Diagnosability of intermittent sensor faults in discrete event systems

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Abstract—We address in this paper the problem of diagnosing intermittent sensor faults. In order to do so, we modify the model of intermittent loss of observations proposed in the literature to account for sensor malfunction only. Using this model together with a modified label automaton, it will be possible to change the problem of detecting intermittent sensor faults into a problem of diagnosing a language generated by an automaton in the presence of intermittent faults, where the fault event will be an unobservable event that models the non-detection of the event to be registered by the sensor under consideration. In this regard, we present necessary and sufficient conditions for diagnosability of intermittent sensor faults and propose a test based on diagnoser automaton to verify intermittent sensor fault diagnosability.

I. INTRODUCTION

Sensors are vital in any feedback controlled systems and so, their operation plays a crucial role in reliability and safety of controlled systems. Sensor failures have been reported as the cause of several accidents that led to either material or life losses [1]. It is, therefore, important to find the ways to distinguish between sensor malfunction from its ordinary (normal) behavior.

There are basically three main approaches to the problem of detecting incorrect sensor readings [2]: the use of simple hardware redundancy with majority voting, model- and knowledge-based approaches. Hardware redundancy with majority voting is the simplest way to improve sensor reliability. Model-based design relies on some model developed for the system under consideration and the decision is made based on comparisons between the outputs of the model and of the real system. Knowledge-based design deploys artificial intelligence techniques such as neural network and fuzzy logic to develop expert systems.

Among the model-based approach, the most relevant works reported in the literature are the incipient work by Clark [3], who proposed the so-called dedicated observer scheme (DOS), in which separate dedicated observers are designed for each one of the sensors — inputs of the observers are the output signals of the corresponding sensors and the plant input signal — the paper by Frank [2], which besides presenting a literature survey, also improved the scheme developed in [3], leading to the so-called generalized observer scheme (GOS), in which more than one output is fed into the observers. Lunze and Schröder [4] who proposed a method for the detection and identification of sensor and actuator faults using discrete event theory by modeling the plant of the system under consideration as a stochastic automaton, and Ding et al. [5], who presented a model-based sensor monitoring scheme for the electronic stability program (ESP) system consisting of an anti-lock break system, a traction control and a yaw torque control. Expert systems were proposed by Athanasopoulou and Chatziathanasiou [6], who developed an intelligent system for identification and replacement of faulty sensor measurements in thermal power plants based on a procedure for identifying sensor faults and reconstructing the erroneous measurements, and da Silva et al. [1], who presented a knowledge-based system for sensor fault diagnosis using a neural network approach.

We propose in this paper a discrete event approach to the problem of diagnosing intermittent sensor faults by modeling the dynamic system as a deterministic automaton. The sensor fault diagnosing system should be built separately from the ordinary failure diagnosing system [7], but must be one of the components of the supervisory control and fault diagnosing systems using discrete event models, as illustrated in Figure 1. To the authors’ knowledge, this is the first time that the detection of intermittent sensor faults is considered using discrete event models.

Sensor faults have already been addressed in the context of supervisory control [8], [9], and as part of the design requirements of fault diagnosing systems [10], [11] — in the latter, the resulting diagnosing systems are said to be robust against intermittent and permanent loss of observations, respectively. Differently from the works in [8], [9],

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In this paper, we are not proposing a system that can cope with sensor faults but a system that actually detects sensor malfunction. In order to do so, we modify a previously introduced model of loss of observations [10] to consider only sensor malfunction, and convert the problem of detecting intermittent sensor faults into a problem of diagnosing intermittent faults similar to that proposed in [12]. In this regard, we present necessary and sufficient conditions for intermittent sensor fault diagnosability and propose a test based on diagnoser automaton to verify intermittent sensor fault diagnosability.

This paper is organized as follows. We present in Section II a brief review of DES theory and, in Section III, we modify the automaton model for intermittent loss of observations proposed in [10] in order to account for sensor fault only. With the help of the model presented in Section III, we convert the problem of sensor fault diagnosis into an equivalent one that consists of diagnosing the language generated by an automaton subject to intermittent faults, where the fault event is the event recorded by the sensor whose fault must be diagnosed. After presenting necessary and sufficient conditions for the diagnosis of intermittent faults, we propose in Section V a test based on a diagnoser automaton to verify language diagnosability in the presence of intermittent faults. We present an example in Section VI to illustrate the results of the paper and, finally, in Section VII we revise the main contributions of the paper.

II. PRELIMINARIES

A deterministic automaton is a five-tuple \( G = (X, \Sigma, f, \Gamma, x_0) \), where \( X \) denotes the state space, \( \Sigma \) the event set, \( f : X \times \Sigma \rightarrow X \) the state transition function that is partial in its domain, \( \Gamma : X \rightarrow 2^\Sigma \) is the active event function, and \( x_0 \) the initial state. For a given event set \( \Sigma \), \( \Sigma^* \) denotes the set of all possible finite length traces that can be formed with the elements of \( \Sigma \), including the empty trace \( \varepsilon \).

Let \( G_1 = (X_1, \Sigma_1, f_1, \Gamma_1, x_{01}) \) and \( G_2 = (X_2, \Sigma_2, f_2, \Gamma_2, x_{02}) \) denote two finite state automata. The parallel composition between \( G_1 \) and \( G_2 \) (denoted as \( G_1 \parallel G_2 \)) produces an automaton with the following behavior: (i) a common event to \( G_1 \) and \( G_2 \) can occur, only when \( G_1 \) and \( G_2 \) are in states whose active event sets both have this event, (ii) private events, i.e., events belonging either to \( \Sigma_1 \setminus \Sigma_2 \) or to \( \Sigma_2 \setminus \Sigma_1 \) can occur as long as they belong to the active event set of the current state.

Let \( \Sigma = \Sigma_0 \cup \Sigma_{\text{uo}} \) be a partition of \( \Sigma \), where \( \Sigma_0 \) and \( \Sigma_{\text{uo}} \) are, respectively, the set of observable and unobservable events. The language generated by \( G \) (denoted as \( L(G) \), or simply, \( L \)) is the set of all traces \( s \) for which \( f(x_0, s) \) is defined. An important language operation is the projection \( P_o \) [13], which is defined as follows.

**Definition 1:** (Projection) The projection \( P_o \) is the mapping

\[
P_o : \Sigma^* \rightarrow \Sigma^*_o \quad \text{satisfying the following properties:}
\]

\[
P_o(\varepsilon) = \varepsilon,
\]

\[
P_o(\sigma) = \begin{cases} \sigma, & \text{if } \sigma \in \Sigma_o, \\ \varepsilon, & \text{if } \sigma \notin \Sigma_o \end{cases},
\]

\[
P_o(s\sigma) = P_o(s)P_o(\sigma), s \in \Sigma^*, \sigma \in \Sigma.
\]

The projection operator can be extended to a language \( L \) by applying the natural projection (2) to all traces of \( L \). Therefore, if \( L \subseteq \Sigma^* \), then \( P_o(L) = \{ t \in \Sigma_o^* : (\exists s \in L)[P_o(s) = t] \} \). The inverse projection \( P_o^{-1} \) is the mapping from \( \Sigma_o^* \) to \( 2^{\Sigma} \), and is defined as \( P_o^{-1}(s) = \{ t \in \Sigma^* : P_o(t) = s \} \).

The dynamic behavior of a deterministic automaton \( G \) with unobservable events, can be described by a deterministic automaton called observer (denoted as \( \text{Obs}(G) \)), whose event set is the set of observable events of the given automaton.

The observer states indicate all possible states where a deterministic automaton with unobservable events can be after the occurrence of a trace formed with observable events only. It is not difficult to verify [14] that \( L(\text{Obs}(G)) = P_o(L(G)) \).

Let \( \Sigma_{\text{uf}} = \sigma \subseteq \Sigma_o \) and define \( \Sigma_{\text{uf}}' = \{ \sigma' : \sigma \in \Sigma_{\text{uf}} \} \) and form the following sets: \( \Sigma_{\text{uf}} = \Sigma \cup \Sigma_{\text{uf}}' \). Another important language operation is the dilation \( D \) [10], defined as follows.

**Definition 2:** (Dilation) The dilation \( D \) is the mapping

\[
D : \Sigma^* \rightarrow 2^{\Sigma_{\text{uf}}},
\]

\[
s \mapsto D(s),
\]

where

\[
D(\varepsilon) = \{ \varepsilon \},
\]

\[
D(\sigma) = \begin{cases} \{ \sigma \}, & \text{if } \sigma \in \Sigma \setminus \Sigma_{\text{uf}}, \\ \{ \sigma, \sigma' \}, & \text{if } \sigma \in \Sigma_{\text{uf}}, \text{ with } \sigma' \in \Sigma_{\text{uf}}', \\ D(s)D(\sigma), & s \in \Sigma^*, \sigma \in \Sigma.
\end{cases}
\]

Notice that the dilation operation \( D \) can be extended from traces to languages by applying it to all sequences in the language, that is, \( L_{\text{dil}} = D(L) \cup \bigcup_{s \in L} D(s) \).

III. MODELING OF DES SUBJECT TO INTERMITTENT SENSOR FAULTS

We will present in this section two automaton models for sensors: the first model considers the sensors as an ideal system, and the second one models the sensor behavior when subject to intermittent faults. In these models, we will suppose that the event to be recorded is unobservable from the system point of view.

A. Sensor model for automaton

Figure 2(a) shows automaton \( S_{\text{id}} \) that models the behavior of an ideal sensor. In this automaton, \( a_u \) denotes the unobservable event whose occurrence must be recorded by the sensor and \( a_f \) denotes the event that represents the sensor reading when it records the occurrence of \( a_u \). Figure 2(b) shows automaton \( S_{\text{if}} \) that models the behavior of a sensor when subject to intermittent fault in recording the occurrence of \( a_u \). The main difference between automata \( S_{\text{id}} \) and \( S_{\text{if}} \) is the addition of the transition labeled with event \( a_f \) that represents the occurrence of a sensor fault, i.e. the sensor fails to record the occurrence of \( a_u \).
The self-loops in the initial states of Figures 2(a) and 2(b) have been added to ensure that other plant events besides $a_u$, whose occurrences must also be recorded by other sensors (or command events), can occur, where $\Sigma$ denotes the plant event set. After the occurrence of events $a_u$, $a_r$, and $a_f$ are the only ones that can occur, since they are private events of automata $S_{ideal}$ and/or $S_{isf}$. The behavior of the system assuming intermittent sensor faults can be obtained by performing the parallel composition between the plant automaton $G$ and $S_{isf}$.

With the view to illustrating the influence of sensor faults in the plant model, let us consider the state transition diagram of part of an automaton model of a plant, as shown in Figure 3(a). Performing the parallel composition between

$$\Sigma \setminus a_u$$

and $S_0 \to S_1 \in \Sigma \setminus a_u$

we obtain automaton $G_{dil} = G_\parallel S_{isf}$ shown in Figure 3(b). Notice that the occurrence of $a_u$ determines the following traces: $a = a_da_d$, $e = a_da_d$. Notice that trace $a$ ends with an observable event (the event that corresponds to sensor recording) whereas both events of $d'$ are unobservable. Therefore, $a$ and $d'$ can be associated, respectively, to two events: an observable event $a$, associated to the occurrence of $a_u$, and event $d'$, that models the sensor fault in recording the occurrence of $a_u$. As a consequence, the model of $G$ of Figure 3(a) must be replaced with the model of Figure 3(c) in order to take into account possible intermittent malfunction in the sensor associated with event $a_u$.

It is important to point out that the model given by Figure 3(c) is identical to that proposed by [10], in the context of fault diagnosis, to approach the problem of intermittent loss of observations. However, differently from [10], we will not consider here fault events in $G$. Another difference between the problem addressed in this paper and that solved in [10], is that the fault in [10] is assumed to be permanent and the sensors are subject to intermittent faults, whereas in this paper, the fault diagnosis problem is formulated so as, in the final model, all sensors associated with events different from the faulty sensor are assumed to be ideal and the fault associated with the faulty event intermittent, i.e., the event to be recorded can occur and to be observed, can occur and not to be observed, occur and not to be observed but when it occurs again, to be or not to be observed.

B. Model for automaton of DES subject to intermittent sensor faults

Let $G$ be a deterministic automaton whose event set is denoted by $\Sigma$ and let $L$ denote the language generated by $G$. Assume that $\Sigma$ is partitioned as $\Sigma = \Sigma_0 \cup \Sigma_{uo}$, i.e., $\Sigma = \Sigma_u \cup \Sigma_a$ and $\Sigma_u \cap \Sigma_{uo} = \emptyset$. Consider a subset $\Sigma_{isf}$ of $\Sigma_u$, whose events are associated with intermittent loss of sensors. Define $\Sigma_{isf} = \{ \sigma' : \sigma \in \Sigma_{isf} \}$ and form the following sets: $\Sigma_{dil} = \Sigma \cup \Sigma_{isf}$ and $\Sigma_{uo,dil} = \Sigma_{uo} \cup \Sigma_{isf}$.

It is not difficult to see [10] that automaton $G_{dil}$, that takes into account intermittent sensor faults, can be obtained from $G$ by adding transitions labeled with $\sigma' \in \Sigma_{isf}$ in parallel with transition $\sigma$. Therefore, $G_{dil}$, can be defined as follows:

$$G_{dil} = (X_{dil}, \Sigma_{dil}, f_{dil}, \Gamma_{dil}, x_{0_{dil}}), \quad (5)$$

where $X_{dil} = X$, $x_{0_{dil}} = x_0$, $\Gamma_{dil}(x) = \Gamma(x) \cup \{ \sigma' \}$, if $\sigma \in \Gamma(x)$ or $\Gamma_{dil}(x) = \Gamma(x)$, otherwise, $f_{dil}(x, \sigma) = f(x, \sigma)$, $\forall \sigma \in \Gamma(x)$ and $f_{dil}(x, \sigma') = f(x, \sigma)$, $\forall \sigma' \in \Gamma_{dil}(x)$.

As proved in [10], the language generated by $G_{dil}$ ($L_{dil}$) is given by

$$L_{dil} = D(L),$$

where $D(\cdot)$ denotes the dilation operation given in Definition 2.

IV. DIAGNOSIS OF INTERMITTENT SENSOR FAULTS

Automaton $A_t$, shown in Figure 4, models the dynamic behavior of a sensor subject to intermittent sensor fault, where $\sigma \in \Sigma_{isf}$ is the event recorded by the sensor whose malfunction we are interested in detecting, and $\sigma' \in \Sigma_{isf}'$ is the corresponding fault event associated with $\sigma$. Note that, while the sensor records correctly the occurrence of event $\sigma$, we can say that the sensor has a normal behavior. However, when a sensor fault occurs, automaton $A_t$ moves to state $F$ and remains there for as long as the sensor continues to fail. If the sensor returns to work, i.e., when event $\sigma$ occurs again, $A_t$ changes to state $R$, where it stays while the sensor continues to record the occurrence of $\sigma$. On the other hand, if, at some time, the sensor fails to record $\sigma$ again, automaton $A_t$ returns to state $F$.

The detection of a sensor fault is carried out by detecting the occurrence of $\sigma'$. Since $\sigma'$ is not an event of $G$, sensor fault diagnosability cannot be stated in terms of the diagnosability of $L$ but must be stated in terms of $L_{dil}$. Therefore,
although the model used here to account for sensor fault is a modification of that presented in [10], the sensor fault diagnosability definition cannot be stated in a similar way to that given in [10]. This is so because in [10] the fault event to be diagnosed was already an event of $G$ and the dilation was used to provide different trace possibilities that involve or not sensor faults. Here the dilation of $L$ will give the language generated by an automaton model that accounts for sensor fault; and although some fault event may appear in the system model, we are not interested in detecting its occurrence. In addition, the dilation makes possible that $\sigma$ may occur even after the occurrence of $\sigma'$; in case the sensor recovers its normal behavior.

We make the following assumptions:

**A1.** The language generated by $G$ is live, i.e., $\Gamma(x_i) \neq \emptyset$ for all $x_i \in X$;

**A2.** Only one intermittent sensor fault is to be detected, i.e., $\Sigma_{\text{sf}} = \{\sigma\}$ and $\Sigma'_{\text{sf}} = \{\sigma'\}$.

Assume that $s_f$ denotes the last event of a trace $s$. Then $\Psi(\Sigma'_{\text{sf}}) = \{s \in L : s_f \in \Sigma'_{\text{sf}}\}$ is the set of all traces of $L$ that end with event $\sigma'$, and $L/s = \{t \in \Sigma^* : st \in L\}$ is the language continuation of $L$ after trace $s$. In addition, let $\overline{s}$ denote the prefix closure of $s$. With a slight abuse of notation, the relationship $\Sigma'_{\text{sf}} \subseteq s$ is used to denote that $\overline{s} \cap \Psi(\Sigma'_{\text{sf}}) \neq \emptyset$. Therefore, we may say that a trace $s \in L$ has a sensor fault event if $\Sigma'_{\text{sf}} \subseteq s$. We present the following definitions.

**Definition 3:** (F-diagnosability and R-diagnosability) Let $L_{\text{dil}}$ be the language generated by automaton $G_{\text{dil}}$ (the automaton that models the behavior of plant subject to intermittent sensor faults). Then:

- $L_{\text{dil}}$ is F-diagnosable with respect to projection $P_{\text{dil},o} : \Sigma_{\text{dil}} \rightarrow \Sigma_o$ and $\Sigma_{\text{sf}}$, if the following holds true:

$$\exists n \in \mathbb{N} \exists s \in \Psi(\Sigma'_{\text{sf}}) \forall t \in L_{\text{dil}}/s, \Sigma_{\text{sf}} \not\subset t \Rightarrow (||t|| \geq n) \Rightarrow D_F,$$

where the diagnosability condition $D_F$ is

$$(\forall \omega = \omega' \omega'' \in P^{-1}_{\text{dil},o}(P_{\text{dil},o}(st)) \cap L_{\text{dil}})((\omega' \in \Psi(\Sigma'_{\text{sf}})) \land (\Sigma'_{\text{sf}} \cup \Sigma_{\text{sf}} \not\subset \omega''))$$

- $L_{\text{dil}}$ is R-diagnosable with respect to projection $P_{\text{dil},o} : \Sigma_{\text{dil}} \rightarrow \Sigma_o$, $\Sigma_{\text{sf}}$, and $\Sigma_{\text{sf}}$, if the following holds true:

$$\exists n \in \mathbb{N} \forall r \in \Psi(\Sigma_{\text{sf}}) \forall s \in L_{\text{dil}}/r, s \in \Psi(\Sigma_{\text{sf}}) \forall t \in L_{\text{dil}}/rs, \Sigma_{\text{sf}} \not\subset t \Rightarrow (||t|| \geq n) \Rightarrow D_R,$$

where the diagnosability condition $D_R$ is

$$(\forall \omega = \omega' \omega'' \in P^{-1}_{\text{dil},o}(P_{\text{dil},o}(rst)) \cap L_{\text{dil}})((\omega' \in \Psi(\Sigma'_{\text{sf}})) \land (\Sigma'_{\text{sf}} \cup \Sigma_{\text{sf}} \not\subset \omega'') \land (\Sigma_{\text{sf}} \not\subset \omega'').$$

**Remark 1:** The idea behind the definitions of F-diagnosability and R-diagnosability presented above is the same as that of the definitions of intermittent fault diagnosability presented in [12]; the main difference is that the definition presented in [12] requires besides a fault event, another unobservable event, the so-called "reset" event. In our problem, there is no "reset" event; the occurrence of event $\sigma$ after some occurrence of $\sigma'$ plays the role of the "reset". In any case, as pointed out in [12], both notions of diagnosability (those presented in [12] and those given in Definition 3) are "natural extensions of the notion of diagnosability for permanent faults introduced in [7]."

**Remark 2:** It is assumed in [12] that there does not exist in $G$ cyclic paths of unobservable events and, furthermore, all the results obtained in [12] required that assumption. We removed this assumption here for the following reason: even if we precluded $G$ from having cyclic paths of unobservable events, these kinds of paths could still appear in $G_{\text{dil}}$. This is so because the dilation operation introduces a transition labeled with $\sigma'$ (which is an unobservable event) in parallel with $\sigma$ (which is an observable event). It may be the case that $\sigma$ be the unique observable event within a cyclic path $s$, and thus, an additional cyclic path of unobservable event $s'$ will be formed in $G_{\text{dil}}$ with the same events as $s$ except that $\sigma$ is replaced with $\sigma'$. Therefore, all the tests proposed in [12] cannot be applied to the problem considered here, thus requiring further development in order to take into account cyclic paths of unobservable events.

V. VERIFICATION OF DIAGNOSABILITY OF INTERMITTENT SENSOR FAULTS USING DIAGNOSERS

In this section we propose a test to verify the diagnosability of intermittent sensor faults using a diagnoser automaton. In order to do so, we first compute automaton $G_{\text{dil},\ell}$ as follows:

$$G_{\text{dil},\ell} = G_{\text{dil}} \parallel A_{\ell},$$

where $A_{\ell}$ is the label automaton shown in Figure 4. Note that the states of $G_{\text{dil},\ell}$ are obtained by adding label $N, F$ or $R$ to the states of the plant to indicate whether the sensor has not failed (or equivalently is in normal operation), if the sensor has failed or if the sensor failed and has recovered from fault. Denoting $L_{\text{dil},\ell}$ and $\Sigma_{\text{dil},\ell} = \{\sigma, \sigma'\}$ as the language generated and the set of events of $G_{\text{dil},\ell}$, respectively, then it is straightforward to see that $L_{\text{dil},\ell} = L_{\text{dil}}$ since $\Sigma_{\text{dil},\ell} \subseteq \Sigma_{\text{dil}}$.

The fact that the diagnosability is based on observable events, only, suggests the computation of the following automaton:

$$G_{\text{dil},d} = (X_{\text{dil},d}, \Sigma_{\text{dil},d}, f_{\text{dil},d}, \Gamma_{\text{dil},d}, o_{\text{dil},d}) = \text{Obs}(G_{\text{dil},\ell}),$$

where $\Sigma_{\text{dil},d} = \Sigma_o$. Noticed that different combinations of labels may appear in the states of $G_{\text{dil},d}$, which leads to the following state classification$^1$:

**Definition 4:** A state $x_{\text{dil},d} \in X_{\text{dil},d}$ is called:

- Normal (or non-faulty) if $\ell = N$ for all $(x, \ell) \in x_{\text{dil},d}$;

$^1$It is important to remark the state classification introduced here is a natural extension of the definitions originally presented by [7].
• Certain of the occurrence of sensor faults (or F-certain), if \( \ell = F \) for all \((x, \ell) \in x_{dil,d}\);
• Certain of the recovery of the sensor after some fault (or R-certain), if \( \ell = R \) for all \((x, \ell) \in x_{dil,d}\);

Moreover, if there exist:

• \((x, \ell), (y, \tilde{\ell}) \in x_{dil,d}\), \(x\) not necessarily distinct from \(y\) such that \(\ell = N\) and \(\tilde{\ell} = F\), then \(x_{dil,d}\) is called NF-uncertain, i.e., uncertain if the sensor has failed or not;

• \((x, \ell), (y, \tilde{\ell}) \in x_{dil,d}\), \(x\) not necessarily distinct from \(y\) such that \(\ell = N\) and \(\tilde{\ell} = R\) then \(x_{dil,d}\) is called NR-uncertain, i.e., uncertain if either the sensor has not failed or if the sensor has failed and recovered from fault;

• \((x, \ell), (y, \tilde{\ell}) \in x_{dil,d}\), \(x\) not necessarily distinct from \(y\) such that \(\ell = F\) and \(\tilde{\ell} = R\) then \(x_{dil,d}\) is called FR-uncertain, i.e., uncertain if either the sensor has failed and has not recovered or failed and has recovered from fault;

With the view to presenting necessary and sufficient conditions for F- and R-diagnosability, we must define indeterminate cycles that express the diagnoser uncertainty with respect to the traces generated by the plant that can be affected by intermittent sensor faults. Notice that the possibility of appearing cyclic paths of unobservable events requires that not only indeterminate observed cycles but also indeterminate hidden cycles [15],[16],[10] must be considered.

**Definition 5:** (Cycle of states) A set of states \(\{x_1, x_2, \ldots, x_n\} \subset X\) forms a cycle in an automaton \(H = (X, \Sigma, f, \Gamma, x_0)\) if there exits a trace \(s = \sigma_1 \sigma_2 \ldots \sigma_n \in L(H, x_1)\) such that \(f(x_k, \sigma_k) = x_{k+1}, k = 1, \ldots, n-1\), and \(f(x_n, \sigma_n) = x_1\), where \(L(H, x) = \{s \in \Sigma^n : f(x, s) \in X\}\).

**Definition 6:** (Indeterminate observed cycle) A set of uncertain states \(\{x_{dil,1}, x_{dil,2}, \ldots, x_{dil,p}\} \subset X_{dil,d}\) forms an indeterminate observed cycle in \(G_{dil,d}\) if the following conditions hold true

1) \(x_{dil,1}, x_{dil,2}, \ldots, x_{dil,p}\) form a cycle in \(G_{dil,d}\);

2) \((x_{dil,k}^k, \ell, x_{dil,k}^\ell, \tilde{\ell}) = x_{dil,d}\), if \((x_{dil,k}^k, \ell) \in x_{dil,d}\) and \(x_{dil,k}^\ell\) not necessarily distinct from \(x_{dil,k}^\ell\), \(k = 1, 2, \ldots, p\).

Moreover, if:

- \((\ell = N, \tilde{\ell} = F)\), then the cycle is an F-indeterminate observed cycle (FR-ioc);
- \((\ell = F, \tilde{\ell} = R)\), then the cycle is an R-indeterminate observed cycle (FR-ioc);

The language \(L_{dil}\) generated by automaton \(G_{dil}\) is F-diagnosable with respect to projection \(P_{dil,o} : \Sigma_{dil}^* \rightarrow \Sigma_{dil}^*\) and \(\Sigma_{dsf}\), if and only if, the diagnoser \(G_{dil,d}\) has neither F- nor R-indeterminate (observed or hidden) cycles.

The language \(L_{dil}\) generated by automaton \(G_{dil}\) is R-diagnosable with respect to projection \(P_{dil,o} : \Sigma_{dil}^* \rightarrow \Sigma_{dil}^*\) and \(\Sigma_{dsf}\), if and only if, the diagnoser \(G_{dil,d}\) has neither R- nor FR-indeterminate (observed or hidden) cycles.

Proof: The proof will be omitted for the lack of space.

**VI. EXAMPLE**

Consider automaton \(G\) whose state transition diagram is shown in Figure 5(a). Assume that the sets of observable and unobservable events are \(\Sigma_o = \{b, c\}\) and \(\Sigma_{uo} = \{a\}\), respectively, and that \(\Sigma_{dsf} = \{b\}\). The model that takes into account intermittent faults in the sensor responsible for recording event \(b\), \(G_{dil}\), and the corresponding diagnoser \(G_{dil,d}\) are depicted in Figures 5(b) and 6, respectively.

From Figure 6, it is possible to see that there exist F-, R-, and FR-ics in \(G_{dil,d}\) and one R-ioc. Therefore, according to Theorem 1, the language generated by \(G_{dil}\) is neither F-diagnosable nor R-diagnosable with respect to projection \(P_{dil,o} : \Sigma_{dil}^* \rightarrow \Sigma_{dil}^*\) and \(\Sigma_{dsf} = \{b\}\). The lack of diagnosability can be justified as follows.

Consider initially, the F-ic in state \(\{2N, 3F, 4F, 7F, 8F\}\). It is not difficult to see that there exist two traces: a faulty
trace $s_F = b'acbb'd^n$, $n \in \mathbb{N}$, and a normal trace $s_N = ac$, for which $P_{dil,o}(s_F) = P_{dil,o}(s_N) = c$, therefore violating the F-diagnosability condition.

Consider, now, the FR-ihc in state $\{3N, 4F, 4R, 8R\}$. In this case, we can find three traces: a normal trace $s_N = acb$, a faulty trace $s_F = acbb'$ and a trace $s_R = b'acbb'n$, for which $P_{dil,o}(s_R) = P_{dil,o}(s_N) = P_{dil,o}(s_F) = cb$, therefore violating both F- and R-diagnosability conditions.

Finally, consider the R-ihc in state $\{4F, 4R, 8R\}$. In this case, we can identify the existence of a faulty trace $s_F = acbb'e^n$ and two traces with occurrences of event $b$ after the occurrence of $b'$, $s_{R1} = acbb'bc^n$ and $s_{R2} = b'acbe^n$, $n \in \mathbb{N}$, with the same projection over $\Sigma_r$, i.e., $P_{dil,o}(s_F) = P_{dil,o}(s_{R1}) = P_{dil,o}(s_{R2}) = cbe^n$, which violates the R-diagnosability condition, since it is not possible to state whether the sensor has either failed permanently or failed and returned to work.

It is worth noting that there is also an R-ioc in state $\{4F, 4R, 8R\}$, which violates the R-diagnosability condition.

VII. CONCLUSION

We addressed in this paper the problem of diagnosing intermittent sensor faults. It is worth remarking that: (i) differently from what one could expect, the use of the the adapted sensor model to deal with malfunction has not made the problem of intermittent sensor fault similar to that presented in [10]; (ii) the formulation of the problem of detection of intermittent fault addressed here is different from that addressed in [12], in the sense that the fault and reset events used in [12], are now the events associated with the malfunction and normal sensor behaviors, respectively, and moreover, the reset event is here observable as opposed to [12], which considers it as an unobservable event; this latter fact has made the diagnosability definitions introduced here different from those presented in [12]; (iii) We allow here cycles of states connected with unobservable events.

REFERENCES