Continuous Nonsingular Terminal Sliding-Mode Control of Shape Memory Alloy Actuators Using Time Delay Estimation

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Abstract—We have developed a continuous nonsingular terminal sliding-mode control with time delay estimation (TDE) for Shape memory alloys (SMA) actuators. The proposed method does not need to describe a mathematical model of a hysteresis effect and other nonlinearities; thus, it is simple and model-free. The proposed control consists of three elements which have clear meaning: a TDE element that cancels nonlinearities in the SMA dynamics, an injection element that specifies desired terminal sliding-mode (TSM) dynamics, and a reaching element using a fast terminal sliding manifold that is activated accordingly when the system trajectory is not confined in the TSM. The proposed control has been successfully implemented in an SMA actuated system and experimental results show the proposed control is easily implementable and highly accurate. Once the TSM and the reaching condition are suitably specified, the tracking performance of the proposed control is improved compared with a conventional time delay control with a linear error dynamics.

Index Terms—Shape memory alloys, Terminal sliding mode, Model-Free control, Time delay estimation

I. INTRODUCTION

SHAPE memory alloy (SMA) actuators have been widely utilized as a power source of devices or mechanisms in diverse scientific and industrial applications such as robotics [1], [2], precision control system [3], [4], aerospace engineering [5], rotary actuator [6] and medical devices [7], [8]. Compared with traditional actuators, the SMA actuator is silent, compact in size, and excellent in power-to-weight ratio; this has recently led a new trend of actuation systems in bio-inspired robotics [9]–[13]. The SMA can be directly driven through a resistive heating from an electrical current exploiting a “shape memory effect.” The shape memory effect is due to a solid-solid phase transformation between two crystallographic phases, i.e., a low temperature martensite and a high temperature austenite. This, however, introduces a hysteresis effect, which is highly nonlinear.

To obtain a high control accuracy for the SMA actuators, the hysteresis effect must be compensated. There are two major research directions to cope with the hysteresis effect: the use of mathematical model which describes hysteresis behavior and the use of artificial intelligence techniques such as a neural network (NN) to establish a “black box” model. The mathematical models, such as Preisach model [14], [15], Duhem model [16], [17], Dutta model [18], Liang model [1], [19], and other empirical models [20], [21], have been successfully used with control methods (proportional-integral-derivative (PID) control, sliding mode control, and adaptive control) to perform a position tracking. However, it is difficult or time-consuming to obtain an exact mathematical model and to identify system parameters of the model due to the highly nonlinear and complicated dynamics of the hysteresis effect. For example, a homogenized energy model is recently developed for SMA actuators [22]. Data-driven techniques [23], [24] are used to identify the homogenized energy model [25]; the number of model parameters is sixteen, and they require five identification experiments with different scenarios in [25]. Understanding of a complex suite of mathematic equations and algorithms is mandatory to realize control of SMA actuators.

The “black box” models, such as NN-based model [26] and neuro-fuzzy model [27], have been proposed to mitigate difficulty of obtaining the mathematical model, and many robust controllers have been developed through the NN based approaches [26], [27]. However, the NN-based approaches introduce a number of tuning parameters (e.g., 320 weighting parameters used in [26], and 512 in [27]); thus, it is not easy to practically implement the NN-based approaches. Consequently, the mathematical models and the “black box” models are only available for expert control engineers due to their complexity of algorithm and difficulty of tuning.

What practicing engineers need is a controller that is simple, robust, and highly accurate. In this paper, we present a controller without a complex physical model or many tuning parameters by using time delay estimation (TDE). The TDE is efficiently and effectively used to cancel the nonlinear dynamics of SMA. It has been applied in various applications and provided satisfactory results even under large disturbances and parameter variations. The controllers based on the TDE [28]–[32] have two elements: the TDE element that cancels nonlinear dynamics and an injecting element that endows desired error dynamics. As for target error dynamics, a linear error dynamics is widely adopted in [28]–[31], and the system states asymptotically converge to the equilibrium point at an infinite time.

It is worth to note that nonlinear sliding mode called termi-
nal sliding mode (TSM) can achieve finite-time stability, while a linear sliding mode makes the system states converge to the equilibrium point at an infinite time. By adjusting parameters of TSM, one can enjoy a faster and finite-time convergence. The TSM is firstly introduced as a terminal attractor [33]. Through extensive research, the TSM [34] has evolved into fast terminal sliding mode (FTSM) [35], nonsingular terminal sliding mode (NTSM) [36], [37], and finally continuous nonsingular fast terminal sliding mode (CNFTSM) [38]. The CNFTSM is a kind of sliding mode that can achieve finite-time convergence, and it is fast, singularity free and chattering free. However, mathematical model of plant is required to apply the CNFTSM in [38].

In this paper, we propose a model-free and accurate controller for SMA actuators using TDE and CNFTSM. First, an NTSM-type sliding surface is selected such that a system trajectory intersects and stays on the manifold; second, reaching law is defined by an FTSM-type dynamic attractor so that the system trajectory intersects and stays on the manifold; third, the nonlinearities of the SMA actuator is estimated and canceled by TDE without parameter identification of the complex SMA dynamics. As a result, the control input of the proposed controller has a transparent structure consist of three elements that have clear meaning. Experimental results show that the proposed approach enables faster and more accurate position tracking compared with time delay control (TDC) with linear error dynamics or PID control.

The rest of the paper is organized as follows. In Section II, the dynamics of an SMA actuated system is investigated. A continuous nonsingular terminal sliding-mode control using TDE is developed for the system in the next section. In Section IV, stability analysis is given. Experimental results are given to verify the highly accurate tracking performance of the proposed control in Section V. Finally, we end this paper with some conclusions in Section VI.

II. SMA ACTUATED SYSTEM

The experimental setup of the SMA actuated system is presented in Fig. 1, which provides a capability to test feedback position control schemes on a single SMA actuator with a pulley and the load of a bias spring.

The dynamics of the SMA actuated system is taken from [1], [39], and described by

\[ J\ddot{\theta} + b\dot{\theta} + k\theta + d = \tau_s(\sigma), \]

where \( \theta, \dot{\theta}, \) and \( \ddot{\theta} \) represent the position, the velocity and the acceleration of the system, respectively, \( J, b, \) and \( k \) denote an effective inertia, an effective damping, and an effective stiffness, respectively, and \( d \) denotes unexpected disturbances, while \( \tau_s \) denotes a torque generated by the SMA actuator, and \( \sigma \) denotes a Piola-Kirchhoff stress.

As described in [18], a mathematical model of the SMA actuator can be expressed as four sub-dynamics which are Joule heating and heat transfer, phase transformation, stain, and electrical resistance. First, the Joule heating and heat transfer equation is given as

\[ \Lambda c_p \dot{T} + c_h A_c (T - T_0) = Ru, \]

where \( \Lambda \) denotes the mass per unit length, \( c_p \) denotes the specific heat capacity, \( T \) denotes the temperature of the SMA, \( c_h \) denotes the convective heat transfer coefficient, \( A_c \) denotes the circumferential area, \( R \) denotes the resistor, and the input \( u = I^2 \). The \( c_p \) and \( c_h \) are functions of the temperature expressed by

\[ c_p = b_1 + b_2 erf((T - m_1)/n_1), \]

and

\[ c_h = \begin{cases} a_1 - a_2 T, & T \geq 0, \\ a_3 + a_4 erf((T - m_2)/n_2), & T < 0 \end{cases} \]

where \( a_1, b_1, m_1 \) and \( n_1 \) with subscriptsed numbers \( i \) are constant parameters. Second, the hysteretic behavior of SMA is related to martensite-to-austenite phase transformation. This can be described by Duhem differential hysteresis model which presents the relationship between the martensite fraction (\( f_m \)) and the temperature as follows:

\[ \dot{f}_m = \begin{cases} g_+(T) h_+(T) + f_m^{-1} h_-(T), & T \geq 0, \\ g_-(T) h_+(T) + f_m^{-1} h_-(T), & T < 0 \end{cases} \]

where \( f_m(0) = 1 \), the subscriptsed \( + \) and \( - \) denote increasing and decreasing phases in hysteresis loops, respectively, and \( g_+ \) and \( h_\pm \) are given by

\[ g_\pm(T) = (c_\pm \sqrt{2\pi})^{-1} \exp[-(u - \mu_\pm)^2/(2c^2_\pm)], \]

\[ h_\pm(T) = 1/2[1 + erf[-(u - \mu_\pm)/(\sqrt{2}c_\pm)]], \]

where \( \mu_\pm \) and \( c_\pm \) denote mean and covariance values which govern the shape of loops. Third, the strain of SMA normally
depends on the condition of stiffness of the bias spring, the pretension stress and physical parameters of the SMA material. The authors in [18] provide a good approximation of the strain \( \epsilon \) to a continuous polynomial form in \( f_m \) as

\[
e = c_0 + k_1 f_m + k_2 f_m^2 + k_3 f_m^3,
\]

\( k_1, k_2, \) and \( k_3 \) are constant parameters. Last, the electrical resistance of the SMA is expressed by SMA temperature and martensite fraction as follows:

\[
R = \rho L_0^2 (1 + 2 \epsilon) \rho_0(T) \rho_m(T) / \Lambda (1 - f_m(T) + f_m\rho_0(T)),
\]

where \( L_0 \) denote the length of undeformed SMA, while \( \rho_0 \) and \( \rho_m \) denote the electrical resistivity of austenite and martensite, respectively, which can be given by

\[
\rho_m(T) = (p_1 + p_2 \exp[-p_3(T - T_0)]) + \sum_{k=1}^9 \omega_i (T - T_0)^k,
\]

where \( p_i, q_i, \) and \( \omega_i \) with subscripted numbers \( i \) are constant parameters. From the above SMA mathematical model, the SMA actuator can be expressed in the form of the second-order dynamics equation as follows:

\[
\begin{bmatrix}
\ddot{f}_m &= f'_{11}(f_m, T) f_m + f'_{12}(f_m, T) T \\
\dot{T} &= f'_{21}(T) f_m + g(f_m, T) u
\end{bmatrix}
\]

(10)

It is worth to note that since \( f_m \) and \( T \) inside SMA are not directly measurable, recent research [16], [40] presented that this SMA actuator dynamics can be converted in a form of a nonlinear second-order system, where the states are displacement \((x)\) and velocity \((\dot{x})\) of the end point of the SMA actuator as follows:

\[
\ddot{x} = f(x, \dot{x}, u) + g(x) u(t),
\]

(11)

where \( f(x) \) and \( g(x) \) terms contain the nonlinear hysteresis effect due to martensite-austenite phase transformation and uncertainties due to the SMA resistance variation. The authors in [41] have also developed a simplified second-order system model through experimental investigation and simulation.

Since the generated torque from the SMA actuator is applied to the system shown in Fig. 1, the overall SMA actuated system dynamics (1) can be described as

\[
J\ddot{\theta} + b\dot{\theta} + k\theta + d = h(\theta, \dot{\theta}, u) + \alpha(\theta)u,
\]

(12)

where \((h(\theta, \dot{\theta}, u)\) denotes the hysteretic nonlinear term and \(\alpha(\theta)\) denotes the input variable. Because \(\alpha\) depends on the SMA resistance and deformation, its value is bounded by the range of the related parameters; thus, \(|\alpha(\theta)| \leq D_\alpha\), where \(D_\alpha\) is a positive constant. Additionally, it is inferred from (2) that the hysteresis term \(h(\theta, \dot{\theta}, u)\) is depending on the SMA temperature \((T)\) but also the room temperature \((T_0)\).

As shown in Fig. 2, we have experimentally compared the response of the mathematical model and the real SMA actuated system, while a sinusoidal current at 1/15 Hz is applied. The parameters of the SMA model are shown in Table I. From the responses shown in Fig. 2 (b), it can be seen that the dynamics model of the overall system fairly describes the complex response of the real system, i.e., the hysteretic behavior.

\[
\ddot{m} + \eta(\theta, \dot{\theta}, \dot{\theta}, u) = u,
\]

(13)

where \(m\) denotes a nominal value of \(J/\alpha(\theta)\) and \(\eta(\theta, \dot{\theta}, \dot{\theta}, u) = (\alpha(\theta) - \bar{m})\dot{\theta} + \alpha(\theta)^{-1}[b(\theta + \dot{\theta} + d - h(\theta, \dot{\theta}, u)],

### TABLE I

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A)</td>
<td>7.41 \times 10^{-4} \text{ kgm}^{-1}</td>
<td>(A_c)</td>
<td>11.2 \degree C</td>
</tr>
<tr>
<td>(A_c)</td>
<td>4.78 \times 10^{-4} \text{ m}^2</td>
<td>(c_e)</td>
<td>5.8 \degree C</td>
</tr>
<tr>
<td>(\rho)</td>
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<td>(p_1)</td>
<td>9.2 \times 10^{-7} \text{ tm}</td>
</tr>
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<td>(L_0)</td>
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<td>(p_2)</td>
<td>8.4 \times 10^{-7} \text{ tm}</td>
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<td>(T_0)</td>
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<td>(p_3)</td>
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<tr>
<td>(a_1)</td>
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<td>(q_1)</td>
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<tr>
<td>(a_2)</td>
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<td>(q_2)</td>
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<td>(k_3)</td>
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<td>(b_2)</td>
<td>1000 \text{ Jkg}^{-1} \text{ C}^{-1}</td>
<td>(\omega_1)</td>
<td>4.8 \times 10^{-8} \text{ tm}^2 \text{ C}^{-1}</td>
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<td>(\omega_5)</td>
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<td>(\omega_6)</td>
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</tr>
<tr>
<td>(n_3)</td>
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<td>(\omega_7)</td>
<td>3.2 \times 10^{-16} \text{ tm}^2 \text{ C}^{-7}</td>
</tr>
<tr>
<td>(\mu_+)</td>
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<td>(\omega_8)</td>
<td>2.2 \times 10^{-18} \text{ tm}^2 \text{ C}^{-8}</td>
</tr>
<tr>
<td>(\mu_-)</td>
<td>34 \degree C</td>
<td>(\omega_9)</td>
<td>6.7 \times 10^{-21} \text{ tm}^2 \text{ C}^{-9}</td>
</tr>
</tbody>
</table>
that is, all the nonlinearities of the SMA actuator system including the hysteresis effect and external disturbances.

To ensure fast and accurate position tracking performance of the SMA actuator system, an NTSM-type sliding surface is defined as follows:

\[ s = e + \kappa \text{sgn}(\dot{e})^a, \]

(14)

where \( e \) denotes a tracking error between a desired position \( \theta_d \) and a real position, \( \kappa > 0 \) and \( 1 < a < 2 \) denote design values, and \( \text{sgn}(x)^\gamma = |x|^\gamma \text{sgn}(x) \) is used for simplicity of expression [42].

Not only to attenuate a control chatter, but also to preserve fast finite-time convergence, the following dynamic attractor of reaching phase is defined, which is a type of fast terminal sliding manifold:

\[ \dot{s} + \lambda_1 s + \lambda_2 \text{sgn}(s)^b = 0, \]

(15)

where \( \lambda_1 \) and \( \lambda_2 > 0 \) denote design constants for the reaching phase and \( 0 < b < 1 \).

For the tracking control of SMA actuator system, when the sliding manifold is chosen as (14) and the reaching law is as (15), a control input can be designed as

\[ u = \bar{m} \nu + \hat{\eta} \]

(16)

with

\[ \nu = \ddot{\theta}_d + (\alpha \kappa)^{-1} \text{sgn}(\dot{\epsilon})^{2-a} + (\lambda_1 s + \lambda_2 \text{sgn}(s)^b). \]

(17)

In (16), \( \hat{\eta} \) is the estimation of \( \eta \) shown in (13) which denotes all the nonlinearities including hysteresis, friction and external disturbances. This term is difficult to model and requires laborious parameter identification process.

In order to endow robustness against uncertainties and simplicity in form to the controller, we incorporate the TDE as follows:

\[ \dot{\eta} = \eta_{(t-L)}, \]

(18)

where the subscript \( (t-L) \) denotes a time-delayed value with a sampling period \( L \). From (13), the TDE can be simply obtained by

\[ \eta_{(t-L)} = u_{(t-L)} - \bar{m} \hat{\dot{\theta}}_{(t-L)}. \]

(19)

Figure 3 shows the nonlinear term \( \eta \) in (13) and its TDE error using (18) and (19) from the previous response of mathematical model. The TDE is effective to estimate \( \eta \), which contains all the nonlinearities of the SMA actuated system.

Finally, with the combination of (16), (17), (18), and (19), CNFTSM control law is expressed by

\[ u = \bar{m}(\ddot{\theta}_d + (\alpha \kappa)^{-1} \text{sgn}(\dot{\epsilon})^{2-a} + \lambda_1 s + \lambda_2 \text{sgn}(s)^b) \]

injecting desired NTSM error dynamics reaching law using FTSM

\[ + u_{(t-L)} - \bar{m} \hat{\dot{\theta}}_{(t-L)}, \]

(20)

Since the sliding surface is the continuous and differentiable NTSM and the dynamic attractor of reaching phase is the fast-type TSM, the proposed control has fast finite-time convergence without control chatter. Furthermore, the TDE simply and effectively cancels hysteresis behavior of the SMA and other nonlinearities of the system, such as friction, and external disturbances; thus it enables model-free control.

Interestingly, if \( a = b = 1 \), the proposed control can be transformed into TDC with linear error dynamics introduced in [28]–[31]. With \( a = b = 1 \), the sliding surface becomes \( s = e + \kappa \dot{e} \) and the control input (20) is described by

\[ u = \bar{m}\{\ddot{\theta}_d + [\kappa^{-1} + \kappa(\lambda_1 + \lambda_2)] \dot{\epsilon} + (\lambda_1 + \lambda_2) \epsilon\}

\[ + u_{(t-L)} - \bar{m} \dot{\theta}_{(t-L)}, \]

(21)

The above equation is equivalent to the control input of TDC [28], as given by

\[ u = \bar{m}(\ddot{\theta}_d + K_D \dot{\epsilon} + K_P e) + u_{(t-L)} - \bar{m} \dot{\theta}_{(t-L)} \]

(22)

with \( K_D = \kappa^{-1} + \kappa(\lambda_1 + \lambda_2) \) and \( K_P = \lambda_1 + \lambda_2 \). It is the condition \( 1 < a < 2 \) and \( 0 < b < 1 \) that activates the nonlinear error dynamics described by CNFTSM.

IV. Stability Analysis

The closed-loop stability of the proposed control is analyzed in this section. By substituting the control equation (16) into the plant dynamics (13), a closed-loop error dynamics is expressed as

\[ v - \ddot{\theta} = \bar{m}^{-1}(\eta - \hat{\eta}). \]

(23)

Since the right-hand side of (23) denotes a remaining error after applying TDE, we define TDE error as \( \varepsilon \Delta = \bar{m}^{-1}(\eta - \hat{\eta}) \) which is bounded by \( |\varepsilon| \leq \varphi \), where \( \varphi \) denotes a positive value. The proof of boundedness of \( \varepsilon \) is provided in Appendix A.

Substituting (17) into (23) yields

\[ \ddot{\epsilon} + (\alpha \kappa)^{-1} \text{sgn}(\dot{\epsilon})^{2-a} + (\lambda_1 s + \lambda_2 \text{sgn}(s)^b) = \varepsilon. \]

(24)
By differentiating the sliding manifold equation shown in (14), we have
\[ s = \dot{e} + a\kappa |\dot{e}|^{a-1}\dot{e}. \] (25)
Consider Lyapunov function \( V = 0.5s^2 \). Substituting (25) into the time derivative of \( V \) yields
\[ \dot{V} = ss = s(\dot{e} + a\kappa |\dot{e}|^{a-1}\dot{e}). \] (26)
By substituting the closed-loop error dynamics, shown in (24), into (26), one can obtain
\[ \dot{V} = -a\kappa s|\dot{e}|^{a-1}(\lambda_1 s + \lambda_2 \text{sign}(s)b - \varepsilon). \] (27)
Equation (27) can be rearranged as the following two forms:
\[ \dot{V} = -a\kappa s|\dot{e}|^{a-1}\left[\lambda_1 s + \left(\lambda_2 - \frac{\varepsilon}{\text{sign}(s)}\right)s + \lambda_2 \text{sign}(s)b\right]. \] (28)
and
\[ \dot{V} = -a\kappa s|\dot{e}|^{a-1}\left[\lambda_1 s + \left(\lambda_2 - \frac{\varepsilon}{\text{sign}(s)}\right)s\right]. \] (29)
For (28), the time derivative of the Lyapunov function is negative, if \(|s| > \lambda_1^{-1}|\dot{e}|\). Thus, by the boundedness of \(|e| \leq \varphi\), the following region can be reached in finite time:
\[ |s| \leq \delta_1, \] (30)
where \( \delta_1 \triangleq \lambda_1^{-1}\varphi \). Note that the finite-time reachability is provided in Appendix B. Similarly, if \(|s|^b > \lambda_2^{-1}|\dot{e}|\), the time derivative of the Lyapunov function (29) is negative and the region
\[ |s| \leq \delta_2 \] (31)
can be reached in finite time, where \( \delta_2 \triangleq (\lambda_2^{-1}\varphi)^{1/b} \). From (30) and (31), the finite-time reachable region of the NTSM is obtained as
\[ |s| \leq \delta_{\min}, \] (32)
where \( \delta_{\min} \triangleq \min\{\delta_1, \delta_2\} \).
By considering (32), the NTSM equation shown in (14) can be expressed as
\[ e + \tilde{\kappa}\text{sign}(\dot{e})a = \tilde{\delta}, \] (33)
where \( |\tilde{\delta}| \leq \delta_{\min} \). Then, (33) is rearranged as
\[ e + \tilde{\kappa}\text{sign}(\dot{e})a = 0, \] (34)
where \( \tilde{\kappa} \triangleq \kappa - \text{sign}(\dot{e})a \). If \( \tilde{\kappa} > 0 \), the error dynamics, (34), can be regarded as the NTSM. Therefore, one can show the convergence of the position- and velocity-tracking errors from the following condition:
\[ \kappa = \tilde{\kappa} - \text{sign}(\dot{e})a = \kappa - \tilde{\delta}/|\dot{e}|^a\text{sign}(\dot{e}) > 0. \] (35)
From (35), the velocity-tracking error has finite-time convergence to
\[ |\dot{e}| \leq (\delta_{\min}/\kappa)^{1/a}. \] (36)
Furthermore, (33) can be expressed as
\[ e = -\kappa|\dot{e}|^a\text{sign}(\dot{e}) + \tilde{\delta}. \] (37)

From (36) and (37), the position-tracking error has finite-time convergence to
\[ |e| \leq \kappa|\dot{e}|^a + \tilde{\delta} \]
\[ \leq \delta_{\min} + \delta_{\min} = 2\delta_{\min}. \] (38)
Therefore, the stability is guaranteed in the sense of Lyapunov and the finite-time reachability is obtained, that is, the system trajectory will converge to the region (36) and (38) in a finite time.

V. EXPERIMENTAL VERIFICATION

A. Experimental Setup

The specification of experimental devices, shown in Fig. 1, is summarized in Table II. The SMA actuator system consists of the NiTi-type SMA actuator and the bias-spring. The position measurement of the SMA actuator is a high precision potentiometer, Copal JT30. The analog signal of the potentiometer is converted into a digital signal by using a 14 bit analog-to-digital convertor (ADC) of a multi-functional data acquisition (DAQ) board, Sensoray S626; the resolution of the sensor is determined by its ADC as 0.0183 deg, and the root-mean squared (RMS) values of the measurement noise is 0.01 deg. The DAQ board also has 13 bit digital-to-analog converters, thus it generates an analog voltage for the control input. The analog voltage is transformed into the current by a voltage-to-current converter, Lord RD-3002-1, with a ratio of 0.4 A/V and a current limit of 2 A. The proposed controller is implemented by using C++ language under Linux-RTAI, a real-time operating system, with a 3.0 GHz CPU mounted on an industrial PC. The sampling period \( L \) is 1 ms.

B. Controller Setting

The implementation procedure is systematic because the proposed controller (20) has a transparent structure and clear meanings for each term. The gains of proposed controller can be tuned as follows. Assumed that \( a = b = 1 \), the target error dynamics (14) and reaching condition (15) become linear, and we can define the slope of error dynamics (\( \kappa \)) and the slope of reaching condition (\( \lambda_1, \lambda_2 \)). Then one can tune \( m \) by increasing from a small positive value, while checking the control performance by trial and error. After that, we twist the
linear sliding mode to the TSM by increasing $a$ ($1 < a < 2$) from 1, and decreasing $b$ ($0 < b < 1$) from 1 to realize an appropriate fast TSM, while checking whether the tracking error is satisfactory and the control input is not too noisy. Finally, fine tuning of parameters and gains is conducted to achieve greater performance.

In this experiment, the design parameters for the sliding surface are selected as $\kappa = 0.3$ and $a = 5/3$ to inject desired NTSM error dynamics into the closed-loop system; and the dynamic attractor is designed by $\lambda_1 = 20.0$, $\lambda_2 = 8.0$, and $b = 0.6$ to ensure the behavior of the fast terminal sliding manifold. The control gain is determined by $\bar{m} = 0.004$ which is tuned by being gradually increased from a small value just before the tracking performance becomes worse.

The acceleration $\ddot{\theta}_{(t-L)}$ in the proposed controller (20) can be calculated by the numerical differentiation,

$$\ddot{\theta}_{(t-L)} = (\theta(t) - 2\theta_{(t-L)} + \theta_{(t-2L)})/L^2. \quad (39)$$

Because the TDE uses the information of the acceleration, it is worth to discuss the noise effect due to the numerical differentiation. The calculation of the acceleration via the differentiation is easily corrupted by the noise of the sensor signal, thus a low pass filter (LPF) is required to attenuate the noise effect. Fortunately, it is revealed that the lowering $\bar{m}$, the controller gain, has the effects of using the first order digital LPF [43]–[45]. Therefore, the effect of attenuating the noise is implicated in the tuning process of $\bar{m}$.

We have compared the proposed control (20) with the conventional TDC with the linear error dynamics (22). The TDC controller (22) is implemented with $K_D = 6.0$, $K_P = 9.0$, and $\bar{m} = 0.006$. The first two parameters are designed to let the closed-loop dynamics of the system have a critically-damped response and the gain $\bar{m}$ is well-tuned by the same manner in [28], [29].

C. Results on Sinusoidal Commands

The SMA actuator system is commanded to follow the sinusoidal trajectory of

$$\theta_d(t) = A \cdot \sin(2\pi ft), \quad (40)$$

where $A$ denotes an amplitude in deg and $f$ denotes a frequency in Hz. During the experiments, sinusoidal waves have the amplitude of 20 deg with frequencies of 1/30, 1/15, and 1/12 Hz.

The position responses, tracking error, and control inputs of TDC with linear error dynamics (22) and the proposed control (20) are depicted in Fig. 4. Both conventional TDC and the proposed control display good tracking performance. As summarized in Table III, the proposed control shows smaller root-mean-squared (RMS) errors than those of TDC throughout the experiments. The control inputs shown in Fig. 4 reveal that the continuous NTSM of the proposed control functions well without noticeable control chatters although control inputs of TDC is less noisy.

Comparing the performance regarding different frequencies with the amplitude of 20 deg, one can find that error ratios (RMS errors/amplitudes) of TDC are degraded from 0.73 % to 2.09 % as frequencies become faster (from 1/30 Hz to 1/12 Hz), while error ratios of the proposed control are less degraded from 0.56 % to 0.77 % and keeps error ratio values within 1 %. Because TDC and the proposed control are using the same technique, TDE, this result confirms that the nonlinear target error dynamics described by the NTSM has faster convergence than the linear target error dynamics.

To investigate the robustness of the proposed control against hysteresis effect of SMA actuators, hysteresis curves of the closed-loop system after applying two controls are depicted in Fig. 5. The hysteresis effect is almost compensated when applying both TDC and the proposed control at low frequency (typically 1/30 Hz) trajectory as shown in Fig. 5 (a). This result implies that the robustness of the proposed control stems from not only the cancellation of nonlinearities (including the hysteresis effect) by using TDE, but also the use of the nonlinear target error dynamics described by the NTSM. Consequently, the excellent performance of the propose control comes from not only the hysteresis compensation but also the good hysteresis compensation because CNFTSM can provide faster convergence compared with the linear error dynamics.

D. Results on Step Commands

To verify the control performance of tracking irregular commands, we have experimented with three model-free controls: PID control, the conventional TDC, and the proposed control. The PID control has the following form:

$$u = K_P \dot{e} + K_D e + K_I \int (e \, dt), \quad (41)$$

where $K_P = 1.0092$, $K_D = 5.1137$, and $K_I = 2.2479$; these gains are tuned by the particle swarm optimization method [16] after 100 iterations of tuning.

Figure 6 presents step responses and control inputs. To clearly investigate the results, the responses of 20 deg step command of the three controllers are analyzed in Table IV, where the steady-state (SS) error denotes its RMS value after each settling time. For the step responses of PID controller, the

| Table III: Summary of experimental results |
|-------------------|-------------------|-------------------|
| Sinusoidal trajectory | RMS tracking errors |
| Freq. (Hz) | Amp. (deg) | TDC (deg) | Proposed (deg) |
| 1/30 | 20 | 0.1459 | 0.1129 |
| 1/15 | 20 | 0.3130 | 0.1718 |
| 1/12 | 20 | 0.4180 | 0.1544 |

| Table IV: Responses of 20 deg step command |
|-------------------|-------------------|-------------------|-------------------|
| PID | Rise time | Overshoot | Settling time | SS error |
| TDC | 0.73 s | 10.02% | 7.83 s | 1.2606 deg |
| Proposed | 0.33 s | 0.44% | 0.70 s | 0.0585 deg |
Fig. 4. The experimental result of position responses, tracking errors and control inputs for sinusoidal trajectories with different frequencies, \( f \), and amplitudes, \( A \); the black solid line indicates results of TDC, the blue dash-dotted line indicates those of the proposed control, and the red dotted line indicates desired trajectories: (a) \( f = 1/30 \) Hz and \( A = 20 \) deg, (b) \( f = 1/15 \) Hz and \( A = 20 \) deg, and (c) \( f = 1/12 \) Hz and \( A = 20 \) deg.

Fig. 5. Hysteresis curves of SMA actuator system after applying two controls with different frequencies, \( f \), and amplitudes, \( A \), where the red-dotted line indicates results of TDC and the black-solid line indicates those of the proposed control.

rise time is fairly fast, 0.73 s, but it is difficult to have a small overshoot in parallel. And this overshoot eventually makes the settling time slower than that of the other controllers. Otherwise, TDC shows small overshoot and state-state error as similar to those of the proposed controller. This implies that the TDE term, embedded in both TDC and the proposed controller, can effectively compensate for nonlinear effects such as hysteresis behavior. However, as observed in the rise time, the proposed controller shows the fastest convergence, and the smallest overshoot and steady-state error. This result indicates the efficacy of the fast NTSM dynamics in the proposed control to achieve fast tracking performance.

E. Robustness against Disturbance

To verify the robustness of the proposed control against external disturbances, an additional load is exerted by stretching the bias spring, where the number of winding threads attached to the spring is adjusted as shown in Fig. 7. The additional load acts as an external disturbance, and the amount of the load is added by 120 % to that of the previous experiment. It is worth to notice that the change in loading conditions also affects the
Fig. 6. The step command experiment and the comparison result to PID, TDC, and proposed controller: (a) step responses and the input current, and (b) magnified step responses plot during t=2-7s.

Fig. 7. Change of the bias spring setting to apply the additional load as an external disturbance (right).

Fig. 8. The sinusoidal tracking responses of proposed control under the external load, where the RMS tracking error with and without the load are 0.1661 deg and 0.1556 deg, respectively.

VI. CONCLUSION

We have developed continuous nonsingular terminal sliding-mode control with time delay estimation for the shape memory alloy actuators. The proposed control consists of three elements that have clear meaning: the TDE element, the injection element which specifies the desired NTSM dynamics, and the reaching element with the FTSM if the system trajectory is not confined in the NTSM. The proposed control is model-free and accurate. Interestingly, the proposed control can be transformed into TDC with linear error dynamics if $a=b=1$. Under the condition $1 < a < 2$ and $0 < b < 1$, the continuous NTSM control is activated, and shows better position-tracking performance than that of TDC with linear error dynamics. The proposed control with properly chosen $a$ and $b$ shows smaller size of errors compared with TDC with linear error dynamics thanks to the use of NTSM and FTSM. The experimental results demonstrate that the TDE can successfully work with nonlinear target error dynamics, and the proposed control is highly accurate and easily implementable without identifying the complicated SMA dynamics.

One of the challenging work for the further research is an exploration of more practical and improved stability considering time delays in the close-loop system [47]. The other research direction will be to automatically tune the $a$, $b$, and $\bar{m}$ with some performance specifications, and to implement the proposed method within data-driven framework for the SMA actuated devices in various industrial applications.
APPENDIX A

BOUNDEDNESS OF TIME DELAY ESTIMATION ERROR

Here, the boundedness of ε is proved for the stability analysis in Section IV. Recalling (23), one can express the closed-loop dynamics applying the proposed control as

$$v - \hat{\theta} = \varepsilon.$$  \hspace{1cm} (42)

Substituting (42) to (12), the dynamics equation of the SMA system can be expressed with respect to ε as follows:

$$J\dot{\varepsilon} = J\nu + \xi - \alpha \varepsilon,$$  \hspace{1cm} (43)

where $\xi \hat{=} b\hat{\theta} + k\theta + d - h(\theta, \hat{\theta}, u)$. By combination of (16) and (43), the closed-loop system is represented as follows:

$$J\dot{\varepsilon} = (J - \alpha\hat{m})\nu + \xi - \alpha \eta.$$  \hspace{1cm} (44)

From (13) and (18), (44) is expressed:

$$J\dot{\varepsilon} = (J - \alpha\hat{m})\nu + \xi - (J_{(t-L)} - \alpha\hat{m})\hat{\theta}_{(t-L)} - \xi_{(t-L)}$$

$$= (J - \alpha\hat{m})\nu - (J_{(t-L)} - \alpha\hat{m})\hat{\theta}_{(t-L)} + \psi,$$  \hspace{1cm} (45)

where

$$\psi \hat{=} \xi_{(t-L)}$$

$$= b\hat{\theta} + k\theta + d - h(\theta, \hat{\theta}, u)$$

$$- b\hat{\theta}_{(t-L)} - k\theta_{(t-L)} - d_{(t-L)} + h(\theta, \hat{\theta}, u)_{(t-L)}.$$  \hspace{1cm} (46)

Note that the hysteresis term $h(\theta, \hat{\theta}, u)$ shown in (46) is a non-smooth nonlinearity, but a Lipschitz-continuous function because it is well known that many hysteresis operators are Lipschitz continuous [14], [48]. The unexpected discontinuous term $d$ can be divided as $d = d_{\text{sm}} + d_{\text{dis}}$, where $d_{\text{sm}}$ is smooth, that is, continuous and differentiable, while $d_{\text{dis}}$ is discontinuous and assumed to be bounded. Then, we can divide $\psi$ into smooth, Lipschitz-continuous, and discontinuous terms as follows:

$$\psi = \psi_{\text{sm}} + \psi_{\text{Lip}} + \psi_{\text{dis}},$$  \hspace{1cm} (47)

where

$$\psi_{\text{sm}} = b\hat{\theta} + k\theta + d_{\text{sm}} - b\hat{\theta}_{(t-L)} - k\theta_{(t-L)} - d_{\text{sm}}_{(t-L)},$$  \hspace{1cm} (48)

$$\psi_{\text{Lip}} = -h(\theta, \hat{\theta}, u) + h(\theta, \hat{\theta}, u)_{(t-L)},$$  \hspace{1cm} (49)

$$\psi_{\text{dis}} = d_{\text{dis}} - d_{\text{dis}}_{(t-L)}.$$  \hspace{1cm} (50)

If the boundedness and smoothness condition of $b\hat{\theta} + k\theta + d_{\text{sm}}$ are satisfied, $\psi_{\text{sm}} = O(L^2)$ as shown in [49]. From the Lipschitz-continuous condition, we can obtain that $\psi_{\text{Lip}} = O(L)$. In addition, it is clear that $|\psi_{\text{dis}}| \leq \beta$, where $\beta$ is a positive constant value. Therefore, $\psi$ is bounded by $|\psi| \leq \beta + O(L)$ for a sufficient small $L$. And, the approximation error can become small by reducing the sampling period. Substituting $\hat{\theta}_{(t-L)} = \nu_{(t-L)} - \varepsilon_{(t-L)}$ from (42) into (45) gives

$$J\varepsilon = (J - \alpha\hat{m})\nu - (J - \alpha\hat{m})\nu_{(t-L)} - \xi_{(t-L)}$$

$$- (J_{(t-L)} - J)\hat{\theta}_{(t-L)} + \psi.$$  \hspace{1cm} (51)

The equation (51) can be rearranged as

$$\varepsilon = [1 - (\alpha\hat{m}/J)]\varepsilon_{(t-L)} + [1 - (\alpha\hat{m}/J)]\kappa_1 + \kappa_2,$$  \hspace{1cm} (52)

where $\kappa_1 = \nu - \nu_{(t-L)}$ and $\kappa_2 = [(J - J_{(t-L)})\hat{\theta}_{(t-L)} + \psi]/J$.

Then, (52) can be expressed in the discrete-time domain as follows:

$$\varepsilon(k) = [1 - \alpha\hat{m}/J(k)]\varepsilon(k-1) + [1 - \alpha\hat{m}/J(k)]\kappa_1(k) + \kappa_2(k),$$  \hspace{1cm} (53)

with $\kappa_1(k) = \nu(k) - \nu_{(k-1)}$ and $\kappa_2(k) = [(J_{(k)} - J_{(k-1)})\hat{\theta}_{(k-1)} + \psi(k)]/J(k)$ where $\bullet(k)$ denotes the $k$th sampling time. Because $\kappa_1(k)$ and $\kappa_2(k)$, forcing functions of $\varepsilon(k)$, are bounded for a sufficient small time-delay $L$, the first-order differential equation shown in (53) is asymptotically bounded if the roots of $[1 - \alpha\hat{m}/J(k)]$ reside inside a unit circle as

$$|\varepsilon| \leq \varphi,$$  \hspace{1cm} (54)

where $\varphi$ is a positive value. Therefore, the boundedness of $\varepsilon$ is proved.

APPENDIX B

FINITE-TIME REACHABILITY

In this section, the finite-time reachability of the closed-loop system is analyzed by the same method of [38], [42]. Recall (28), defining a positive value $\lambda_1 \hat{=} \lambda_1 - \varepsilon/s$, the equation can be simplified as follows:

$$\dot{V} = -\hat{\lambda}_1 s^2 - \hat{\lambda}_2 |s|^{b+1},$$  \hspace{1cm} (55)

where $\hat{\lambda}_1 = \lambda_1 |s|^{b+1}$ and $\hat{\lambda}_2 = \lambda_2 |s|^{b+1}$. Referring to the inequality condition $(x^y)^x \leq (x^y)^2$ holds if $x > 0$ and $0 < y < 2$, one can express (55) as

$$\dot{V} \leq -\hat{\lambda}_1 s^2 - \hat{\lambda}_2 |s|^{2(b+1)/2},$$  \hspace{1cm} (56)

and it can be rearranged as a form of extended Lyapunov description as follows:

$$\dot{V} + (2\hat{\lambda}_1)V + (2|s|^{b+1}/2\hat{\lambda}_2)V^{(b+1)/2} \leq 0.$$  \hspace{1cm} (57)

Then, the closed-loop system reaches to the terminal sliding surface (15) in finite-settling time by

$$T \leq \frac{1}{\hat{\lambda}_1 (1 - b)} \ln [2^{(b+1)/2} \hat{\lambda}_2/\hat{\lambda}_1 V^{(1-b)/2} + 1].$$  \hspace{1cm} (58)

Further, the same method can be applied for (29). Therefore, the finite-time reachability of the closed-loop system is guaranteed.

REFERENCES

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