads to a set of simultaneous equations which are linear in the unknown source strength of the free surface sources which is solved by standard techniques. This solves the unknown potential $\Phi_w^{(2)}$. Having solved $\Phi_w^{(2)}$ the right-hand side of the boundary condition for $\Phi_d^{(2)}$ at the mean position of the hull of the vessel equation (IV-15) can be evaluated and $\Phi_d^{(2)}$ solved by the method described in Appendix A. Having solved both potentials $\Phi_w^{(2)}$ and $\Phi_d^{(2)}$ the contribution of these potentials to the second order forces can be evaluated by means of equation (IV-5).

The procedure given here indicates that for the second order potentials results can be obtained using numerical approximation techniques. It will be appreciated, however, that the computational effort to obtain results for this contribution will be considerable since the above procedure must be repeated for all relevant combinations of frequencies $\omega_1$ and $\omega_2$. Advantage can, however, be gained of the fact that the basic solution of $\Phi_d^{(2)}$ need only be obtained once for every series of combinations of $\omega_1$ and $\omega_2$ which yield the same difference frequency $\omega_1 - \omega_2$. For the present work a different, more simplified approach has been followed in order to approximate the contribution of the second order potential to the second order forces.

### IV.2.3.2. Approximation for the contribution of the second order potential

The approximation is based on the assumption that the major part of the low frequency second order force due to the second order potential is the wave exciting force component due to the contribution $\Phi_{ww}^{(2)}$ of the undisturbed incoming waves to the second order potential, thus assuming that the first order diffraction and body motion potentials $\Phi_d^{(1)}$ and $\Phi_b^{(1)}$ are small relative to the undisturbed wave potential $\Phi_w^{(1)}$. This means that in the right-hand side of the free surface boundary condition of equation (IV-13) only terms involving the first order velocity potential $\Phi_w^{(1)}$ of the undisturbed incoming waves remain. The second order potential which satisfies this boundary condition and the boundary condition at the sea floor of equation (III-29) as well as the equation of continuity (III-25) has been given by Bowers [IV-3]. We now consider a regular wave group travelling in the positive $X_1$-direction consisting of two regular waves with frequencies $\omega_i$ and $\omega_j$ with $\omega_i > \omega_j$. The first order velocity potential associated with these waves is:

$$\Phi_w^{(1)} = - \sum_{i=1}^{2} \frac{\zeta_i^{(1)} g}{\omega_i} \frac{\cosh k_i (X_3 + h)}{\cosh k_i h} \sin(k_i X_1 - \omega_i t + \underline{\varepsilon}_i) \qquad \text{(IV-17)}$$

The low frequency component of the second order velocity potential associated with these waves is as follows:

$$\Phi_{ww}^{(2)} = - \sum_{i=1}^{2} \sum_{j=1}^{2} \zeta_i^{(1)} \zeta_j^{(1)} A_{ij} \frac{\cosh\{(k_i - k_j)(X_3 + h)\}}{\cosh(k_i - k_j)h} \cdot$$

$$\cdot \sin\{(k_i - k_j)X_1 - (\omega_i - \omega_j)t + (\underline{\varepsilon}_i - \underline{\varepsilon}_j)\} \qquad \text{(IV-18)}$$

in which $A_{ij}$ is a coefficient depending on $\omega_i$, $\omega_j$ and the water depth h:

$$A_{ij} = \tfrac{1}{2} g^2 \frac{B_{ij} + C_{ij}}{(\omega_i - \omega_j)^2 - (k_i - k_j) g \tanh(k_i - k_j)h} \qquad \text{(IV-19)}$$

in which:

$$B_{ij} = \frac{k_i^2}{\omega_i \cosh^2 k_i h} - \frac{k_j^2}{\omega_j \cosh^2 k_j h} \qquad \text{(IV-20)}$$

$$C_{ij} = \frac{2 k_i k_j (\omega_i - \omega_j)(1 + \tanh k_i h \tanh k_j h)}{\omega_i \omega_j} \qquad \text{(IV-21)}$$

The low frequency component of this second order potential represents a long wave which is induced by the presence of the regular wave group. The phase of this long wave relative to the regular wave group is such that it has a trough where the wave group attains its maximum wave height. This is shown in Figure IV-1.

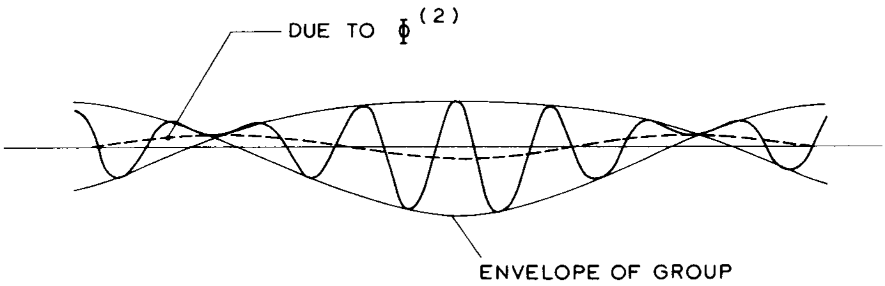



Fig. IV-1 Wave due to second order potential of a wave group.

The potential associated with such a wave does not satisfy the boundary condition on the body which for the simplified case is assumed to be equivalent to the normal first order boundary condition. This means that in the right-hand side of equation (IV-15) first order contributions are neglected.

As shown in chapter III the second order diffraction potential $\Phi_d^{(2)}$ satisfies the equation of continuity, the boundary condition at the sea floor, the radiation condition and the homogeneous free surface condition:

$$g\Phi_{x_3} + \Phi_{tt} = 0 \quad \text{(IV-22)}$$

This last condition gives rise to the well known dispersion relationship:

$$\omega^2 = kg \tanh kh \quad \text{(IV-23)}$$

The incoming waves due to the low frequency second order potential have a wave number equal to $k_i - k_j$ and wave frequency equal to

$\omega_i - \omega_j$. These waves do not obey the dispersion equation (IV-23).

If the incoming waves have a frequency of $\omega_i - \omega_j$ then the diffracted waves have the same frequency but the wave number will be according to the relationship:

$$(\omega_i - \omega_j)^2 = kg \tanh kh \quad \ldots\ldots\ldots\ldots \quad \text{(IV-24)}$$

In order to simplify the situation we allow the diffracted waves to have the same wave number $k_i - k_j$ as the incoming waves. This means that differences will occur in the diffracted waves further away from the vessel. Close to the body the situation will be similar since the boundary condition at the body still has to be satisfied. The reason for this alteration in wave number of the diffracted waves will be apparent from the following.

We have reduced the problem to the situation where we have to determine the wave exciting force on the body due to a wave which has a velocity potential as given by equation (IV-18) while we allow the diffracted waves to have the same wave number and frequency as the incoming waves. This is solved by considering the ordinary first order wave exciting force $F^{(1)}$ on the body in a regular wave with wave number k equal to $k_i - k_j$ in an ordinary gravity field with g as constant of gravity. For such a case the associated wave frequency $\omega$ will be in accordance with the dispersion relationship of equation (IV-23). The frequency of this wave can be made equal to the frequency $\omega_i - \omega_j$ of the second order waves by selecting a different value for the constant of gravity:

$$g_{ij} = \frac{(\omega_i - \omega_j)^2}{(k_i - k_j)\tanh(k_i - k_j)h} \quad \ldots\ldots\ldots \quad \text{(IV-25)}$$

Since the wave exciting force is proportional to the constant of gravity the initial force $F^{(1)}$ with wave with frequency $\omega$, which follows from equation (IV-23), becomes a second order force with frequency $\omega_i - \omega_j$ by simply applying the factor:

$$n_{ij} = \frac{g_{ij}}{g} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad \text{(IV-26)}$$

to the initial force. This does not complete the transformation, however, since, besides satisfying the requirement that wave number and wave frequency be equal, the amplitudes of the potentials must be equal. After the alteration of the constant of gravity the transformed potential of the first order regular wave is:

$$\Phi = - \frac{\zeta_a^{(1)} g_{ij}}{(\omega_i - \omega_j)} \frac{\cosh\{(k_i - k_j)(X_3 + h)\}}{\cosh(k_i - k_j)h} \sin\{(k_i - k_j)X_1 + - (\omega_i - \omega_j)t + (\underline{\varepsilon}_i - \underline{\varepsilon}_j)\} \qquad \text{(IV-27)}$$

The amplitude of the second order potential is given in equation (IV-18). Equality of the amplitudes means that:

$$\frac{\zeta_a^{(1)} g_{ij}}{(\omega_i - \omega_j)} = \zeta_i^{(1)} \zeta_j^{(1)} A_{ij} \qquad \text{(IV-28)}$$

This means that the first order wave amplitude must be selected so that:

$$\zeta_a^{(1)} = \zeta_i^{(1)} \zeta_j^{(1)} \frac{A_{ij}(\omega_i - \omega_j)}{g_{ij}} \qquad \text{(IV-29)}$$

The first order force $F^{(1)}$ is determined for a value of unity for $\zeta_a^{(1)}$. Since forces are proportional to the wave amplitude equation (IV-29) gives a second correction factor which has to be applied to the force $F^{(1)}$ in order to give the required second order force $F^{(2)}$:

$$F_{ij}^{(2)} = n_{ij} \frac{\zeta_i^{(1)} \zeta_j^{(1)} A_{ij}(\omega_i - \omega_j)}{g_{ij}} F^{(1)} \qquad \text{(IV-30)}$$

which taking into account equation (IV-22) gives:

$$F_{ij}^{(2)} = f_{ij} . F^{(1)} \qquad \text{(IV-31)}$$

where:

$$f_{ij} = \frac{\zeta_i^{(1)} \zeta_j^{(1)} A_{ij}(\omega_i - \omega_j)}{g} \qquad \text{(IV-32)}$$

### IV.2.3.3. Comparison between the exact results and the approximation

It can be shown that this method of approximation gives exact results in two simple cases and gives a reasonable approximation for a third, more practical, case.

The first case concerns the second order pressure due to the second order potential in undisturbed irregular waves in a point $X_3 = -a$ below the still water level. The second order pressure is:

$$p^{(2)} = -\rho\Phi_t^{(2)} \quad \text{(IV-33)}$$

For the low frequency component given in equation (IV-18) the amplitude of the pressure is:

$$p_{ij}^{(2)} = \rho\zeta_i^{(1)}\zeta_j^{(1)}A_{ij}(\omega_i - \omega_j)\,\frac{\cosh\{(k_i - k_j)(-a+h)\}}{\cosh(k_i - k_j)h} \quad \text{(IV-34)}$$

For the first order potential the pressure follows from:

$$p^{(1)} = -\rho\Phi_t^{(1)} \quad \text{(IV-35)}$$

The amplitude of the pressure using a first order potential component of the type given in equation (IV-17), unit wave amplitude $\zeta_a^{(1)}$ and wave number $k_i - k_j$ is:

$$p_a^{(1)} = \rho g\,\frac{\cosh\{(k_i - k_j)(-a+h)\}}{\cosh(k_i - k_j)h} \quad \text{(IV-36)}$$

Using the coefficient $f_{ij}$ given in equation (IV-32) gives the following approximation for the second order pressure amplitude:

$$p_{ij}^{(2)} = \rho\zeta_i^{(1)}\zeta_j^{(1)}A_{ij}(\omega_i - \omega_j)\,\frac{\cosh\{(k_i - k_j)(-a+h)\}}{\cosh(k_i - k_j)h} \quad \text{(IV-37)}$$

which equals the exact value given in equation (IV-34). The reason for this is that other contributions to the exact value which are neglected in the approximation (those due to diffraction and body motions) are in this case zero.

The second case concerns the horizontal low frequency wave drifting force, due to the second order potential, acting on a vertical wall in deep water. It can be shown that the approximation is also equal to the exact result in this case. The reason for this is that the first order incoming waves and the first order outgoing waves are identical (total reflection) and the low frequency component of the total second order potential consists of a contribution associated with the undisturbed incoming waves and a contribution due to the outgoing diffraction waves. Since the approximation gives the exact value for the second order potential associated with the incoming waves it also gives the exact value for the second order potential associated with the outgoing waves and hence the approximation is also the exact value.

The third example concerns the two-dimensional case of a free floating cylinder in beam waves as presented by Faltinsen and Løken [IV-4]. These authors solved the second order problem and gave numerical results on the contribution of the total second order potential to the low frequency second order sway force in regular wave groups in deep water. The method of approximation presented here was applied to the same case using results given by Vugts [IV-5] on the first order sway force in regular beam waves. The coefficient $f_{ij}$ of equation (IV-32) becomes for deep water:

$$f_{ij} = -\frac{\omega_i(\omega_i - \omega_j)}{g}\zeta_i^{(1)}\zeta_j^{(1)} \quad \ldots\ldots\ldots\ldots \quad \text{(IV-38)}$$

The results are presented in the form of the amplitude of the low frequency second order forces due to the second order potential for a range of combinations of $\omega_i$ and $\omega_j$ which are the frequencies of two waves making up a regular wave group. Faltinsen's results are compared with the approximation in Table IV-1.


FALTINSEN

| $\omega_j\sqrt{d/g}$ \ $\omega_i\sqrt{d/g}$ | 0.59 | 0.72 | 0.84 | 0.95 | 1.12 |
|---|---|---|---|---|---|
| 0.59 | 0 | 0.02 | 0.01 | 0.02 | 0.13 |
| 0.72 | | 0 | 0.03 | 0.01 | 0.10 |
| 0.84 | | | 0 | 0.03 | 0.04 |
| 0.95 | | | | 0 | 0.06 |
| 1.12 | | | | | 0 |

$\dfrac{F^{(2)}}{\rho g L \zeta_i^{(1)} \zeta_j^{(1)}}$

APPROXIMATION

| $\omega_j\sqrt{d/g}$ \ $\omega_i\sqrt{d/g}$ | 0.59 | 0.72 | 0.84 | 0.95 | 1.12 |
|---|---|---|---|---|---|
| 0.59 | 0 | 0.03 | 0.11 | 0.20 | 0.33 |
| 0.72 | | 0 | 0.03 | 0.11 | 0.26 |
| 0.84 | | | 0 | 0.04 | 0.18 |
| 0.95 | | | | 0 | 0.10 |
| 1.12 | | | | | 0 |

$\dfrac{F^{(2)}}{\rho g L \zeta_i^{(1)} \zeta_j^{(1)}}$

Table IV-1 Low frequency drifting forces on a cylinder in beam seas due to the second order potential.

Comparison of the results shows that near the line $\omega_i = \omega_j$ the approximation is good but for larger differences between $\omega_i$ and $\omega_j$ the approximation is considerably higher than Faltinsen's values. Further study will be required to determine the reason for the large differences which occur at higher frequencies. At the present, however, it may be tentatively concluded that the method of approximation gives the right order of magnitude to the low frequency forces due to the second order potential for difference frequencies which are not too large. For the cylinder in beam seas large differences between Faltinsen's results and the approximation occurred for values of the non-dimensional difference frequency greater than about 0.1.

For floating bodies the approximation will give best results when the contribution to the second order potential of the first order diffraction waves and waves due to body motions are negligible. This requirement is satisfied more by vessels such as semi-submersibles than by ordinary ship or barge shapes. It will be found, however, that when first order diffraction and body motion effects increase the influence of the first order contributions to the total second order forces will become dominant, so that the increase in the error of the component due to the second order potentials still remains small relative to the total force.

## IV.2.4. Symmetry of the quadratic transfer functions

The first order wave elevation in a regular wave group consisting of two regular waves with frequency $\omega_i$ and $\omega_j$ is as follows:

$$\zeta^{(1)}(t) = \sum_{i=1}^{2} \zeta_i^{(1)}.\cos(\omega_i t + \varepsilon_i)$$

$$= \zeta_1^{(1)}.\cos(\omega_1 t + \varepsilon_1) + \zeta_2^{(1)}.\cos(\omega_2 t + \varepsilon_2) \qquad \text{(IV-39)}$$

The second order force associated with such a wave train has the following form:

$$F^{(2)}(t) = \sum_{i=1}^{2}\sum_{j=1}^{2} \zeta_i^{(1)}\zeta_j^{(1)}P_{ij}.\cos\{(\omega_i - \omega_j)t + (\varepsilon_i - \varepsilon_j)\} +$$

$$+ \sum_{i=1}^{2}\sum_{j=1}^{2} \zeta_i^{(1)}\zeta_j^{(1)}Q_{ij}.\sin\{(\omega_i - \omega_j)t + (\varepsilon_i - \varepsilon_j)\}$$

$$= \zeta_i^{(1)^2}.P_{11} + \zeta_2^{(1)^2}.P_{22} +$$

$$+\zeta_1^{(1)}\zeta_2^{(1)}(P_{12}+P_{21}).\cos\{(\omega_1 - \omega_2)t + (\varepsilon_1 - \varepsilon_2)\} +$$

$$+\zeta_1^{(1)}\zeta_2^{(1)}(Q_{12}-Q_{21}).\sin\{(\omega_1 - \omega_2)t + (\varepsilon_1 - \varepsilon_2)\}$$

. . . . . . . . . . . . . . (IV-40)

From equation (IV-40) it is seen that the second order force contains two constant components. Each of these components represents the constant force which would be found if the wave train consisted of a single regular wave with frequency $\omega_1$ or $\omega_2$ respectively. This shows that, although the force is a non-linear phenomenon, the constant or mean second order force in a wave train consisting of a superposition of regular waves is the sum of the mean forces found for each of the component waves. The quadratic transfer function:

$$P_{11} = P(\omega_1,\omega_2) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad \text{(IV-41)}$$

gives the mean second order force in regular waves with frequency $\omega_1$. In literature dealing with the mean second order forces on floating objects in regular or irregular waves this is often expressed as a function dependent on one frequency $\omega_1$. The above equations show that the transfer function for the mean or constant part is, however, only a specific case of the general quadratic transfer function $P(\omega_1,\omega_2)$ for the force in regular wave groups.

Besides the constant parts the second order force contains low frequency parts with a frequency corresponding to the difference frequency $\omega_1 - \omega_2$ of the component regular waves. It is seen that the amplitudes of the in-phase and out-of-phase parts depend on the sum of the in-phase quadratic transfer functions $P_{12}$ and $P_{21}$ and the difference of the out-of-phase functions $Q_{12}$ and $Q_{21}$. From equation (IV-40) it is seen that the transfer functions do not appear in isolation but rather in pairs. In general, the in-phase and out-of-phase components of the quadratic transfer functions as determined from equations (IV-1) through (IV-4) for combinations of $\omega_1$ and $\omega_2$ will be so that, for instance:

$$P(\omega_1,\omega_2) \neq P(\omega_2,\omega_1) \quad \ldots\ldots\ldots\ldots\ldots \quad \text{(IV-42)}$$

However, since the force as given in equation (IV-40) depends on the sum or difference of the components of the quadratic transfer functions these may be so reformulated that the following symmetry relations are valid:

$$P(\omega_1,\omega_2) = P(\omega_2,\omega_1) \quad \ldots\ldots\ldots\ldots\ldots \quad \text{(IV-43)}$$

$$Q(\omega_1,\omega_2) = -Q(\omega_2,\omega_1) \quad \ldots\ldots\ldots\ldots\ldots \quad \text{(IV-44)}$$

The approximation for the force due to the second order potential $\Phi^{(2)}$ as given in equation (IV-31) is only defined for $\omega_i > \omega_j$. In order to conform with the definition given in equations (IV-43) and (IV-44) for the quadratic transfer function the in-phase and out-of-phase parts of the force component due to $\Phi^{(2)}$ become:

$$P_{ij} = \tfrac{1}{2}P_{ij}^{(2)} \qquad \omega_i > \omega_j \quad \ldots\ldots\ldots\ldots \quad \text{(IV-45)}$$

$$P_{ji} = P_{ij} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad \text{(IV-46)}$$

$$Q_{ij} = \tfrac{1}{2}Q_{ij}^{(2)} \qquad \omega_i > \omega_j \quad \ldots\ldots\ldots\ldots \quad \text{(IV-47)}$$

$$Q_{ji} = -Q_{ij} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad \text{(IV-48)}$$

In these equations $P_{ij}^{(2)}$ and $Q_{ij}^{(2)}$ represent the in-phase and out-of-phase components of the second order force as determined by means of equation (IV-31). This transformation was applied to the example concerning the free floating cylinder in beam waves. The in-phase component $P(\omega_1,\omega_2)$ of the quadratic transfer function of the total second order force takes the form of a matrix which is symmetrical about the diagonal for which $\omega_1$ is equal to $\omega_2$ while the out-of-phase component $Q(\omega_1,\omega_2)$ is asymmetrical about the diagonal. The second order force given in equation (IV-40) may also be written as follows:

$$F^{(2)}(t) = \zeta_1^{(1)^2}.P_{11} + \zeta_2^{(1)^2}.P_{22} + $$
$$+2\zeta_1^{(1)}\zeta_2^{(1)}.T_{12}.\cos\{(\omega_1-\omega_2)t + (\varepsilon_1-\varepsilon_2) + \varepsilon_{12}\}$$

. . . . . . . . . . . . . . (IV-49)

in which $T_{12}$ is the amplitude of the quadratic transfer function:

$$T_{12} = T(\omega_1,\omega_2) = \sqrt{P^2(\omega_1,\omega_2) + Q^2(\omega_1,\omega_2)} \quad . . . . \text{(IV-50)}$$

It also follows that:

$$T(\omega_1,\omega_2) = T(\omega_2,\omega_1) \quad . . . . . . . . . . . . . . . \text{(IV-51)}$$

$\varepsilon_{12}$ is a phase angle defined by:

$$\tan\varepsilon_{12} = \tan\varepsilon(\omega_1,\omega_2) = -\frac{Q(\omega_1,\omega_2)}{P(\omega_1,\omega_2)} \quad . . . . . . \text{(IV-52)}$$

$\varepsilon_{12}$ gives the phase angle of the low frequency part of the second order force relative to the low frequency part of the square of the wave elevation of equation (IV-39), which is as follows:

$$\zeta^{(1)^2}(t) = \sum_{i=1}^{2}\sum_{j=1}^{2} \tfrac{1}{2}\zeta_i^{(1)}\zeta_j^{(1)}.\cos\{(\omega_i-\omega_j)t + (\varepsilon_i-\varepsilon_j)\}$$
$$= \tfrac{1}{2}\zeta_1^{(1)^2} + \tfrac{1}{2}\zeta_2^{(1)^2} + \zeta_1^{(1)}\zeta_2^{(1)}.\cos\{(\omega_1-\omega_2)t +$$
$$+(\varepsilon_1-\varepsilon_2)\}$$

. . . . . . . . . . . . . . (IV-53)

## IV.3. Frequency domain representation of the mean and low frequency forces in irregular waves

We consider the case of a body floating in irregular waves which are characterized by a spectral density or energy density $S_\zeta(\omega)$ where:

$$S_\zeta(\omega).d\omega = \tfrac{1}{2}\zeta^{(1)^2} \quad \ldots \quad \text{(IV-54)}$$

The wave elevation in a point is a normally distributed process with zero mean and a probability density given by:

$$p(\zeta^{(1)}) = \frac{1}{\sqrt{2\pi m_0}} \cdot e^{-\frac{\zeta^{(1)^2}}{2m_0}} \quad \ldots \quad \text{(IV-55)}$$

where:

$$m_0 = \int_0^\infty S_\zeta(\omega).d\omega = \text{area of the spectrum} \quad \ldots \quad \text{(IV-56)}$$

The mean second order force is found by putting $\omega_i = \omega_j$ in equation (IV-9):

$$F^{(2)}_{1_{mean}}(t) = \sum_{i=1}^{N} \zeta_i^{(1)^2}.P_{ii} = \sum_{i=1}^{N} \zeta_i^{(1)^2}.P(\omega_i,\omega_i)$$

$$\ldots \quad \text{(IV-57)}$$

Passing from a discrete to a continuous formulation taking into account equation (IV-54) gives:

$$F^{(2)}_{1_{mean}}(t) = 2\int_0^\infty S_\zeta(\omega).P(\omega,\omega).d\omega \quad \ldots \quad \text{(IV-58)}$$

The spectral density of the low frequency components of the force in equation (IV-9) is found to be as follows:

$$S_F(\mu) = 8\int_0^\infty S_\zeta(\omega+\mu).S_\zeta(\omega).T^2(\omega+\mu,\omega).d\omega$$

$$\ldots \quad \text{(IV-59)}$$

where:

$$T^2(\omega+\mu,\omega) = P^2(\omega+\mu,\omega) + Q^2(\omega+\mu,\omega) \quad \ldots\ldots\ldots \quad \text{(IV-60)}$$

and:

| | |
|---|---|
| $\mu$ | = frequency of the low frequency second order force |
| $P(\omega+\mu,\omega)$, $Q(\omega+\mu,\omega)$ | = in-phase and out-of-phase components of the quadratic transfer function |
| $T(\omega+\mu,\omega)$ | = amplitude of the quadratic transfer function |
| $S_F(\mu)$ | = spectral density of the force. |

Besides knowledge of the mean and spectral density of the second order forces, knowledge of the distribution function of the force is of interest. In general, however, the distribution function of a second order force in irregular waves of the type as given by equation (IV-9) cannot be given. An indication can, however, be given of the type of probability distribution function involved by inspection of a quantity which is closely related to the second order force. This is the low frequency part of the square of the wave elevation as given in equation (IV-12). It can be shown (see ref. [IV-6]) that the low frequency part of the square of a normally distributed signal is of the exponential type with a probability density function:

$$p(\zeta_{low}^{(1)^2}) = \frac{1}{m_0} \cdot e^{-\frac{\zeta_{low}^{(1)^2}}{m_0}} \quad \ldots\ldots\ldots\ldots \quad \text{(IV-61)}$$

In this equation $m_0$ is the area of the wave spectrum given by equation (IV-56). The distribution function as given by equation (IV-61) is shown in Figure IV-2.

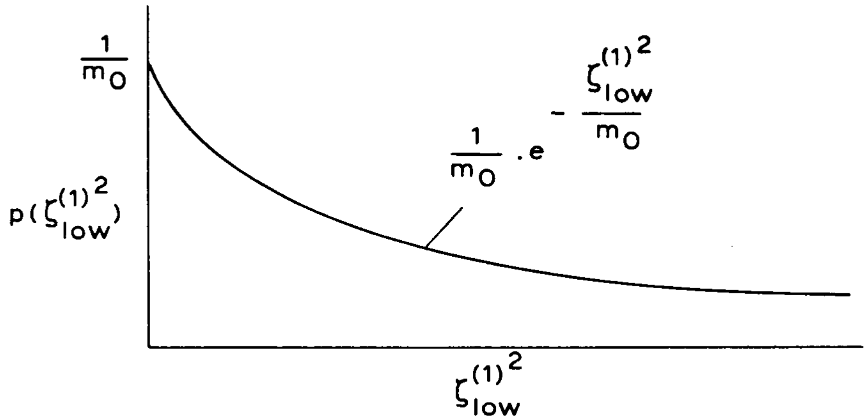



Fig. IV-2 Probability density of low frequency part of the square of the wave elevation.

## IV.4. Time domain representation of the mean and low frequency second order forces

According to Dalzell [IV-7] the low frequency second order forces can be computed given the quadratic transfer function and the time record of the wave elevation using the following relationship:

$$F^{(2)}(t) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} g^{(2)}(t_1,t_2).\zeta^{(1)}(t-t_1) \; . \; \zeta^{(1)}(t-t_2).dt_1.dt_2 \qquad \text{(IV-62)}$$

where:

$g^{(2)}(t_1,t_2)$ = quadratic impuls response function

$\zeta^{(1)}(t)$ = time dependent wave elevation

$t_1,t_2$ = time shifts.

The quadratic impuls response function $g^{(2)}(t_1,t_2)$ is derived from the following expression:

$$g^{(2)}(t_1,t_2) = \frac{1}{2\pi^2} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} e^{(i\omega_1 t_1 - i\omega_2 t_2)} \cdot$$

$$\cdot\ G^{(2)}(\omega_1,\omega_2).d\omega_1.d\omega_2$$

$$\ldots \ldots \ldots \ldots \quad \text{(IV-63)}$$

in which:

$G^{(2)}(\omega_1,\omega_2)$ = complex quadratic transfer function

$= P(\omega_1,\omega_2) + iQ(\omega_1,\omega_2)$ . . . . . . . . . . . . (IV-64)

and:

$P(\omega_1,\omega_2)$, $Q(\omega_1,\omega_2)$ = in-phase and out-of-phase components of the quadratic response function.

From equation (IV-62) it is seen that if the quadratic impuls response function $g^{(2)}(t_1,t_2)$ is known the time record of the low frequency second order forces can be computed for arbitrary wave elevation records. The applicability of this technique has been demonstrated extensively and convincingly by Dalzell [IV-7] using quadratic transfer functions for the second order forces obtained from tests in irregular waves using cross-bi-spectral analysis techniques.

## IV.5. Conclusions

In this chapter it is shown that the mean and low frequency second order forces and moments may be expressed in the frequency domain in terms of quadratic transfer functions which are dependent on two frequencies. Physically, the quadratic transfer functions contain information on the mean and amplitude of the low frequency second order forces on the body floating in regular wave groups consisting of two regular waves. It has been indicated how these quadratic transfer functions are evaluated and an approximation for the contribution due to second order potential effects has been discussed. Dalzell [IV-7] has demonstrated that on the basis of the quadratic transfer functions time records of the second order forces can be generated. In this study therefore attention will be focussed on the frequency domain results only. If such results can be accurately predicted by computations then time domain results, which are often required for simulations, can also be generated with good accuracy.

# V. COMPARISON BETWEEN RESULTS OF COMPUTATIONS AND ANALYTICAL RESULTS ON THE MEAN WAVE DRIFT FORCE IN REGULAR WAVES

## V.1. Introduction

In chapter III and chapter IV the theory and method of computation of the mean and low frequency second order wave drift forces on floating objects through direct integration of pressure has been treated. In this section results of computation of the mean wave drift forces in regular waves will be compared with analytical results obtained by Kudou [V-1] who made use of an expression given by Maruo [V-2]. This expression is based on energy and momentum considerations. The results concern the three-dimensional case of a free floating hemisphere in infinitely deep water. In order to make a complete comparison results of first order wave loads, added mass and damping and motions are also compared.

## V.2. Computations

### V.2.1. General

For the computations the mean wetted surface of the body is approximated by 206 facets as shown in Figure V-1.

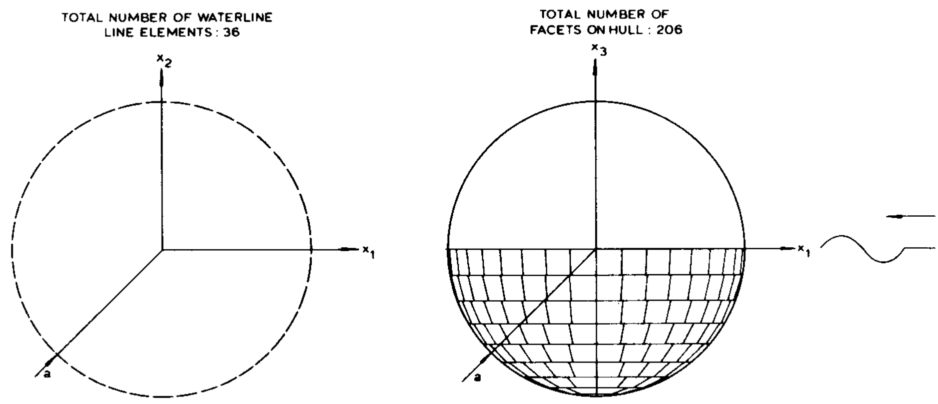



Fig. V-1 Distribution of facets on the sphere and the distribution of line elements on the waterline.

For the computation of the second order force component dependent on the relative wave elevation the waterline is approximated by 36 line elements. The relative wave elevation is computed in the centre of each line element. The first order wave loads, added mass and damping and first order motions in long-crested regular waves are computed in accordance with the method given in Appendix A. The computations cannot be carried out for infinitely deep water due to restriction of a numerical nature. For this case the water depth was different for each wave frequency. In all cases the water depth was larger than one wave length. The effect of the restriction in water depth then becomes negligible and results are comparable to deep water results. Having solved the first order problem the mean second order wave drift forces are calculated according to chapter IV. In regular waves only the four components given in equations (IV-1) through (IV-4) give contributions to the mean second order forces. The contribution of equation (IV-5) due to second order potentials is always zero in regular waves. See Salvesen [V-3]. In Figures V-2 and V-3 the transfer functions for the amplitudes of the first order wave exciting forces in surge and heave are compared. The computed values of the phase angles are also given. No analytical values were available. The phase angle is positive when the quantity under consideration reaches its maximum positive value before the crest of the undisturbed incoming wave passes the mean position of the centre of the hemisphere. The added mass and damping coefficients in surge and heave are given in Figures V-4 and V-5. The first order surge and heave motion transfer functions including the computed phase angles are given in Figures V-6 and V-7, while the mean second order horizontal drift force is given in Figure V-8. The components of the computed mean horizontal drift force are given in Figure V-9. The numerals refer to the contributions given in equations (IV-1) through (IV-4). In Figure V-10 the total and the components of the computed mean vertical drift forces are given. No analytical results have been given by Kudou on this quantity. The results in these figures are given to a base of the product of wave number k and radius a of the sphere.

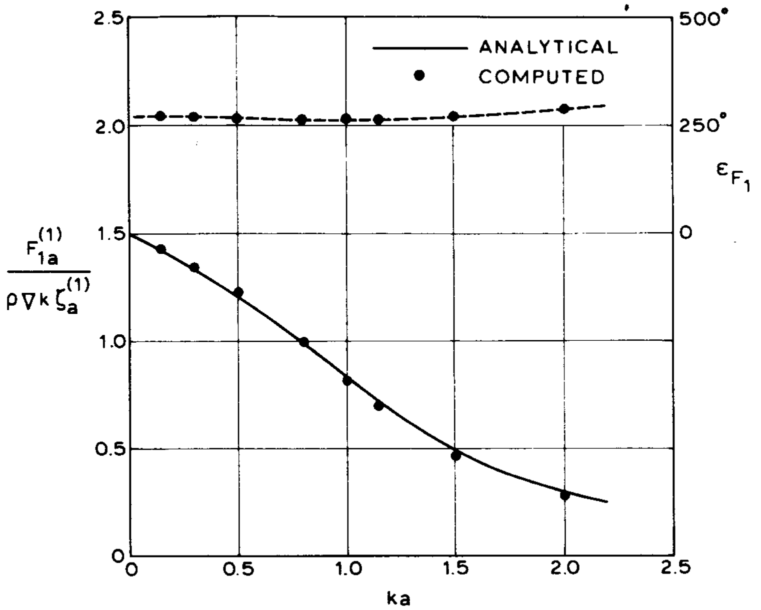



Fig. V-2  First order wave exciting force in surge.

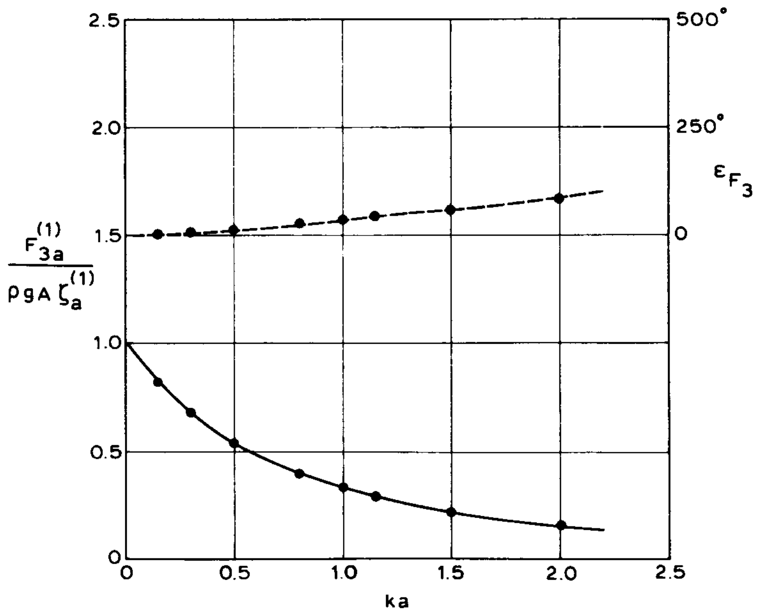



Fig. V-3  First order wave exciting force in heave.

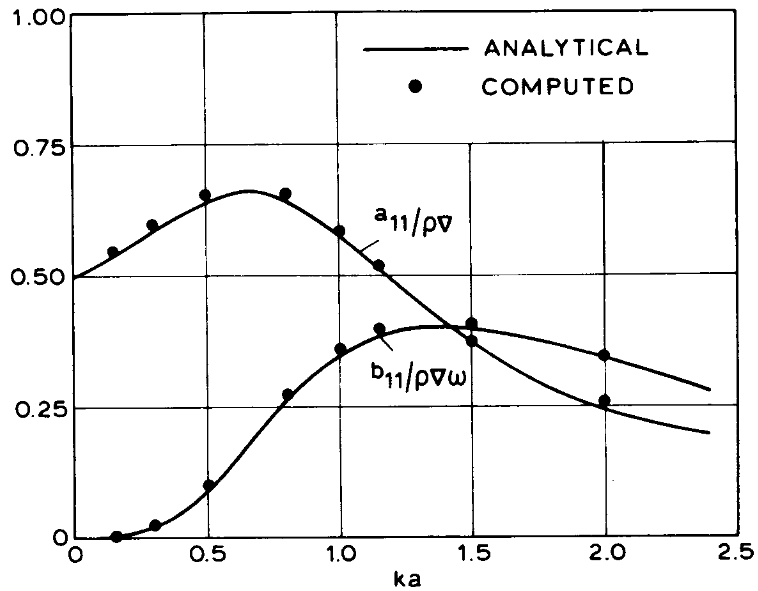



Fig. V-4  Added mass and damping coefficient for surge.

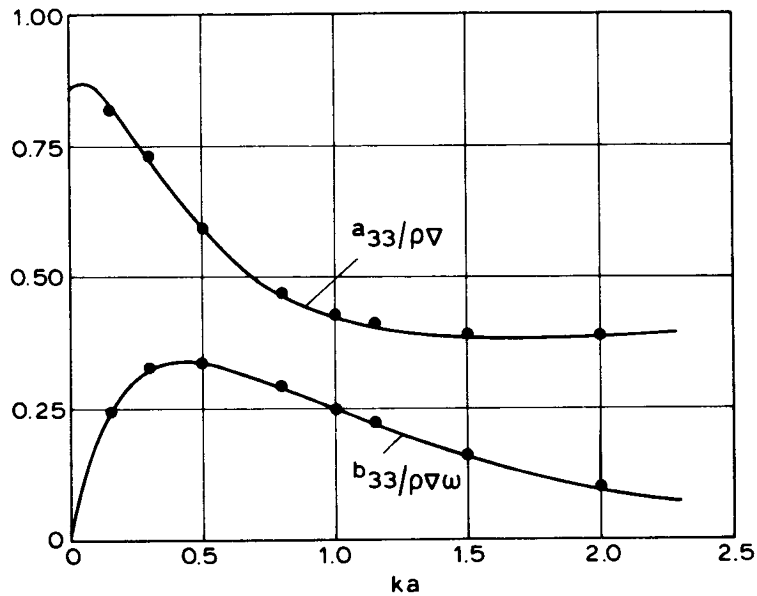



Fig. V-5  Added mass and damping coefficient for heave.

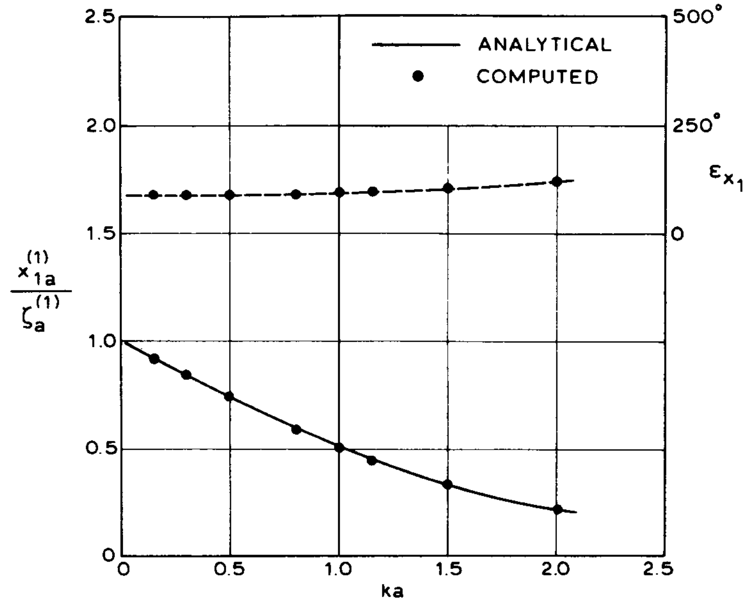



Fig. V-6 First order surge motion transfer function.

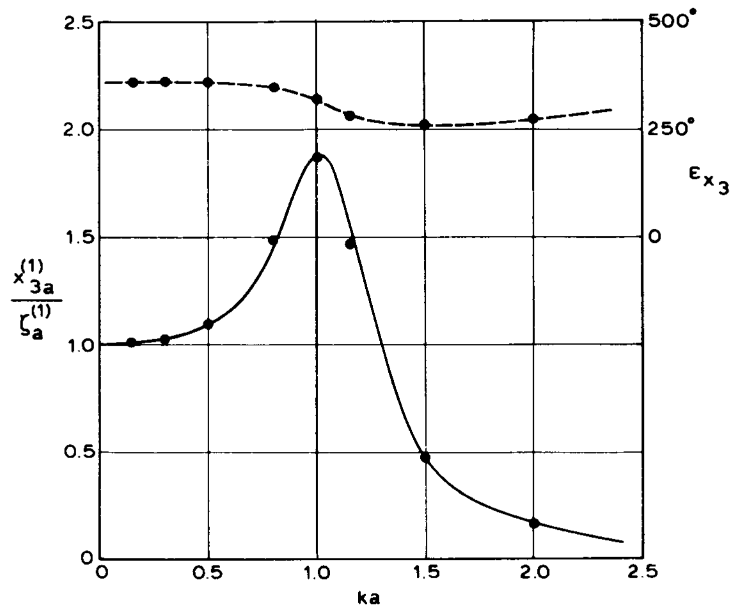



Fig. V-7 First order heave motion transfer function.

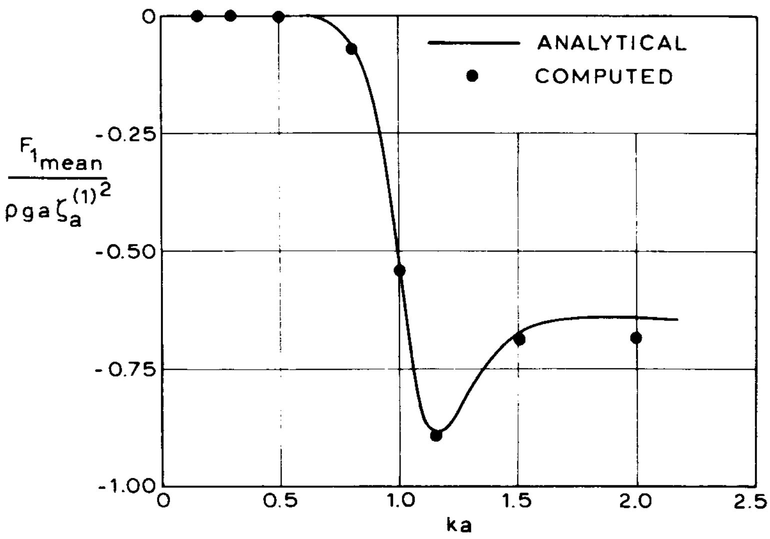



Fig. V-8 Mean second order horizontal drift force.

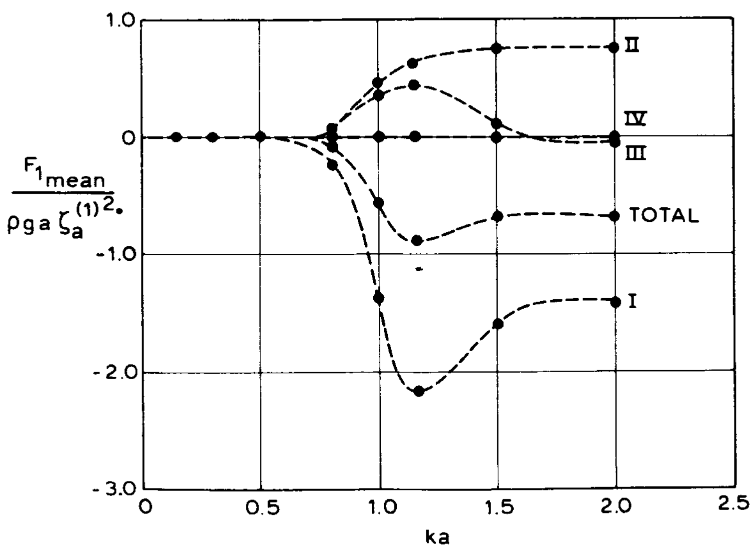



Fig. V-9 Components of the computed mean second order horizontal drift force.

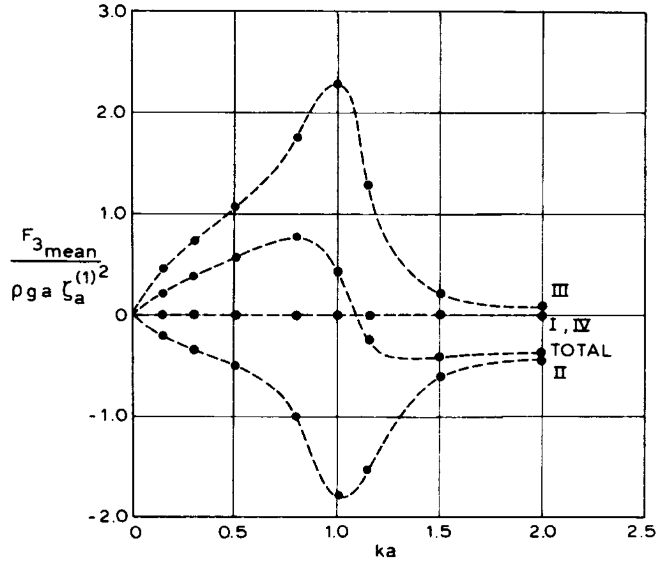



Fig. V-10 Components of the computed mean second order vertical force.

### V.2.2. Motions and mean horizontal drift force

From these results it is seen that the computed results of both first and second order quantities agree well with analytical results. When viewing the general characteristics of the mean horizontal drift force given in Figure V-8 it is seen that for low wave frequencies the force is zero. In this condition the sphere has a first order heave amplitude equal to the wave amplitude as seen in Figure V-7. The sphere is following the wave motion completely without creating noticeable disturbance. As the wave frequency increases the heave motion increases and at the same time the mean horizontal drift force increases. The maximum heave motion occurs at a slightly lower wave frequency than the maximum horizontal drift force. In this range of frequencies the effects due to diffraction and body motions on the wave drift forces are increasing. At higher frequencies the body motions decrease continuously to become zero at frequencies tending to infinity. In the range of these frequencies the effects of body motions on the drift force decrease rapidly and in the limit only effects due to diffraction remain. As can be seen from Figure V-8 the mean drift force at high frequencies is of the same order of magnitude as the peak value and at increasing frequency tends to some limit.

The value of the mean horizontal drift force for wave frequencies tending to infinity is easily found by taking into consideration that for very high wave frequencies the wave action, due to the very small wave length, is confined to a thin layer near the waterline of the stationary object. In this case the form of the hull may be replaced by a vertical wall which totally reflects the short waves. The circumference of the waterline may be considered as short sections of straight vertical walls by which the incoming wave is reflected as is shown in Figure V-11.

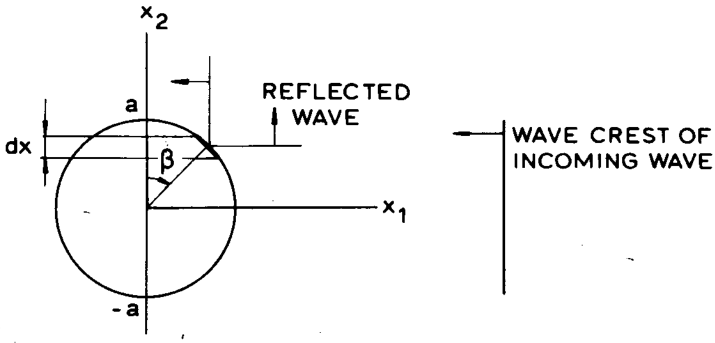



Fig. V-11 The mean horizontal drift force in short waves.

The contribution of such a section of the circumference of the waterline to the mean horizontal drift force is due to components I and II of equations (IV-1) and (IV-2) and is found to be as follows:

$$dF_{1_{mean}} = -\tfrac{1}{2}\rho g \zeta_a^{(1)^2} . (\sin^2 \beta) . dX_2 \quad \ldots\ldots\ldots\ldots \quad (V-1)$$

The total mean force is found to be:

$$F_{1_{mean}} = -\tfrac{1}{2}\rho g \zeta_a^{(1)^2} . \int_{-a}^{+a} (\sin^2 \beta) . dX_2 \quad \ldots\ldots\ldots \quad (V-2)$$

This equation has been given previously by a number of authors. See for instance Maruo [V-2]. It is applicable to floating objects of arbitrary shape for determining the high frequency limit of the horizontal wave drifting forces. Taking into account that at high frequencies there are no waves behind the body the following results are found for the sphere:

$$F_{1_{mean}} = -\frac{2}{3}\rho g a \zeta_a^{(1)^2} \quad \ldots\ldots\ldots\ldots\ldots \quad (V\text{-}3)$$

which means that the high frequency limit of the non-dimensional force given in Figure V-8 equals $-\frac{2}{3}$.

V.2.3. Components of the mean horizontal drift force

The computed components I through IV of the mean horizontal drift force are given in Figure V-9. These components cannot be compared with analytical results since the analytical results given by Kudou were determined using a different theory and are given only as a total force. Inspection of Figure V-9 reveals that the total force consists of components which differ considerably both in sign and magnitude. Contribution I, due to the relative wave elevation, is about twice as large and of the same sign as the total force. Contribution II, due to the square of the velocity, is different in sign and of about the same magnitude as the total. Contribution III is smaller and different in sign to the total force and also different in character since it tends to zero at high frequencies. Contribution IV, due to products of angular motions and accelerations, remains zero for all frequencies. At high wave frequencies only contribution I and II remain. This is in agreement with the previous discussion on the high frequency limit of the total force. It is of interest to discuss some aspects of the various components.

Contribution I

The general equation for this contribution is as follows:

$$F_{I_{mean}} = -\int_{WL} \tfrac{1}{2}\rho g \zeta_r^{(1)^2} . \bar{n} . d\ell_{mean} \quad \ldots\ldots\ldots\ldots \quad (V\text{-}4)$$

The integrand in this case always represents a pressure increase acting inwardly at the waterline. The sign of this contribution in Figure V-11 is in the direction of propagation of the waves indicating that, as may be expected, the relative wave elevation on the incoming wave side is larger than on the shadow side of the sphere. In general, therefore, if it is seen that an object floating in waves exhibits a large difference in relative wave height on both sides of the object, it may be expected that this contribution will be large.

Contribution II

The general equation for this contribution is as follows:

$$F_{II_{mean}} = -\iint_{S_0} -\tfrac{1}{2}\rho|\overline{\nabla}\Phi^{(1)}|^2.\overline{n}.dS_{mean} \quad \ldots\ldots\ldots \quad \text{(V-5)}$$

The integrand in this case always represents a pressure decrease acting outwardly on the mean wetted surface of the hull. In general, the fluid velocity $\overline{\nabla}\Phi^{(1)}$ tends to be largest on the incoming wave side. This results in a mean force component which contrary to intuition, is directed <u>into</u> the waves.

Contribution III

The general equation in this case is:

$$F_{III_{mean}} = -\iint_{S_0} -\rho(\overline{X}^{(1)}.\overline{\nabla}\Phi_t^{(1)}).\overline{n}.dS_{mean} \quad \ldots\ldots\ldots \quad \text{(V-6)}$$

Since this is a mixed product of first order motion and pressure gradient it is not possible, in general, to predict the sign of this quantity. The sign depends on the phase angles of both quantities. In the case of the sphere the mean force due to this component was also directed <u>into</u> the waves. This force component is dependent on the motions of the body and on pressure gradients. At very high wave frequencies the motions and hence this component tend to zero, while at very low frequencies it is the pressure gradients which tend to zero and consequently the force also.

Contribution IV

The general equation for this component is:

$$F_{IV_{mean}} = \overline{\alpha}^{(1)} \times (M.\ddot{\overline{X}}_g^{(1)})_{mean} \qquad \text{(V-7)}$$

As may be seen this component depends on angular motions and body accelerations. In the case of the sphere no angular motions occur and hence this force component is zero at all frequencies. In general, at high wave frequencies, motions cease and as a result this contribution tends to zero. At very low frequencies accelerations tend to zero and consequently this component also. At intermediate frequencies the sign of this force component is not generally known since it is a product of two quantities. The sign in such cases depends on the phase angles of the first order quantities involved.

V.2.4. Mean vertical drift force

The computed value of the mean vertical drift force is given in Figure V-10 along with its components. In this case the component due to the relative wave elevation is always zero. This is due to the fact that at the waterline the body lines run vertically. As is also the case with the mean horizontal force, the component due to the product of angular motions and body accelerations is zero. The component due to the square of the first order velocity acts vertically downwards while the component due to the product of motion and pressure gradient acts upwards. The total mean force is vertically upwards for wave frequencies below the peak in the vertical motions and downwards for wave frequencies above the peak in the vertical motions. For low wave frequencies the mean vertical force tends to zero due to the fact that velocities and pressure gradients tend to zero. At high wave frequencies the wave action is confined to a thin layer at the waterline. Since at the waterline the body lines run vertically no mean vertical force component is generated.

## V.3. Conclusions

From the agreement obtained between numerical and analytical results it may be concluded that the potential accuracy of numerical methods is sufficient for the prediction of both first order oscillatory quantities and the mean second order drift forces.

The agreement between results obtained using the method of direct integration of pressure and the results given by Kudou based on Maruo's theory demonstrates the equivalence of these formulations with respect to the total force.

Inspection of the various contributions to the mean second order force reveals that the total force consists of contributions which differ both in sign and magnitude. From the results found for the mean horizontal force on the sphere it is seen that the general characteristics of the total force are also present in contribution I due to the relative wave elevation (see Figure V-9) although contribution I is greater than the total by about a factor 2.

In the next section, in which experimental data on the mean horizontal drift forces on stationary vessels are compared with computed results, attention will also be paid to the relative importance of the various contributions.

# VI. COMPARISON BETWEEN COMPUTATIONS AND MEASUREMENTS OF THE MEAN SECOND ORDER FORCE IN REGULAR WAVES

## VI.1. Introduction

In this chapter experimental results on the first order oscillatory motions and mean second order forces and moments for different vessel forms floating in regular waves will be compared with results of computations. The purpose of this comparison is to demonstrate the validity of the theory with respect to physical reality. For this purpose the following vessels were selected:

- a tanker;
- a semi-submersible;
- a rectangular barge;
- a submerged horizontal cylinder.

Main particulars and body plans of these vessels are given in Table VI-1 and in Figure VI-1.

The hull forms of the first three vessels encompass the majority of vessels in use by the offshore industry either as permanent storage vessels, drill ships, drilling platforms, semi-submersible crane vessels, derrick barges and lay barges. For these vessels the first order motions and mean horizontal drift forces and yawing moment were determined for a range of wave frequencies and three wave directions, i.e. head waves ($180^{\circ}$), bow quartering waves ($135^{\circ}$) and beam waves from the starboard side ($90^{\circ}$).

The submerged horizontal cylinder is representative of a submersible hovering just below the surface or of horizontal submerged elements of a semi-submersible. For this vessel the first order vertical motions and the mean second order vertical forces were determined for a range of wave frequencies and the same wave directions as mentioned above.

| Designation | Symbol | Unit | Tanker | | Semi-submersible | | Rectangular barge | | Submerged horizontal cylinder | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | Tested | Calculated | Tested | Calculated | Tested | Calculated | Tested | Calculated |
| Scale ratio | - | - | 1:82.5 | | 1:40 | | 1:50 | | 1:30 | |
| Length (between perpendiculars) | $L_{PP}$ | m | 310.00 | | 100.00 | | 150.00 | | 75.60 | |
| Breadth | B | m | 47.17 | | 76.00 | | 50.00 | | 8.40 | |
| Draft | T | m | 18.90 | | 20.00 | | 10.00 | | 12.60* | |
| Displacement volume | $\nabla$ | $m^3$ | 234,826 | | 35,925 | | 73,750 | | 4,034 | |
| Centre of gravity above base | $\overline{KG}$ | m | 13.32 | | 8.64 | 7.92 | 10.00 | | 4.20 | |
| Metacentric height | $\overline{GM}$ | m | 5.78 | | 16.76 | 17.48 | 16.23 | | 0.00 | |
| Transverse gyradius in air | $k_{xx}$ | m | - | 14.77 | - | 30.55 | - | 20.00 | 2.94 | 2.94 |
| Transverse gyradius in water | $k_{xx}$ | m | 17.02 | 17.02 | 43.47 | 46.60 | 24.16 | 24.20 | - | - |
| Longitudinal gyradius in air | $k_{yy}$ | m | 77.47 | 77.50 | 30.89 | 30.89 | 39.00 | 39.00 | 16.80 | 16.80 |
| Vertical gyradius in air | $k_{zz}$ | m | - | 79.30 | - | 41.74 | - | 39.00 | 16.80 | 16.80 |
| Natural period of heave | $T_z$ | sec. | 11.8 | 11.7 | 21.3 | 21.8 | - | 10.4 | - | - |
| Natural period of roll | $T_\phi$ | sec. | 14.2 | 14.2 | 21.3 | 22.4 | 12.0 | 12.1 | - | - |
| Natural period of pitch | $T_\Theta$ | sec. | 10.8 | 10.6 | 19.5 | 19.8 | 9.3 | 9.4 | - | - |
| Water depth | Wd | m | 82.5 | | 40.0 | | 50.0 | | 75.0 | |

* Distance between base and mean still water surface.

Table VI-1 Main particulars and stability data.

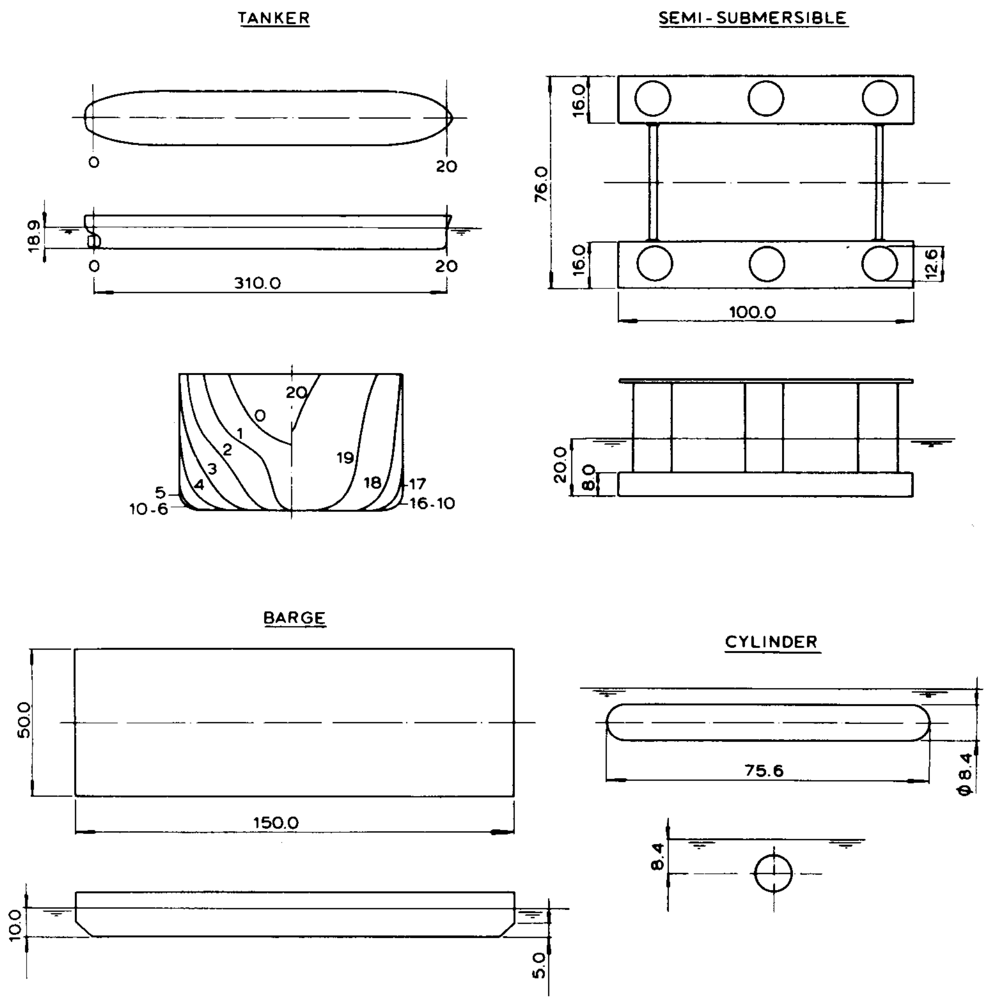



Fig. VI-1 The vessels.

## VI.2. Model tests

### VI.2.1. General

In order to measure the mean second order wave drift forces on the model of a vessel in regular waves, attention has to be paid to the measuring system employed. This generally takes the form of a mooring system in which force transducers are incorporated which measure the mooring force necessary to keep the vessel in a mean position. The mean mooring force is equated to the mean second order drift force. In chapter II and chapter III it was shown that the second order forces are dependent on first order quantities. First order quantities are, for instance, the body motions with frequency equal to the wave frequency.

If it is required to measure the mean second order forces on a free floating vessel the mooring system must be such that the first order motion, etc. are not influenced by the system. In theory this requires that the only force which the mooring system applies to the vessel is a constant force equal and opposite to the mean second order wave drift force.

The mooring system shown in Figure VI-2(a) makes use of a falling weight to counteract the mean wave drift forces.

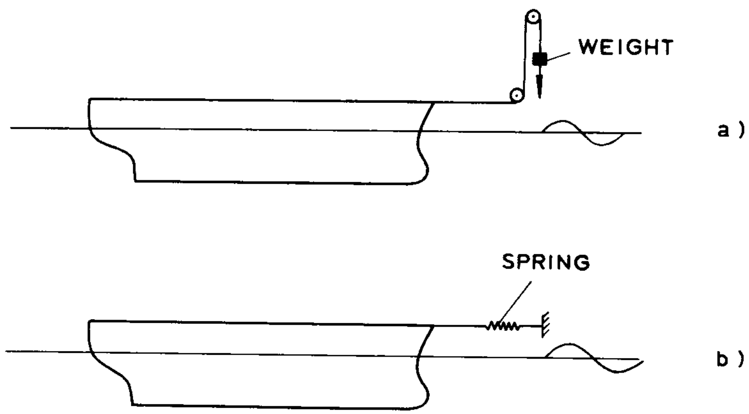



Fig. VI-2(a) and 2(b) Systems of restraint.

This system has characteristics which conform well with the above given requirement and it has been used by several investigators in the past. This system, being of the constant tension type, has certain disadvantages, however. In a test in regular waves the falling weight has to be selected so that the model remains in the same mean position. If the weight is not quite correct, or the wave amplitude varies slightly in time, the equilibrium of forces is upset and the model will commence to drift away from the mean position. Since this mooring system is of the constant tension type the drift motion will not be checked unless the weight is altered. In practice it appears to be difficult to find the correct value for the weight due to this effect.

A mooring system which is used extensively is the soft spring mooring system shown in Figure VI-2(b). This system was also employed for the model tests of which results are given in this chapter. In this case the vessel is moored by means of lines incorporating soft springs. In regular waves this system applies a force which contains a constant part equal to the mean second order drift force and an oscillating part commensurate with the first order motions and the spring characteristics of the mooring system. This oscillating part may modify the first order motions if the spring is too stiff. This in turn affects the mean second order force. In practice the spring constants are chosen so that the natural frequencies of the motions of the vessel, which are induced by the presence of the springs in the mooring system, are sufficiently far removed from the frequencies of interest of the regular waves. Generally, spring constants are chosen such that the natural frequencies induced by the mooring system are in the order of 5 times lower than the frequencies of the waves. This ensures that the effect of the mooring system on the first order motions and hence the mean second order forces will be negligible.

Once the method of restraint for the model has been chosen, the question arises as to where the force in the mooring system must be measured in order to obtain the correct value of the mean second order forces. In general, three positions of the force transducers are possible, i.e.: in the mooring line, fixed to the vessel and fixed to the mooring point. The three positions are shown in Figure VI-3(a). In this figure only one mooring line is shown.

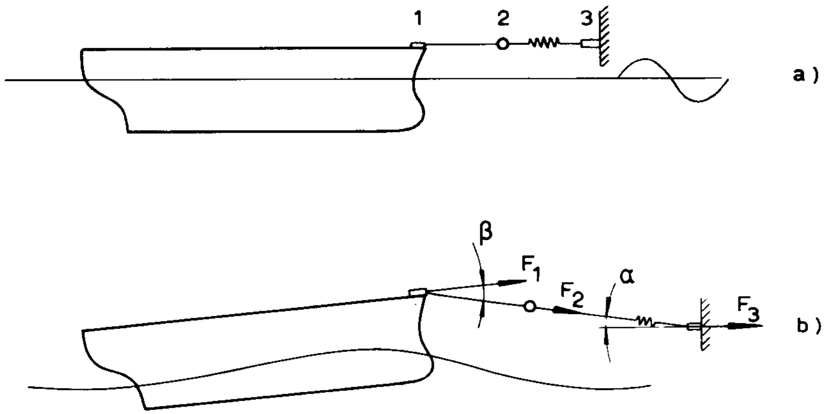



Fig. VI-3(a) and 3(b) Drift force measuring set-up.

This situation applies to a vessel for which it is required to determine the mean horizontal drift forces. The mooring line runs horizontally from the vessel to the mooring point with the vessel in the rest position. The three force transducers shown in the figure are:

- a force transducer measuring the longitudinal force component of the mooring line force relative to the body axes (transducer 1);
- a force transducer measuring the tension in the mooring line (transducer 2);
- a force transducer measuring the horizontal component of the mooring line force relative to an earth-bound system of axes (transducer 3).

Since it is required to determine the horizontal drift force the true force will be measured by transducer 3.

In order to determine the validity of measuring the force by transducer 1 or transducer 2 we assume that the vessel is carrying out oscillatory motions which induce the angles α and β between the mooring line and the horizontal plane and the longitudinal axis of the vessel (see Figure VI-3(b)). If the force in the mooring line is measured by transducer 2 is $F_2$, then the relationship between this force and the force $F_3$ is:

$$F_3 = F_2 \cos \alpha \quad \ldots \ldots \ldots \quad \text{(VI-1)}$$

or:

$$F_2 = F_3 \frac{1}{\cos \alpha} \quad \ldots \ldots \ldots \quad \text{(VI-2)}$$

Assuming that the angle $\alpha$ is small we may write:

$$\cos \alpha \approx 1 - \tfrac{1}{2}\alpha^2 \quad \ldots \ldots \ldots \quad \text{(VI-3)}$$

from which it follows that:

$$F_2 \approx F_3(1 + \tfrac{1}{2}\alpha^2) \quad \ldots \ldots \ldots \quad \text{(VI-4)}$$

From this it is seen that the error in $F_2$ is in the order of $F_3 \cdot \tfrac{1}{2}\alpha^2$. Normally the angle $\alpha$ will be of the same order as the angular motions of the vessel (pitch, roll and yaw) which, except for the case of resonant roll motions, will normally have amplitudes less than about 0.1 rad. Assuming this value of $\alpha$ the error in $F_2$ will, at most, be of the order of $F_3 \times 0.005$ or about one half percent of $F_3$. In the same way it can be shown that the error in $F_1$ is of about the same magnitude from which it can be concluded that the forces may be measured in any of the three mentioned positions provided the angular motions of the vessel are not large.

### VI.2.2. Model test conditions

Model tests were carried out in regular waves for three directions, i.e. head waves, bow quartering waves and beam waves for a range of regular wave frequencies. The test conditions with respect to the wave direction, wave frequencies and wave amplitudes are given in Table VI-2 through Table VI-5.

| Wave frequency | | Wave amplitude $\zeta_a$ in metres | | |
|---|---|---|---|---|
| $\omega$ in rad./sec. | $\omega\sqrt{\frac{\nabla^{1/3}}{g}}$ | Wave direction 90 degrees | Wave direction 135 degrees | Wave direction 180 degrees |
| 0.178 | 0.446 | 2.14 | 2.14 | 2.14 |
| 0.267 | 0.669 | 1.21 | 1.21 | 1.21 |
| 0.267 | 0.669 | 2.11 | 2.11 | 2.11 |
| 0.267 | 0.669 | 2.86 | 2.86 | 2.86 |
| 0.357 | 0.895 | 1.86 | 1.86 | 1.86 |
| 0.443 | 1.110 | 1.23 | 1.23 | 1.23 |
| 0.443 | 1.110 | 1.98 | 1.98 | 1.98 |
| 0.443 | 1.110 | - | 2.83 | 2.83 |
| 0.532 | 1.333 | 2.02 | 2.02 | 2.02 |
| 0.623 | 1.562 | 2.19 | 2.19 | 2.19 |
| 0.713 | 1.787 | 1.20 | 1.20 | 1.20 |
| 0.713 | 1.787 | 2.06 | 2.06 | 2.06 |
| 0.713 | 1.787 | - | 2.76 | 2.76 |
| 0.804 | 2.016 | 2.11 | 2.11 | 2.11 |
| 0.887 | 2.223 | 1.76 | 1.76 | 1.76 |

Table VI-2 Test conditions for model tests with a tanker.

| Wave frequency | | Wave amplitude $\zeta_a$ in metres | | |
|---|---|---|---|---|
| $\omega$ in rad./sec. | $\omega\sqrt{\frac{\nabla^{1/3}}{g}}$ | Wave direction 90 degrees | Wave direction 135 degrees | Wave direction 180 degrees |
| 0.300 | 0.550 | - | - | 2.61 |
| 0.300 | 0.550 | - | - | 4.49 |
| 0.300 | 0.550 | - | - | 5.09 |
| 0.455 | 0.834 | 0.97 | 0.97 | 1.02 |
| 0.455 | 0.834 | 2.15 | 2.15 | 1.96 |
| 0.455 | 0.834 | 3.18 | 3.18 | 2.79 |
| 0.525 | 0.963 | 0.91 | 0.91 | 0.98 |
| 0.630 | 1.155 | 1.01 | 1.01 | 1.02 |
| 0.630 | 1.155 | 1.99 | 1.99 | 1.97 |
| 0.630 | 1.155 | 2.83 | 2.83 | - |
| 0.727 | 1.333 | 1.10 | 1.10 | - |
| 0.785 | 1.440 | - | - | 1.03 |
| 0.805 | 1.476 | 1.18 | 1.18 | - |
| 0.910 | 1.668 | 1.14 | 1.14 | - |
| 0.920 | 1.687 | - | - | 1.04 |
| 0.920 | 1.687 | - | - | 1.76 |
| 1.047 | 1.920 | - | - | 0.98 |
| 1.047 | 1.920 | - | - | 1.61 |
| 1.060 | 1.944 | 1.04 | 1.04 | - |
| 1.168 | 2.142 | 0.94 | 0.94 | - |
| 1.200 | 2.200 | - | - | 0.78 |
| 1.200 | 2.200 | - | - | 0.81 |
| 1.278 | 2.344 | 0.98 | 0.98 | - |
| 1.339 | 2.455 | 0.80 | 0.80 | - |
| 1.350 | 2.476 | - | - | 0.64 |
| 1.398 | 2.564 | 0.83 | 0.83 | - |
| 1.570 | 2.879 | - | - | 0.43 |

Table VI-3 Test conditions for model tests with a semi-submersible.

| Wave frequency | | Wave amplitude $\zeta_a$ in metres | | |
|---|---|---|---|---|
| $\omega$ in rad./sec. | $\omega\sqrt{\frac{\nabla^{1/3}}{g}}$ | Wave direction 90 degrees | Wave direction 135 degrees | Wave direction 180 degrees |
| 0.300 | 0.620 | 0.81 | 0.81 | 0.81 |
| 0.400 | 0.827 | 0.94 | 1.07 | 1.07 |
| 0.500 | 1.033 | 0.96 | 0.96 | 0.96 |
| 0.600 | 1.241 | 0.98 | 0.98 | 0.98 |
| 0.700 | 1.447 | 1.09 | 1.09 | 1.09 |
| 0.800 | 1.654 | 1.16 | 1.16 | 1.16 |
| 0.900 | 1.861 | 1.08 | 1.08 | 1.08 |
| 1.000 | 2.067 | 0.94 | 0.94 | 0.94 |
| 1.100 | 2.274 | 0.79 | 0.79 | 0.79 |

Table VI-4 Test conditions for model tests with a rectangular barge.

| Wave frequency | | Wave amplitude $\zeta_a$ in metres | | |
|---|---|---|---|---|
| $\omega$ in rad./sec. | $\omega\sqrt{\frac{\nabla^{1/3}}{g}}$ | Wave direction 90 degrees | Wave direction 135 degrees | Wave direction 180 degrees |
| 0.760 | 0.968 | 0.55 | 0.55 | 0.55 |
| 0.881 | 1.122 | 0.73 | 0.73 | 0.94 |
| 1.081 | 1.377 | 1.06 | 1.06 | 1.06 |
| 1.260 | 1.605 | 1.08 | 1.08 | 1.08 |
| 1.530 | 1.949 | 1.20 | 1.20 | 1.20 |
| 1.880 | 2.395 | 0.75 | 0.75 | 0.75 |
| 2.167 | 2.760 | 0.50 | 0.50 | 0.50 |

Table VI-5 Test conditions for model tests with a submerged horizontal cylinder.

VI.3. Computations

The distribution of facet elements on the mean wetted part of the hulls of the vessels and, for the surface vessels, the distribution of the line elements around the waterline are given in Figures VI-4 through VI-8.

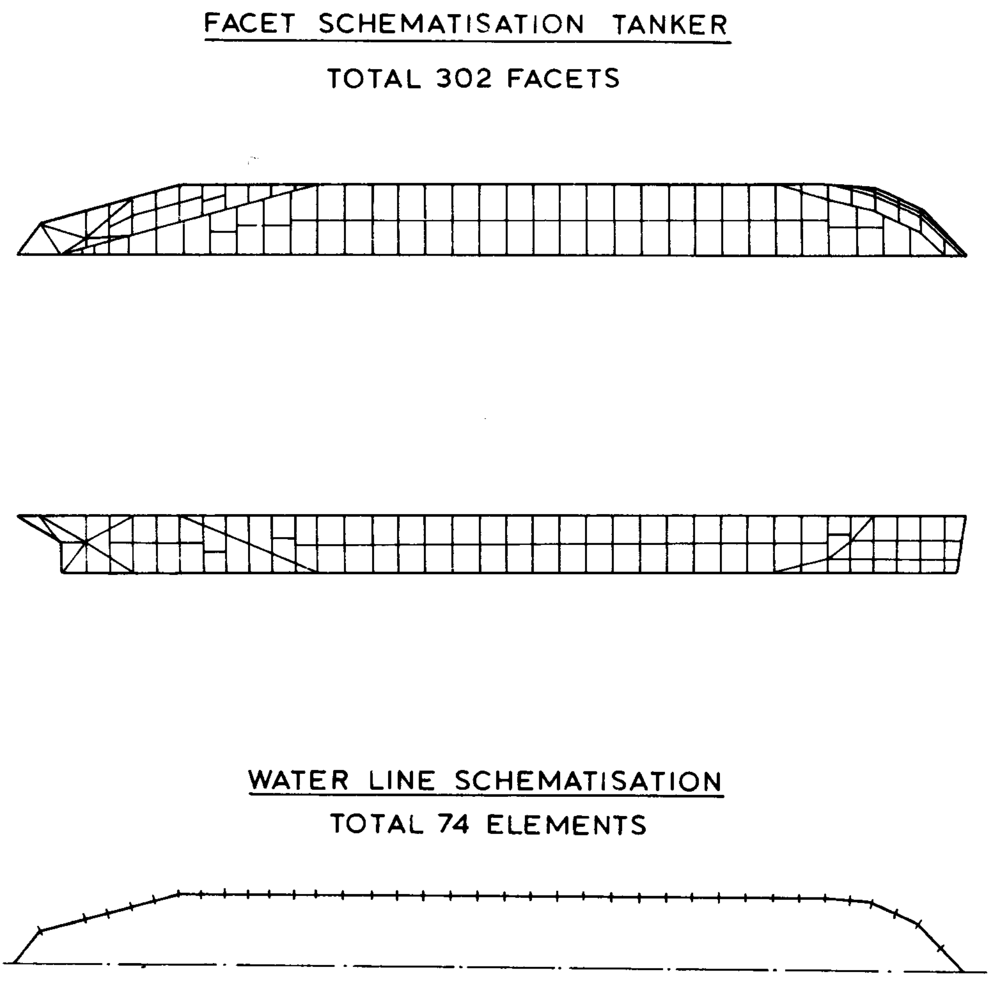



Fig. VI-4 Facet and waterline element distribution of a tanker.

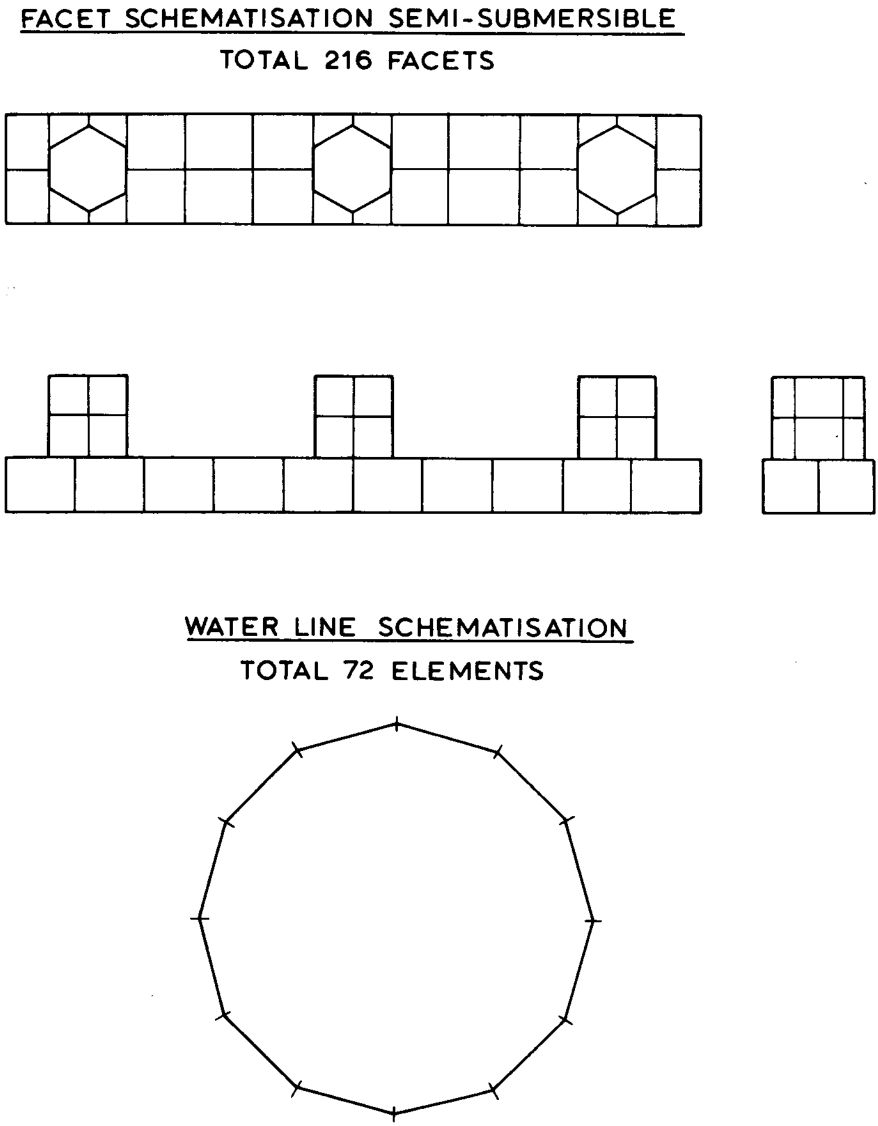
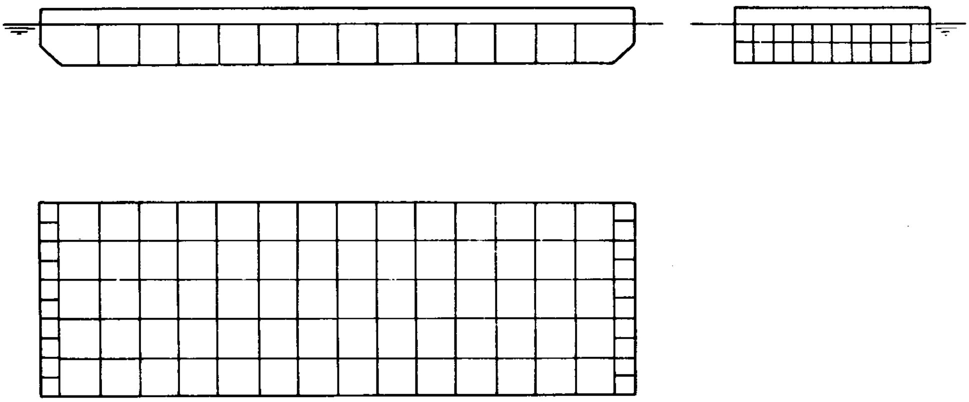



Fig. VI-5 Facet and waterline element distribution of the semi-submersible: 216 facets.

FACET SCHEMATISATION BARGE
TOTAL 138 ELEMENTS



WATER LINE SCHEMATISATION
TOTAL 48 ELEMENTS

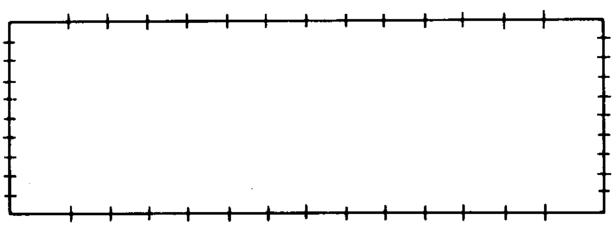


Fig. VI-6 Facet and waterline element distribution of the rectangular barge

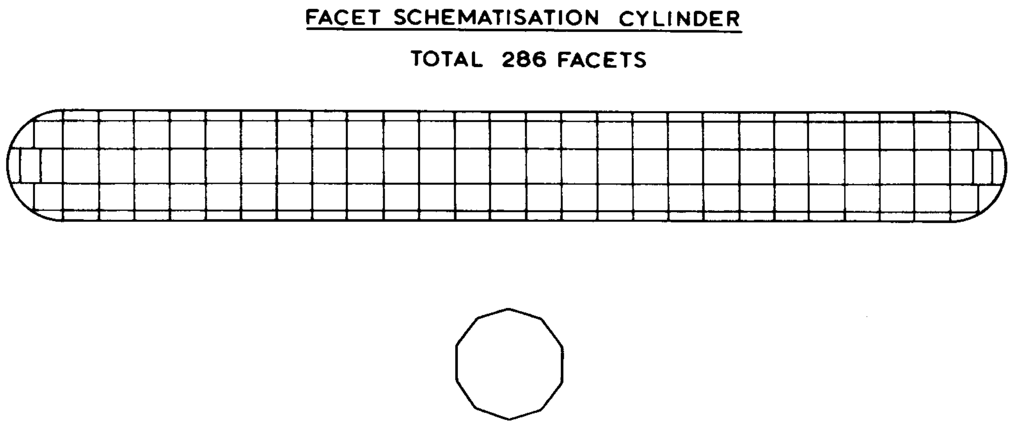



Fig. VI-7 Facet distribution of the submerged horizontal cylinder.

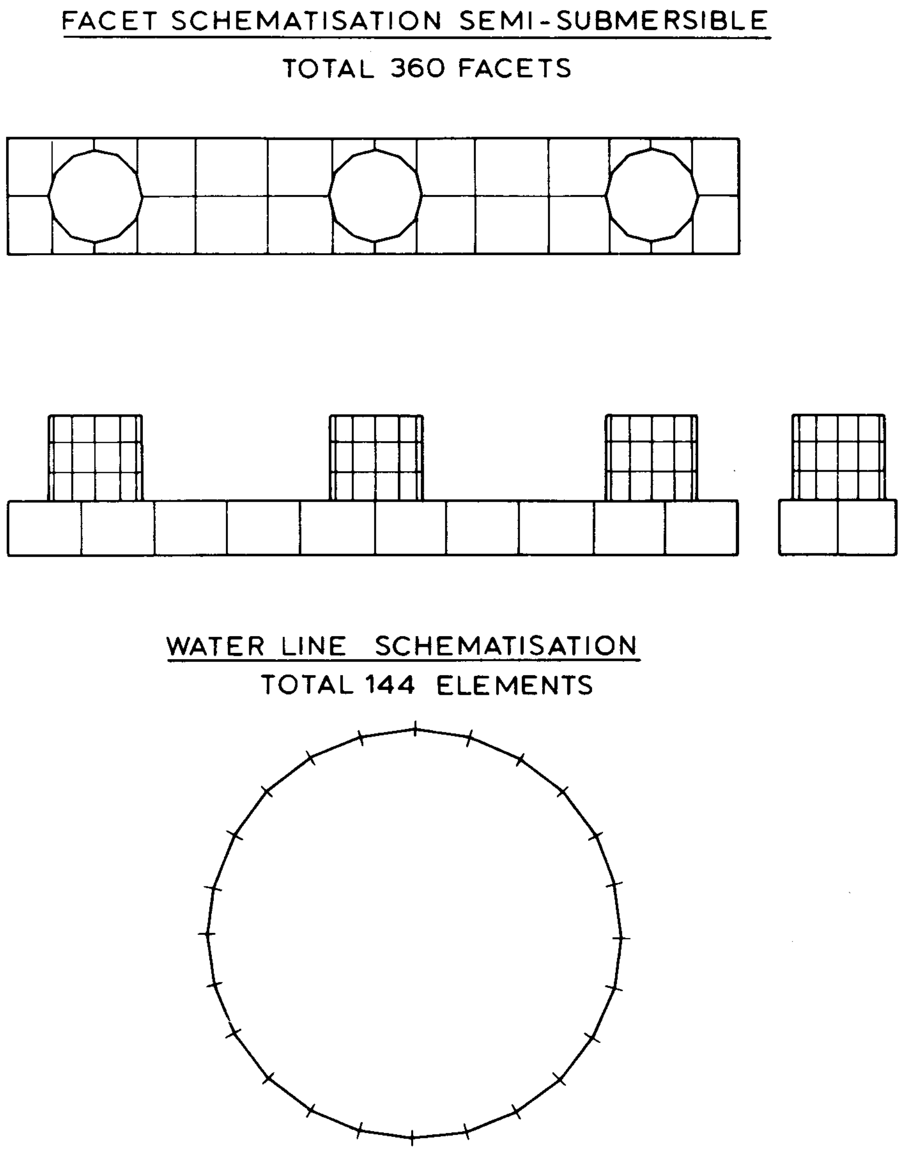



Fig. VI-8 Facet and waterline element distribution of the semi-submersible: 360 facets.

The computations were carried out for the same wave directions as the model tests and covered the range of wave frequencies tested. The wave frequencies for which computations were carried out for the four vessels are given in Table VI-6.

| Vessel | Wave frequency | | Vessel | Wave frequency | |
|---|---|---|---|---|---|
| | ω in rad./sec. | $\omega\sqrt{\frac{\nabla^{1/3}}{g}}$ | | ω in rad./sec. | $\omega\sqrt{\frac{\nabla^{1/3}}{g}}$ |
| | | | | 0.300 | 0.620 |
| | | | | 0.450 | 0.930 |
| | 0.079 | 0.198 | | 0.480 | 0.990 |
| | 0.112 | 0.281 | | 0.500 | 1.035 |
| | 0.189 | 0.474 | | 0.516 | 1.070 |
| | 0.266 | 0.667 | | 0.539 | 1.116 |
| | 0.354 | 0.887 | | 0.560 | 1.160 |
| Tanker | 0.444 | 1.113 | Rectangular barge | 0.600 | 1.240 |
| | 0.523 | 1.312 | | 0.650 | 1.340 |
| | 0.600 | 1.505 | | 0.700 | 1.450 |
| | 0.713 | 1.788 | | 0.780 | 1.610 |
| | 0.758 | 1.901 | | 0.800 | 1.660 |
| | 0.887 | 2.224 | | 0.900 | 1.860 |
| | | | | 1.000 | 2.070 |
| | | | | 1.100 | 2.270 |
| | 0.098 | 0.180 | | | |
| | 0.146 | 0.268 | | | |
| | 0.200 | 0.367 | | | |
| | 0.300 | 0.550 | | | |
| | 0.400 | 0.733 | | | |
| | 0.500 | 0.916 | | | |
| | 0.600 | 1.100 | | 0.421 | 0.536 |
| | 0.702 | 1.290 | | 0.698 | 0.889 |
| Semi-submersible | 0.800 | 1.466 | Submerged horizontal cylinder | 0.836 | 1.064 |
| | 0.900 | 1.650 | | 1.080 | 1.376 |
| | 1.000 | 1.830 | | 1.325 | 1.687 |
| | 1.050 | 1.925 | | 1.528 | 1.946 |
| | 1.100 | 2.020 | | 1.872 | 2.385 |
| | 1.150 | 2.108 | | 2.160 | 2.752 |
| | 1.200 | 2.200 | | | |
| | 1.250 | 2.292 | | | |
| | 1.300 | 2.380 | | | |
| | 1.400 | 2.570 | | | |
| | 1.500 | 2.750 | | | |

Table VI-6 Frequencies used for computations.

Additional input data for the computations with respect to the vessels are given in Table VI-1. In the case of the semi-submersible some differences occur between the model data and input data for computations with respect to the position of the centre of gravity and the transverse radius of gyration in air. The influence of these differences on the results of computations was small, however.

## VI.4. Results of computations and measurements

The results of computations and measurements with respect to the first order oscillatory motions of and about the centre of gravity are given in Figures VI-9 through VI-13. The results are presented in the form of non-dimensional frequency response functions of the amplitudes of the motions to a base of the non-dimensional wave frequency. The phase angles of the motions are given in degrees, also to a base of the non-dimensional wave frequency. A positive phase angle indicates that a motion reaches its positive maximum value before the crest of the undisturbed incoming regular wave passes the centre of gravity of the vessel. The positive direction of motions and forces are in accordance with the rectangular system of axes $G-X_1'-X_2'-X_3'$ shown in Figure III-1.

Comparison of computed and experimental data on the first order motions shows that the motion amplitudes of all four vessels are generally well predicted by the computations. Significant differences occur in roll and sway motion amplitudes near the natural frequency of the roll motions of the barge and the tanker. These are mainly due to the fact that the computations predict a larger roll motion due to the omission of viscous effects in roll damping in the computations. Due to the sway-roll coupling computed sway motions also differ somewhat from the results of measurements.

At very low wave frequencies non-dimensional amplitudes of the measured roll motions of the tanker become increasingly larger than the computed values. The differences between computations and measurements at these frequencies are, however, exaggerated due to the way angular motions are made non-dimensional.

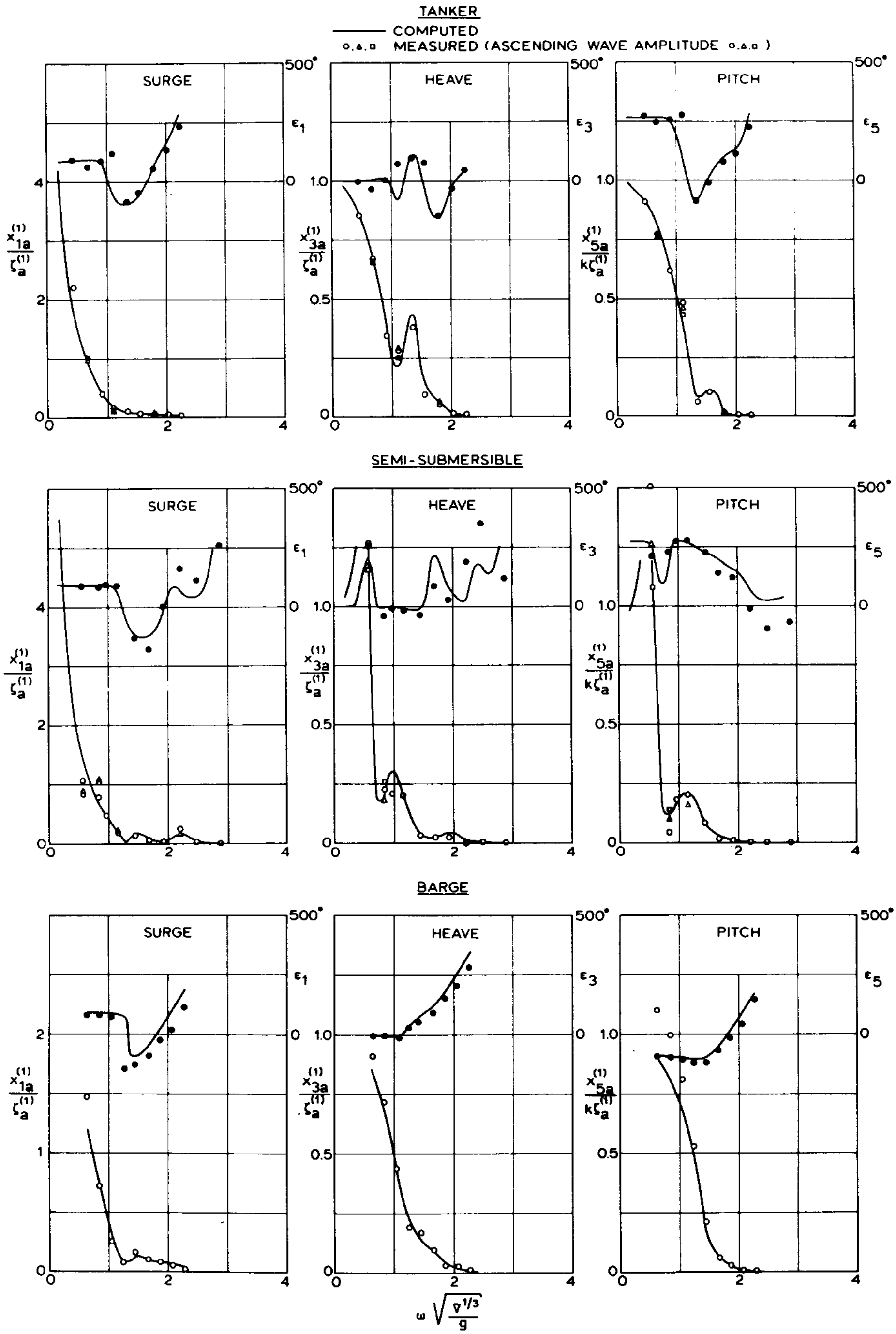



Fig. VI-9 First order motions in head waves.

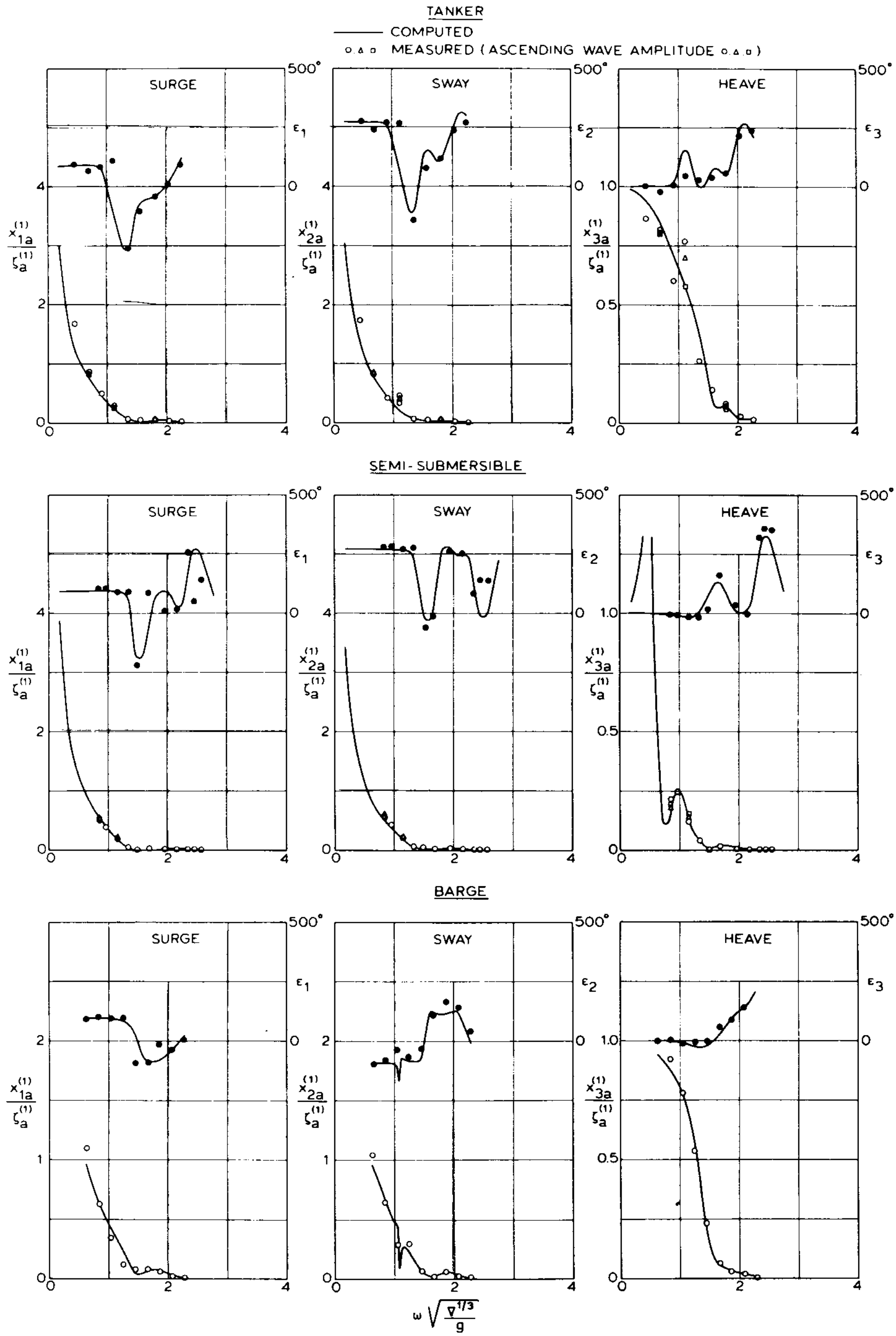



Fig. VI-10 First order linear motions in bow quartering waves.

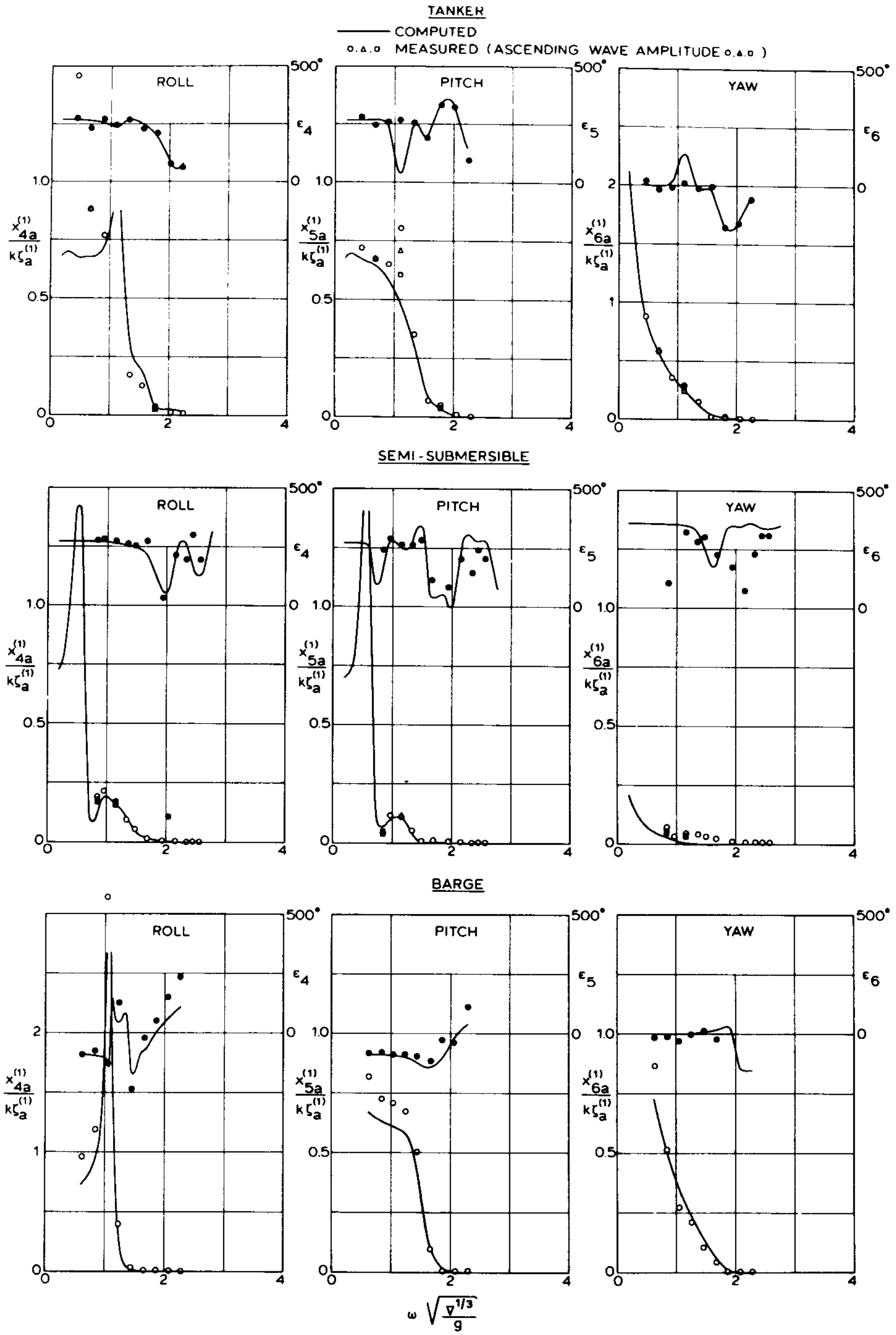



Fig. VI-11 First order angular motions in bow quartering waves.

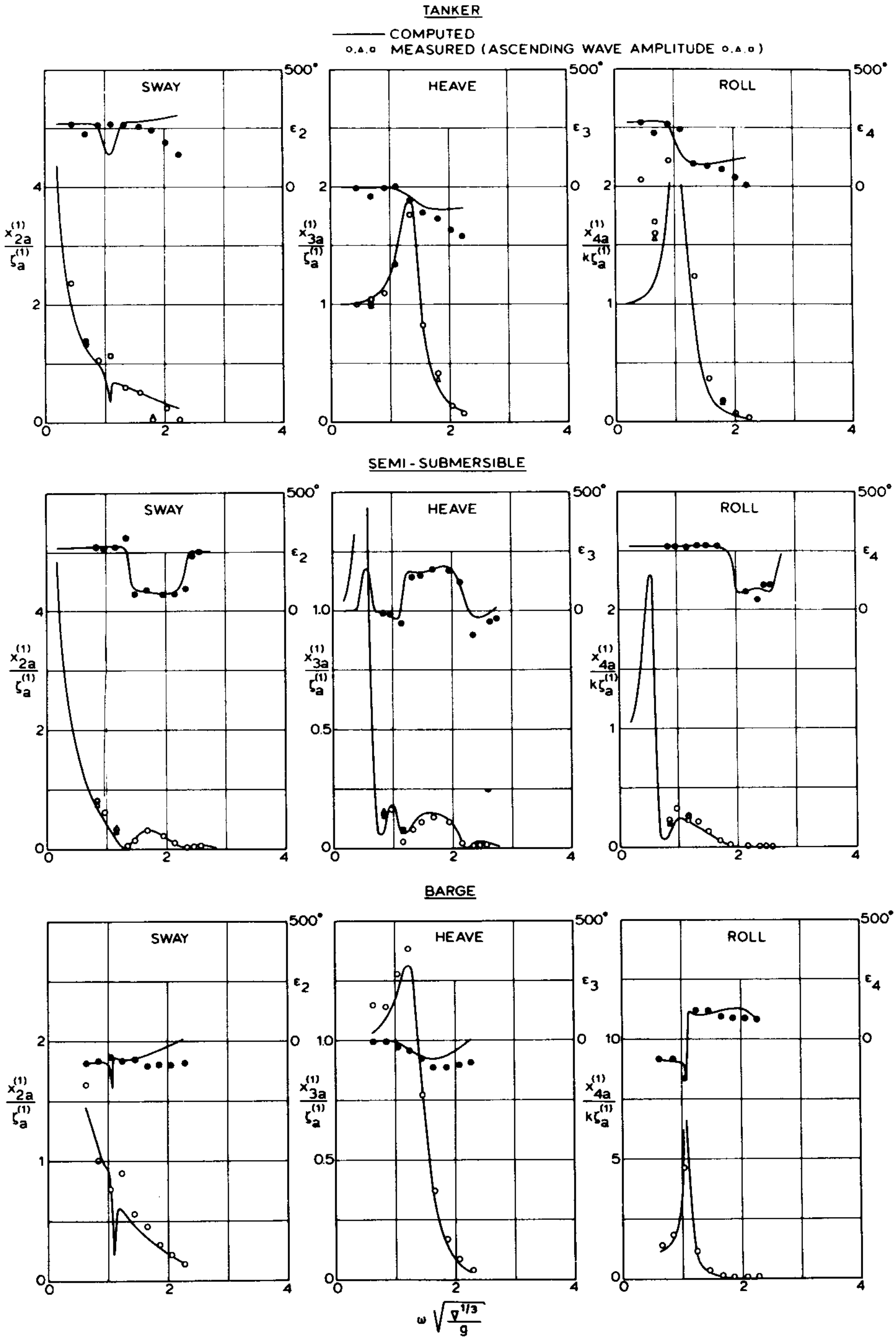



Fig. VI-12 First order motions in beam waves.

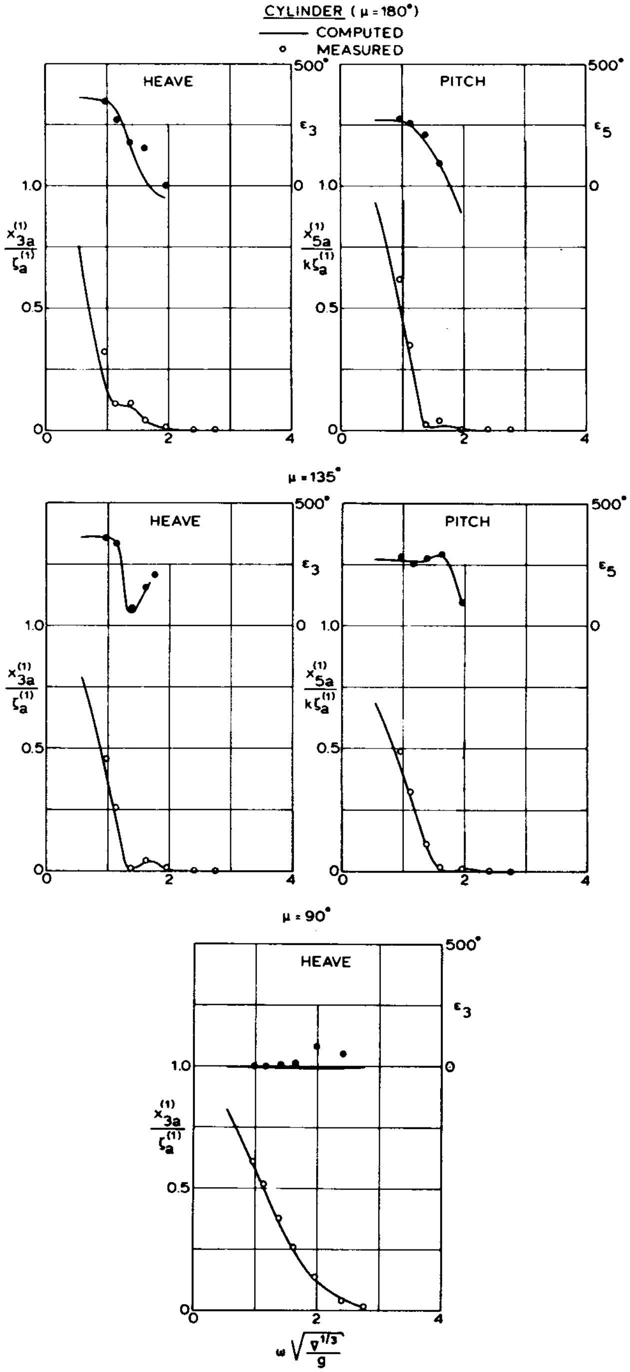



Fig. VI-13 First order vertical motions of the cylinder in head waves, bow quartering waves and beam waves.

In terms of absolute differences between the measured roll angle and the roll angle computed for the wave amplitude concerned the differences are comparable to those found at higher frequencies.

The phase angles of the first order motions are generally reasonably well predicted. When motion amplitudes are small differences in the phase angles are somewhat increased. In such cases the harmonic analysis technique, by means of which the amplitude and phase angle of the motions are determined from measured data, tends to give less correct results since at low motion amplitudes the influence of errors in the measurements become more important.

The results of computations and experiments on the mean second order forces are given in Figures VI-14 through VI-17. These results are given in non-dimensional form making use of the displaced volume of the vessels. For equal displacement volumes the results for the various vessels are directly comparable. The forces and moments are given to a base of the non-dimensional wave frequency.

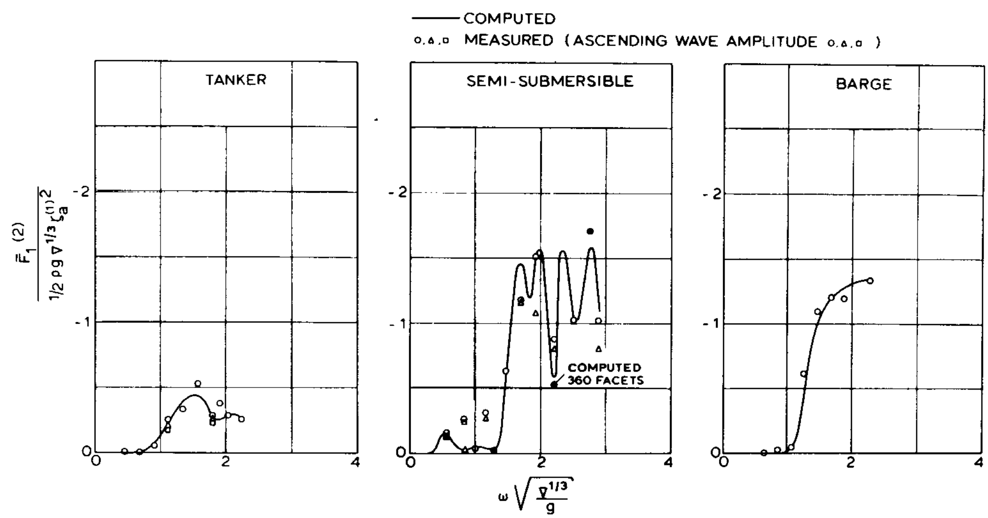



Fig. VI-14 Mean longitudinal drift forces in head waves.

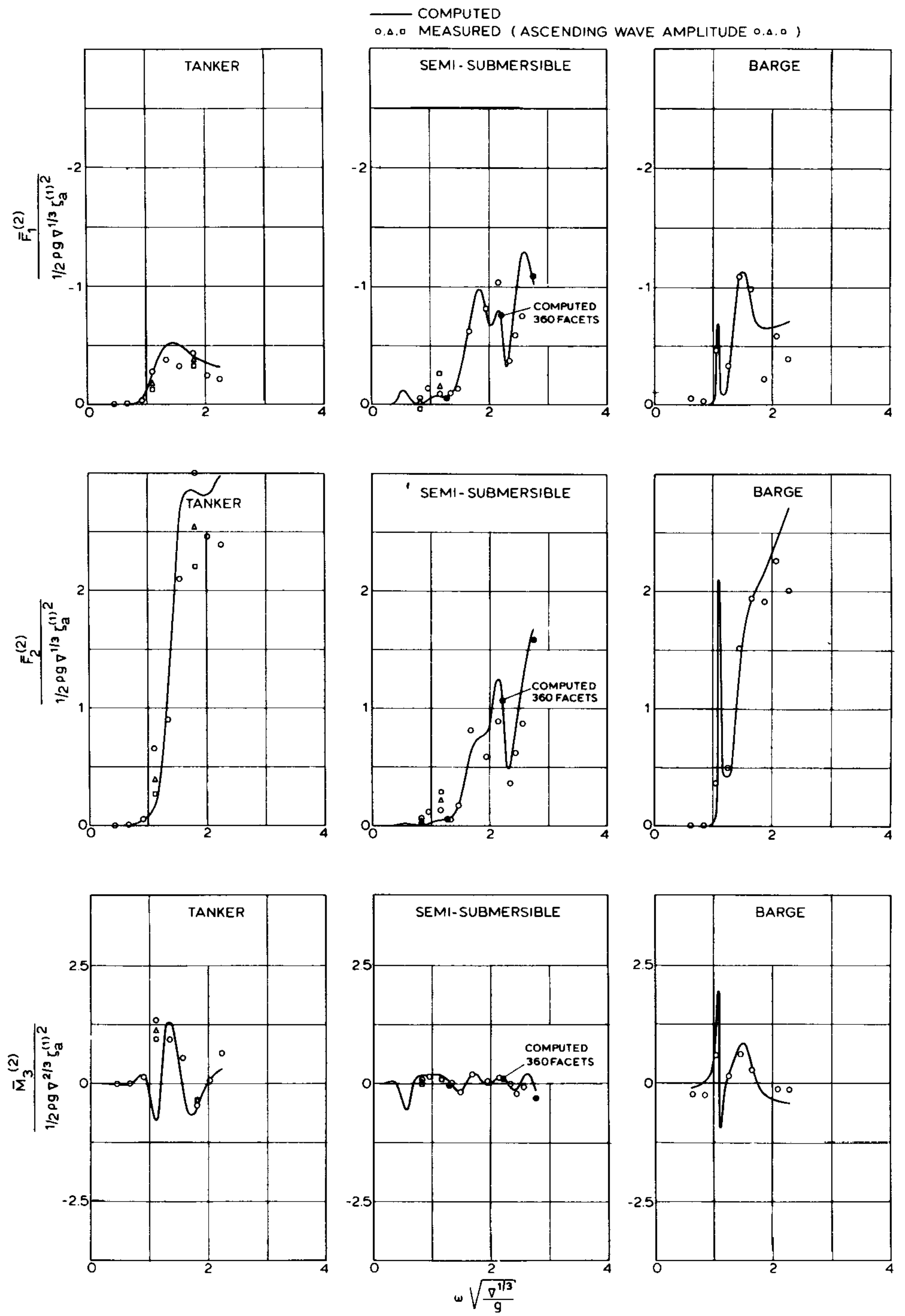



Fig. VI-15 Mean longitudinal and transverse drift forces and yawing moment in bow quartering waves.

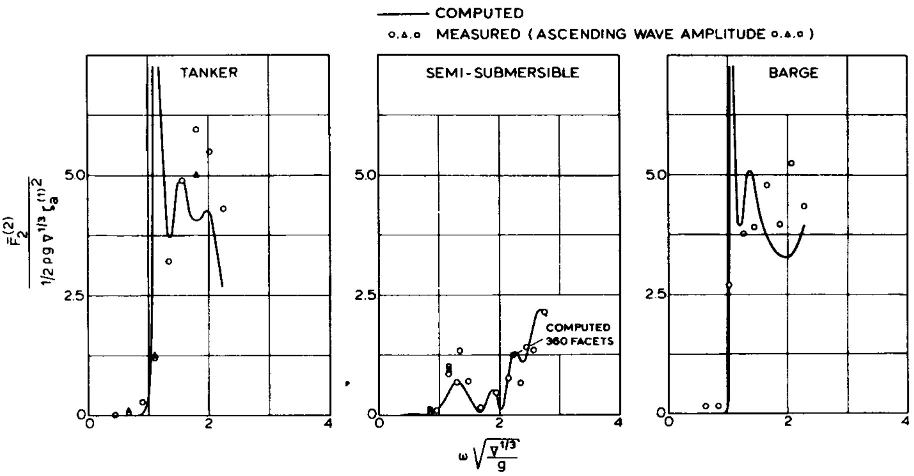



Fig. VI-16 Mean transverse drift forces in beam waves.

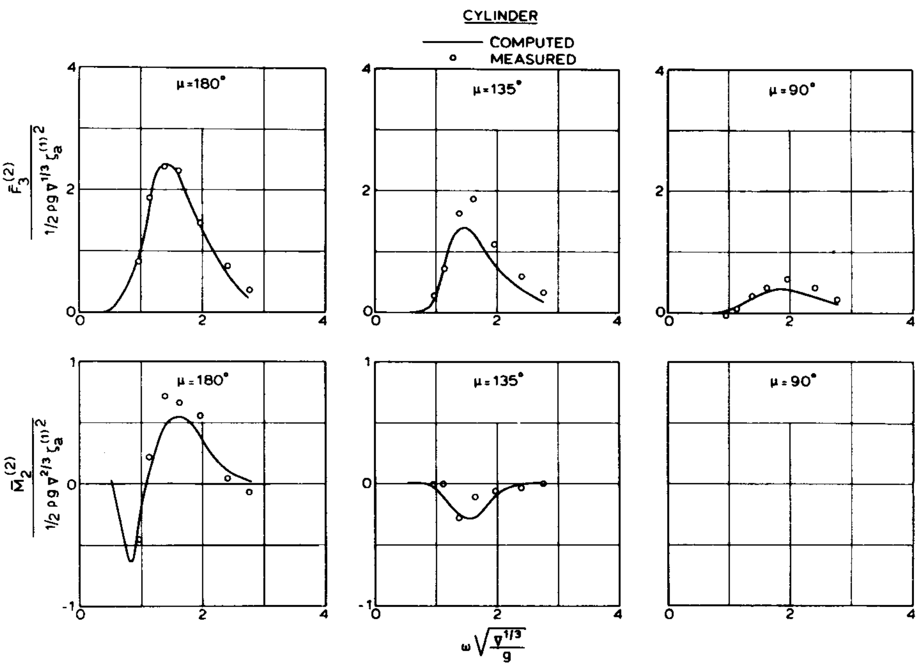



Fig. VI-17 Mean vertical drift force and trim moment on the cylinder in head waves, bow quartering waves and beam waves.

The results given in Figure VI-14 through Figure VI-17 indicate that in general the mean second order forces and moments are reasonably well predicted with some notable exceptions. For the tanker the mean transverse force in beam waves given in Figure VI-16 and the mean yaw moment in bow quartering waves in Figure VI-15 show a large difference between computations and measurements at the non-dimensional wave frequency corresponding to the natural roll frequency. At this frequency the mean transverse force given in Figure VI-16 shows a large peak in the computed results which is not present in the experimental results. The mean yaw moment given in Figure VI-15 shows that at the natural roll frequency computed and experimental results are opposite in sign. The computed results show a marked negative peak which again is not present in the experimental results. For the barge similar abrupt changes in the computed drift forces and yawing moment are found near the natural roll frequency. It is felt that, as is the case with the overprediction by the computations of the first order roll motions, the large discrepancies in the mean drift forces may be also due to the fact that viscous effects in the roll damping are not included.

In order to check, qualitatively, the influence of the roll damping on the results of the mean drift forces, computations were repeated for different values of the linear roll damping at the natural roll frequency of the tanker. Computations were carried out for roll damping values of 1, 3 and 5 times the potential roll damping. The results of computations are shown in Table VI-7 in which the mean values of the measured data, given at this frequency in the figures, are also given. From the results given in this table it may be concluded that increasing the roll damping tends to effect both first and second order quantities in such a way that better agreement with experimental data is obtained. The results also show, however, that if the roll damping is increased to such an amount that the computed roll motions agree with the measured roll motions this does not necessarily result in agreement in the computed and measured drift forces. This discrepancy may be a result of the phenomenon that viscous damping effects are less linear than has been assumed here.

FREQUENCY: $\omega = 0.444$ rad./sec.

$$\omega\sqrt{\frac{\nabla^{1/3}}{g}} = 1.113$$

| Description | Wave direction in deg. | Unit | Calculations | | | | | | Measure-ments | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | Roll damping multiplication factor | | | | | | | |
| | | | 1 | | 3 | | 5 | | | |
| | | | $u_a/..$ | $\varepsilon_{u\zeta}$ | $u_a/..$ | $\varepsilon_{u\zeta}$ | $u_a/..$ | $\varepsilon_{u\zeta}$ | $u_a/..$ | $\varepsilon_{u\zeta}$ |
| MOTIONS | | | | | | | | | | |
| Sway | 90 | $x_{2a}^{(1)}/\zeta_a^{(1)}$ | 0.35 | 140 | 0.63 | 255 | 0.70 | 263 | 1.12 | 267 |
| Heave | 90 | $x_{3a}^{(1)}/\zeta_a^{(1)}$ | 1.38 | 353 | 1.38 | 353 | 1.38 | 353 | 1.35 | 2 |
| Roll | 90 | $x_{4a}^{(i)}/k\zeta_a^{(1)}$ | 62.50 | 141 | 15.19 | 172 | 8.46 | 176 | 9.36 | 248 |
| Surge | 135 | $x_{1a}^{(1)}/\zeta_a^{(1)}$ | 0.25 | 91 | 0.25 | 91 | 0.25 | 91 | 0.28 | 112 |
| Sway | 135 | $x_{2a}^{(1)}/\zeta_a^{(1)}$ | 0.19 | 290 | 0.25 | 268 | 0.26 | 268 | 0.41 | 273 |
| Heave | 135 | $x_{3a}^{(1)}/\zeta_a^{(1)}$ | 0.56 | 9 | 0.56 | 9 | 0.56 | 9 | 0.68 | 33 |
| Roll | 135 | $x_{4a}^{(1)}/k\zeta_a^{(1)}$ | 7.12 | 85 | 1.73 | 117 | 0.97 | 121 | 5.18 | 253 |
| Pitch | 135 | $x_{5a}^{(1)}/k\zeta_a^{(1)}$ | 0.48 | 253 | 0.48 | 253 | 0.48 | 253 | 0.71 | 271 |
| Yaw | 135 | $x_{6a}^{(1)}/k\zeta_a^{(1)}$ | 0.25 | 348 | 0.26 | 356 | 0.26 | 357 | 0.26 | 8 |
| MEAN DRIFT FORCES AND MOMENT | | | | | | | | | | |
| Transverse force | 90 | $\bar{F}_2/\frac{1}{2}\rho g\nabla^{1/3}\zeta_a^{(1)^2}$ | 33.628 | | 9.650 | | 5.431 | | 1.210 | |
| Longitudinal force | 135 | $\bar{F}_1/\frac{1}{2}\rho g\nabla^{1/3}\zeta_a^{(1)^2}$ | -0.274 | | -0.203 | | -0.193 | | -0.203 | |
| Transverse force | 135 | $\bar{F}_2/\frac{1}{2}\rho g\nabla^{1/3}\zeta_a^{(1)^2}$ | -0.152 | | 0.233 | | 0.221 | | 0.443 | |
| Yaw moment | 135 | $\bar{M}_3/\frac{1}{2}\rho g\nabla^{2/3}\zeta_a^{(1)^2}$ | -0.783 | | 0.405 | | 0.505 | | 1.143 | |

Table VI-7 Influence of linear roll damping on the motions and mean drift forces and moment on a tanker at the natural roll frequency.

With respect to the results obtained for the semi-submersible it is noted that the mean drift force in head waves given in Figure VI-14 shows rapid fluctuations with the wave frequency, which are not present in the corresponding results for the tanker and the barge. Fluctuations are also present in the results for the tanker and barge at other headings. In that case they are mainly due to the additional peak at the natural roll frequency of the tanker and the barge. For the semi-submersible these fluctuations are not due to such effects since the frequencies at which resonance in roll, pitch or heave occurs are restricted to quite low frequencies as can be seen from the results on the first order motions given in Figures VI-9 through VI-12.

For the semi-submersible the fluctuations in the mean drift forces appear to be related to interaction effects between the columns. In head waves the results given in Figure VI-14 show a marked reduction in the mean drift force at a non-dimensional wave frequency of 2.2. In beam waves the results given in Figure VI-16 show a similar reduction at a non-dimensional wave frequency of about 1.8. The wave lengths corresponding to these frequencies for the head waves and the beam wave case amount to 43 m and 62 m respectively for the vessel size as given in Figure VI-1. These values are quite close to the distance between the columns as measured in the direction of the wave propagation which amount to 38 m and 60 m respectively. In such cases standing wave effects may occur between the columns.

In order to check the quadratic relationship between the mean second order forces and the wave amplitude, experiments with the tanker and the semi-submersible were carried out in waves with different amplitudes. The influence of the wave amplitude on the various quantities at a number of wave frequencies can be seen in the figures. The wave amplitudes are given in Table VI-2 and Table VI-3. In general the quadratic relationship between the mean forces and the wave amplitude is conformed with to a reasonable degree. For the tanker there is a trend, however, which indicates that the non-dimensional forces and motions reduce with increasing wave amplitudes. For the semi-submersible the influence of the wave amplitude is less consistent.

For the computations the surface of the hull of the vessel is approximated by a distribution of plane elements or facets. These facets represent a distribution of source singularities which each contribute to the potential of the flow about the vessel. The choice of the number of facets used for the computations is a compromise between the quality of the results obtained and the costs of computations. Increasing the number of facets is expected to increase the quality of the results but the costs also increase. The question as to whether a given number of facets will yield satisfactory results can only be checked by repeating computations using more elements and comparing the results. In order to show the influence of the number of facets on the results additional computations were carried out for the semi-submersible at three wave frequencies. For these computations 360 facets were used instead of 216 facets. The distributions are shown in Figure VI-5 and Figure VI-8. From these figures it can be seen that for the case of 360 facets the additional number of facets arises from the finer distribution on the columns. The results of the additional computations are indicated in the figures. In general the influence of the number of facets is small. Some influence is found at the highest wave frequency. The differences in the results using more or less elements are, however, less than the differences found between measurements and computations, so that for this case 216 facets were sufficient to give satisfactory results.

In Figure VI-18 the computed components of the mean second order longitudinal drift forces on the tanker, semi-submersible and barge are given. The numerals indicate the components given by equations (IV-1) through (IV-4). From the results given in this figure it is seen that, as was the case with the hemisphere treated in chapter V, contribution I due to the relative wave elevation is dominant in all cases. Contribution II, due to the pressure drop as a consequence of the fluid velocity, is in a direction opposite to contribution I. Contributions III and IV are generally less important. The sign of these contributions is different for the three vessels. For the tanker they are predominantly of the same sign as the total force. For the barge the opposite is true. For the semi-submersible contribution IV is practically zero except for the very low frequencies near heave and pitch resonance. For this vessel

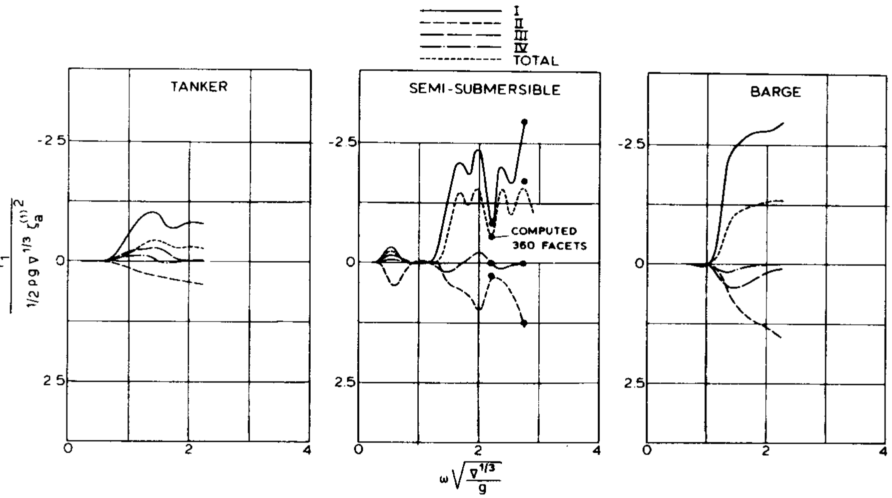



Fig. VI-18 Components of the computed mean longitudinal drift forces in head waves.

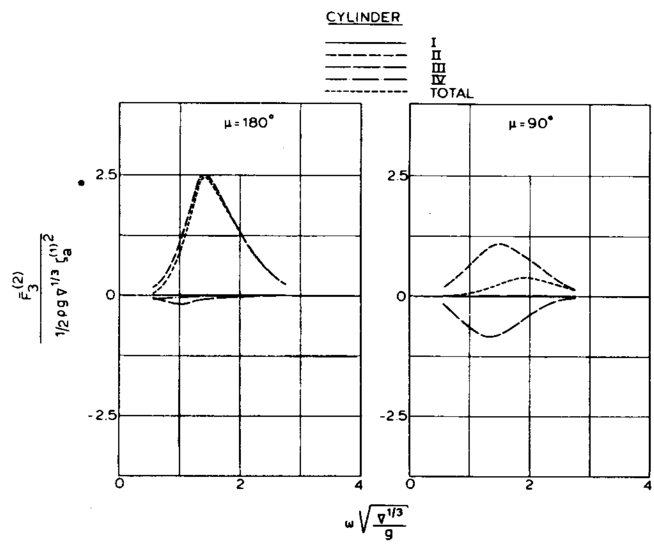



Fig. VI-19 Components of the computed mean vertical drift forces on the cylinder in head waves and beam waves.

contribution III oscillates in sign with the wave frequency.

For the submerged cylinder the contributions to the total mean vertical drift force are given in Figure VI-19 for the head wave and beam wave case. Since the vessel is fully submerged no contribution arises from relative wave elevations around a waterline. Hence contribution I is zero. Since the cylinder is circular with the centre of gravity in the centre of the cylinder no roll motions occur. Due to this effect contribution IV is zero in beam waves. The results shown in Figure VI-19 show that in this case the relative importance of the contributions to the total force can vary quite significantly. In head waves contribution II is dominant and contributions III and IV have only minor effect on the total. In beam waves, however, contributions II and III are of the same order but opposite in sign. In both cases the total mean force is directed upwards. The sign of contribution II is also upwards in both cases. It will be recalled that for surface vessels the mean horizontal force due to contribution II was directed opposite to the total force.

## VI.5. Conclusions

From the results presented in this section it can be concluded that the mean wave forces on bodies of arbitrary shape can be computed with reasonable accuracy based on the method presented in this study. It was generally recognized that the mean forces acting on full forms such as barges and ships could be computed with the aid of potential theory and the supposition that the forces are basically a second order phenomenon. The correlation shown here between the results of computations and measurements of the mean wave forces acting on a semi-submersible and a submerged cylinder indicates that the same theory can also be used to predict these forces on more slender forms.

From the results some interesting observations with respect to the mean horizontal forces acting on a semi-submersible can be made. The results shown in Figure VI-15 on the mean horizontal drift force in head waves indicate that, contrary to expectation, for equal displaced volumes the mean force acting on the semi-submersible can be as large as the force acting on a rectangular barge and considerably larger than the force acting on a tanker. Furthermore, in beam waves the reverse is true. From Figure VI-16 it is seen that the mean forces on the semi-submersible are less than on the barge or on the tanker. The mean force on the semi-submersible in beam waves is of the same magnitude as in head waves; however, the frequencies at which the mean forces are large or small are different in head waves and beam waves. This means that in irregular waves, if it is required to minimize the mean forces acting on the vessel, it may be possible to let the peak of the wave spectrum coincide with a minimum in the response function of the mean force by altering the heading of the semi-submersible. For the case of this semi-submersible, for instance, if the irregular wave spectrum is such that the non-dimensional frequency of the peak of the spectrum is about equal to 1.8, the mean force on the vessel will be smallest with the vessel turned beam-on to the waves. If the non-dimensional frequency of the peak of the spectrum is appreciably lower, say about equal to 1.2, then the mean force is smallest with the vessel head-on to the waves.

From the aforegoing discussion it can also be concluded that, if a semi-submersible is to operate in a specific location and under specific design conditions with respect to the irregular waves, it is in principle meaningful and possible to investigate the influence of the dimensions and layout of the vessel on the mean second order drift forces with the aim of optimizing a design from this point of view.

# VII. DETERMINATION OF THE QUADRATIC TRANSFER FUNCTION OF THE LOW FREQUENCY SECOND ORDER FORCES

## VII.1. Introduction

Experimental determination of the low frequency components of the second order wave forces acting on vessels in waves places unusual demands on the system of restraint and the measuring system employed. In this chapter the ideal characteristics of the system of restraint are discussed and two possible realizations of such systems are introduced. The ideal characteristics of the system of restraint are only partly obtained by these two systems so that it must be borne in mind that further development in this field is necessary.

In order to demonstrate the type of results which may be obtained experimentally and to verify the results of computations some experimental results on the quadratic transfer functions for the amplitude of the low frequency second order longitudinal force on two vessels in head waves will be compared with results of computations.

As indicated in chapter IV the quadratic transfer function for the low frequency force corresponds with the low frequency component of the second order force when a vessel is floating in a regular wave group consisting of two regular waves with frequencies $\omega_1$ and $\omega_2$. There are two methods by means of which these results may be obtained from experiments:

1. From model tests in regular wave groups; the results are directly comparable with computed results.
2. From model tests in irregular waves; the time records of the second order forces are analyzed by means of cross-bi-spectral analysis techniques. The results of this analysis are directly comparable with results of computations.

In this chapter some results obtained by both methods will be given. The cross-bi-spectral analysis technique employed here was based on the method developed by Dalzell [VII-1]. The computations of the quadratic transfer function are in accordance with the method discussed in chapter IV.

Experiments and computations were carried out for the following vessels, which were also treated in chapter VI:

- a tanker;
- a semi-submersible.

The main particulars of these vessels are given in Table VI-1. For the tanker model tests were carried out in regular wave groups and in irregular waves. For the semi-submersible model tests were carried out in irregular waves only. In all cases results are given for head waves. For many practical cases, for instance in very high seas, head waves represent the most important wave direction for moored vessels.

## VII.2. Model test set-up

### VII.2.1. General

In this section the specific requirements, which must be met in order to be able to measure the low frequency second order wave forces on a vessel, will be discussed.

From the expressions derived for the second order forces in chapter III it can be concluded that, in order to arrive at the correct value of the forces, the model restraint must be so that first order motions are not affected by the method of restraint. In case the mean second order forces are to be determined no other requirements have to be met by the mooring system. When it is required to measure the low frequency second order force an additional requirement must be fulfilled, namely that the model does not carry out motions with frequencies which coincide with the frequency of the second order forces. This requirement is analogous to the case where it is required to measure first order wave loads. In that case the captive model must be rigidly held so that it does not carry out motions at wave frequencies. Failure to comply with this requirement results in incorrect values of the forces due to dynamic magnification effects following from the elasticity of the system of restraint. For the case under consideration the system of restraint must allow the model to move freely at wave frequencies while at the same time low frequency motions corresponding to the low frequency wave drift forces must be fully suppressed. In that case the forces in the mooring system will possess only mean and low frequency components which will correspond to the required forces.

The response of the mooring system is shown schematically in Figure VII-1.

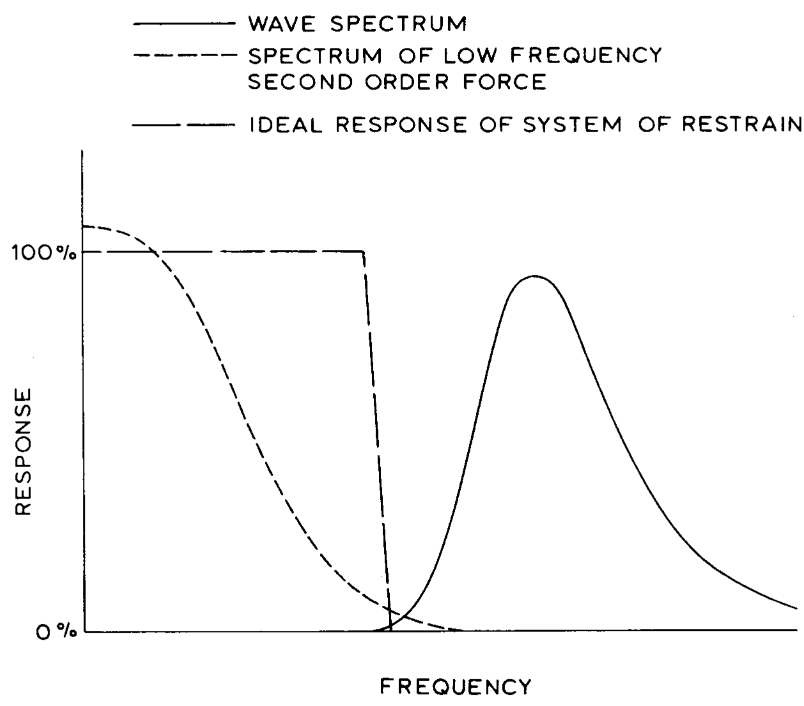



Fig. VII-1 Ideal characteristic of the system of restraint.

In this figure the wave spectrum and the spectrum of the low frequency second order forces in irregular waves are given schematically to a base of frequency. Also shown in this figure is a line which indicates the idealized characteristics of the system of restraint of the vessel. In the range of frequencies of the second order forces the system is at 100%, indicating that it must suppress fully motion components of these frequencies. At wave frequencies the system is at 0%, indicating that for these frequencies the system should, ideally, not exert any forces on the vessel. The system of restraint, in effect, must possess low-pass characteristics. The high frequencies to be filtered out in this case being the frequencies of the waves. Such characteristics may be approximated by the application of a dynamic system of restraint incorporating control and servo systems which react in the required manner to the motions of the vessel in waves. In Figure VII-2 a block diagram of such a system is given.

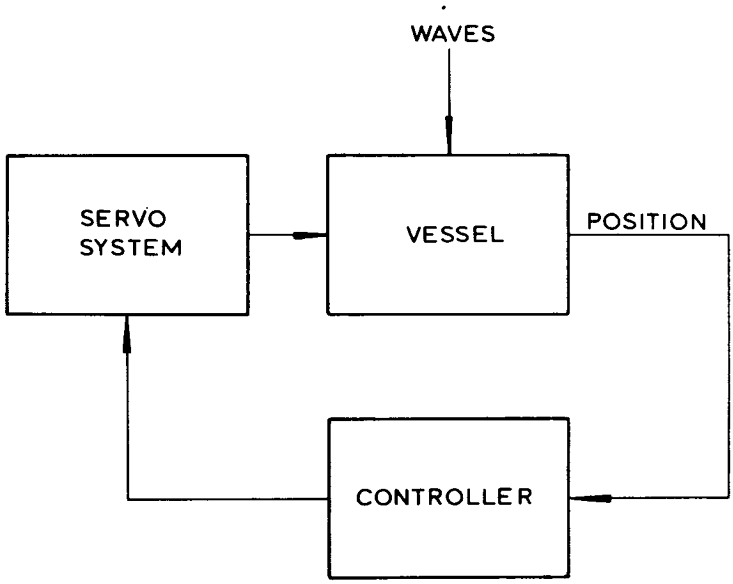



Fig. VII-2 Block diagram of dynamic system of restraint.

This diagram represents a possible control system for a vessel moored in waves. The waves exert forces on the vessel which in turn cause motions which contain wave frequencies and low frequencies due to the drift forces. The motions are measured and fed to the control unit which filters out the high frequency part of the motion signal and gives only low frequency force commands to the servo system which exerts the required force on the vessel. In some cases, instead of using an active system of restraint as described here, a simple passive system consisting of linear springs may be employed. Consider the case of a vessel restrained in the longitudinal direction by means of linear springs. For simplicity we assume the virtual mass and damping to be constant. The system of restraint is shown schematically in Figure VII-3.

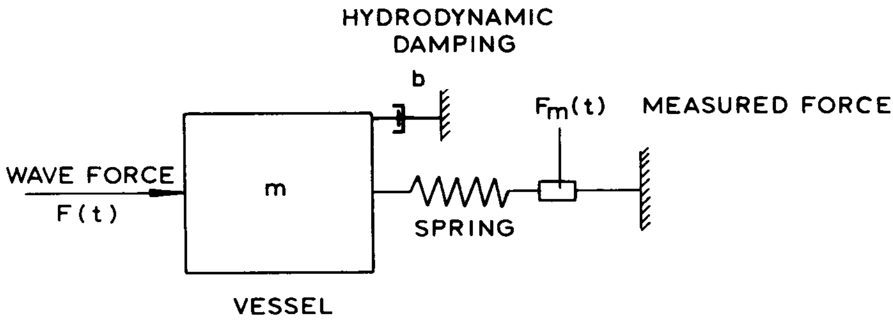



Fig. VII-3 Schematic representation of a passive mooring system.

The waves exert first and second order forces on the vessel which cause it to execute motions. The force is denoted by F(t) in Figure VII-3. In the mooring system a force transducer is mounted which measures the mooring force $F_m(t)$. If the ship-mooring-system is assumed to be linear the amplitude response function of the measured force in ratio to the wave force is:

$$\frac{F_{ma}}{F_a} = \frac{1}{\sqrt{(1-\lambda^2)^2 + \nu^2\lambda^2}} \qquad \text{(VII-1)}$$

where:

$\lambda$ = $\omega/\omega_e$

$\omega$ = frequency of force excitation

$\omega_e$ = natural frequency of the horizontal motion of the moored vessel = $\sqrt{c/m}$

c = stiffness of the mooring system

m = virtual mass of the vessel

$\nu$ = non-dimensional damping factor = $b/\sqrt{cm}$

b = damping coefficient.

The amplitude response function is shown in Figure VII-4.

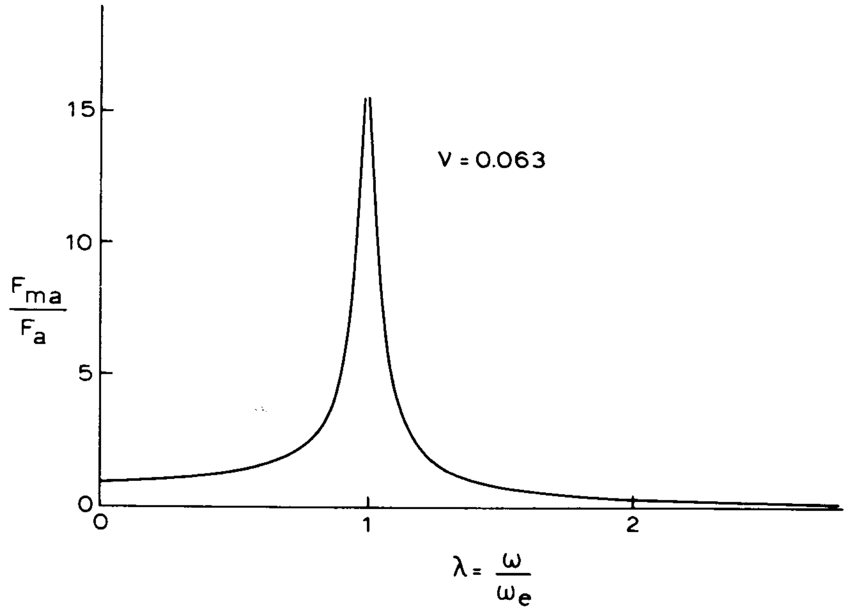



Fig. VII-4 Amplitude response of measured force.

From the figure we can see that at low frequencies of excitation the measured force equals the wave force while for frequencies above the natural frequency the measured force becomes progressively smaller in ratio to the wave forces. In between these regions considerable dynamic magnification is evident. We now consider the case that the vessel is moored in irregular waves. The wave spectrum and the spectrum of the low frequency second order forces are shown schematically in Figure VII-5. Superimposed on these spectra is the amplitude response function of the measured force (mooring force) in ratio to the wave forces. As can be seen from this figure, if the stiffness of the mooring system is sufficiently large so that the peak of the response function is between the frequencies of the second order forces and the wave frequencies, the dynamic magnification of the measured force is small in the range of frequencies of the low frequency second order forces. At the same time the ratio of measured or mooring force to wave force in the range of the first order motions and first order wave frequencies can be small. This means that, although a stiff system is employed, the influence of the mooring system on the first order motions, although present, need not affect appreciably the first order motions.

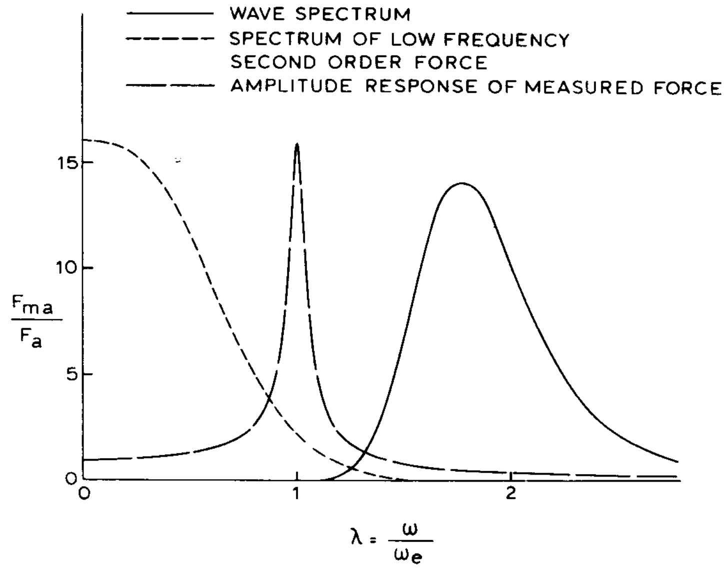



Fig. VII-5 Schematic representation of wave spectrum, drift force spectrum and amplitude response of the measuring system.

It will be evident that the applicability of such a system of restraint becomes greater in irregular waves with a narrow spectrum and with decreasing mean periods. In such cases the separation between the high frequency wave spectrum and low frequency force spectrum becomes larger so that the influence of the peak of the response function of the measured force is reduced. In that case both the dynamic magnification of the measured low frequency force and the effect of the mooring system on the first order motions are reduced. A drawback in this system is that, although the first order motions at wave frequencies may be relatively unaffected, the measured force $F_m(t)$ will nevertheless contain force components with wave frequencies which are commensurate with the first order motions and the mooring stiffness. These wave frequency force components can be large relative to the second order forces. Before the results of such measurements are analyzed further the high frequency force components are filtered out of the force signals.

VII.2.2. Realizations of two systems of restraint

In the aforegoing the basic principles of two systems of restraint of a vessel have been discussed. The experimental data on the low frequency second order horizontal forces acting on the tanker were obtained using the dynamic system of restraint. The vessel was positioned in the horizontal plane by means of three servo units arranged as shown in Figure VII-6.

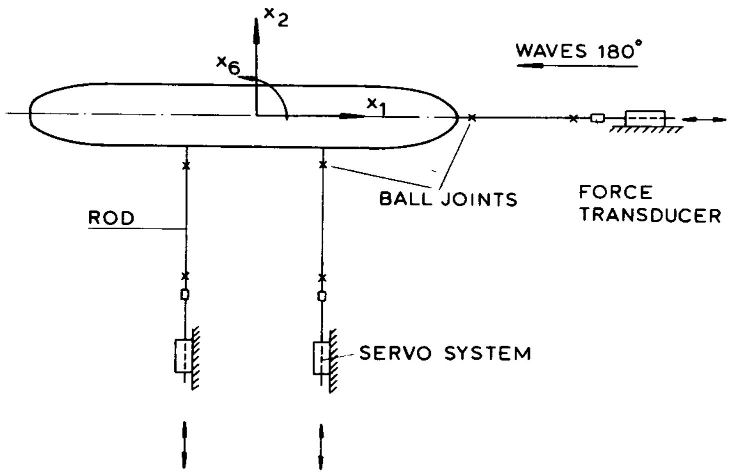



Fig. VII-6 Dynamic system of restraint.

One servo unit was used to control the surge motion while two additional units were used to control the sway and yaw motion.

Each servo unit could exert forces on the model through the lightweight metal rod connecting the model to the servo units. Force transducers, from which the required forces were obtained, were incorporated in the connecting rods.

Basically, each servo unit worked independently as a position control system with characteristics based on feed-back of the motions of the vessel in the direction of the axis of the connecting rods. The motion and motion velocity were measured within each of the units and fed to analog control units with proportional-differential type characteristics. The output of these units governed the forces applied by the servo units on the vessel. The proportional-differential characteristics were so adjusted that the control systems applied small restoring forces and heavy damping to the vessel motions for the low motion frequencies. Through the use of a low-pass filter on the motion velocities the damping characteristics were filtered out for higher (wave) frequencies. The proportional part of the control was independent of the wave frequency.

The final settings chosen for the control systems allowed determination of low frequency wave forces on the vessel with frequencies up to approximately 0.05 rad./sec. full scale. Above this frequency dynamic magnification effects became evident. This was due to phase lag introduced by analog filtering of the velocity feed-back which introduced a resonance peak in the response of the system at a frequency of about 0.11 rad./sec. At higher frequencies than 0.11 rad./sec. the forces due to the system of restraint rapidly reduced until only the relatively weak proportional restoring forces remained in the range of normal wave frequencies. At these frequencies the effect of the system of restraint on the first order motions was negligible.

The system of restraint used for the tanker did not fully conform with the requirements discussed in section 2 of this chapter. The discrepancies are evident from the relatively narrow band (0 - 0.05 rad./sec.) of the low frequencies for which the low frequency second order wave forces may be measured without appreciable dynamic magnifications. This problem will probably be reduced by the application of more sophisticated control systems than used for this investigation. From the point of view of practicality of results obtainable by the present system it can be stated that, since the natural frequencies of the horizontal motions of a moored tanker are generally lower than 0.05 rad./sec., it is possible to measure accurately those low frequency horizontal wave forces which are important for the low frequency horizontal behaviour of such vessels.

In the aforegoing the feed-back control characteristics of the dynamic system of restraint used for the model tests with the tanker are discussed. In order to enhance the position keeping characteristics of the system in the range of the low frequency horizontal motions an additional control signal, based on the instantaneous relative wave elevation measured at a number of points around the tanker model, was generated and used as an additional command signal for the servo units. This control signal constituted an approximation for the instantaneous value of the low frequency second order horizontal wave forces on the vessel as derived from instantaneous evaluation of equation (IV-1). The effect of including this additional control signal will be discussed in chapter VIII in connection with dynamic positioning of a vessel at sea and will not be treated further here.

As stated previously the model tests with the semi-submersible were carried out using a passive system of restraint based on stiff linear spring characteristics. The model tests were carried out in head waves only, so that only in the longitudinal direction a stiff spring system of restraint was used. The set-up is shown in Figure VII-7. The system of restraint consisted of a forward and aft mooring line, each incorporating a force transducer and a linear spring. The restoring force of the mooring system in the longitudinal or surge direction amounted to 513 tf per metre displacement for full scale. From a surge motion decay test in still water the natural frequency of the surge motion as induced by the system of restraint (mooring system) amounted to 0.4 rad./sec. full scale.

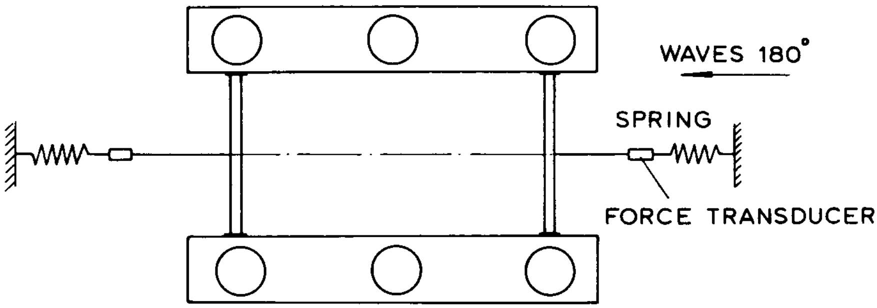



Fig. VII-7 Passive system of restraint.

In order to show the influence of the stiffness of the mooring system on the wave frequency motions of the semi-submersible the amplitude response functions of the heave, surge and pitch motions in head waves were computed for the stiff mooring system and compared with the results computed for the free floating case, i.e. for the case with zero mooring stiffness. The motion response functions are compared in Figure VII-8.

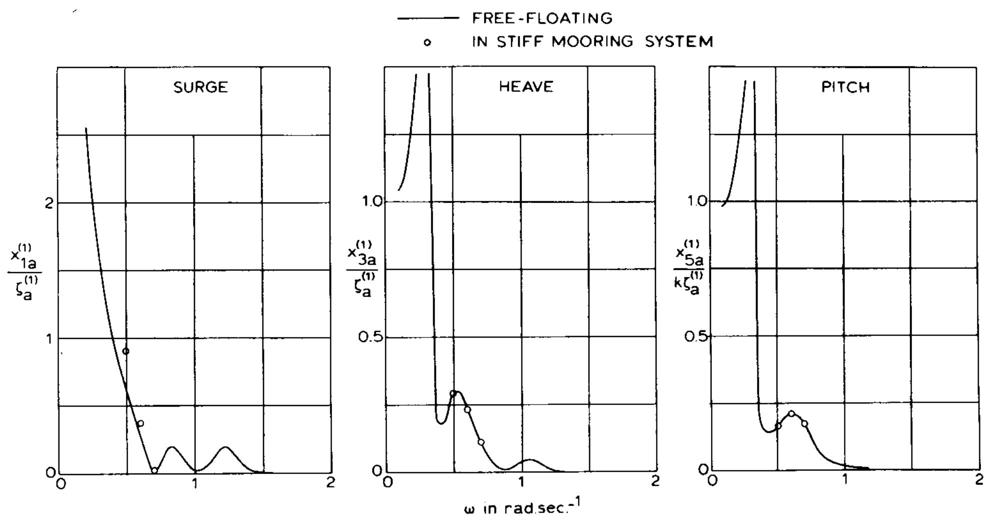



Fig. VII-8 Influence of mooring system on the amplitudes of the motions of the semi-submersible in regular head waves.

From this figure it is seen that the heave and pitch motions are practically unaffected by the stiffness of the mooring system. At lower wave frequencies the surge response is increased due to the stiffness of the mooring system. This indicates that from this point of view the stiffness of the mooring was somewhat too great. It will be seen, however, that the second order wave exciting forces are practically unaffected. Computations of the mean second order forces in regular head waves on the free floating semi-submersible were presented in chapter VI. For some wave frequencies the computations were repeated taking into account the stiffness of the mooring system in the longitudinal direction.

In Figure VII-9 the mean longitudinal force in regular waves is compared for the stiff mooring system and the free floating vessel.

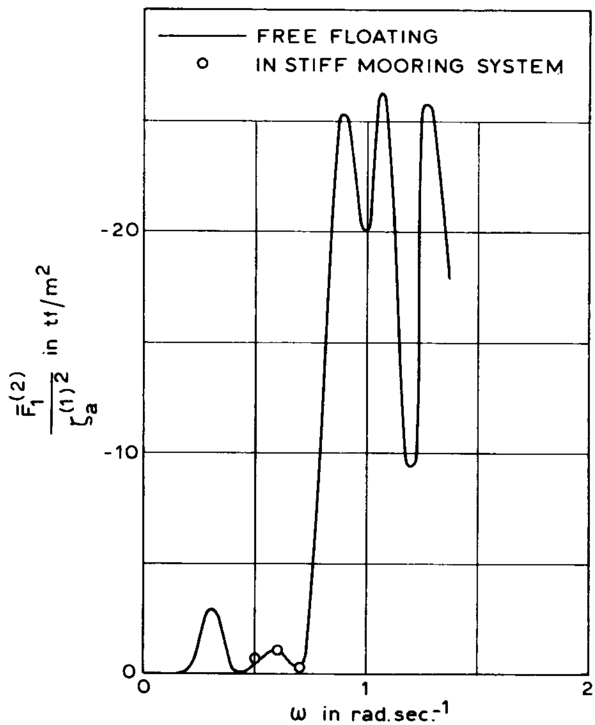



Fig. VII-9 Influence of mooring system on the mean longitudinal drift force on the semi-submersible in regular head waves.

The results show that the influence of the stiff mooring system on the mean second order force is small. This result seems to be in contradiction with the statement made previously that if the first order motions are affected the second order forces will be affected as well. In the case of the semi-submersible, however, the force level at the lower wave frequencies is already small indicating that the vessel is only causing slight diffraction of the incoming waves due to the small size of the members, such as the columns relative to the wave length at these frequencies. In such cases the disturbance created by the vessel motions also contributes little to the forces in which cases, even though the motions increase, the second order forces still remain small. On the basis of this result it is concluded that also the low frequency forces will be only slightly affected by the stiff mooring system.

## VII.3. Model tests

### VII.3.1. Generation of waves

For both vessels model tests were carried out in the Wave and Current Laboratory of the Netherlands Ship Model Basin. This basin measures 60 m by 40 m with a variable water depth from 0 to 1.10 m. Wave generators of the fixed stroke, variable frequency type are disposed on two sides of the basin; see Figure VII-10. For the tests only the wave generators on the short side of the basin were used.

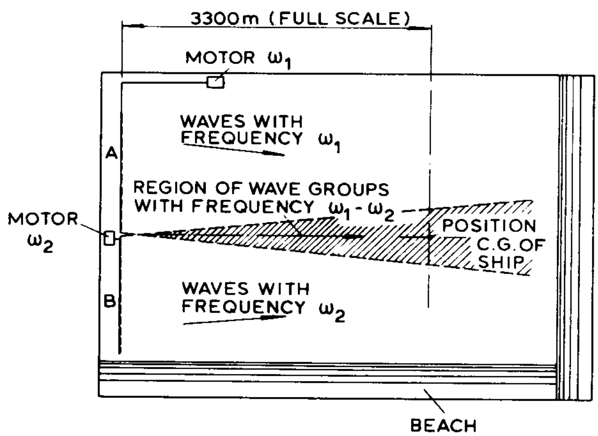



Fig. VII-10 Wave and Current Basin.

## Regular wave groups

Since the wave generators are of the fixed stroke, variable frequency type it is not possible to generate regular wave groups in a straightforward way. This would require wave generators of the type which are variable both in frequency and stroke. In order to generate regular wave groups consisting of two regular waves with small difference frequency the bank of wave generators was split into two independently driven sections denoted by A and B in Figure VII-10. By driving section A at a constant frequency to produce regular waves of frequency $\omega_1$ and section B to produce regular waves of frequency $\omega_2$ regular wave groups are created at the common edge between these two wave fields. This is shown in Figure VII-10. The frequency of the wave groups is equal to the difference frequency $\omega_1 - \omega_2$. The width of the overlap region between the two fields of regular waves increases with the distance from the wave generators. For the tests in regular wave groups the tanker model was situated as indicated in Figure VII-10. Examples of the wave elevation in wave groups measured at the location of the tanker model are given in Figure VII-11. Due to the method of generating wave groups these were not long-crested.

## Irregular waves

Irregular waves are generated by varying in a random manner the frequency of the wave generators at a fixed stroke of the wave paddles. The stroke of the wave paddles and the variations of the frequency are chosen so that irregular waves are generated which conform with a given spectral density distribution or wave spectrum. The irregular waves were in all cases long-crested. For the tests with the tanker four wave spectra were used. These are shown in Figure VII-12. For the tests with the semi-submersible only one wave spectrum was used. This spectrum is shown in Figure VII-13.

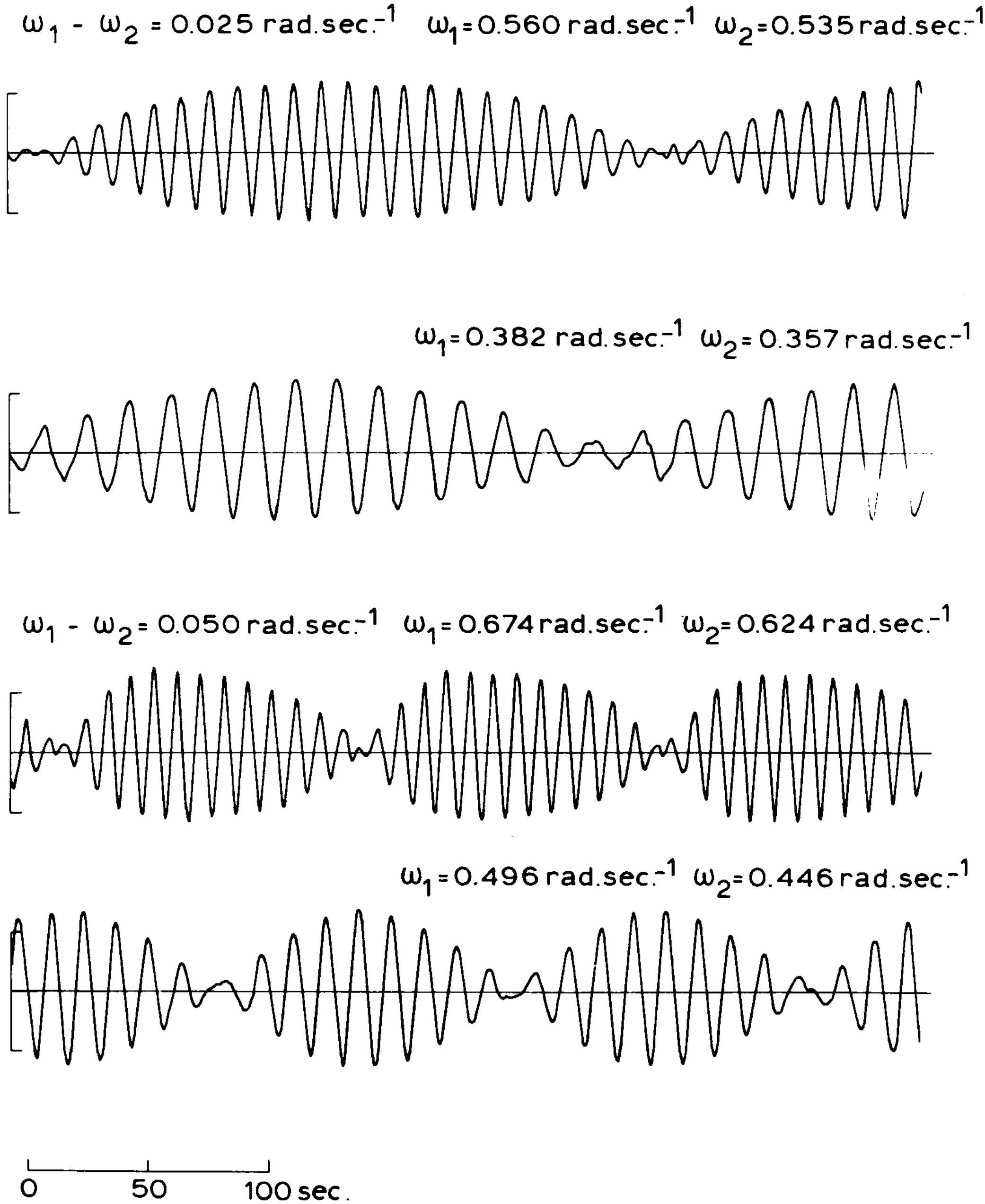



Fig. VII-11 Regular wave groups.

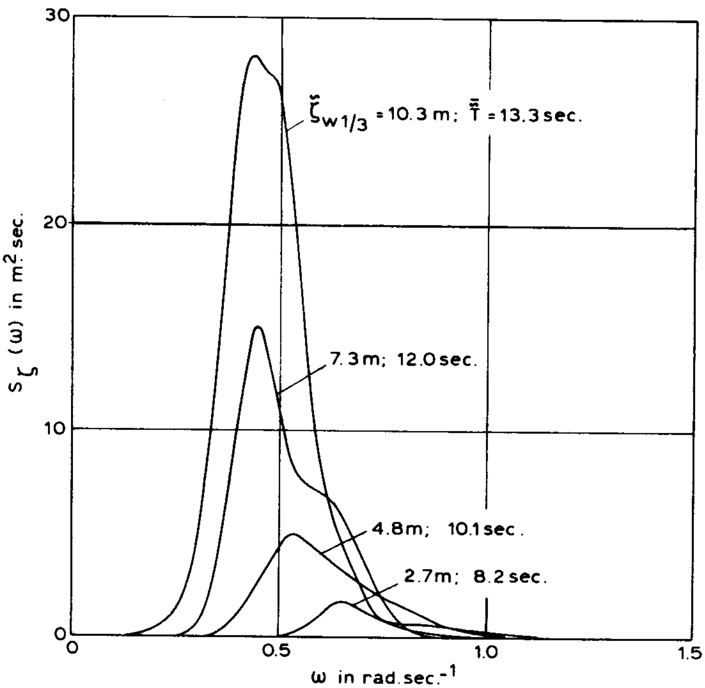



Fig. VII-12 Spectra of irregular waves for tests with the tanker.

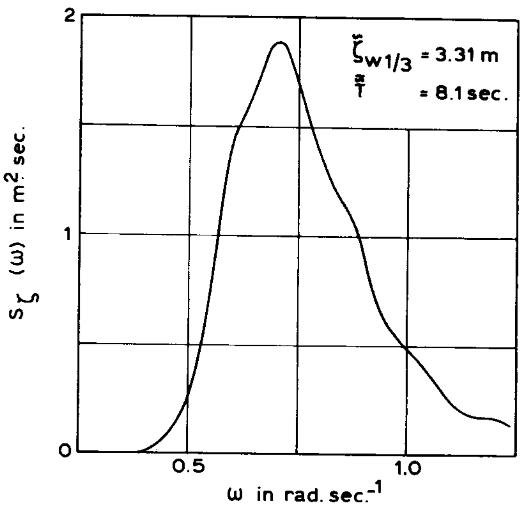



Fig. VII-13 Spectrum of irregular waves for test with the semi-submersible.

## VII.3.2. Test procedure and duration of measurements

The general procedure during the tests in regular wave groups and irregular waves was as follows:

- in still water the static value of the forces and motions was recorded;
- the wave generators are switched on;
- after transient motion behaviour had decayed measurements were carried out.

For the tests with the tanker in regular wave groups the test duration corresponded to at least 8 oscillations at the wave group frequency. During the tests with the tanker in irregular waves the test duration corresponded with 3.5 hours full scale. The tests with the semi-submersible in irregular waves lasted a time corresponding with 6 hours full scale.

The long duration of the tests in irregular waves is necessary due to the low frequencies of the second order forces. In order to analyze these signals by means of spectral methods the signals must contain a certain number of oscillations at the frequencies of interest. When first order motions at wave frequencies are considered a minimum number of about 80 oscillations are sufficient for the normal spectral analysis. The low frequency second order forces were analyzed by means of cross-bi-spectral methods. The frequencies of interest ranged from zero up to 0.05 rad./sec. full scale for the tanker and from zero up to 0.1 rad./sec. full scale for the semi-submersible. It will be clear that in such cases, if a minimum number of oscillations is required at zero frequency, the required test duration becomes infinite. Since this requirement cannot be met it can be concluded that results obtained by cross-bi-spectral methods on the low frequency second order forces for frequencies tending to zero will be questionable as far as the accuracy of the results is concerned. Knowledge of the influence of the test duration on the accuracy of results of cross-bi-spectral analyses is at present still lacking. Dalzell [VII-1] suggests that test durations corresponding to about 1200 - 1600 oscillations at wave frequency may be sufficient. The test duration for the tests in irregular waves with the tanker corresponds to 950 to 1500 oscillations at wave frequency depending on the mean period of the irregular waves. For the test with the semi-submersible the test duration corresponded with about 2700 oscillations at wave frequency.

## VII.4. Analysis of results of measurements of the low frequency longitudinal force in head waves

### VII.4.1. Regular wave groups

In Figure VII-14 some typical results of the low frequency longitudinal force on the tanker in regular wave groups are shown. In the same figure the corresponding wave elevation is also shown.

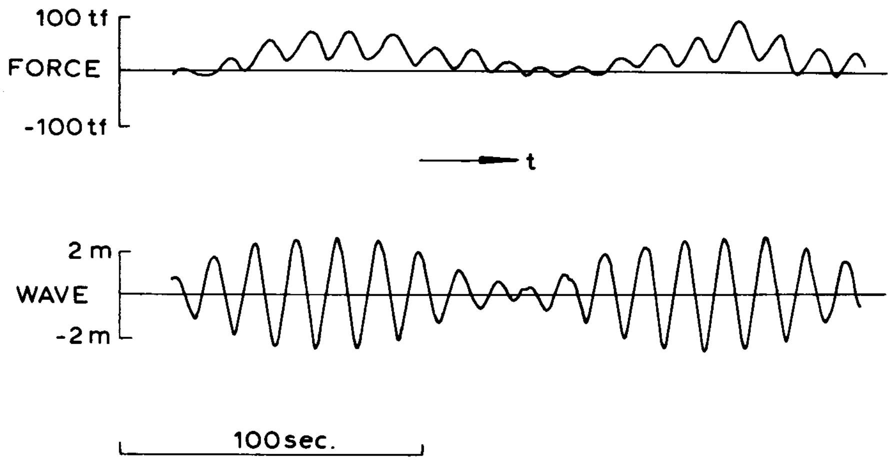



Fig. VII-14 Longitudinal drift force on the tanker in regular wave group.

The time record of the longitudinal force contains a constant part corresponding to the sum of the mean second order force due to each of the regular wave components and a low frequency oscillatory part arising from the combined action of the regular wave components; see equation (IV-49). From the results of measurements the quadratic transfer function of the amplitude of the low frequency force $T_{12}$ is found by simply dividing the amplitude of the measured low frequency force by $2\zeta_1^{(1)}\zeta_2^{(1)}$, where $\zeta_1^{(1)}$ and $\zeta_2^{(1)}$ are the amplitudes of the regular wave components.

VII.4.2. Irregular waves

In Figure VII-15 typical results on the low frequency components of the longitudinal force in irregular waves on the tanker and the semi-submersible are given with the corresponding wave elevations. The time records of the longitudinal forces contain a constant part corresponding to equation (IV-58) and low frequency components.

In order to obtain results on the quadratic transfer function for the low frequency longitudinal forces cross-bi-spectral analysis was applied based on methods developed by Dalzell [VII-1]. Due to the specialized nature a full discussion on cross-bi-spectral methods is outside the scope of this work. For this we refer to the above mentioned author. In Appendix B a brief discussion on the method and some details on the analyses are given. The model test results obtained for the tanker were analyzed in full accordance with Dalzell's method. The results of the model tests with the semi-submersible were analyzed using a slightly modified version of Dalzell's method. The modification is discussed in Appendix B.

VII.5. Computations

Computations of the quadratic transfer function of the amplitude of the longitudinal force in head waves were carried out in accordance with the theory set forth in chapter II through chapter IV.

For the tanker and the semi-submersible the quadratic transfer functions are given in Table VII-1 and Table VII-2 respectively.

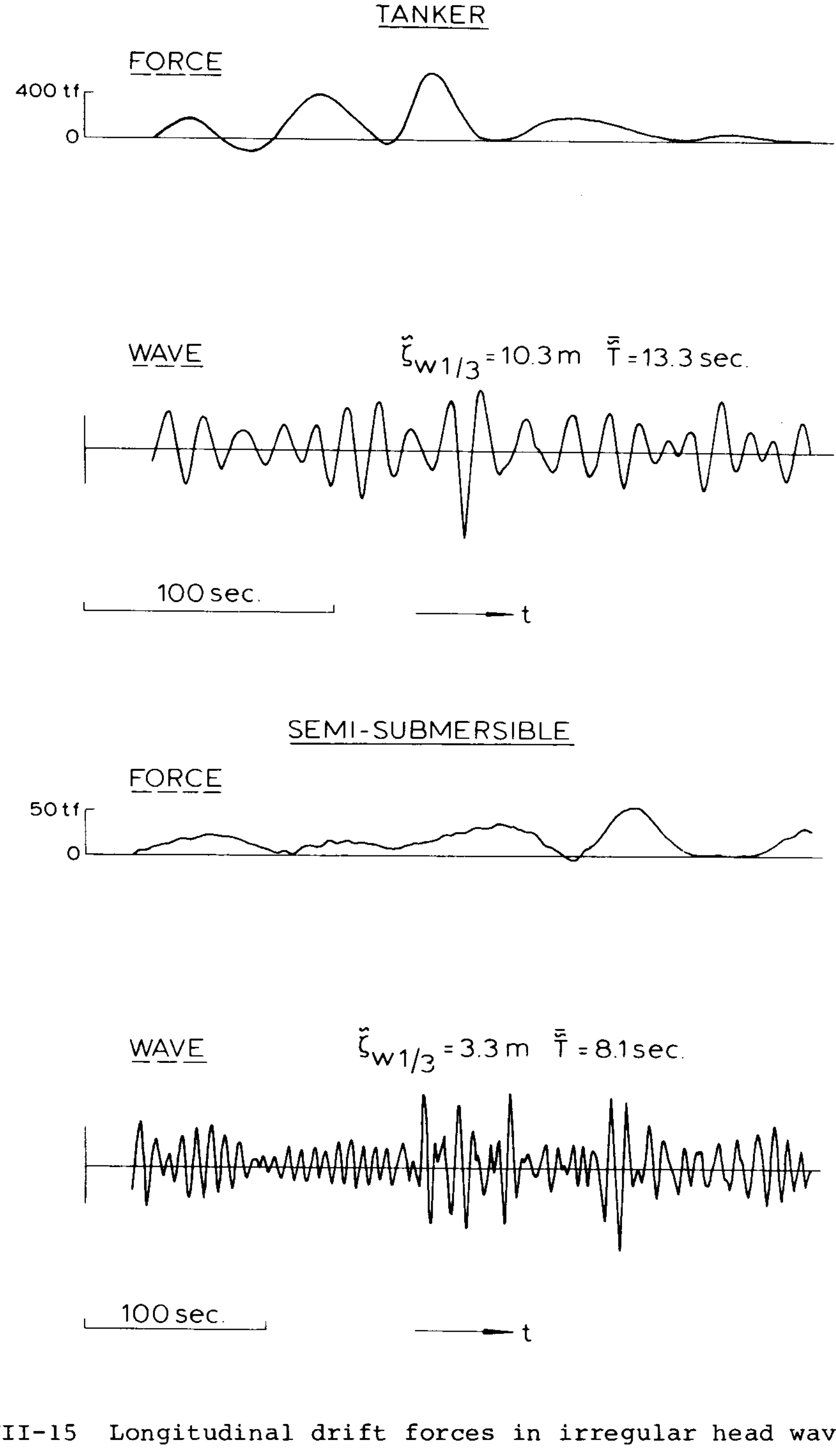



Fig. VII-15  Longitudinal drift forces in irregular head waves.

The transfer functions are given in matrix form of which the two axes represent the two frequency components of a regular wave group. The data given in these tables represent the amplitude of the low frequency forces as computed based on equation (IV-50). The data on the diagonal ($\omega_1 = \omega_2$) represent the amplitudes of the forces for zero difference frequency which, except for the sign, correspond to the mean drift force in regular waves. Since the transfer functions are symmetrical about the diagonal, values are only given for $\omega_1 > \omega_2$. The results give the quadratic transfer function for the force in $tf/m^2$ to a base of wave frequency for the full scale.

As can be seen from these tables the quadratic transfer function for the tanker has been computed for frequency combinations of which the smallest difference frequency $\omega_1 - \omega_2$ is greater than 0.05 rad./sec. The results obtained from model tests apply to frequencies of 0 rad./sec., 0.025 rad./sec. and 0.05 rad./sec. respectively. In order to be able to compare results of computations with experimental results the computed data were cross-faired and interpolated at the difference frequencies of 0.025 rad./sec. and 0.05 rad./sec. respectively. For a difference frequency of 0 rad./sec. no problem exists since at this frequency computed data are also available. These are the computed data on the diagonal of the matrix of the quadratic transfer functions.

| $\omega_2$ \ $\omega_1$ | 0.354 | 0.444 | 0.523 | 0.600 | 0.713 | 0.803 | 0.887 |
|---|---|---|---|---|---|---|---|
| 0.354 | 2.0 | 8.7 | 10.4 | 24.5 | 10.0 | 38.4 | 37.5 |
| 0.444 | | 7.0 | 20.8 | 19.4 | 25.7 | 12.1 | 35.2 |
| 0.523 | | | 12.4 | 16.4 | 8.3 | 14.3 | 14.2 |
| 0.600 | | | | 14.0 | 9.5 | 18.0 | 14.9 |
| 0.713 | | | | | 8.6 | 4.3 | 6.7 |
| 0.803 | | | | | | 9.2 | 4.7 |
| 0.887 | | | | | | | 8.6 |

$T_{12}$ in $tf/m^2$

Frequencies in rad./sec.

Table VII-1 Quadratic transfer function of longitudinal force on the tanker in head waves.

| $\omega_2$ \ $\omega_1$ | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 | 1.1 |
|---|---|---|---|---|---|---|---|
| 0.5 | 0 | 8 | 11 | 7 | 8 | 20 | 15 |
| 0.6 | | 1 | 6 | 7 | 5 | 14 | 19 |
| 0.7 | | | 0 | 7 | 15 | 3 | 18 |
| 0.8 | | | | 11 | 23 | 20 | 13 |
| 0.9 | | | | | 25 | 21 | 9 |
| 1.0 | | | | | | 20 | 21 |
| 1.1 | | | | | | | 24 |

$T_{12}$ in $tf/m^2$

Frequencies in rad./sec.

Table VII-2 Quadratic transfer function of longitudinal force on the semi-submersible in head waves.

For the semi-submersible the experimental data obtained from cross-bi-spectral analysis are valid for difference frequencies of 0 rad./sec. and 0.1 rad./sec. The data for these frequencies are found on the diagonal and the first row next to the diagonal of the computed data given in Table VII-2.

## VII.6. Comparison between computations and experiments

The quadratic transfer functions for the low frequency longitudinal force in head waves on the tanker as obtained from computations and experiments in regular wave groups and irregular wave groups are compared in Figures VII-16(a), (b) and (c). In Figures VII-17(a) and (b) the computed data and experimental data for the semi-submersible in irregular head waves are compared. For each value of the difference frequency the data are given in one figure. In each figure the data are given to a base of the mean frequency of the regular wave components.

The experimental data for the tanker obtained by cross-bi-spectral analysis of data from tests in irregular waves show a rather irregular character. This is ascribed to the parameter settings used during the cross-bi-spectral analysis.

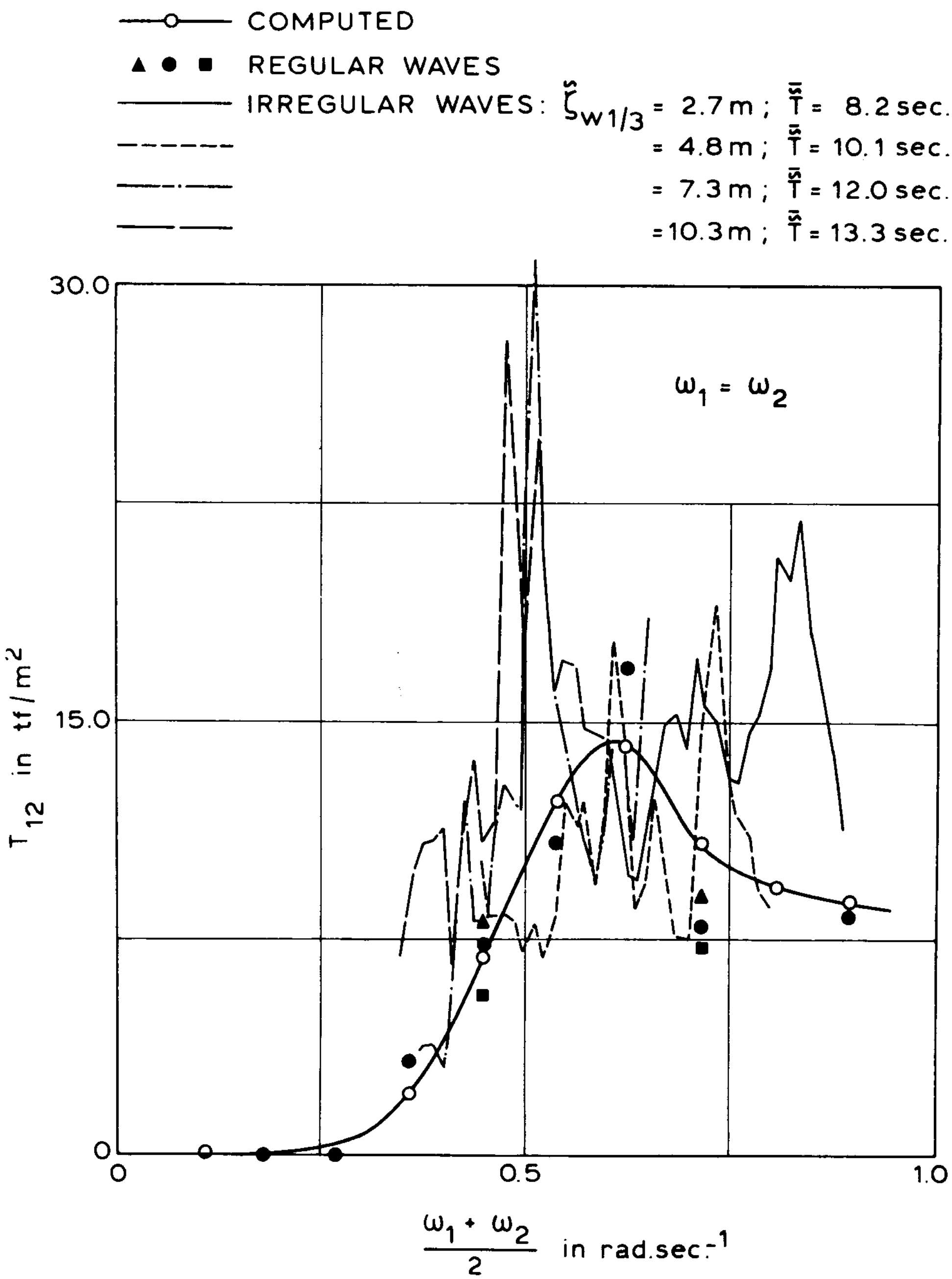



Fig. VII-16(a) Quadratic transfer function of the low frequency longitudinal drift force on the tanker.

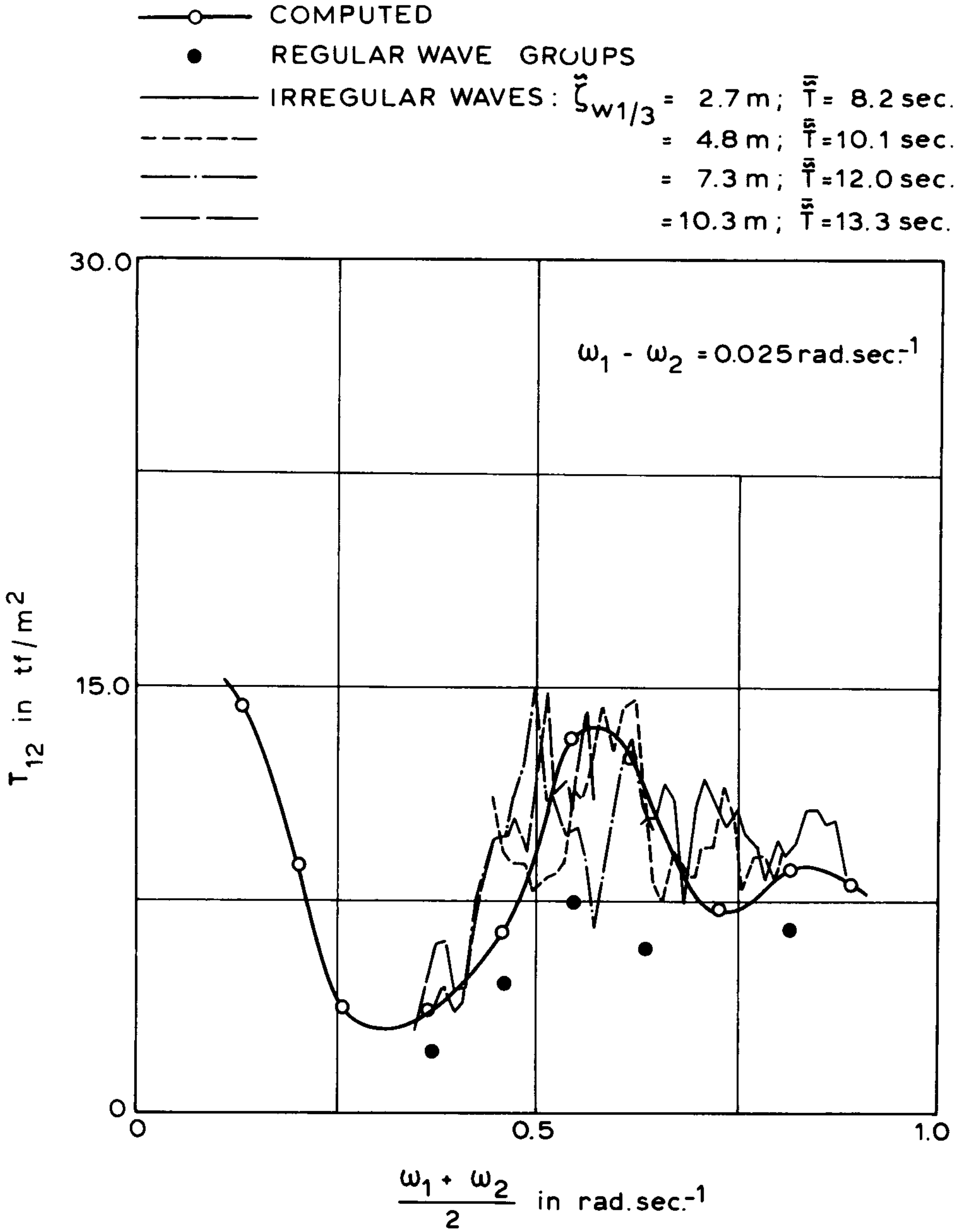



Fig. VII-16(b) Quadratic transfer function of the low frequency longitudinal drift force on the tanker.

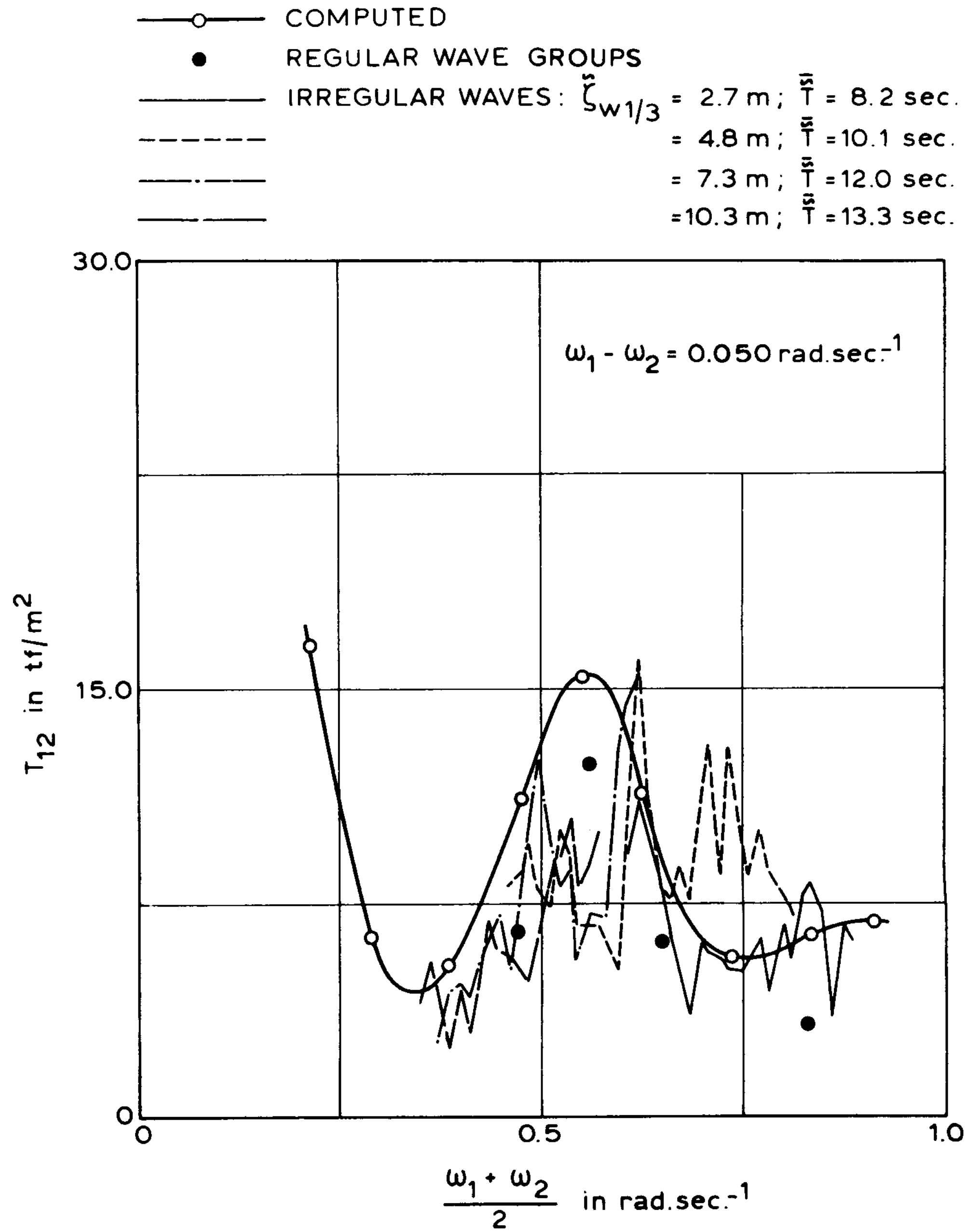
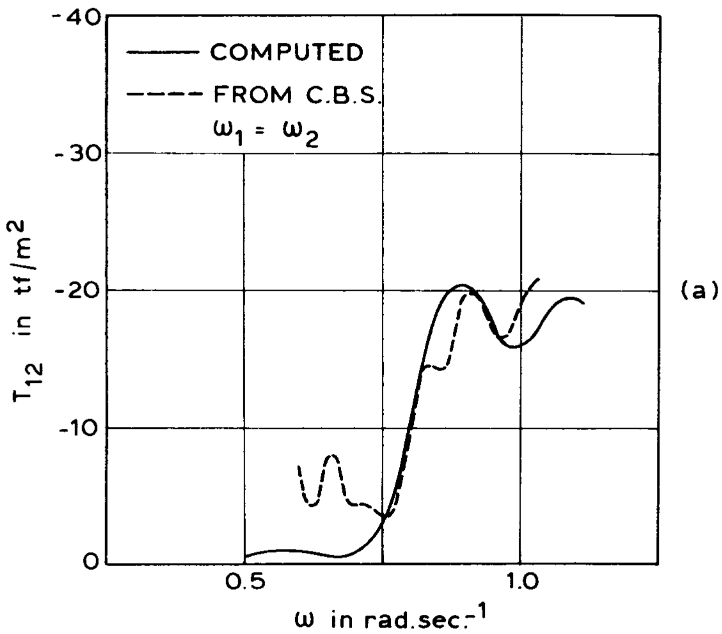



Fig. VII-16(c) Quadratic transfer function of the low frequency longitudinal drift force on the tanker.



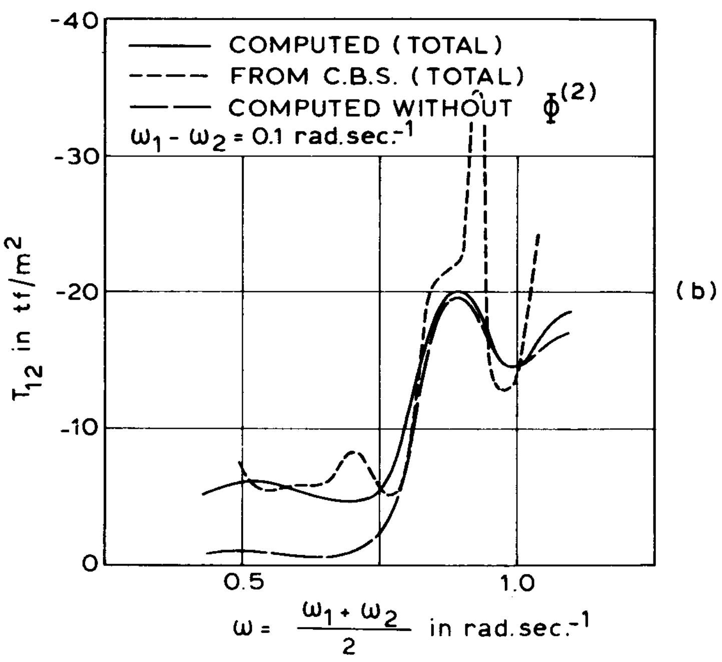




Fig. VII-17(a) and 17(b) Quadratic transfer function of the low frequency longitudinal drift force on the semi-submersible.

By, for instance, changing the filter settings for these computations a smoother set of data could be obtained. Essentially, the results would, however, be the same. The tanker data also show that the scattering of the experimental data from tests in irregular waves is less for difference frequencies of 0.025 rad./sec. and 0.05 rad./sec. than for 0 rad./sec. This is probably related to the fact that the number of oscillations in the low frequency force components with frequencies tending to zero also become zero. This will tend to decrease the reliability of the results of the cross-bi-spectral analysis.

Generally, the experimental data for the tanker from the different tests in irregular waves show reasonable correlation considering the complexity in both the test set-up and the method of analysis. It should also be remembered that the level of the actual force varies considerably between the tests in the lowest and the highest irregular waves. Since the low frequency forces vary quadratically with the wave height the force level in the highest irregular sea state is about ten times larger than in the lowest sea state. Bearing this in mind it can be concluded that the results obtained from the various tests in irregular waves on the quadratic transfer function agree reasonably well. The experimental data obtained for the semi-submersible are smoother than those obtained for the tanker. This is in part due to the increased test duration and in part due to the different parameters used during cross-bi-spectral analysis of these results (see Appendix B).

In general the experimentally obtained data from tests in irregular waves compare reasonably well with the computed data for both the tanker and the semi-submersible. The data from tests with the tanker in regular wave groups are somewhat lower than computed data and data from tests in irregular waves. This may be due to the method used to generate the regular wave groups in the basin. The wave groups were only realized in a relatively narrow field as indicated in Figure VII-10 while the computations assume that these will be long-crested. As a result of this it is possible that the corresponding second order forces were not fully developed during the experiments.

The computed data for the tanker for difference frequencies of 0.025 rad./sec. and 0.05 rad./sec. given in Figures VII-16(b) and 16(c) show a sharp increase for lower values of the mean wave frequency. No experimental data are available at these frequencies to confirm this trend. Examination of the contributions due to the five components to the force given in equations (IV-1) through (IV-5) shows that this effect is due to component V which is caused by the non-linear second order potential.

Examination of the computed data for the semi-submersible reveals that also here some influence of the non-linear second order potential is found. In this case for a difference frequency of 0.1 rad./sec. the experimental data appear to confirm the existence of the contribution due to the non-linear second order potential.

In Table VII-3 the computed quadratic transfer function for the semi-submersible is given without the influence of component V. The contribution due to component V is given in Table VII-4.

| $\omega_2$ \ $\omega_1$ | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 | 1.1 |
|---|---|---|---|---|---|---|---|
| 0.5 | 0 | 1 | 1 | 5 | 6 | 3 | 7 |
| 0.6 | | 1 | 1 | 3 | 6 | 5 | 12 |
| 0.7 | | | 0 | 3 | 10 | 3 | 8 |
| 0.8 | | | | 11 | 21 | 18 | 8 |
| 0.9 | | | | | 25 | 21 | 11 |
| 1.0 | | | | | | 20 | 20 |
| 1.1 | | | | | | | 24 |

$T_{12}$ in tf/$m^2$

Frequencies in rad./sec.

Table VII-3 Quadratic transfer function of longitudinal force on the semi-submersible in head waves without contribution due to second order potential.

| $\omega_2$ \ $\omega_1$ | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 | 1.1 |
|---|---|---|---|---|---|---|---|
| 0.5 | 0 | 7 | 12 | 11 | 3 | 19 | 12 |
| 0.6 | | 0 | 6 | 10 | 6 | 12 | 15 |
| 0.7 | | | 0 | 6 | 9 | 0 | 15 |
| 0.8 | | | | 0 | 6 | 8 | 6 |
| 0.9 | | | | | 0 | 7 | 6 |
| 1.0 | | | | | | 0 | 7 |
| 1.1 | | | | | | | 0 |

$T_{12}$ in tf/$m^2$

Frequencies in rad./sec.

Table VII-4 Contribution of second order potential to the quadratic transfer function for the longitudinal force on the semi-submersible in head waves.

Comparison of the results given in these tables and the total given in Table VII-2 shows that for lower values of the frequency of the regular wave components $\omega_1$ and $\omega_2$ the total low frequency force at difference frequencies greater than zero are dominated by component V. At higher wave frequencies components I through IV tend to dominate the results. These components are due to products of first order quantities which become large when ship motions increase and/or first order diffraction effects increase. When components I through IV dominate the results the first of these components generally is the largest of these, as was already indicated in chapter VI.

## VII.7. Approximation for the low frequency force in irregular waves

In this chapter some results have been given on the low frequency second order longitudinal force in head waves. The results of computations will be used in a discussion concerning a method for approximating the low frequency components of the second order forces in irregular waves.

It is generally believed that the low frequency components of the second order forces in irregular waves can be predicted using the mean forces in regular waves only; see ref. [I-2], [I-3], [I-10], [I-14] and [VII-2]. Within the framework of this study this assumption implies that in a regular wave group with frequency components $\omega_1$ and $\omega_2$ the correct amplitude $T_{12}$ of the low frequency component of the second order forces can be replaced by the value for the amplitude found on the diagonal of the matrix of the quadratic transfer function at the mean value of the two frequencies. Thus:

$$T_{12} = T(\omega_1,\omega_2) \approx T(\frac{\omega_1+\omega_2}{2}, \frac{\omega_1+\omega_2}{2}) \quad \ldots\ldots\ldots \quad \text{(VII-2)}$$

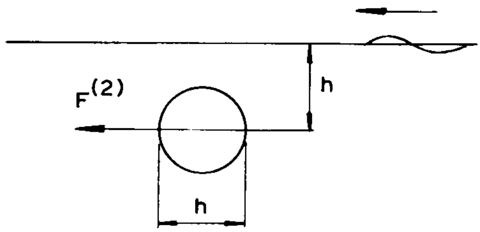
We may check the assumption for the case of the tanker and the semi-submersible by inspection of Figures VII-16(a), (b) and (c) and Figures VII-17(a) and (b).

Equation (VII-2) implies that in Figures VII-16(a), (b) and (c) the computed curves of $T_{12}$ given for the difference frequencies of 0.025 rad./sec. and 0.05 rad./sec. should be the same as the computed curve given for zero difference frequency. For mean frequencies below 0.4 rad./sec. this is clearly not the case. At higher mean frequencies the discrepancies are less but nevertheless increase for the larger value of the difference frequency. The differences below a mean frequency of about 0.4 rad./sec. are mainly caused by the force contribution V due to the second order non-linear potential. This contribution is zero in regular waves.

In Figures VII-17(a) and (b) in which results are given for the semi-submersible it is seen that for mean frequencies below about 0.8 rad./sec. the correct value of $T_{12}$ for a difference frequency of 0.1 rad./sec. would be underestimated if it were replaced by the value found for zero difference frequency. At mean frequencies higher than 0.8 rad./sec. the differences are small. In Figure VII-17(b) computed results are also shown for the case that the contribution V is neglected. Comparison with the total force including this contribution shows that for mean frequencies higher than about 0.8 rad./sec. the influence of this contribution is small. At these mean frequencies the low frequency force is dominated by contributions I through IV which is typically a result of the fact that in shorter waves first order diffraction effects are large.

From the above discussion it is concluded that the low frequency second order forces in irregular waves can be predicted using only the mean force in regular waves provided that the low frequencies (typically the natural frequencies of the horizontal motions of the moored vessels) are not too large. At the same time, the dominant part of the spectrum of the irregular waves must be at wave frequencies where the major part of the second order excitation is due to first order diffraction effects. For the tanker this means that the low frequencies of interest should not exceed about 0.025 rad./sec. The dominant part of the wave spectrum should be at frequencies greater than about 0.4 rad./sec. For the semi-submersible these frequencies are about 0.1 rad./sec. and 0.8 rad./sec. respectively. It may be expected that for the horizontal forces on other types of surface vessels similar conclusions will be reached. It can be concluded therefore that, taking into account some restrictions such as discussed here, for many practical cases the mean second order force on surface vessels in regular waves can be used to predict the low frequency components of the force in irregular waves.

The same assumption cannot, however, be made for all bodies or modes of the second order force. When, for instance, contribution V due to the second order non-linear potential is dominant in the low frequency force at high mean frequencies this will not be true. For example, Ogilvie [II-21] derived analytically the mean second order horizontal force on a submerged horizontal cylinder in regular beam waves. He found that for all wave frequencies the mean force was equal to zero. Using the computation method described in this study his results were rederived and extended to include the low frequency force in regular wave groups. The results are given in Table VII-5. On the diagonal the quadratic transfer function $T_{12}$ for the force is zero. This confirms Ogilvie's analytical results. The values outside the diagonal are, however, non-zero. From this result it can be concluded that for this case the mean force in regular waves cannot be used to predict the low frequency components of the force in irregular waves.




| $2k_2h$ \ $2k_1h$ | 2 | 4 | 6 | 8 |
| --- | --- | --- | --- | --- |
| 2 | 0.00 | 0.08 | 0.09 | 0.08 |
| 4 | | 0.00 | 0.06 | 0.07 |
| 6 | | | 0.00 | 0.05 |
| 8 | | | | 0.00 |

Values: $\frac{T_{12}}{\pi \rho g L}$

Table VII-5 Amplitude of low frequency second order transverse force in regular wave groups on a submerged cylinder in beam waves.

## VII.8. Conclusions

In this chapter aspects were discussed of the system of restraint necessary to determine experimentally the low frequency second order wave forces on vessels in waves. Two possible realizations of such a system were used for model tests. For the frequency ranges in which the system behaved as required, reasonable correlation between results of computations and measurements was found. It should be stated, however, that further development of such systems of restraint are necessary in order to increase the frequency range of application.

On the basis of results of computations on the mean and low frequency second order wave exciting forces, the applicability of a method for approximating the low frequency forces in irregular waves based on knowledge of the mean forces in regular waves was discussed. The results of this discussion indicate that for surface vessels the low frequency horizontal forces which are of importance from the point of view of the low frequency motions (i.e.: force components with frequencies near the natural frequency of the horizontal motions of a moored vessel) can be predicted with reasonable accuracy based on knowledge of the mean forces in regular waves. Certain conditions with respect to the natural frequency of the horizontal motions and the dominant period of the irregular waves must be satisfied however.

In the case of a submerged horizontal cylinder it was found that in regular beam waves the mean horizontal force is equal to zero. The low frequency force in regular wave groups was, however, found to be non-zero, thus indicating that in some cases the mean force in regular waves cannot be used to predict the low frequency force in irregular waves.

# VIII. APPLICATION OF THEORY TO DYNAMIC POSITIONING OF A VESSEL IN IRREGULAR WAVES

## VIII.1. Introduction

In the previous chapters it has been shown that the mean and low frequency second order horizontal forces acting on floating vessels are dominated by the contribution from the relative wave elevation at the waterline of the vessel. In this section it will be shown that this knowledge may be put to practical use to improve the positioning accuracy of dynamically positioned vessels at sea.

Dynamic positioning or station keeping of vessels is a technique which employs ship mounted propulsion units to counteract environmental forces due to wind, waves and current acting on the vessel, thereby maintaining as closely as possible some desired position in the horizontal plane.

The last decade has seen a steady increase in the number of vessels which are stationed at sea by means of dynamic positioning systems. Up to now most dynamic positioning systems were used for positioning drilling ships in deep water where conventional anchoring systems were considered to be too cumbersome. Nowadays dynamic positioning is also being used for diving support vessels and maintainance and survey vessels. This increasing interest in dynamic positioning systems stems from the need for a means of maintaining the vessel's positioning which is quick, accurate and versatile and does not interfere with systems, such as pipelines, lying on the sea floor.

The propulsion units used for dynamic positioning can be either fixed, tunnel mounted controllable pitch propellers, azimuthing right-angle drive units with propellers externally fixed to the vessel, or vertical axis propellers. The magnitude and direction of the thrust produced by such units are governed by a control system which has as input the position error of the vessel relative to the required position and heading in the horizontal plane (feed-back control). Also, in most cases, the instantaneous wind speed and direction measured from the vessel are used to estimate the instantaneous value of the wind force to be counteracted by the propulsion units (wind-feed-forward or wind force compensation).

The position error signals, which form the input to the feed-back control, contain components with wave frequencies and low frequency components. The first order wave forces which induce motions at wave frequencies are generally too large in magnitude to be compensated effectively by ship mounted propulsion units. Even if this was not the case the frequencies involved would require rapid variations in the thrust of the propulsion units which in turn leads to excessive wear and tear of mechanical components. For these reasons the control signals to propulsion units may only contain low frequencies. This requires filtering of the position error signals so that only low frequency components are used for control. One of the major problems involved in filtering the position error signals is due to the occurrence of phase lag in the low frequency output of the filter. This phase lag reduces the quality of the control system from the point of view of its ability to maintain the vessel's position. In order to obtain high quality control signals Kalman filter techniques are often employed, see ref. [VIII-1].

As has been shown by Sjouke and Lagers [VIII-2] including a wind-feed-forward signal in the control system can enhance the position keeping performance considerably, especially in gusty weather conditions. This is a consequence of the fact that the propulsion units counteract the instantaneous force thus preventing the vessel from moving instead of reacting when the vessel has already moved off position as would be the case with feed-back control only.

This chapter is concerned with the capability to determine the instantaneous mean and low frequency second order horizontal wave drift forces and yawing moment acting on a vessel in arbitrary irregular wave conditions. This signal can be used as a control signal to the ship mounted propulsion units which will act to compensate the mean and low frequency forces thus reducing the mean and low frequency horizontal motions of the vessel. In analogy with the term wind-feed-forward this additional capability may be termed as "wave-feed-forward".

In the previous chapters it was shown that in the case of surface vessels, such as a sphere, a tanker, a barge and a semi-submersible, the mean and low frequency wave drift forces are dominated by the contribution arising from the relative wave elevation around the waterline of the vessel. Generally, this contribution has the same sign as the total force and differs mainly in magnitude. In this chapter the effectiveness of wave-feed-forward with respect to the accuracy of station keeping of a dynamically positioned vessel will be investigated. The effectiveness of wave-feed-forward will first be demonstrated based on the results of computations. Experimental data confirming the conclusion based on the results of computations will also be treated. Computations and experimental data apply to the same tanker as used in previous chapters. The main particulars of this vessel are given in Table VI-1 while general and body plans are given in Figure VI-1.

## VIII.2. Theoretical prediction of the effect of wave-feed-forward

The effectiveness of wave-feed-forward may be judged by regarding the nett mean and low frequency force acting on the vessel, that is the mean and low frequency wave drift forces minus the generated thruster forces based on wave-feed-forward control signals. We assume that the time lag between the wave-feed-forward control signals and the corresponding thruster forces is negligible at the low frequencies of interest and that the required thrust is obtained at all times. In that case we may judge the effect of wave-feed-forward on the nett low frequency force on a vessel by regarding the computed quadratic transfer functions of the total low frequency wave drift forces, the contribution due to the relative wave elevation and the nett force which is the total wave drift force minus some fraction of the contributions due to the relative wave elevations.

In Table VIII-1 computed results are shown for the longitudinal force on the tanker in head seas ($180^{\circ}$). The results are given in full scale values in the form of quadratic transfer functions for the amplitude of the longitudinal force. Since the transfer function for the amplitude is symmetrical about the diagonal, see equation (IV-51), values are only given for $\omega_1 > \omega_2$. The nett low frequency longitudinal force was obtained by multiplying the in-phase and out-of-phase components of the transfer functions due to the relative wave elevation contribution by a factor of 0.4 and subtracting these from the corresponding components of the total wave drift force. The quadratic transfer function for the amplitude of the nett force was obtained from the in-phase and out-of-phase components using equation (IV-50).

In Tables VIII-2 through VIII-4 corresponding data are given on the longitudinal force, transverse force and yaw moment for bow quartering waves ($135^o$). In this case the contributions due to the relative wave elevation were multiplied by a gain factor 0.4 for the longitudinal force, a gain factor 0.3 for the transverse force and a gain factor 0.3 for the yawing moment. The above mentioned gain factors were chosen so that the nett forces at zero difference frequency would be about minimal over the frequency range under consideration. From these results it can be seen that for low difference frequencies (data near the diagonal) the nett low frequency forces on the vessel are substantially reduced relative to the total wave drift forces. For larger values of the difference frequencies wave-feed-forward is less effective for reducing the nett forces on the vessel.

From the results given here it may be concluded that application of wave-feed-forward will serve to compensate effectively part of the instantaneous low frequency wave forces. Experimentally this was verified by carrying out model tests in irregular waves with a dynamically positioned tanker using a feed-back control system with and without the application of wave-feed-forward. However, at the time of execution of the model tests the results of computations given here were not available. The gain factors used during the model tests were adjusted on a trial and error basis. The effect of this additional control signal was judged from the reduction obtained in the low frequency horizontal motions of the vessel. In the following sections the experimental set-up and the model tests will be discussed.

TOTAL

| $\omega_2$ \ $\omega_1$ | 0.354 | 0.444 | 0.523 | 0.600 | 0.713 | 0.803 | 0.887 |
|---|---|---|---|---|---|---|---|
| 0.354 | 2 | 9 | 10 | 25 | 10 | 38 | 37 |
| 0.444 | | 7 | 21 | 19 | 26 | 12 | 35 |
| 0.523 | | | 12 | 16 | 8 | 14 | 14 |
| 0.600 | | | | 14 | 10 | 18 | 15 |
| 0.713 | | | | | 9 | 4 | 7 |
| 0.803 | | | | | | 9 | 5 |
| 0.887 | | | | | | | 9 |

$T_{12}$ in $tf/m^2$
Frequencies in rad./sec.

CONTRIBUTION I

| $\omega_2$ \ $\omega_1$ | 0.354 | 0.444 | 0.523 | 0.600 | 0.713 | 0.803 | 0.887 |
|---|---|---|---|---|---|---|---|
| 0.354 | 10 | 19 | 27 | 23 | 21 | 20 | 21 |
| 0.444 | | 25 | 32 | 31 | 26 | 28 | 28 |
| 0.523 | | | 32 | 34 | 27 | 30 | 27 |
| 0.600 | | | | 31 | 29 | 29 | 27 |
| 0.713 | | | | | 23 | 27 | 25 |
| 0.803 | | | | | | 25 | 28 |
| 0.887 | | | | | | | 25 |

$T_{12}$ in $tf/m^2$
Frequencies in rad./sec.

TOTAL - 0.4 * CONTRIBUTION I

| $\omega_2$ \ $\omega_1$ | 0.354 | 0.444 | 0.523 | 0.600 | 0.713 | 0.803 | 0.887 |
|---|---|---|---|---|---|---|---|
| 0.354 | 2 | 9 | 15 | 20 | 15 | 31 | 37 |
| 0.444 | | 3 | 10 | 10 | 19 | 5 | 24 |
| 0.523 | | | 1 | 13 | 5 | 3 | 15 |
| 0.600 | | | | 2 | 6 | 11 | 10 |
| 0.713 | | | | | 1 | 7 | 6 |
| 0.803 | | | | | | 1 | 7 |
| 0.887 | | | | | | | 1 |

$T_{12}$ in $tf/m^2$
Frequencies in rad./sec.

Table VIII-1 Longitudinal force in head waves.

TOTAL

| $\omega_2$ \ $\omega_1$ | 0.354 | 0.444 | 0.523 | 0.600 | 0.713 | 0.803 | 0.887 |
|---|---|---|---|---|---|---|---|
| 0.354 | 1 | 18 | 25 | 32 | 14 | 39 | 29 |
| 0.444 | | 9 | 19 | 38 | 14 | 16 | 5 |
| 0.523 | | | 16 | 38 | 36 | 31 | 27 |
| 0.600 | | | | 17 | 11 | 10 | 15 |
| 0.713 | | | | | 13 | 8 | 3 |
| 0.803 | | | | | | 12 | 4 |
| 0.887 | | | | | | | 10 |

$T_{12}$ in tf/$m^2$
Frequencies in rad./sec.

CONTRIBUTION I

| $\omega_2$ \ $\omega_1$ | 0.354 | 0.444 | 0.523 | 0.600 | 0.713 | 0.803 | 0.887 |
|---|---|---|---|---|---|---|---|
| 0.354 | 5 | 10 | 21 | 22 | 18 | 14 | 17 |
| 0.444 | | 25 | 33 | 24 | 17 | 13 | 8 |
| 0.523 | | | 54 | 50 | 47 | 36 | 43 |
| 0.600 | | | | 40 | 42 | 33 | 35 |
| 0.713 | | | | | 22 | 34 | 26 |
| 0.803 | | | | | | 23 | 34 |
| 0.887 | | | | | | | 22 |

$T_{12}$ in tf/$m^2$
Frequencies in rad./sec.

TOTAL - 0.4 * CONTRIBUTION I

| $\omega_2$ \ $\omega_1$ | 0.354 | 0.444 | 0.523 | 0.600 | 0.713 | 0.803 | 0.887 |
|---|---|---|---|---|---|---|---|
| 0.354 | 1 | 21 | 17 | 32 | 16 | 39 | 30 |
| 0.444 | | 1 | 6 | 30 | 10 | 12 | 5 |
| 0.523 | | | 6 | 19 | 19 | 18 | 10 |
| 0.600 | | | | 1 | 18 | 13 | 6 |
| 0.713 | | | | | 5 | 13 | 8 |
| 0.803 | | | | | | 2 | 9 |
| 0.887 | | | | | | | 2 |

$T_{12}$ in tf/$m^2$
Frequencies in rad./sec.

Table VIII-2 Longitudinal force in bow quartering waves.

TOTAL

| $\omega_2$ \ $\omega_1$ | 0.354 | 0.444 | 0.523 | 0.600 | 0.713 | 0.803 | 0.887 |
|---|---|---|---|---|---|---|---|
| 0.354 | 1 | 35 | 9 | 13 | 34 | 19 | 24 |
| 0.444 | | 5 | 22 | 50 | 42 | 69 | 116 |
| 0.523 | | | 33 | 63 | 3 | 24 | 21 |
| 0.600 | | | | 79 | 57 | 15 | 8 |
| 0.713 | | | | | 91 | 58 | 6 |
| 0.803 | | | | | | 90 | 60 |
| 0.887 | | | | | | | 95 |

$T_{12}$ in $tf/m^2$
Frequencies in rad./sec.

CONTRIBUTION I

| $\omega_2$ \ $\omega_1$ | 0.354 | 0.444 | 0.523 | 0.600 | 0.713 | 0.803 | 0.887 |
|---|---|---|---|---|---|---|---|
| 0.354 | 5 | 43 | 35 | 49 | 11 | 17 | 15 |
| 0.444 | | 61 | 159 | 44 | 134 | 41 | 79 |
| 0.523 | | | 102 | 165 | 73 | 26 | 40 |
| 0.600 | | | | 259 | 192 | 50 | 41 |
| 0.713 | | | | | 298 | 221 | 69 |
| 0.803 | | | | | | 297 | 230 |
| 0.887 | | | | | | | 309 |

$T_{12}$ in $tf/m^2$
Frequencies in rad./sec.

TOTAL - 0.3 * CONTRIBUTION I

| $\omega_2$ \ $\omega_1$ | 0.354 | 0.444 | 0.523 | 0.600 | 0.713 | 0.803 | 0.887 |
|---|---|---|---|---|---|---|---|
| 0.354 | 0 | 46 | 6 | 17 | 31 | 21 | 21 |
| 0.444 | | 13 | 43 | 47 | 29 | 65 | 103 |
| 0.523 | | | 2 | 15 | 20 | 19 | 17 |
| 0.600 | | | | 1 | 13 | 21 | 5 |
| 0.713 | | | | | 2 | 17 | 23 |
| 0.803 | | | | | | 1 | 16 |
| 0.887 | | | | | | | 2 |

$T_{12}$ in $tf/m^2$
Frequencies in rad./sec.

Table VIII-3 Transverse force in bow quartering waves.

TOTAL

| $\omega_2$ \ $\omega_1$ | 0.354 | 0.444 | 0.523 | 0.600 | 0.713 | 0.803 | 0.887 |
|---|---|---|---|---|---|---|---|
| 0.354 | 364 | 7085 | 9284 | 8809 | 1433 | 4448 | 2544 |
| 0.444 | | 1527 | 16655 | 7551 | 3052 | 5067 | 12269 |
| 0.523 | | | 2554 | 7577 | 5870 | 2937 | 2994 |
| 0.600 | | | | 782 | 5124 | 2363 | 2445 |
| 0.713 | | | | | 1193 | 4576 | 3574 |
| 0.803 | | | | | | 200 | 4998 |
| 0.887 | | | | | | | 562 |

$T_{12}$ in tfm/m$^2$
Frequencies in rad./sec.

CONTRIBUTION I

| $\omega_2$ \ $\omega_1$ | 0.354 | 0.444 | 0.523 | 0.600 | 0.713 | 0.803 | 0.887 |
|---|---|---|---|---|---|---|---|
| 0.354 | 1881 | 1993 | 3637 | 1611 | 3742 | 1669 | 1872 |
| 0.444 | | 11633 | 16368 | 26015 | 3456 | 9644 | 7040 |
| 0.523 | | | 9066 | 5377 | 11610 | 6747 | 2808 |
| 0.600 | | | | 1909 | 14844 | 12883 | 2498 |
| 0.713 | | | | | 4366 | 13830 | 14818 |
| 0.803 | | | | | | 678 | 13751 |
| 0.887 | | | | | | | 1003 |

$T_{12}$ in tfm/m$^2$
Frequencies in rad./sec.

TOTAL - 0.3 * CONTRIBUTION I

| $\omega_2$ \ $\omega_1$ | 0.354 | 0.444 | 0.523 | 0.600 | 0.713 | 0.803 | 0.887 |
|---|---|---|---|---|---|---|---|
| 0.354 | 200 | 7589 | 8223 | 8634 | 2515 | 3989 | 2685 |
| 0.444 | | 1963 | 15856 | 8282 | 2434 | 3250 | 11398 |
| 0.523 | | | 166 | 6081 | 2626 | 3330 | 2292 |
| 0.600 | | | | 209 | 677 | 2854 | 3146 |
| 0.713 | | | | | 117 | 1402 | 2861 |
| 0.803 | | | | | | 404 | 1196 |
| 0.887 | | | | | | | 261 |

$T_{12}$ in tfm/m$^2$
Frequencies in rad./sec.

Table VIII-4 Yaw moment in bow quartering waves.

## VIII.3. Generation of the wave-feed-forward control signal

The wave-feed-forward control signal is derived from the continuous evaluation of the following equations for the longitudinal force, the transverse force and the yaw moment:

- longitudinal force:

$$F_1(t) \approx -C_1 \int_{WL} \tfrac{1}{2}\rho g \zeta_r^{(1)^2} . n_1 . d\ell \quad \ldots\ldots\ldots\ldots \quad \text{(VIII-1)}$$

- transverse force:

$$F_2(t) \approx -C_2 \int_{WL} \tfrac{1}{2}\rho g \zeta_r^{(1)^2} . n_2 . d\ell \quad \ldots\ldots\ldots\ldots \quad \text{(VIII-2)}$$

- yaw moment:

$$M_3(t) \approx -C_3 \int_{WL} \tfrac{1}{2}\rho g \zeta_r^{(1)^2} . (x_1 n_2 - x_2 n_1) . d\ell \quad \ldots\ldots \quad \text{(VIII-3)}$$

in which $C_1$, $C_2$ and $C_3$ are gain factors which express the ratio between the total wave drift forces and the contribution due to the relative wave elevation.

The above equations contain line integrals around the waterline which involve the geometry of the hull form at the waterline and the instantaneous values of the first order relative wave elevation. It must be remembered that $\zeta_r$ is the wave elevation as measured relative to the vessel at the waterline. Its value at any point can therefore be measured directly by means of a wave probe fixed to the side of the vessel. This is shown in Figure VIII-1.

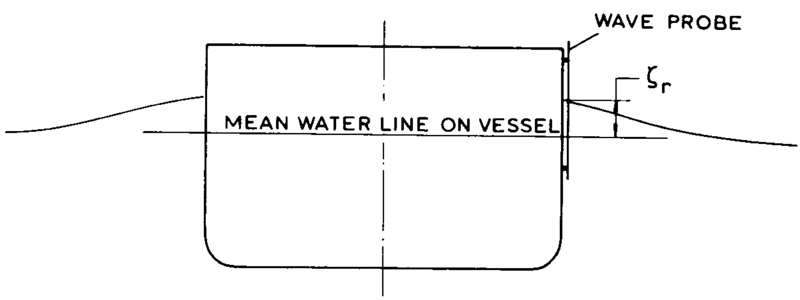



Fig. VIII-1 Wave probe measuring relative wave elevation $\zeta_r$.

Another method to determine the relative wave elevation is to install pressure pick-ups in the side of the vessel just below the waterline. The pressure variations may be easily converted into relative wave elevations making use of the fact that near the wave surface pressure variations are due to hydrostatic effects. For the experiments use was made of wave probes fixed to the side of the vessel.

The measured relative wave elevation at a point along the waterline contains first and higher order components:

$$\zeta_r = \varepsilon\zeta_r^{(1)} + \varepsilon^2\zeta_r^{(2)} + \ldots \qquad \text{(VIII-4)}$$

The square of $\zeta_r$ gives:

$$\zeta_r^2 = \varepsilon^2\zeta_r^{(1)^2} + \varepsilon^3\zeta_r^{(1)}\zeta_r^{(2)} + 0(\varepsilon^4) + \ldots \qquad \text{(VIII-5)}$$

From this it follows that the lowest order in the square of $\zeta_r$ involves only $\zeta_r^{(1)}$ as is required for the elevation of equations (VIII-1) through (VIII-3). Components of order $\varepsilon^3$ and higher are generally small and of high frequency and will be neglected hereafter.

In irregular waves the first order relative wave elevation in a point along the waterline may be written as:

$$\zeta_r^{(1)}(t) = \sum_{i=1}^{N} \zeta_{r_i}^{(1)}.\cos(\omega_i t + \underline{\varepsilon}_i) \qquad \text{(VIII-6)}$$

Squaring this expression gives:

$$\zeta_r^{(1)^2}(t) = \sum_{i=1}^{N}\sum_{j=1}^{N} \zeta_{r_i}^{(1)}\zeta_{r_j}^{(1)}.\cos(\omega_i t + \underline{\varepsilon}_i).\cos(\omega_j t + \underline{\varepsilon}_j)$$

$$= \sum_{i=1}^{N}\sum_{j=1}^{N} \tfrac{1}{2}\zeta_{r_i}^{(1)}\zeta_{r_j}^{(1)}.\cos\{(\omega_i - \omega_j)t + (\underline{\varepsilon}_i - \underline{\varepsilon}_j)\} +$$

$$+ \sum_{i=1}^{N}\sum_{j=1}^{N} \tfrac{1}{2}\zeta_{r_i}^{(1)}\zeta_{r_j}^{(1)}.\cos\{(\omega_i + \omega_j)t + (\underline{\varepsilon}_i + \underline{\varepsilon}_j)\}$$

$$\qquad \text{(VIII-7)}$$

This shows that the square of the measured wave elevation will in general contain mean and low frequency components corresponding to the difference frequencies and high frequency components corresponding to the sum frequency of the frequency components in equation (VIII-6). In Figure VIII-2 time records of $\zeta_r^{(1)}(t)$ and the square of $\zeta_r^{(1)}(t)$ are shown schematically. In this figure the dotted line indicates the low frequency part of the square of the wave elevation.

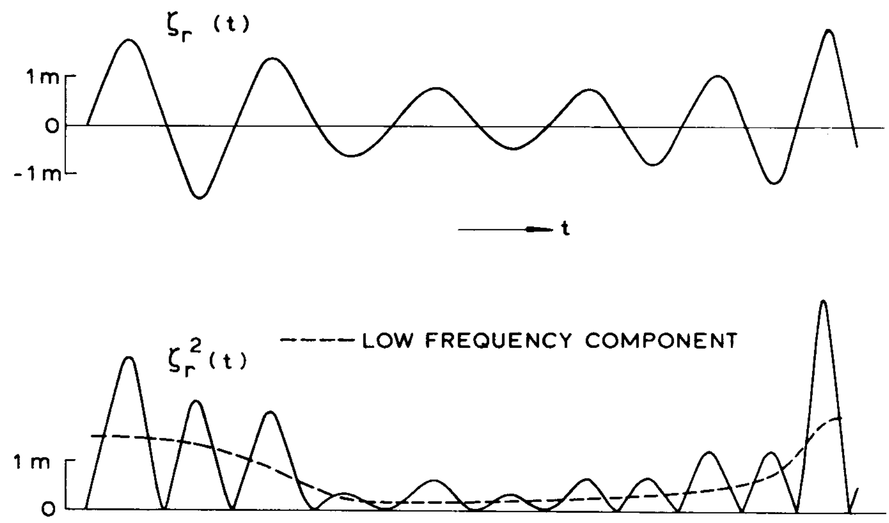



Fig. VIII-2 Relative wave elevation and square of relative wave elevation.

For the model tests equations (VIII-1) through (VIII-3) were evaluated by replacing the integrals by simple summations of the following type:

$$F_1(t) \approx -C_1 \sum_{n=1}^{K} \tfrac{1}{2}\rho g \zeta_{rn}^{\;2}(t) . n_{1n} . \Delta\ell_n \quad . \; . \; . \; . \; . \; . \; . \; . \quad \text{(VIII-8)}$$

$$F_2(t) \approx -C_2 \sum_{n=1}^{K} \tfrac{1}{2}\rho g \zeta_{rn}^2(t) . n_{2n} . \Delta \ell_n \quad \ldots\ldots\ldots \text{(VIII-9)}$$

$$M_3(t) \approx -C_3 \sum_{n=1}^{K} \tfrac{1}{2}\rho g \zeta_{rn}^2(t) . (x_{1n} n_{2n} - x_{2n} n_{1n}) . \Delta \ell_n \quad \ldots\ldots\ldots\ldots \text{(VIII-10)}$$

in which:

$K$ = number of wave probes

index $n$ = denotes wave probe under consideration

$\zeta_{rn}(t)$ = measured relative wave elevation of $n^{th}$ wave probe

$x_{1n}$, $x_{2n}$ = co-ordinates of $n^{th}$ wave probe and the centre of a straight line element approximating the local waterline form

$\Delta \ell_n$ = length of $n^{th}$ waterline element

$n_{1n}$, $n_{2n}$ = direction cosines of the $n^{th}$ waterline element

$C_1$, $C_2$, $C_3$ = gain factors.

For the model tests eight wave probes were used. The position of the wave probes are shown in Figure VIII-3. Equation (VIII-8) through equation (VIII-10) were evaluated continually by means of an analog computer.

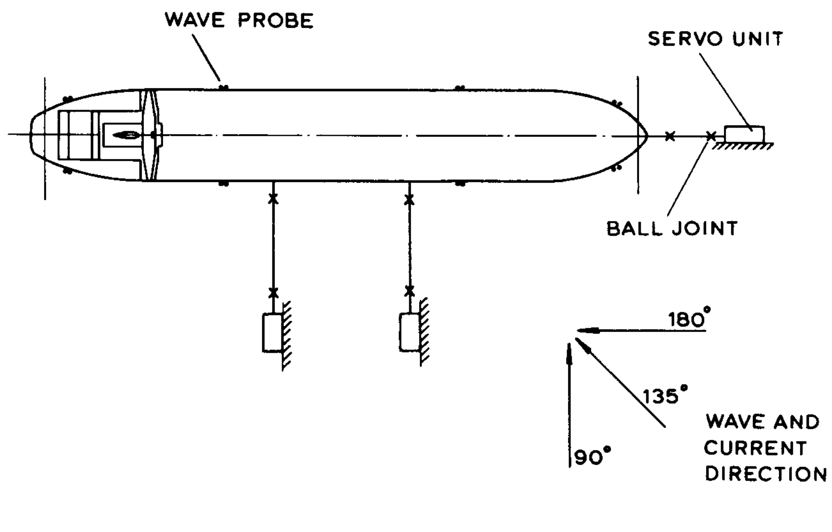



Fig. VIII-3 Position of wave probes measuring relative wave elevation.

From equation (VIII-7) it will be clear that the output of the control signals as generated by equations (VIII-8) through (VIII-10) will contain mean and low frequency components corresponding to the difference frequencies of the irregular waves and high frequencies corresponding to the sum frequencies of the irregular waves. As indicated in the introduction of this chapter thruster control signals may not contain high frequencies from the point of view of wear and tear of mechanical components. The wave-feed-forward thrust control signals must therefore be filtered to eliminate the high frequencies which in this case are sum frequencies. As was already indicated in the introduction care must be taken to select an analog filter which, while removing the sum frequency component, does not cause appreciable phase lag in the low frequency components. With this type of signal this does not form a problem because the high frequencies are in the order of twice the wave frequencies. This means that the demands placed on the filter are more easily met in this case than in the case of a normal feed-back control system based on the position error signal. In such cases the high frequencies coincide with the wave frequencies.

The amplitude and phase characteristics of the analog filter through which the wave-feed-forward signals generated by equations (VIII-8) through (VIII-10) were passed are given in Figure VIII-4. From this figure it is seen that the phase lag remains less than 45 degrees for frequencies up to 0.21 rad./sec. full scale. The spectra of the irregular waves in which model tests were carried out are given in Figure VIII-5. From this figure it can be deduced that the sum frequency components in the unfiltered wave-feed-forward signals range from about 0.6 rad./sec. upwards, which is twice the lowest frequencies present in the irregular waves with the longest mean period. The amplitude of the filter has at this frequency reduced to 50%. For the wave spectrum with the lowest mean period the sum frequency components have frequencies higher than 1.0 rad./sec. At this frequency the filter amplitude is 40%.

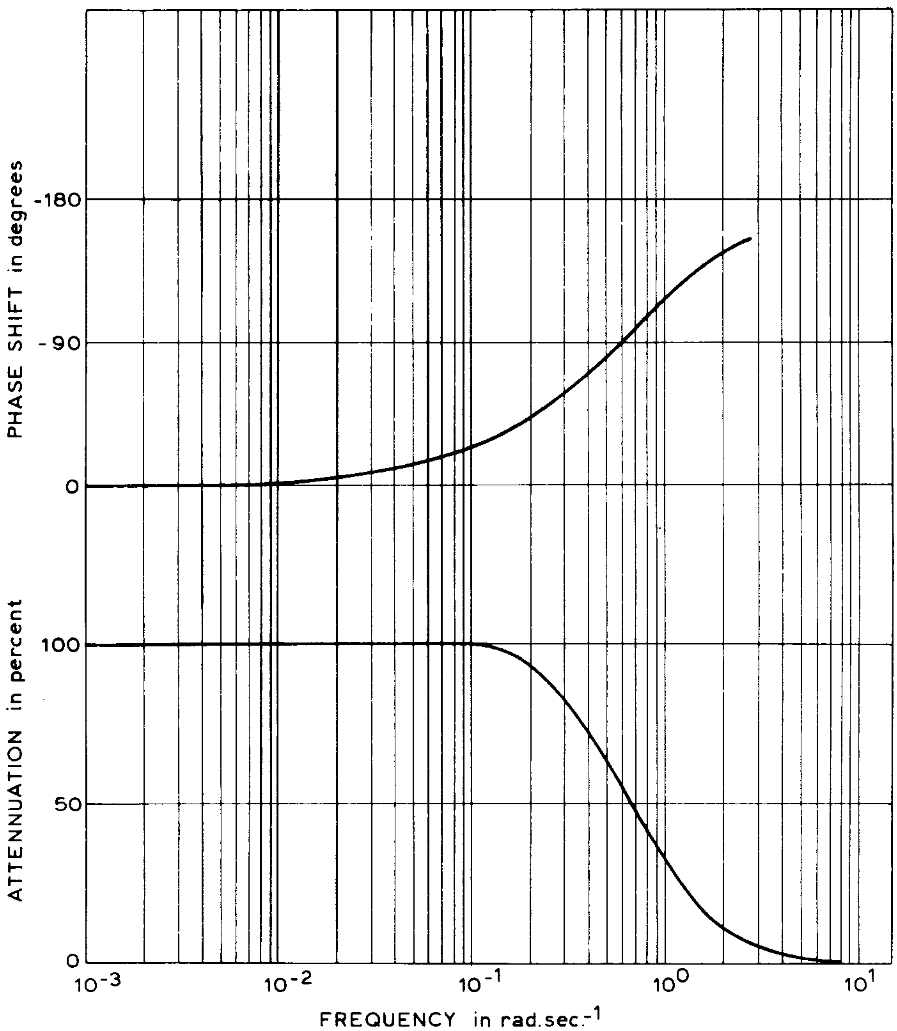



Fig. VIII-4 Low-pass frequency characteristic of the wave-feed-forward.

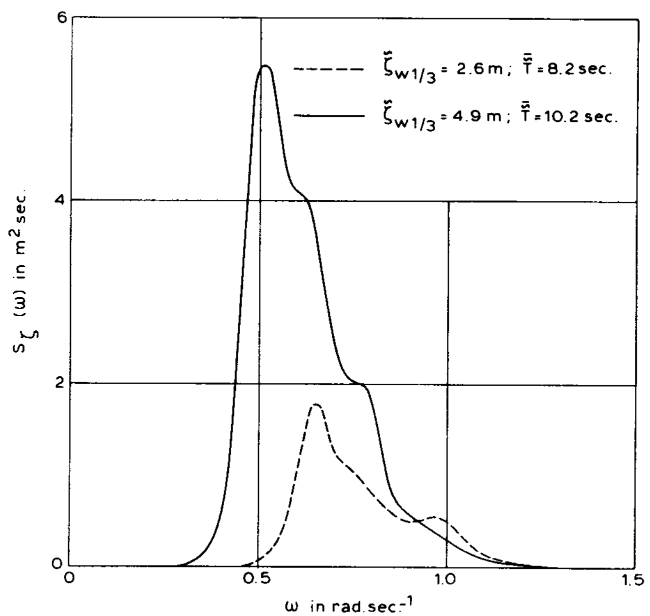



Fig. VIII-5 Spectra of irregular waves.

## VIII.4. Positioning system

Instead of using ship mounted propulsion units the horizontal position and heading angle of the vessel were governed by three servo units which could apply a horizontal longitudinal force and two horizontal transverse forces. This set-up is shown in Figure VIII-3. The light-weight rods connecting the vessel to the servo units incorporated axial force transducers. The servo units applied forces on the vessel in response to two control signals, i.e. the feed-back control signal and the wave-feed-forward control signal.

The feed-back control signal was generated within each of the three servo units independently. The control was of the proportional-differential type and acted on the low frequency horizontal displacement and displacement velocity of each connecting rod relative to the servo unit. To this end the horizontal displacement velocity signals of the horizontal rods, which contained both wave frequencies and low frequencies, were filtered to remove as much as possible the wave frequencies. The amplitude and phase characteristics of these filters are shown in Figure VIII-6.

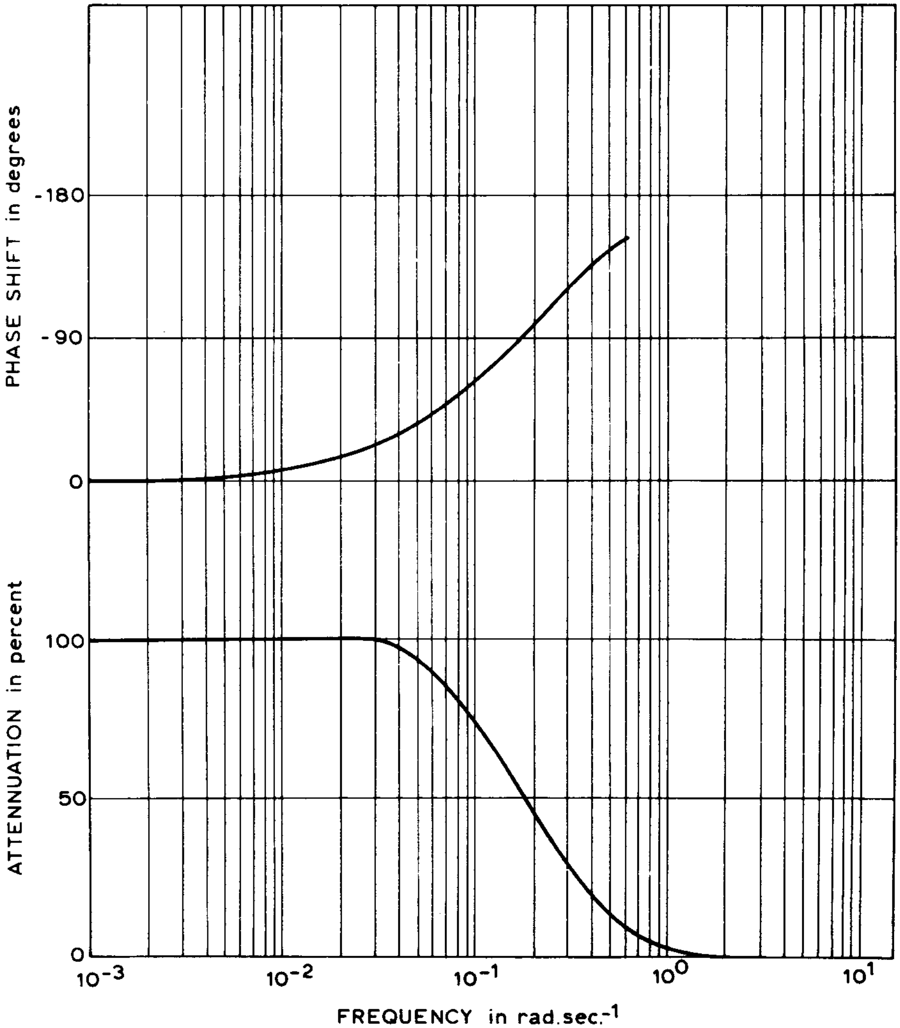



Fig. VIII-6 Low-pass frequency characteristic of the damping term.

The part of the control signal which was proportional to the displacement remained unfiltered. The contribution of this part to the total feed-back control signal was weak and mainly served to limit the mean displacement of the vessel.

The overall characteristics of the feed-back control system were such that high damping was achieved in the low frequency region only. The horizontal motions of the vessel in still water after an initial displacement out of the equilibrium position are given in Figure VIII-7 and demonstrate the high damping introduced by the feed-back system. During the tests which were all carried out in irregular waves the feed-back system parameters remained unchanged, except for the test in head seas (180$^{o}$) where the damping for surge motion was reduced in order to show more clearly the effect of wave-feed-forward.

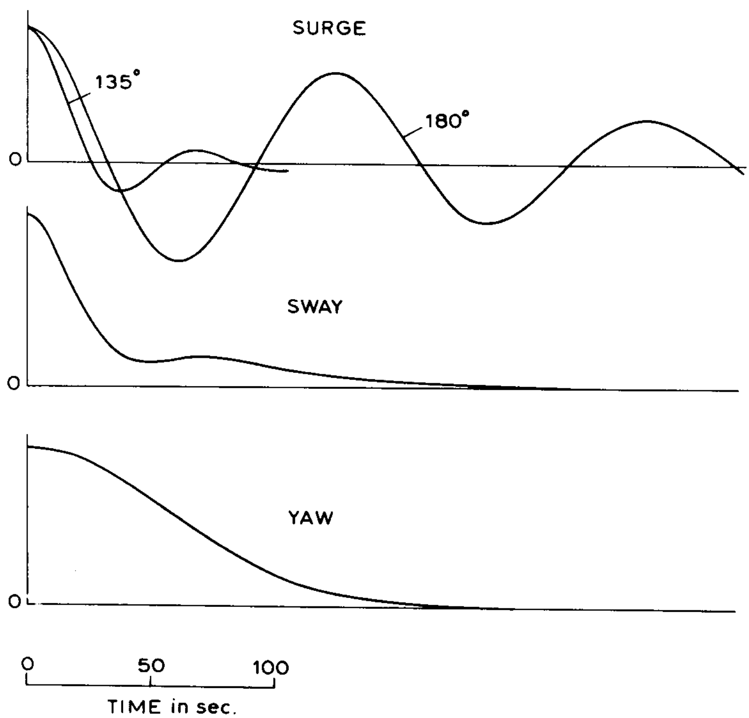



Fig. VIII-7 Surge motion decay after an initial displacement in still water.

For each sea condition a model test was carried out twice in the same wave train, once with and once without the wave-feed-forward control signals. A block diagram of the control system is shown in Figure VIII-8.

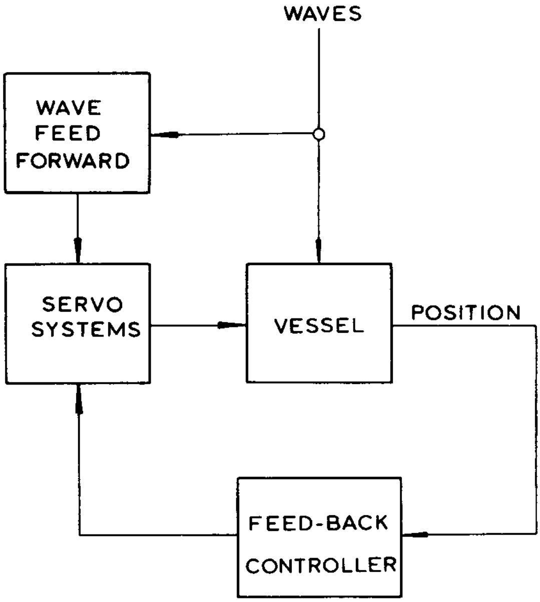



Fig. VIII-8 Block diagram of dynamic positioning system.

The values of the gain factors $C_1$, $C_2$ and $C_3$ of the wave-feed-forward signals were adjusted on a trial and error basis as at the time of execution of model tests it was not possible to predict these values on the basis of computations. Due to limitation in the test set-up it also was not possible to determine afterwards which value had actually been used.

## VIII.5. Model tests

### VIII.5.1. General

The model tests were carried out in irregular waves for the following sea conditions:

- In head waves ($180^{o}$) with a significant wave height of 4.9 m and a mean period of 10.2 sec.
- In bow quartering waves ($135^{o}$) with a significant wave height of 4.9 m and a mean period of 10.2 sec. with and without a current of 1 knot from 45 degrees.
- In bow quartering waves ($135^{o}$) with a significant wave height of 2.6 m and a mean period of 8.2 sec.

The spectra of the irregular waves are shown in Figure VIII-5. The wave and current directions are defined in Figure VIII-3.

During a model test, which was carried out for a time duration corresponding to 35 minutes full scale, the surge, sway and yaw motions and the total longitudinal and transverse forces and yaw moment exerted on the vessel by the servo units were measured and recorded on F.M.-tape. The results of measurements were analyzed to determine the spectra of the low frequency components of the motions and the forces as well as the mean values of the forces. The mean values of the motions are not of importance in this case, since this can be easily rectified in reality by the inclusion of an additional control signal based on a time integral of the displacements.

### VIII.5.2. Results of tests in irregular waves

The results of the tests are presented in the form of examples of time traces of the horizontal motions (Figures VIII-9 and VIII-10) and in the form of spectra of the low frequency components of forces, moment and horizontal motions (Figures VIII-11 through VIII-13).

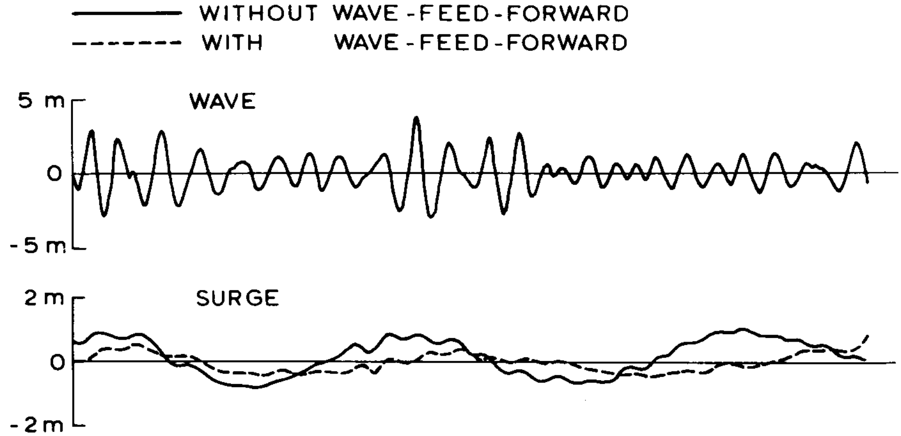



Fig. VIII-9 Surge motions in irregular head waves. Significant height 4.9 m.

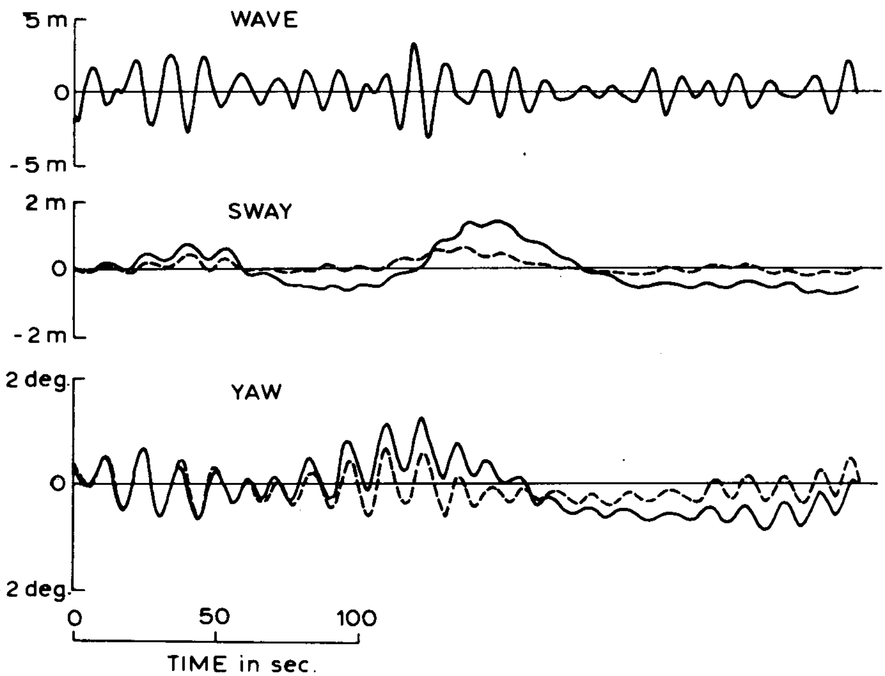



Fig. VIII-10 Sway and yaw motions in irregular bow quartering waves. Significant height 4.9 m.

From the results it is seen that, except for the surge motions in bow quartering waves (135°), the low frequency parts of the horizontal motions are significantly reduced when applying wave-feed-forward. It appears that a reduction in the motions need not necessarily result in a corresponding increase in the thrust to be applied to the vessel.

The mean wave drifting forces are not affected by the control system used as is demonstrated from the results given in Figures VIII-11 through VIII-13. In Figure VIII-12 it is seen that the low frequency component of the sway force $F_2$ does not change significantly even though the sway motion itself is considerably smaller when using wave-feed-forward. In the same figures it is seen that the spectral density of the yaw moment is increased. In terms of lateral forces applied at the end of the vessel the absolute value of the moment is small.

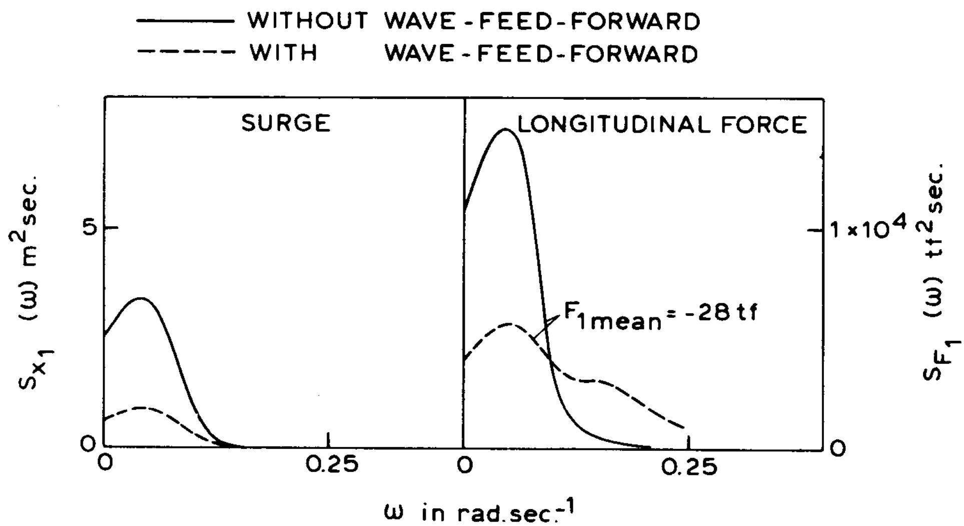



Fig. VIII-11 Spectra of low frequency surge motion and force in irregular head waves. Significant height 4.9 m.

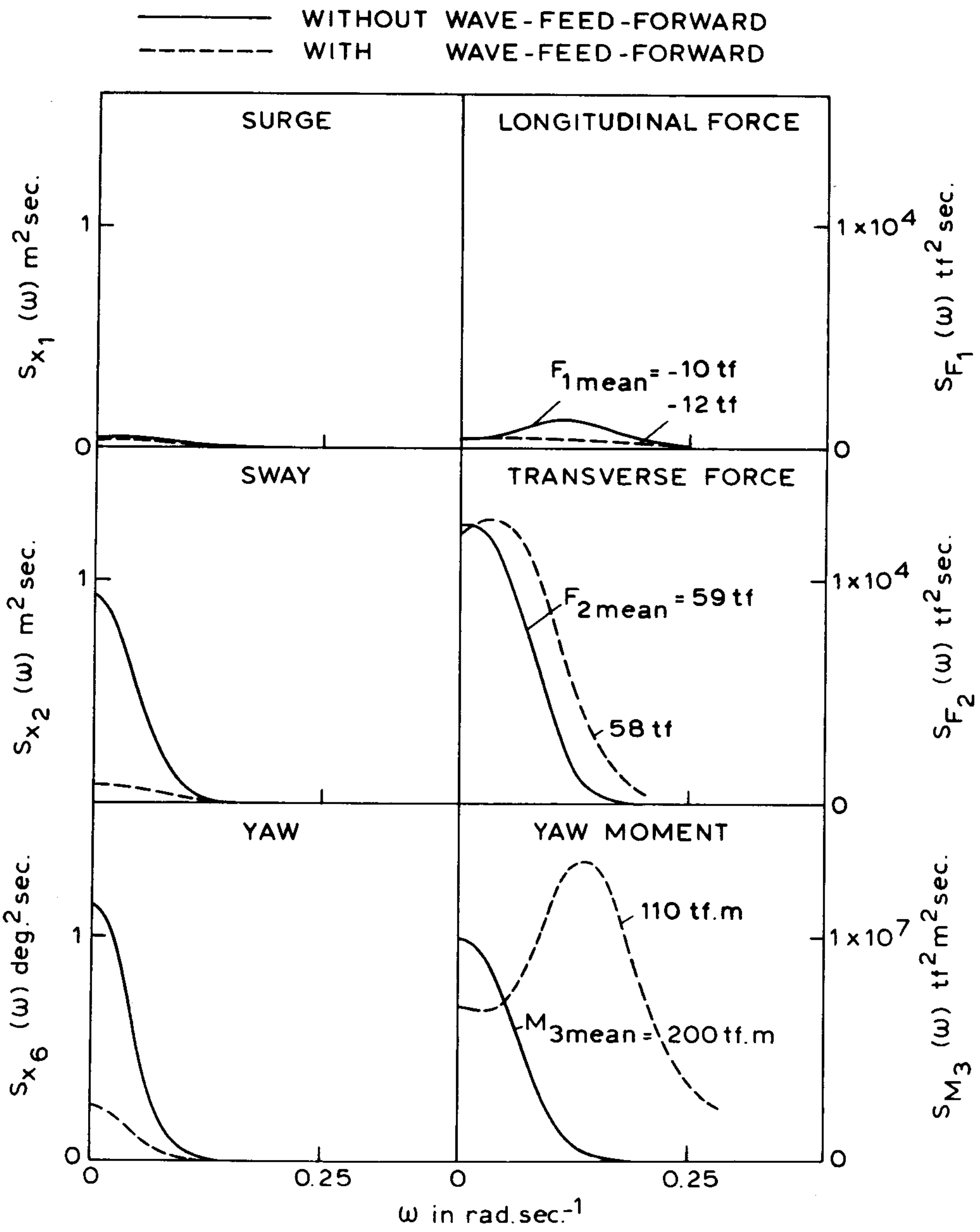



Fig. VIII-12 Spectra of low frequency forces and motions in irregular bow quartering waves. Significant height 2.6 m.

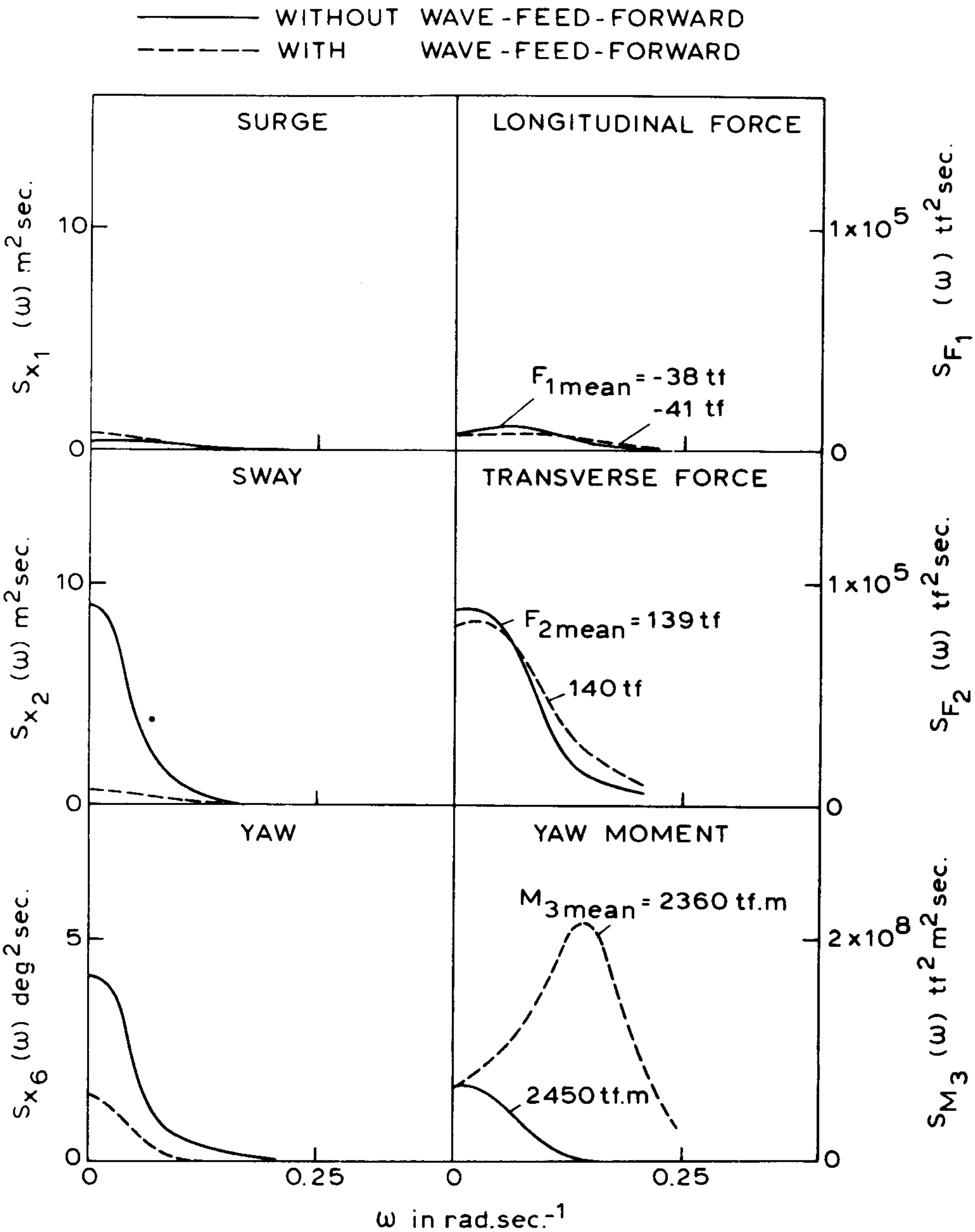



Fig. VIII-13 Spectra of low frequency forces and motions in irregular bow quartering waves. Significant height 4.9 m.

In Figures VIII-12 and VIII-13 it is seen that the surge motion is hardly affected by wave-feed-forward. It appeared that the damping of the low frequency surge motion was so large that the feed-back system alone could reduce low frequency surge motions to almost minimal values. Adding the wave-feed-forward control signal brought about only minimal changes. The corresponding surge motion decay test in still water is shown in Figure VIII-7, indicated by $135^{o}$, being the wave direction in the tests shown in Figure VIII-12 and Figure VIII-13. For the test in irregular head waves, shown in Figure VIII-11, the surge motion damping was reduced (see surge motion decay test in Figure VIII-7: $180^{o}$), showing more clearly the effect of wave-feed-forward on low frequency surge motions.

VIII.5.3. Results of tests in irregular waves and current

Tests were carried out with and without wave-feed-forward in bow quartering irregular waves ($135^{o}$) with a significant wave height of 4.9 m and a mean period of 10.2 sec. and a stern quartering current ($45^{o}$) of about 1 knot. The spectra of the low frequency parts of the motions and forces are shown in Figure VIII-14. The results are similar to the results shown in Figure VIII-13. The irregular waves are in both cases according to the wave spectrum given in Figure VIII-5.

The results given in Figure VIII-14 show that current does not affect the control signal. This is to be expected since the wave-feed-forward signal is determined from wave elevation signals which do not change appreciably for the normal values of the current speeds encountered. From the results it is also seen that the low frequency forces and motions are not appreciably different from the results given in Figure VIII-13, which indicates that the influence of current on the low frequency wave drift forces is, in this case, not great.

From the results of the tests in irregular waves with and without current it appears that it is possible to reduce the low frequency part of the sway motion by about 70% and the low frequency yaw and surge motion by about 50% through the use of wave-feed-forward.

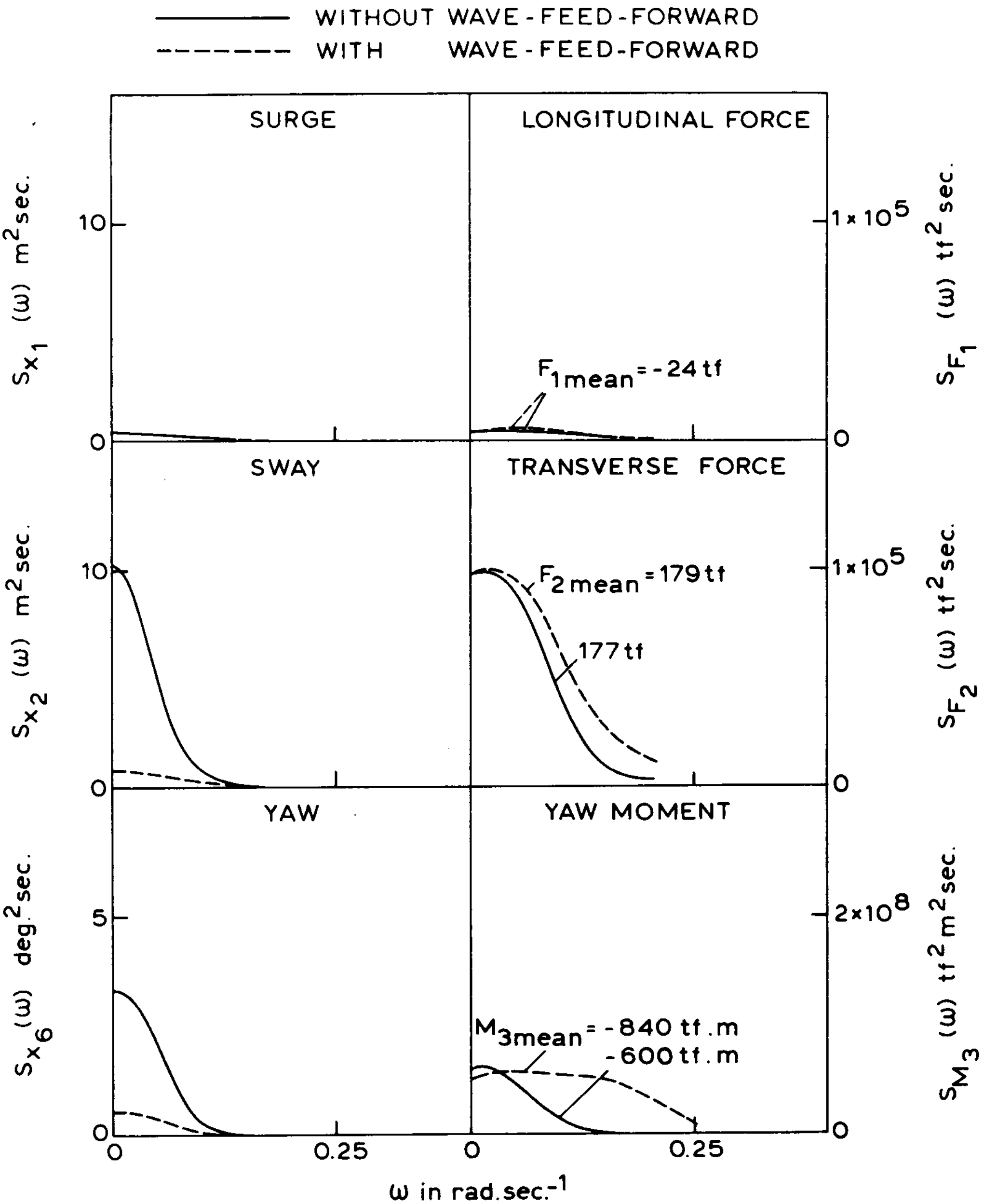



Fig. VIII-14 Spectra of low frequency forces and motions in irregular bow quartering waves and 1 knot stern quartering current. Significant height 4.9 m.

This is less than could be expected on the basis of results of computations given in section 2 of this chapter. It should be borne in mind, however, that for the experimental determination of the wave-feed-forward signals only eight wave probes were used. It may be expected that increasing this number will lead to more accurate evaluation of the low frequency component of the contribution due to the relative wave elevation, thereby increasing the accuracy of the control signals.

## VIII.6. Conclusions

In this chapter it has been shown that, as a result of the theory developed in this study, whereby the mean and low frequency wave drift forces on floating objects are determined through direct integration of all pressure contributions to the second order forces over the wetted part of the hull, expressions are derived which, after numerical evaluation, lead to conclusions regarding the applicability of hitherto unknown methods to improve the accuracy of dynamic positioning of vessels in waves.

It is possible to predict on the basis of the results of computations the effectiveness of a wave-feed-forward control signal with respect to the degree in which such a signal can compensate the instantaneous mean and low frequency wave drift forces acting on a vessel. Theoretical computations can be used to determine the values of the gain factors which are inherent to the wave-feed-forward method. The results of model tests indicate that, even without prior knowledge of such gain factors, wave-feed-forward can function effectively.

## IX. CONCLUSIONS

As a result of the investigations presented in this study the following conclusions can be drawn:

1. The total low frequency hydrodynamic forces acting on a vessel in waves may be considered as the sum of two parts:
   - the mean and low frequency second order wave exciting forces;
   - the hydrodynamic reaction forces resulting from the low frequency motions induced by the low frequency wave forces.

   Furthermore the mean and low frequency second order wave exciting forces are independent of the low frequency motions. The hydrodynamic reaction forces may be expressed in terms of added mass and damping coefficients which are determined by means of existing linear potential theory methods (chapter III).

2. Based on the method of direct integration of fluid pressure acting on the instantaneous wetted part of the hull of a body it is shown that the total second order wave exciting forces contain five components. Four of these components may be evaluated using existing computation methods based on linear potential theory. The fifth contribution depends on the solution of the second order non-linear velocity potential. This contribution may be approximated using results on the first order wave exciting forces (chapters III and IV).

3. The second order wave exciting forces acting on a body in irregular waves are the sum of second order force components due to regular wave groups present in irregular waves. The second order wave exciting forces may be expressed in the form of quadratic transfer functions which may be used to compute the second order forces in irregular waves in the frequency domain or the time domain (chapter IV).

4. The comparison between results of computations using the method of direct integration of pressure and analytical results obtained using an existing method based on energy and momentum considerations demonstrates the equivalence of both methods

with respect to the total force (chapter V).

5. Except at wave frequencies and wave directions which result in large amplitude resonant roll motions, the mean second order horizontal wave forces and yaw moment on a tanker and a rectangular barge can be predicted with good accuracy by means of computations based on potential theory (chapter VI).

6. The good comparison obtained between experimental results and computed results on the mean second order horizontal forces and yaw moment acting on a semi-submersible and the mean second order vertical force and pitch moment on a submerged cylinder indicates that viscous effects are small even for bodies consisting of slender elements (chapter VI).

7. The mean second order horizontal forces acting on surface vessels such as a hemisphere, a tanker, a barge and a semi-submersible are dominated by the contribution due to the relative wave elevation around the waterline. This contribution determines the sign of the total force. The second most important contribution is due to the non-linear pressure contribution in the Bernoulli equation. This contribution is generally of the same order as the total force, but of opposite sign. The contribution of the second order non-linear velocity potential to the mean forces is at all times equal to zero. The remaining two contributions, due to products of local pressure gradients and motions and due to products of angular body motions and inertia forces, vary in sign and are generally smaller in magnitude (chapter VI).

8. The mean second order vertical forces on a submerged horizontal cylinder are dominated by the non-linear pressure contribution in the Bernoulli equation. The contribution due to products of local pressure gradients and motions are also of importance. Other contributions are zero or small compared to the first two contributions (chapter VI).

9. The low frequency second order longitudinal force in irregular waves on a tanker and a semi-submersible contains contributions

arising from products of first order quantities and a contribution due to the second order velocity potential. The relative importance of these contributions depends on the wave frequencies and the low frequencies of interest of the second order force. At high wave frequencies, where first order diffraction effects are large, the low frequency forces are dominated by contributions arising from products of first order quantities. At low wave frequencies, where first order diffraction effects are small, the contribution due to the second order velocity potential is relatively of greater importance. The importance of this contribution becomes greater as the frequency of the second order forces increases (chapter VII).

10. Existing methods for computing the low frequency second order forces in irregular waves on floating structures, which rely solely on the mean forces in regular waves, are applicable for surface vessels provided that:

    - the wave frequencies are sufficiently high to ensure that the second order exciting forces are dominated by products of first order quantities;
    - the frequencies of interest of the second order forces are low (chapter VII).

11. In the case of a long submerged horizontal cylinder in beam waves computations of the low frequency horizontal force in irregular waves cannot be based on the mean force in regular waves (chapter VII).

12. The accuracy of station keeping of a dynamically positioned vessel can be improved through the application of a wave-feed-forward control signal, which is based on the relative wave elevation measured around the vessel (chapter VIII).

## APPENDIX A - COMPUTATION OF THE FIRST ORDER SOLUTION FOR THE VELOCITY POTENTIAL AND BODY MOTIONS

### Introduction

This appendix gives a short account of the underlying theory and method of computation of the first order velocity potential and first order body motions for an arbitrarily shaped body floating in regular, long-crested waves as given by Van Oortmerssen [A-1]. A brief review of the method is given here for the sake of completeness and due to its importance with respect to the computation of the mean and low frequency second order forces. Since this appendix deals only with first order quantities the affix $^{(1)}$, which is used in the main body of this work to distinguish between first and second order quantities, is deleted. Furthermore, in keeping with ref. [A-1] use is made of the complex notation $e^{-i\omega t}$ to denote oscillatory quantities instead of sin $\omega t$ and cos $\omega t$.

### Description of the theory

#### First order wave loads and motions

The ship is considered as a rigid body, oscillating sinusoidally about a state of rest, in response to excitation by a long-crested regular wave. The amplitudes of the motions of the ship as well as of the wave are supposed to be small while the fluid is assumed to be ideal and irrotational. A right-handed, fixed system of co-ordinates $O\text{-}X_1\text{-}X_2\text{-}X_3$ is defined with the origin in the mean position of the centre of gravity of the body and the $O\text{-}X_3$ axis vertically upwards. The oscillating motion of the ship in the $j^{th}$ mode is given by:

$$x_j = \zeta_j \, e^{-i\omega t} \qquad j = 1,....,6 \qquad \text{. . . . . . . (A-1)}$$

in which $\zeta_j$ is the amplitude of the motion in the $j^{th}$ mode and $\omega$ the circular frequency. The motion variables $x_1$, $x_2$ and $x_3$ stand for the translations surge, sway and heave, while $x_4$, $x_5$ and $x_6$ denote rotations around the $O\text{-}X_1$, $O\text{-}X_2$ and $O\text{-}X_3$ axes respectively.

The free surface at great distance from the ship is defined by:

$$\zeta = \zeta_0 \, e^{ik(x_1 \cos\alpha + x_2 \sin\alpha) - i\omega t} \qquad \ldots\ldots \text{(A-2)}$$

where:

$\zeta_0$ = amplitude of the wave
k = wave number = $2\pi/\lambda$, where $\lambda$ is the wave length
$\alpha$ = angle of incidence.

The flow field can be characterized by a first order velocity potential:

$$\Phi(x_1,x_2,x_3,t) = \phi(x_1,x_2,x_3)e^{-i\omega t} \qquad \ldots\ldots\ldots \text{(A-3)}$$

The potential function $\Phi$ can be separated into contributions from all modes of motion and from the incident and diffracted wave fields:

$$\phi = i\omega\,\zeta_0(\phi_0 + \phi_7) - i\omega \sum_{j=1}^{6} \phi_j\,\zeta_j \qquad \ldots\ldots\ldots \text{(A-4)}$$

The incident wave potential is given by:

$$\phi_0 = \frac{1}{\nu}\,\frac{\cosh k(x_3 + c)}{\cosh kd}\, e^{ik(x_1 \cos\alpha + x_2 \sin\alpha)} \qquad \ldots \text{(A-5)}$$

in which:

$\nu = \omega^2/g$
c = the distance from the origin to the sea bed
d = water depth
$\alpha$ = angle of incidence of the waves.

The cases j = 1,....,6 correspond to the potentials due to the motion of the ship in the $j^{th}$ mode, while $\phi_7$ is the potential of the diffracted waves. The individual potentials are all solutions of the Laplace equation which satisfy the linearized free surface condition and the boundary conditions on the sea floor, on the body's surface and at infinity. The potential function $\phi$ can be represented by a continuous distribution of single sources on the boundary surface S:

$$\phi_j(x_1,x_2,x_3) = \frac{1}{4\pi} \iint_S \sigma_j(a_1,a_2,a_3).\gamma_j(x_1,x_2,x_3,a_1,a_2,a_3)dS$$

for $j = 1, 2, \ldots, 7$

$$\ldots \ldots \ldots \ldots \ldots \quad \text{(A-6)}$$

where:

$\gamma_j(x_1,x_2,x_3,a_1,a_2,a_3)$ = the Green's function of a source, singular in $a_1, a_2, a_3$

$a_1, a_2, a_3$ = the vector describing S

$\sigma_j(a_1,a_2,a_3)$ = the complex source strength.

For the Green's function a function is chosen which satisfies the Laplace equation and the boundary conditions on the sea bottom, in the free surface and at infinity. This function is given by (see Wehausen and Laitone [A-2]):

$$\gamma = \frac{1}{r} + \frac{1}{r_1} + PV \; .$$

$$\int_0^\infty \frac{2(\xi+\nu)e^{-\xi d}.\cosh \xi(a_3+c).\cosh \xi(x_3+c)}{\xi \sinh \xi d - \nu \cosh \xi d} J_0(\xi R)d\xi +$$

$$+ i \frac{2\pi(k^2-\nu^2).\cosh k(a_3+c).\cosh k(x_3+c)}{k^2 d - \nu^2 d + \nu} J_0(kR)$$

$$\ldots \ldots \ldots \ldots \ldots \quad \text{(A-7)}$$

in which:

$$\left.\begin{aligned} r &= \sqrt{(x_1-a_1)^2 + (x_2-a_2)^2 + (x_3-a_3)^2} \\ r_1 &= \sqrt{(x_1-a_1)^2 + (x_2-a_2)^2 + (x_3+2c+a_3)^2} \\ R &= \sqrt{(x_1-a_1)^2 + (x_2-a_2)^2} \end{aligned}\right] \ldots \quad \text{(A-8)}$$

John [A-3] has derived the following series for $\gamma$, which is the analogue of (A-7):

$$\gamma = 2\pi \frac{\nu^2 - k^2}{k^2 d - \nu^2 d + \nu} \cosh k(a_3+c).\cosh k(x_3+c) \; .$$

$$. \{Y_0(kR) - i\, J_0(kR)\} + \sum_{i=1}^{\infty} \frac{4(\mu_i^2 + \nu^2)}{d\mu_i^2 + d\nu^2 - \nu} .$$

$$. \cos \mu_i(x_3 + c) . \cos \mu_i(a_3 + c) . K_0(\mu_i R)$$

$$\ldots\ldots\ldots\ldots \quad \text{(A-9)}$$

where $\mu_i$ are the positive solutions of:

$$\mu_i \tan(\mu_i d) + \nu = 0 \quad \ldots\ldots\ldots\ldots \quad \text{(A-10)}$$

Although these two representations are equivalent, one of the two may have preference for numerical computations depending on the values of the variables. In general, equation (A-9) is the most convenient representation for calculations. When R = 0 the value of $K_0$ becomes infinite; therefore equation (A-7) must be used when R is small or zero.

The unknown source strength function $\sigma$ must be determined such that the boundary condition on the body's surface S is fulfilled. Due to the linearization this boundary condition is applied to the surface in its equilibrium position $S_0$.

$$n_j = -\tfrac{1}{2}\sigma_j(x_1,x_2,x_3) + \frac{1}{4\pi} \iint_{S_0} \sigma_j(a_1,a_2,a_3) .$$

$$. \frac{\partial}{\partial n} \gamma_j(x_1,x_2,x_3,a_1,a_2,a_3)dS \qquad \text{for } j = 1,\ldots.,6$$

$$n_j = - \frac{\partial \phi_0}{\partial n} \qquad \text{for } j = 7$$

$$\ldots\ldots\ldots\ldots \quad \text{(A-11)}$$

$n_1$ through $n_6$ are the generalized direction cosines on $S_0$, defined by:

$$\left.\begin{aligned} n_1 &= \cos\,(n,x_1) \\ n_2 &= \cos\,(n,x_2) \\ n_3 &= \cos\,(n,x_3) \\ n_4 &= x_2n_3 - x_3n_2 \\ n_5 &= x_3n_1 - x_1n_3 \\ n_6 &= x_1n_2 - x_2n_1 \end{aligned}\right] \quad \ldots\ldots\ldots\ldots\ldots \quad \text{(A-12)}$$

To solve equation (A-6) numerically the surface S is subdivided into a number of finite, plane elements on which the source strength is constant. The boundary condition is applied in one control point on each element being the centre of the element. The integral equation (A-6) then reduces to a set of algebraic equations in the unknown source strengths. In general, the Green's function $\gamma$ may be computed with sufficient accuracy as if the source strength is concentrated in the centre (control point) of each element. When, however, the influence of an element on its own control point is evaluated $\gamma$ has a singularity of the type $1/r$, which can be removed by spreading the source uniformly over the panel. When the influence of a panel on a control point, which is at a close distance of this panel and not lying in the same plane, is considered the source is spread uniformly and integrated numerically to obtain its contribution to $\phi$ or $\frac{\partial \phi}{\partial n}$.

After solving the equations for the source strengths the first order potential function is known. The pressure on the surface S can then be found from Bernoulli's theorem. The linearized hydrodynamic pressure is given by:

$$p(x_1,x_2,x_3,t) = -\rho\,\frac{\partial \Phi}{\partial t} =$$

$$= \{\rho\omega^2\,\zeta_0(\phi_0 + \phi_7) + \rho\omega^2 \sum_{j=1}^{6} \phi_j\,\zeta_j\}e^{-i\omega t}$$

$$\ldots\ldots\ldots\ldots\ldots \quad \text{(A-13)}$$

Subsequently, the first order wave exciting forces and moments can be found from:

$$X_k = -\rho\omega^2 \zeta_0 e^{-i\omega t} \iint_{S_0} (\phi_0 + \phi_7) n_k \, dS$$

. . . . . . . . . . . . . . (A-14)

The oscillating hydrodynamic forces (k = 1, 2, 3) and moments (k = 4, 5, 6) in the $k^{th}$ direction are:

$$F_k = -\rho\omega^2 \sum_{j=1}^{6} \zeta_j e^{-i\omega t} \iint_{S_0} \phi_j n_k \, dS$$

. . . . . . . . . . . . . . (A-15)

According to common practice the hydrodynamic forces are represented by means of added mass and damping coefficients:

$$a_{kj} = -\rho \text{ Re } \{\iint_{S_0} \phi_j n_k \, dS\} \quad \text{. . . . . . . . . . . . . . (A-16)}$$

$$b_{kj} = -\rho\omega \text{ Im } \{\iint_{S_0} \phi_j n_k \, dS\} \quad \text{. . . . . . . . . . . . . . (A-17)}$$

where:

$a_{kj}$ = the added mass coefficient in the k-mode due to motion in the j-mode

$b_{kj}$ = the damping coefficient in the k-mode due to motion in the j-mode.

Finally, the motion response to first order excitation is computed by means of the well known equations of motion in the frequency domain:

$$\sum_{j=1}^{6} \{-\omega^2 (M_{kj} + a_{kj}).\sin(\omega t + \varepsilon_j) + b_{kj}.\omega.\cos(\omega t + \varepsilon_j) +$$

$$+ C_{kj}.\sin(\omega t + \varepsilon_j)\}\zeta_j = X_k.\sin(\omega t + \delta_k)$$

for k = 1,....,6

. . . . . . . . . . . . . . (A-18)

in which:

$X_k$ = wave excited force in the $k^{th}$ mode

$\varepsilon_j$, $\delta_k$ = phase angles.

$M_{kj}$ is an inertia matrix. Since the origin of the system of axes coincides with the centre of gravity of the ship in its rest position it is found that:

$$M_{kj} = \begin{vmatrix} m & 0 & 0 & 0 & 0 & 0 \\ 0 & m & 0 & 0 & 0 & 0 \\ 0 & 0 & m & 0 & 0 & 0 \\ 0 & 0 & 0 & I_4 & 0 & -I_{46} \\ 0 & 0 & 0 & 0 & I_5 & 0 \\ 0 & 0 & 0 & -I_{64} & 0 & I_6 \end{vmatrix} \qquad \text{(A-19)}$$

where:

$m$ = mass of the ship

$I_k$ = moment of inertia in the $k^{th}$ mode

$I_{kj}$ = product of inertia.

## APPENDIX B - CROSS-BI-SPECTRAL ANALYSIS

### Introduction

The computer program which was used to analyze the input-output relationship between the waves (input) and the low frequency mooring or restraining forces (output) is based on the cross-bi-spectral method as given by Dalzell [B-1], [B-2] and [B-3]. For a complete description of the method we refer to his works.

The intention of this appendix is to give a "feel" for the processes involved, rather than to give a condensed version of the specialist's point of view as given by Dalzell.

We assume that the input (wave elevation) can be written as follows:

$$\zeta^{(1)}(t) = \sum_{i=1}^{N} \zeta_i^{(1)} . \sin(\omega_i t + \underline{\varepsilon}_i) \qquad \text{. . . . . . . . . (B-1)}$$

in which:

$\omega_i$ = frequency in rad./sec.

$\underline{\varepsilon}_i$ = random phase, uniformly distributed from 0 - $2\pi$

$\zeta_i^{(1)}$ = amplitude of component with frequency $\omega_i$

t = time

N = a large number.

The foregoing expression represents a zero-mean, normally distributed, stationary random signal. The square of the wave elevation is:

$$\zeta^{(1)^2}(t) = \{ \sum_{i=1}^{N} \zeta_i^{(1)} . \cos(\omega_i t + \underline{\varepsilon}_1) \}^2$$

$$= \sum_{i=1}^{N} \sum_{j=1}^{N} \zeta_i^{(1)} \zeta_j^{(1)} . \cos(\omega_i t + \underline{\varepsilon}_i) . \cos(\omega_j t + \underline{\varepsilon}_j)$$

$$= \sum_{i=1}^{N} \sum_{j=1}^{N} \tfrac{1}{2}\zeta_i^{(1)}\zeta_j^{(1)}.\cos\{(\omega_i - \omega_j)t + (\underline{\varepsilon}_i - \underline{\varepsilon}_j)\} +$$

$$+ \sum_{i=1}^{N} \sum_{j=1}^{N} \tfrac{1}{2}\zeta_i^{(1)}\zeta_j^{(1)}.\cos\{(\omega_i + \omega_j)t + (\underline{\varepsilon}_i + \underline{\varepsilon}_j)\}$$

. . . . . . . . . . . . . . (B-2)

The low frequency part of the square of the wave elevation is:

$$\zeta_{low}^{(1)^2}(t) = \sum_{i=1}^{N} \sum_{j=1}^{N} \tfrac{1}{2}\zeta_i^{(1)}\zeta_j^{(1)}.\cos\{(\omega_i - \omega_j)t + (\underline{\varepsilon}_i - \underline{\varepsilon}_j)\}$$

. . . . . . . . . . . . . . (B-3)

It is assumed that the output (wave drift force or moment) contains only low frequencies and is closely related to the low frequency part of the square of the wave elevation:

$$F^{(2)}(t) = \sum_{i=1}^{N} \sum_{j=1}^{N} \zeta_i^{(1)}\zeta_j^{(1)}P_{ij}.\cos\{(\omega_i - \omega_j)t + (\underline{\varepsilon}_i - \underline{\varepsilon}_j)\} +$$

$$+ \sum_{i=1}^{N} \sum_{j=1}^{N} \zeta_i^{(1)}\zeta_j^{(1)}Q_{ij}.\sin\{(\omega_i - \omega_j)t + (\underline{\varepsilon}_i - \underline{\varepsilon}_j)\}$$

. . . . . . . . . . . . . . (B-4)

in which $P_{ij}$ and $Q_{ij}$ are in-phase and out-of phase quadratic transfer functions dependent on the frequencies $\omega_i$ and $\omega_j$.

The problem is to determine $P_{ij}$ and $Q_{ij}$ for arbitrary values of $\omega_i$ and $\omega_j$ given that the input $\zeta^{(1)}(t)$ and the output $F^{(2)}(t)$ are only known as time records. The ouput is a signal with low frequency oscillatory components.

From equation (B-4) it can be shown that the time record of any component of $F^{(2)}(t)$ with chosen frequency $\Delta\omega$ is the sum of contributions from components of which the difference frequencies are equal to the chosen frequency $\Delta\omega$:

$$F^{(2)}_{\Delta\omega}(t) = \sum_{i=1}^{N} \zeta_i^{(1)}\zeta_j^{(1)}P_{ij}.\cos\{\Delta\omega t + (\underline{\varepsilon}_i - \underline{\varepsilon}_j)\} +$$

$$+ \sum_{i=1}^{N} \zeta_i^{(1)}\zeta_j^{(1)}Q_{ij}.\sin\{\Delta\omega t + (\underline{\varepsilon}_i - \underline{\varepsilon}_j)\} \qquad \text{(B-5)}$$

in which i and j are chosen such that:

$$\omega_i - \omega_j = \Delta\omega \qquad \text{(B-6)}$$

Equation (B-5) becomes:

$$F^{(2)}_{\Delta\omega}(t) = \left[\sum_{i=1}^{N} \zeta_i^{(1)}\zeta_j^{(1)}\{P_{ij}.\cos(\underline{\varepsilon}_i - \underline{\varepsilon}_j) + Q_{ij}.\sin(\underline{\varepsilon}_i - \underline{\varepsilon}_j)\}\right]\cos\Delta\omega t + \left[\sum_{i=1}^{N} \zeta_i^{(1)}\zeta_j^{(1)}\{Q_{ij}.\cos(\underline{\varepsilon}_i - \underline{\varepsilon}_j) - P_{ij}.\sin(\underline{\varepsilon}_i - \underline{\varepsilon}_j)\}\right]\sin\Delta\omega t \qquad \text{(B-7)}$$

From equation (B-7) it can be seen that the amplitude of a frequency component of $F^{(2)}(t)$ contains information on a range of $P_{ij}$ and $Q_{ij}$ values. It is not possible to determine the value of individual $P_{ij}$'s or $Q_{ij}$'s from the signal. This is a result of the fact that $F^{(2)}(t)$ is a double summation. The foregoing indicates that in order to determine the quadratic transfer functions for required combinations of $\omega_i$ and $\omega_j$ it is necessary to find a way to extract from the time record of the output information which is essentially in the form of a single summation of oscillatory components with amplitudes which are in themselves not a summation of components. For instance, it is possible to generate a time signal U(t) of the following type:

$$U(t) = \sum_{i=1}^{N} U_{ij}.\cos\{(\omega_i - \omega_j)t + (\underline{\varepsilon}_i - \underline{\varepsilon}_j)\} \quad \ldots\ldots \quad (B-8)$$

where:

$\omega_i + \omega_j = \omega_k$ = some chosen fixed frequency

$\underline{\varepsilon}_i, \underline{\varepsilon}_j$ = random phases of $i^{th}$ and $j^{th}$ frequency components of wave elevations of equation (B-1).

If the signal is cross-correlated with the output $F^{(2)}(t)$ of equation (B-4) it follows that:

$$R_{FU}(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{+T/2} U(t).F^{(2)}(t+\tau)dt \quad \ldots\ldots \quad (B-9)$$

By this operation only those components of $F^{(2)}(t)$ will be identified which correspond with the components of equation (B-8). Contributions from all other components will disappear since they are not correlated to the components of U(t).

The actual computation of the transfer functions starts with a transformation of the input signal $\zeta(t)$ according to:

$$\int_{-\tau_m}^{+\tau_m} \cos \omega_k\tau.\zeta^{(1)}(t-\tau).\zeta^{(1)}(t+\tau)d\tau \quad \ldots\ldots \quad (B-10)$$

This represents the Fourier transformation of the product:

$$\zeta^{(1)}(t-\tau).\zeta^{(1)}(t+\tau)$$

in which:

$\tau$ = time shift

$\omega_k$ = some chosen fixed frequency

$\tau_m$ = maximum time shift (maximum number of lags multiplied by sampling interval).

It will be clear that the output is a function of the chosen frequency $\omega_k$ and time t. Substitution of the expression (B-1) for the wave elevation gives the following result for the inner product:

$$\zeta^{(1)}(t-\tau).\zeta^{(1)}(t+\tau) =$$

$$= \sum_{i=1}^{N} \sum_{j=1}^{N} \zeta_i^{(1)} \zeta_j^{(1)}.\cos\{\omega_i(t-\tau) + \underline{\varepsilon}_i\}.\cos\{\omega_j(t+\tau) + \underline{\varepsilon}_j\}$$

$$= \sum_{i=1}^{N} \sum_{j=1}^{N} \tfrac{1}{2}\zeta_i^{(1)} \zeta_j^{(1)}.\cos\{(\omega_i+\omega_j)\tau + (\omega_i-\omega_j)t +$$

$$+ (\underline{\varepsilon}_i - \underline{\varepsilon}_j)\} + \text{high frequency components}$$

. . . . . . . . . . . . . . (B-11)

Multiplication by cos $\omega_k\tau$ gives:

$$\cos \omega_k\tau.\zeta^{(1)}(t-\tau).\zeta^{(1)}(t+\tau) =$$

$$= \sum_{i=1}^{N} \sum_{j=1}^{N} \tfrac{1}{4}\zeta_i^{(1)} \zeta_j^{(1)}.\cos[\{\omega_k - (\omega_i+\omega_j)\}\tau + (\omega_i-\omega_j)t +$$

$$+ (\underline{\varepsilon}_i - \underline{\varepsilon}_j)] + \text{high frequency components}$$

. . . . . . . . . . . . . . (B-12)

Disregarding the high frequency components expression (B-10) becomes:

$$\sum_{i=1}^{N} \sum_{j=1}^{N} \tfrac{1}{2}\zeta_i^{(1)} \zeta_j^{(1)}.\cos\{(\omega_i-\omega_j)t +$$

$$+ (\underline{\varepsilon}_i - \underline{\varepsilon}_j)\} \int_{-\tau_m}^{+\tau_m} \cos\{\omega_k - (\omega_i+\omega_j)\}\tau.d\tau +$$

$$- \sum_{i=1}^{N} \sum_{j=1}^{N} \tfrac{1}{2}\zeta_i^{(1)} \zeta_j^{(1)}.\sin\{(\omega_i-\omega_j)t +$$

$$+ (\underline{\varepsilon}_i - \underline{\varepsilon}_j)\} \int_{-\tau_m}^{+\tau_m} \sin\{\omega_k - (\omega_i+\omega_j)\}\tau.d\tau$$

. . . . . . . . . . . . . . (B-13)

Inspection of this expression shows that the contributions which arise for the cases that $\omega_k = \omega_i + \omega_j$ will dominate, so that the outcome is of the following type:

$$2\tau_m \sum_{i=1}^{N} \tfrac{1}{2}\zeta_i^{(1)}\zeta_{j(i,k)}^{(1)} \cdot \cos\{(\omega_i - \omega_{j(i,k)})t + (\underline{\varepsilon}_i - \underline{\varepsilon}_{j(i,k)})\} \qquad \text{(B-14)}$$

The outcome is a signal which contains only those difference frequency components of the wave elevation which have as sum frequency:

$$\omega_k = \omega_i + \omega_j \qquad \text{(B-15)}$$

which is what was needed in order to be able to identify corresponding components in the output $F^{(2)}(t)$. The above expression appears to increase as $\tau_m$ increases. It must be remembered, however, that the processes involved are stochastic. The output of the above expression is finite.

Some examples are given of the output of the above expression in Figure B-2. The input is the wave elevation of which the ordinary wave spectrum is given in Figure B-1. The output of expression (B-14) is given for three values of the sum frequency $\omega_k$.

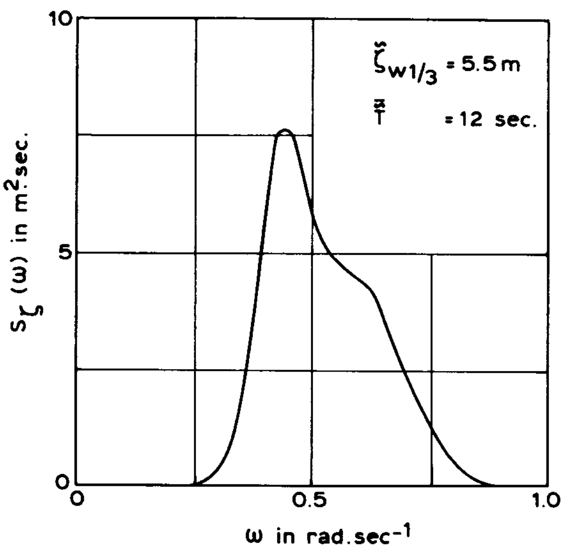



Fig. B-1 Wave spectrum.

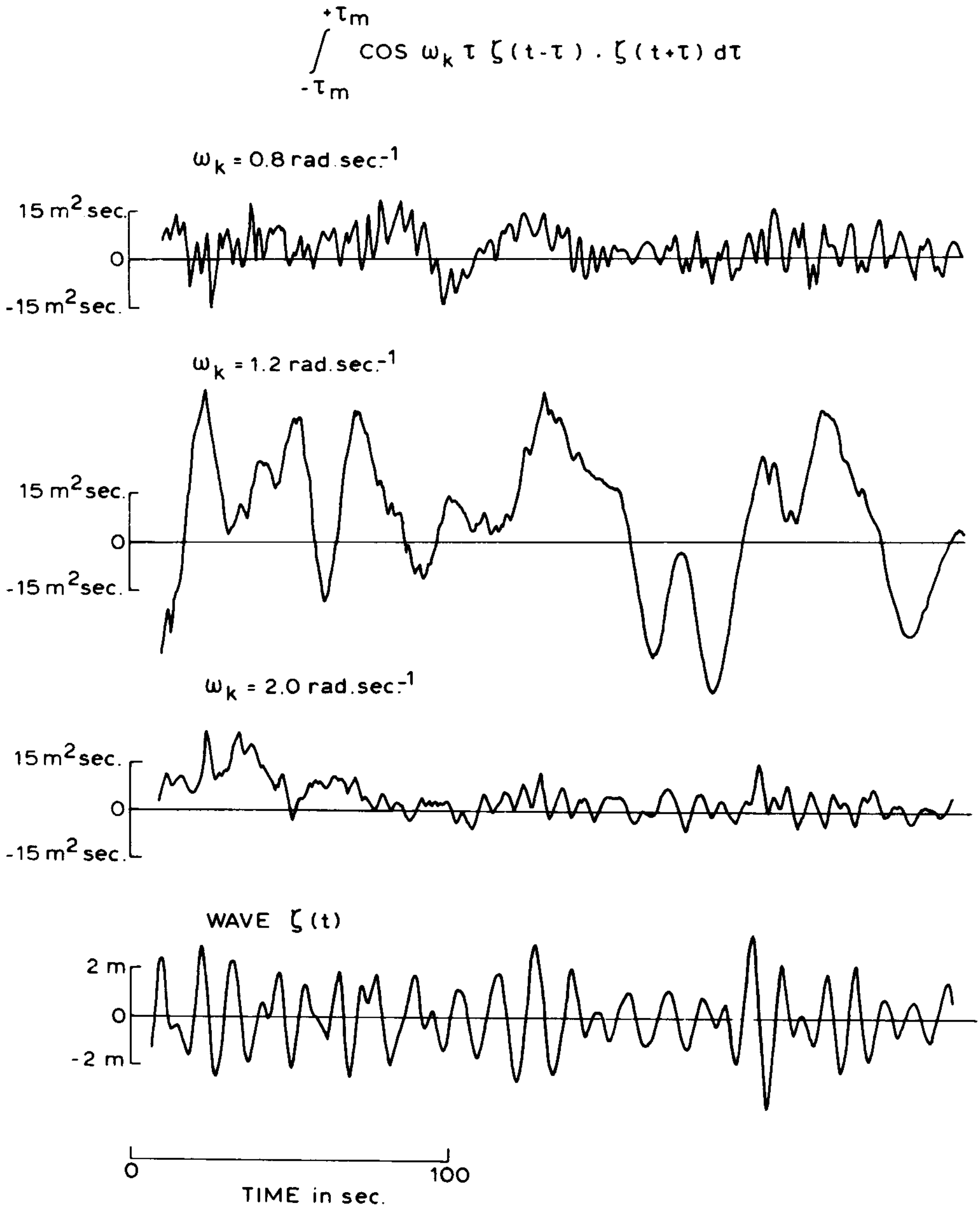



Fig. B-2  Time records of wave and transformed wave.

From the plots given in Figure B-2 it is seen that the middle value of the sum frequency results in a signal which contains large amplitude low frequency components and that the other $\omega_k$ values result in signals with much less low frequency components. This is explained by examination of the wave spectrum given in Figure B-1. From this figure it can be deduced that the low frequency part of the square of the wave elevation, which is related to the occurrence of wave groups, has little energy for the highest and lowest sum frequencies since in those cases the spectral density of the waves and consequently the amplitude of wave groups is smaller than for the middle value of $\omega_k$.

This operation results in a signal which for arbitrary sum frequency $\omega_k$ supplies the time record containing the low frequency part of the square of the wave elevation (only for those difference frequencies which have $\omega_k$ as sum frequency). This solves the problem of determining the transfer functions $P_{ij}$ and $Q_{ij}$. By performing cross-correlation between this signal and the low frequency output only those frequency components of the output corresponding to the input will be identified. This means that the cross-correlation function contains only information for those combinations of $\omega_i$ and $\omega_j$ which have $\omega_k$ as sum frequency. The final result after Fourier transformation of the cross-correlation function is the cross-spectrum of the transformed input and output or, in cross-bi-spectral terminology, the cross-bi-spectrum of input, input and output. The cross-bi-spectrum is valid for the chosen sum frequency $\omega_k$ and contains information for the range of difference frequencies from zero upwards. Any chosen difference frequencies contain information on the transfer function $P_{ij}$ and $Q_{ij}$ for unique values of $\omega_i$ and $\omega_j$. The values of the transfer functions are finally determined from the following type of expression:

$$G(\omega_i,\omega_j) = \frac{C_{F\zeta}(\omega_i,\omega_j)}{S_\zeta(\omega_i)S_\zeta(\omega_j)} \quad \ldots\ldots\ldots\ldots\ldots \quad \text{(B-16)}$$

where:

$C_{F\zeta}(\omega_i,\omega_j)$ = cross-bi-spectrum of input-input-output

$S_\zeta(\omega_i)$ = wave spectrum (scalar spectrum)

$G(\omega_i,\omega_j)$ = quadratic transfer function of output.

The afore given explanation on the cross-bi-spectral analysis method can be used to indicate how the computational method can be altered in order to save computation time and increase the accuracy for identifying the quadratic transfer function for the low frequency wave drift forces.

Since we are restricting ourselves to low frequencies in the output the computations which take place after the initial transformation of the input (see equation (B-10)) can be performed using a considerably increased sampling interval. This can be done after the output of the transformation examples, which are given in Figure B-2, are low-pass filtered. Subsequent computation of cross-correlation functions between the transformed input and the output can be carried out using a greater value of the maximum time shift between the signals thus allowing for potentially more accurate determination of the quadratic transfer functions for very low frequencies.

Analysis of model tests with the tanker and the semi-submersible

The duration of tests in irregular waves with the model of the tanker corresponded to 210 minutes full scale. The input (wave) and output (forces) records were sampled at 0.8 sec. intervals. The bi-spectral analysis of the digitized data were carried out using 75 lags for the cross-correlation functions. In the analysis the above described process by which the sampling interval is increased was not applied.

The duration of the tests with the semi-submersible corresponded with 360 minutes full scale. The time records of input and output were digitized using a sampling interval of 1.5 sec. full scale. The number of lags used for the cross-correlations amounted to 30. After the initial transformation of the input (see equation (B-10)), the transformed input and the output were low-pass filtered and the remaining computations were carried out using 30 lags and a sampling interval corresponding to 7.5 sec. full scale.

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NOMENCLATURE

| | |
|---|---|
| affix$^{(0),(1),(2),(3)}$ | affix denotes whether a quantity is of first, second, third order, etc. |
| $A_{ij}$, $B_{ij}$, $C_{ij}$ | coefficients dependent on wave frequencies $\omega_i$ and $\omega_j$ |
| C(t) | pressure contribution independent of the co-ordinates of the point under consideration |
| $\overline{F}$ | force vector relative to the $G-X_1'-X_2'-X_3'$ system of axes |
| $F_1$, $F_2$, $F_3$ | components of the force vector |
| $\overline{F}_1^{(2)}$, $\overline{F}_2^{(2)}$, $\overline{F}_3^{(2)}$ | mean values of the second order force components |
| G | centre of gravity of a body |
| $G^{(2)}$ | complex quadratic transfer function |
| $G-x_1-x_2-x_3$ | body axes with origin in centre of gravity G, $x_1$-axis positive in forward direction, $x_2$-axis positive to port side and $x_3$-axis positive upwards |
| $O-X_1-X_2-X_3$ | fixed system of axes with origin in the mean free surface, $X_1$-axis and $X_2$-axis in the horizontal plane and $X_3$-axis positive vertically upwards |
| $G-X_1'-X_2'-X_3'$ | system of axes with origin in centre of gravity of the body and axes parallel to $O-X_1-X_2-X_3$ system of axes |
| I | mass moment of inertia matrix $= \begin{bmatrix} I_4 & 0 & -I_{46} \\ 0 & I_5 & 0 \\ -I_{46} & 0 & I_6 \end{bmatrix}$ |
| $I_4$, $I_5$, $I_6$ | mass moments of inertia of the body about the $G-x_1$, $G-x_2$ and $G-x_3$ axes respectively |
| $I_{46}$ | product of inertia |
| L | length of a vessel or cylinder |

| | |
|---|---|
| $M$ | mass matrix $= \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{bmatrix}$ |
| $\overline{M}$ | moment vector relative to the $G$-$X_1'$-$X_2'$-$X_3'$ system of axes |
| $M_1$, $M_2$, $M_3$ | components of the moment vector |
| $\overline{N}$ | outward pointing normal vector of a point on the surface of a body relative to the $O$-$X_1$-$X_2$-$X_3$ or $G$-$X_1'$-$X_2'$-$X_3'$ systems of axes |
| $P_{ij}$, $Q_{ij}$ | components of the quadratic transfer function dependent on $\omega_i$ and $\omega_j$ |
| $T_{ij}$ | amplitude of the quadratic transfer function dependent on $\omega_i$ and $\omega_j$ |
| $S_\zeta$ | wave spectrum |
| $S$ | total wetted surface of the hull |
| $S_0$ | mean wetted surface of the hull |
| $dS$ | surface element of $S_0$ or $S$ |
| $\overline{V}$ | velocity vector relative to fixed system of axes |
| WL | static or mean waterline on the hull of a body |
| $\overline{X}$ | position vector relative to the fixed system of axes |
| $X_1$, $X_2$, $X_3$ | components of $\overline{X}$ |
| $\overline{X}_g$ | position of the centre of gravity of the body relative to the fixed system of axes |
| $\overline{\nabla}$ | vector operator |
| $\nabla$, $V$ | volume of the mean submerged part of a body |
| $\nabla^2$ | Laplace operator |
| PV | Principle Value |
| $A$ | area of waterline of a sphere |
| $X'$ | position vector relative to the $G$-$X_1'$-$X_2'$-$X_3'$ system of axes |

| | |
|---|---|
| a | radius of sphere |
| d | radius of cylinder |
| e | 2.718 (constant of natural logarithm) |
| g | constant of gravity |
| $g^{(2)}$ | second order impuls response function |
| h | water depth |
| i | $\sqrt{-1}$ |
| k | wave number = $2\pi/\lambda$ |
| $d\ell$ | line element of the waterline |
| m | mass of body in vacuum |
| $m_0$ | area of wave spectrum |
| $\overline{n}$ | outward pointing normal vector of a point on the body relative to the body axes $G$-$x_1$-$x_2$-$x_3$ |
| $n_1$, $n_2$, $n_3$ | components of $\overline{n}$ |
| $n_{ij}$, $f_{ij}$ | coefficients depending on $\omega_i$ and $\omega_j$ |
| p | pressure |
| t | time |
| $u_a$ | amplitude of an oscillatory quantity |
| $\overline{x}$ | position vector of a point on the hull of the body relative to the body axes |
| $x_1$, $x_2$, $x_3$ | components of $\overline{x}$ |
| $\alpha$ | chapter VI: angle of mooring line<br>appendix A: angle of incidence of waves; $0^o$ represents stern waves, $90^o$ represents waves from starboard beam and $180^o$ represents head waves |
| $\overline{\alpha}$ | angular motion vector |
| $\beta$ | mooring line angle |
| $\varepsilon$ | a small quantity << 1 |
| $\underline{\varepsilon}_i$ | random phase angle of $i^{th}$ frequency component |

| | |
|---|---|
| $\varepsilon_{u\zeta}$ | phase angle between wave and some oscillatory quantity u |
| $\zeta$ | wave elevation |
| $\zeta_a$ | wave amplitude of a regular wave |
| $\zeta_r$ | relative wave elevation |
| $\zeta_i$ | amplitude of $i^{th}$ frequency component |
| $\tilde{\zeta}_{w1/3}$ | significant wave height |
| $\bar{T}$ | mean wave period |
| $\lambda$ | wave length |
| $\Lambda$ | non-dimensional frequency |
| $\mu$ | frequency of low frequency part of the second order forces |
| $\rho$ | mass density of water |
| $\tau$ | time lag |
| $\Phi$ | velocity potential dependent on co-ordinates and time t |
| $\phi$ | part of velocity potential independent of time |
| $\omega$ | wave frequency |
| $\omega_i$ | $i^{th}$ frequency component |
| $\sigma$ | source strength |
| $\gamma$ | Green's function |
| $a_{11}$, $a_{33}$ | added mass fρr surge and heave motions |
| $b_{11}$, $b_{33}$ | damping for surge and heave motions |
| $x_1^{(1)}$, $x_2^{(1)}$, $x_3^{(1)}$ | first order surge, sway and heave motions |
| $x_4^{(1)}$, $x_5^{(1)}$, $x_6^{(1)}$ | first order roll, pitch and yaw motions |

SUMMARY

In this thesis the mean and low frequency second order wave drift forces on bodies moored or stationed in waves are analyzed. Expressions are derived for the second order forces based on direct integration of pressure acting on the wetted part of the body. It is shown that the second order forces in irregular waves may be determined from knowledge of the mean forces in regular waves and the low frequency forces in regular wave groups.

In order to calculate the mean and low frequency forces on bodies of arbitrary shape use is made of a three-dimensional linear potential theory computer program. The form of the body is approximated by a distribution of plane facet elements representing a source distribution. For a hemisphere the results of computations of the mean second order horizontal forces in regular waves are compared with analytical results. This comparison demonstrates the accuracy of the computations and the equivalence of the expressions for the second order forces developed in this thesis with respect to an already existing expression based on momentum and energy considerations.

In order to demonstrate the validity of the present theory with respect to realistic hull forms, results of computations of the mean second order forces in regular waves are compared with results of experiments on a tanker, a semi-submersible, a rectangular barge and a submerged horizontal cylinder.

For the first three hull forms the mean horizontal forces are compared. For the submerged cylinder the mean vertical forces are compared. The correlation found between results of computations and experiments confirms the general applicability of the theory for predicting the second order forces on a wide range of hull forms.

A detailed analysis of components of the mean second order forces shows that for floating vessels the horizontal forces are dominated by a contribution dependent on the relative wave elevation around the waterline of these vessels.

For the tanker and the semi-submersible results of computations of the low frequency horizontal force in regular wave groups are compared with experimental results obtained from model tests in regular wave groups and irregular waves. The experimental results from tests in irregular waves are analyzed by means of cross-bi-spectral methods. Results of this comparison indicate that, provided certain conditions are fulfilled, the mean second order force in regular waves may be used to approximate the low frequency force in irregular waves.

Finally, for a dynamically positioned vessel the results of computations are used to demonstrate the effectiveness of a wave-feed-forward control signal based on relative wave elevation measurements for reducing low frequency horizontal motions induced by drift forces in irregular waves. The results show that model tests confirm the theoretical predictions.

SAMENVATTING

In dit proefschrift wordt een analyse gegeven van de gemiddelde en laag frekwente tweede orde golfdriftkrachten op een lichaam afgemeerd of gepositioneerd in golven. Uitgaande van integratie van drukken over het natte oppervlak van een lichaam worden uitdrukkingen voor de tweede orde krachten gegeven. Aangetoond wordt dat de tweede orde krachten in onregelmatige golven bepaald kunnen worden uitgaande van kennis van gemiddelde krachten in regelmatige golven en de laag frekwente krachten in regelmatige golfgroepen.

Voor het berekenen van de gemiddelde en laag frekwente krachten op willekeurig gevormde lichamen wordt gebruik gemaakt van een rekenprogramma gebaseerd op drie-dimensionale lineaire potentiaal theorie. De vorm van het lichaam wordt benaderd door middel van een aantal elementen die een verdeling van bronnen voorstelt.

Voor een halve bol worden de resultaten van berekeningen van de gemiddelde horizontale golfdriftkracht in regelmatige golven vergeleken met reeds bekende, langs analytische weg verkregen resultaten. De nauwkeurigheid van de berekeningsmethode wordt hiermee aangetoond, alsmede dat de in dit proefschrift gegeven uitdrukking voor de driftkrachten qua resultaat equivalent is aan een reeds bekende uitdrukking die gebaseerd is op impuls en energie beschouwingen.

Met het doel de geldigheid aan te tonen van de in dit proefschrift gegeven theorie met betrekking tot meer realistische rompvormen worden resultaten van berekeningen van de gemiddelde tweede orde golfkrachten in regelmatige golven vergeleken met experimenteel bepaalde resultaten voor een tanker, een semi-submersible, een rechthoekig ponton en een ondergedompelde horizontale cilinder.

Voor de drie eerstgenoemde rompvormen worden de resultaten voor de horizontale krachten vergeleken. Voor de ondergedompelde cilinder worden de resultaten voor de gemiddelde vertikale kracht vergeleken. De overeenkomst tussen de resultaten van berekeningen en metingen bevestigt de algemene toepasbaarheid van de theorie voor het voorspellen van de tweede orde krachten op een grote verscheidenheid van rompvormen.

Een nadere analyse van de komponenten van de gemiddelde tweede orde krachten toont aan dat voor drijvende konstrukties de horizontale krachten gedomineerd worden door een bijdrage die afhankelijk is van de relatieve golfhoogte ter plaatse van de waterlijn van de konstruktie.

Voor de tanker en de semi-submersible worden de resultaten van berekeningen van de laag frekwente driftkracht in regelmatige golfgroepen vergeleken met resultaten verkregen uit modelproeven in regelmatige golfgroepen en onregelmatige golven. De meetresultaten verkregen uit proeven in onregelmatige golven zijn geanaliseerd door middel van kruis-bi-spektrale methoden. Uit de vergelijking blijkt dat, mits aan bepaalde voorwaarden wordt voldaan, de gemiddelde driftkrachten in regelmatige golven gebruikt kunnen worden om de laag frekwente krachten in onregelmatige golven te benaderen.

Tenslotte worden voor een dynamisch gepositioneerd schip de resultaten van berekeningen gebruikt om een voorspelling te geven van de effektiviteit van een wave-feed-forward regelsignaal die gebaseerd is op metingen van de relatieve golfhoogte voor het verminderen van de laag frekwente horizontale bewegingen, die opgewekt worden door de driftkrachten in onregelmatige golven. Resultaten laten zien dat de modelproeven in overeenstemming zijn met de theoretische voorspellingen.

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