

## Utilization of Markov Model and Non-Parametric Belief Propagation for Activity-Based Indoor Mobility Prediction in Wireless Networks

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**Abstract**—A foremost objective in wireless networks is to facilitate the communication of mobile users and the widespread tracking and prediction of their mobility regardless of their point of attachment to the network. In indoor environments the effective users' motion prediction system and wireless localization technology play an important role in all aspects of people's daily lives, including e.g. living assistant, navigation, emergency detection, surveillance/tracking of target-of-interest, evacuation purposes, and many other location-based services. Prediction techniques that are currently used do not consider the motivation behind the movement of mobile nodes and incur huge overheads to manage and manipulate the information required to make predictions.

In this paper we propose an activity-based continuous-time Markov model to define and predict the human movement patterns. Then we demonstrate the utility of Nonparametric Belief Propagation (NBP) technique in particle filtering, for both estimating the node locations and representing location uncertainties, and for prediction of the areas that would be visited and those that would not in the future. NBP method admits a wide variety of statistical models, and can represent multi-modal uncertainty. This prediction system may be used as an additional input into intelligent building automation systems.

**Keywords**—Indoor Environment, Wireless Network, Markov Chain, Markov Jump Process, Nonparametric Belief Propagation, Data Partitioning

### I. INTRODUCTION

One of the foremost objectives of a wireless network is to facilitate the communication of mobile users and the widespread tracking and prediction of their mobility regardless of their point of attachment to the network. In indoor environments the effective users' motion prediction system and wireless localization technology play an important role in all aspects of people's daily lives, including e.g. living assistant, navigation, emergency detection, surveillance/tracking of target-of-interest, evacuation purposes, and many other location-based services. Prediction techniques that are currently used do not consider the motivation behind the movement of mobile nodes and incur huge overheads to manage and manipulate the information required to make predictions. However, experience from many studies have

shown that mobility of users is in fact done not at random but is activity-based.

User's mobility prediction is an important maneuver that aims to determine the location of the user in the network by the manipulation of the available information about the user's activity. The prediction accuracy depends on the user mobility model and the prediction methodology. Many models currently in use assume a basically random movement of the user. While this is sufficient to simulate the performance of network level protocols, this assumption is not suitable for application level evaluation, even when assuming some specific distributions.

To overcome such limitations we propose in this paper an extension of the *Activity based Mobility Prediction algorithm using Markov modeling (AMPuMM)*, presented in [Mathivaruni, et al., 2008 ], by the implementation of the Markov jump continuous-time process framework to predict the future location of the users. We have limited our study of indoor environments. The presented user's mobility model is a component of the general activity-based model and the user's movement patterns are defined as the paths in a multi-graph representing the physical environment. We support the prediction methodology by the *Non-parametric Belief Propagation* algorithm, which is just briefly studied in this work.

The remainder of the paper is organized as follows. In Sec. II we define the activity-based model for the indoor user's behavior. In Sec. III the main idea of the AMPuMM model and its generalization by using the continuous-time Markov process is presented. The idea of the belief and non-parametric belief propagation support in the user's location prediction is demonstrated in Sec. IV. We end the paper in Section V with some conclusions and indications for future work.

### II. ACTIVITY-BASED MODEL

In activity-based modeling a typical user's daily behavior is defined as sequences of activities derived from a set of parameters. The model's type can be distinguished by the

way it illustrates the users decisions of when and how an activity is carried out. This model is thus driven by the user's activity.

In [Breyer et al., 2004] and [König-Ries et al., 2006] the authors propose an activity-based user-centered approach to various types of wireless networks (in particular, mobile ad-hoc networks). In this model the user's mobility is not considered as a primary criterion, it is defined as a result of the user's decisions and activities and enables the execution of activities by connecting the locations of two consecutive activities. The model derives an integrated view on mobility and network usage from a users real-world activity and thereby obtains mobility patterns and service usage preferences in a natural way.

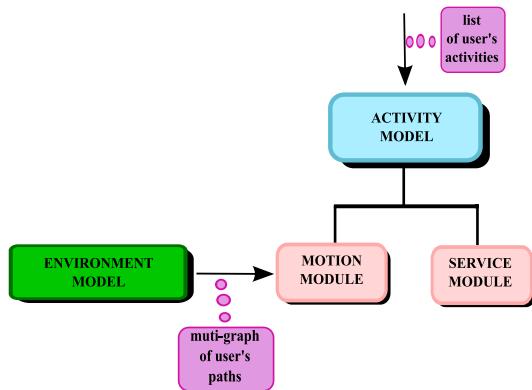


Figure 1. Activity-based model modules

The main idea of the model is present in Fig. 1. It consists of the four following modules (see also [Breyer et al., 2004]):

- **Activity Model:** This module calculates the 'timetables' for the user's non-networking activities;
- **Motion Module:** This module derives the necessary movements for a given user's activity;
- **Service Module:** This module supports the needful services for the user's activity;
- **Environment Model:** This module, which is usually represented as a multi-graph, provides the necessary information about the paths and activity location in the simulation area.

In the following two sub-sections we briefly describe two main modules in the system, namely **Activity** and **Environment** Models.

#### A. Activity Model

The main objective of the activity model is to transform an abstract list of possible non-networking activities into a concrete schedule (list of the activities). Each activity is characterized by the following parameters:

- **Starting time:** it is defined as a fixed point of time, in which the activities either start or are scheduled;

- **Duration:** the time, in which the activities finish or they are predicted to be completed. In the later case the activities' duration is usually defined by using some specified random distribution;
- **Priority:** the criterion, which can be specified by the user in order to make some preferences in the activities. The user's activities are scheduled in the order of their priority when time conflicts arise. Each user can specify its own set of priorities.

To define the activity model we have to specify a list of the non-networking user's actions. The user's actions are classified into several groups. The concept of an action class leads to the collective term of an activity. It represents the entirety of parameter sets for the activity class.

Let us suppose an university department as a hypothetical scenario. The typical actions of the staff members can be providing the lectures, seminars and lab work, providing the exams, preparation to the classes, research work, going to the library, attending departmental meetings. There is another class of possible social activities, which must be also analyzed, i.e. going to the cafeteria, eating lunch, relaxing. In the case of lack of the detailed specification of the teaching topics (low level of details), the seminars, labs and lectures can be generalized to an activity of a course. These actions typically share common locations and are both regular and recurring events over time.

For the detailed characterization of the user's activities, their starting and duration times must be specified. According to such a criterion we can divide the activities into two groups: those with a fixed starting time like providing the lecture, and free-floating activities like borrowing a book from the library. The duration of an activity can be also fixed or variable within a certain range. Usually, free-floating activities have durations that adhere to random distributions. To avoid the overlapping of the activities in time, the users can classify the activities according the priority criterion. The activities of higher priority take precedence over activities of lower priority when there are conflicting starting or ending times.

The activity model is used mainly for the calculation of the concrete activity schedules for a user. It can be achieved by the specification of an optimal plan by placing freely scheduling activities around the ones with fixed starting time.

#### B. Environment Model

To define a proper environment model it is necessary to specify firstly the constraints of the movements in the real-life mobility models. The mobile nodes in such systems have to adhere to the constraints given by buildings, vegetation or route sections.

As an example of such model we can define the pedestrians movements in the departmental scenario presented in the previous sub-section. It is represented as an *undirected multigraph*  $G = (V, E(w))$ , where:

- $V$  - is a set of the vertices in the graph. We define it as sum  $V = T \cup L$ , where  $T$  denotes the transition nodes and  $L$  - a set of the location nodes;
- $E(w)$  - is a set of graph multiple edges. The parameter  $w$  denotes the *width* of the path, which is expressed as a number of edges for the paths in the department between two location or transition nodes.

The example graph structure is presented in Figure 2.

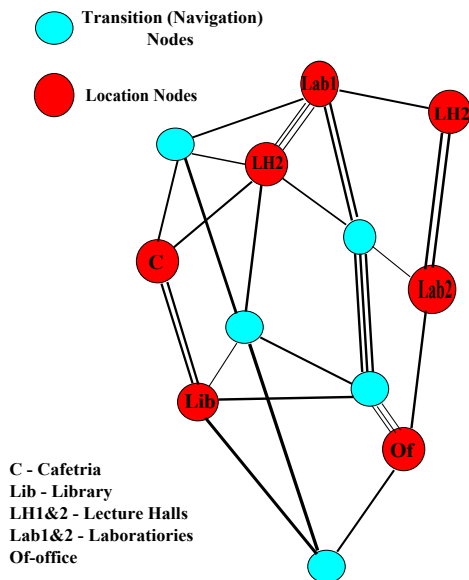


Figure 2. An environment multi-graph model

The edges in the graph define the walkable paths in and around the department. By using the edge 'width' parameter we can model a realistic group movement where several people are moving at the same time. It means that individuals are able to move side by side on one of these parallel paths. The vertices of the environment graph typically represent places where several trails meet, we call them *transition nodes*, and the users' activity locations like cafeteria, library, lecture room, lab room, etc. Each of these locations is suitable for certain activities and can be additionally characterized by its own mobility profile. We can call them *location nodes* and they are often interpreted as the targets when a person is moving and equate to likely destinations during any movement sequence. The sequence of nodes from one location node to another one are defined as the paths in the environment multi-graph. We call as *preferred paths* the paths, which are most commonly 'generated' by a given user during the daily activity. The paths in the graph generates the users' activity and, in particular, mobility patterns.

The environmental multi-graph model and the activity model are the basis for the mobility modeling [Bettstetter, 2001], [Scourias et al., 1999]. The use of the analytical mobility models is facilitated by the particular location node shape and type. Reaching a vertex connected to an

activity location is equivalent to beginning the execution of an activity. Moreover, it involves a different type of mobility than the one connecting two consecutive activity locations. In the following section we present the user's mobility model based on the Markov process.

### III. ACTIVITY-BASED MOBILITY MODELS

Activity Markov-based models accumulate and store information about the mobility behavior of the users in terms of the time sequence in which activities are performed. In this section we firstly highlight the main idea of the *Activity-based Mobility Prediction model using Markov modeling (AMPuMM)* [Mathivaruni, et al., 2008 ] used for the prediction of the future location of the mobile nodes in wireless networks. Then we propose a modification of such model to use the environmental multi-graph as a 'road map'-like tool for the users' mobility prediction and define a continuous-time Markov jump process [Ait-Sahalia et al., 2009] for the user's movement prediction. Next, we prove that AMPuMM model is a simple approximation to the Markov jump process that is the actual dynamics of the system. Finally, we use a *Non-parametric Belief Propagation (NBP)* [Sudderth et al., 2003] for prediction of the propagation of further user's activities and location.

#### A. Activity-based User's Mobility Prediction using Markov Modeling (AMPuMM)

The aim of the Activity-based user's Mobility Prediction using Markov Modeling algorithm (AMPuMM), presented in [Mathivaruni, et al., 2008 ], is to define the user's activity patterns, based on the monitoring of user's past activity provided in a specified time interval, to make a prediction of his next activity. The transition between activities is modeled as a Markov chain to predict  $(n + 1)$ -th day location using  $n$  days information.

The user's activities are classified into two groups: navigation activities and location activities. The location activities are performed while user is staying at a location (cafeteria, lecture room, lab in the department scenario presented in Sec. II). Navigation activities are associated with the physical motion of the users between the locations. The authors in [Mathivaruni, et al., 2008 ] propose a minimum threshold value  $t_{min}$ , which is set for an activity to be considered as a location activity.

The model is based on the observation of the users' everyday activities. Each period in this model is segmented into different time slots. For each slot time, the activity data is gathered and stored separately. The probability of occurrence of the activity and the probability to move from one activity to next one can be extracted from users' trace (which is in fact the path in an environment multi-graph).

Formally, a vector  $A = [a_1, a_2, \dots, a_7]$  defines the vector of seven activity states considered in [Mathivaruni, et al., 2008 ], and let us denote by  $N(a_i)$  the number of time

slots in which the activity  $a_i$  is provided. The transition probability from an activity  $i$  to activity  $j$  is given by the relative frequency of the sequence  $a_i, a_j$  ( $a_i, a_j \in A$ ). The activity transition is modeled as a Markov chain with the states from the set  $A$  and the transition probability matrix  $P$  defined as follows:

$$P = \begin{bmatrix} p_{11} & \dots & p_{17} \\ p_{21} & \dots & p_{27} \\ \vdots & \vdots & \vdots \\ p_{71} & \dots & p_{77} \end{bmatrix} \quad (1)$$

where

$$p_{ij} = \frac{N(a_i - a_j)}{N(a'_i - a_j)}, \quad (2)$$

and  $N(a_i - a_j)$  denotes the number of time slots in which the activity  $a_j$  follows activity  $a_i$ , and  $N(a'_i - a_j)$  is the number of time slots in which the activity  $a_j$  doesn't follow the activity  $a_i$ .

The user's activity (location) pattern is used to develop a model that predicts the user's  $(n+1)$ -th period's movements given  $n$  time intervals of information. The Markov chain model uses the current activity on the  $(n+1)$ -th time interval to predict the next activity for a single interval.

An initial state probability vector is defined by  $\Lambda(a_i) = N(a_i)/N(a'_i)$ , next state probability vector is defined by  $A(t) = A(t-1) \times P$ , where  $A(t)$  is the 'state' of the activity vector  $A$  in the  $t$ -th time slot. It means that each day (time period) of the observation can be additionally divided into the several time segments, and, instead of one transition matrix for the whole day, few transition matrices for *each* individual activity can be defined and considered. Thus, the generated Markov chain has a multidimensional transition matrix, which in fact increases the model complexity.

### B. Generalization of AMPuMM model - Markov jump process approach

There are a few drawbacks with the AMPuMM model presented in Sec. III-A. One main drawback of this model is the difficulty in the implementation of the multidimensional transition matrix. It is very hard to extract the information about the joint actions of the users. It can be concluded from the experimental evaluation of the model performed in [Mathivaruni, et al., 2008] that this approach can be effective just for a small area model (not so many activities and users) and in the case of ignoring of the 'transition (navigation)' nodes. The activity in AMPuMM model is restricted just to an action of the user, which terminates in a specified location. Another drawback is the restriction of such model just to the discrete time case, which means that to achieve a good prediction it is necessary to consider many time slots, which again increases the complexity of the model.

To overcome such disadvantages we propose to define the user's mobility prediction as the *Markov continuous-time*

*jump process* [Ait-Sahalia et al., 2009]. Let us denote by  $A$  a set of all states of a system and by  $a, b \in A$  the states of the system. The states can be interpreted as the user's locations. A jump process is a random variable  $X(t)$  parameterized by time  $t \in [0; \infty)$ . This random variable starts from an initial state  $a_0$  at time  $t = 0$  and stays in this state until some time  $t_1$  when it makes a transitions to a different state  $a_1$ . Similarly, it stays in this state until a later time  $t_2 > t_1$  at which it jumps to a different state  $a_2$ . Then, if  $t_1, t_2, \dots$  are the set of jump times, then  $X(t) = a_0$  for  $t \in [0; t_1)$ ,  $X(t) = a_1$  for  $t \in [t_1; t_2)$ , and so on.

We assume that the jump process  $X(t)$  can be defined for all non-negative values of  $t$ . The probability of changing (jumping) from the state  $a$  to the state  $b$  is defined as  $r(a, b)$ , such that:

- (1)  $r(a, a) = 0$ , and
- (2) for all  $a \in A$   $\sum_a r(a, b) = 1$ .

Once the process is in a state  $a$ , the time period that it stays in this state is a random variable governed by the distribution function  $F_a(t)$  (for each state a different distribution function can be specified). The transition probability distribution can be then defined in the following way:

$$P(\tau \leq t, X(\tau) = b, x(0) = a) = r(a, b) \cdot F_a(t). \quad (3)$$

We denote by  $p(b, a|t)$  as the conditional probability that the jump process is in state  $b$  at time  $t$  given that it was in state  $a$  at time 0. Given times  $0 < t_1 < t_2 < \dots < t_n < s$  and  $t > 0$ , the Markov property for a jump process is defined as follows:

$$P(X(t+s) = b | X(s) = a; X(t_n) = a_n; \dots; X(t_1) = a_1) = p(b, a|t) \quad (4)$$

In such processes the time is modeled by the exponential distribution:

$$F_a(t) = 1 - e^{-\gamma_a t} \quad (5)$$

with the following probability density function:  $f_a(t) = \gamma_a e^{-\gamma_a t}$ . Given a set of previous states at earlier times, the Markov jump process 'forgets' all but the state at the most recent time.

With the Markov property, joint probabilities for Markov jump process can be written as

$$P(X(t+s) = a; X(s) = b; X(0) = c) = p(a, b|t)p(b, c|s) \cdot P(X(0) = c) \quad (6)$$

for  $t, s > 0$ . Hence the Chapman-Kolmogorov equation [Ait-Sahalia et al., 2009] for the Markov jump process is defined in the following way:

$$p(a, c|(t+s)) = \sum p(a, b|t)p(b, c|s) \quad (7)$$

Usually a geometrically distributed discrete random variable is an approximation to the continuous time exponentially distributed random variable (see [Ait-Sahalia et



al., 2009)). Therefore, we can demonstrate now that the AMPuMM model can be defined as a simple approximation to our Markov jump process.

Let us denote by  $\hat{A} = [a_1, \dots, a_k]$  the vector of all possible activities of the users, which is in fact a simple generalization of the vector  $A$  specified in Sec. III-A. We can also consider a generalization of the Markov chain modeling the user's activity transition with the transition probability matrix  $\hat{P}$  defined as follows:

$$P = \begin{bmatrix} p_{11} & \dots & p_{1k} \\ p_{21} & \dots & p_{2k} \\ \vdots & \vdots & \vdots \\ p_{k1} & \dots & p_{kk} \end{bmatrix} \quad (8)$$

The probability that a Markov chain remains in state  $i$  for  $n$  steps can be calculated as follows: if  $p_{ii} = 0$ , then the only possibility is  $n = 0$ ; it must always make a transition and never stay in the same state. Otherwise, the probability is calculated using the following formulae:

$$p_{ii}^n = e^{nl \ln(p_{ii})} = e^{nl \ln(1-p_g(i))} \quad (9)$$

where  $p_g(i) \equiv \sum_{j \neq i} p_{ji}$  is the probability of leaving the state  $i$ . For  $p_g(i) \ll 1$ , we achieve  $\ln(1-p_g(i)) \approx -p_g(i)$ , so then this probability is exponentially distributed with rate  $p_g(i)$ . This suggests that a Markov jump process can be defined as follows:

$$r(j, i) = \frac{p_{ji}}{p_g(i)} \quad (10)$$

for  $i \neq j$  so that this quantity is normalized and  $\gamma_a \equiv \gamma p_g(a)$  where  $\gamma^{-1}$  expresses time units. This defines the time scale of the jump process, and the choice  $\gamma = 1$  makes the  $n$  step of the Markov chain equivalent to  $t = n$  in the Markov jump process.

#### IV. NON-PARAMETRIC BELIEF PROPAGATION SUPPORT

The monitoring of the user's daily activities generates as the result many user's patterns, which in graph terms can be modeled as the paths in the environment multi-graph. For an efficient mobility and users' location prediction some fast and accurate filtering method is needed to be implemented. Additionally, such kind of filtering method can support rather complex Markov mechanism by 'distributing' the prediction into the set of the coherent processes of the low complexity and can be sufficient for both estimating the users' locations and representing the location uncertainties. We utilized for this purpose the nonparametric belief propagation method (NBP) as a recent generalization of particle filtering and a variant of the popular belief propagation (BP) algorithm [Sudderth et al., 2003]. NBP has the advantage that it is easily implemented in a distributed fashion, admits a wide variety of statistical models, and can represent multi-modal uncertainty. NBP has been successfully applied in self-localization of the nodes in the sensor networks (see [Noureddine et al., 2010], [Ihler et al., 2004]).

#### A. Belief Propagation(BP) Method

To introduce the Belief Propagation method, for the undirected graph  $G = (V, E)$  we specify the *neighborhood*  $\Gamma(a), a \in V$  of a node  $a$  using the following formulae:

$$\Gamma(a) = \{b | (a, b) \in E\}, \quad (11)$$

which means that  $\Gamma(a)$  is a set of nodes adjacent to the node  $a$ .

The undirected graphs are usually the basis for more complex graphical models, which can be also used in the users' mobility prediction in wireless networks. Graph-theoretic models associate each node  $a \in V$  with an unobserved random variable  $x_a$  and some noisy local observation  $y_a$ . The graph properties describe the statistical relationship in the set of all hidden and observed variables, denoted by  $x = \{x_a | a \in V\}$  and  $y = \{y_a | a \in V\}$  respectively. In particular, the graph encodes the Markov properties of the random variables through the possible graph separation (we can consider the neighborhoods, which are separated by the sets of nodes or, like in the departmental scenario presented in Sec. II, by the transition nodes (see Fig. 2)).

Using the Hammersley-Clifford theorem [Clifford, 1990] we can specify the distribution of the variables  $x_a$  and  $y_a$  using the following formulae:

$$p(x, y) = \prod_{(a,b) \in E} \psi_{a,b}(x_a, x_b) \prod_{a \in V} \psi_a(x_a, y_a) \quad (12)$$

where  $\psi$  denotes the *potential* function. The Eq. (12) quantifies the relationship between an environment graph and the joint distribution of its random variables.

The main goal of the users' mobility prediction is the specification of the conditional marginal distributions  $p(x_a | y)$  for all nodes  $a \in V$ , which can be directly calculated by Belief Propagation (BP) method. BP takes the form of a *message-passing algorithm* between nodes, the most common of which is a parallel update algorithm, where each node calculates outgoing messages (the provided user's activities) to its neighbors simultaneously. Each iteration of the BP algorithm can be expressed in terms of an update to the outgoing message at iteration  $n$  from each node  $b \in V$  to each neighboring node  $a \in \Gamma(b)$  in terms of the previous iteration's incoming messages from  $b$  neighbors  $\Gamma(b)$ , not including  $a$  itself, i.e.:

$$m_{ba}^n(x_a) = \alpha \int_{x_b} \psi_{a,b}(x_a, x_b) \psi_b(x_b, y_b) \times \prod_{u \in \Gamma(b) \setminus a} m_{ub}^{n-1}(x_b) dx_b \quad (13)$$

where,  $\alpha$  denotes an arbitrary proportionality constant. At any iteration, each node can produce an approximation  $\hat{p}^n(x_a | y)$  to the marginal distribution  $p(x_a | y)$  by combining the incoming messages with the local observation:

$$\hat{p}^n(x_a | y) = \alpha \psi(x_a, y_a) \prod_{b \in \Gamma(a)} m_{ba}^n(x_a) \quad (14)$$

Each iteration of the BP algorithm involves only local message updates, thus it can be successfully applied to any class of undirected graphs.

### B. Non-parametric belief propagation support - preliminary analysis

For graph-based models with continuous hidden variables, analytic evaluation of the BP update integral in Eq. (13) is often intractable. In *Non-parametric belief propagation (NBP)*, which is a modification of the BP method, the resulting message is represented using a sample-based density estimate. In classical NBP approach the mixture of Gaussian densities is considered (however, the formal description of the NBP methodology is not restricted to this specific distribution). We modified the NBP framework by using a mixture of density functions of the negative exponential distribution in a Markov jump process defined in Sec. III-B.

Let  $f_a(t) = \gamma_a e^{-\gamma_a t}$  be the density function of the specified for the Markov jump process. We accept it as a potential function  $\psi$  in Eq. (12). Let us also define an  $N$ -component mixture approximation of  $m_{ba}(x_a)$  by the following formulae:

$$m_{ba}(x_a) = \sum_{i=1}^N w_a^{(i)} f_a^{(i)}(t) \quad (15)$$

where  $w_a^{(i)}$  is the weight associated with the  $i$ -th mixture component  $f_a^{(i)}(t)$ . If we assume the independency of the variables the product of the messages in Eq. (12) can be modeled by the direct product of the negative exponential distribution, which is negative exponential joint distribution for the mixture defined in Eq. (15). Thus, the propagation of the user's messages (beliefs) according the rule specified in Eq. (15) can be realized in two steps: (a) by sampling the updated user's message from the joint density; (b) by propagating each sample from the message product by approximating the belief update integral in Eq. (12). The sampling procedures can be realized by using the Gibbs sampler ([Geman et al., 1984]).

### V. CONCLUSIONS AND FUTURE WORK

In this paper we propose an activity-based continuous-time Markov model to define and predict the human movement patterns. We used the Markov jump method for the specification of the user's mobility as the stochastic continuous-time process with a negative exponential time distribution. We demonstrate the utility of Nonparametric Belief Propagation (NBP) technique in user's pattern filtering, for both estimating the user's locations and representing location uncertainties, and for prediction of the areas that will, or will not, be visited in the future. NBP method admits a wide variety of statistical models, and can represent multimodal uncertainty. We have shown that our model is able to overcome several limitations of existing models in the literature.

This prediction system may be used as an additional input into intelligent building automation systems. We plan to define the detailed properties of the NBP model for the proposed negative exponential distribution mixture and provide the experimental evaluation of the model.

In our future work we plan to implement the users' messages propagation model with the NBP approach.

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