A Cost Function Inspired by Human Arms Movement for a Bimanual Robotic Machining

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Abstract—This paper focuses on a kinematic redundancy resolution of bimanual robotic system for a machining task as a part of factory automation. Inspired by a study of human bimanual action, called Guiard’s principles, a cost function is proposed by using task-compatibility indices. An acceleration-level redundancy resolution is provided via optimization of the cost function in order to reflect the role of human arm movement: one arm performs coarse motion, and the other fine motion. A dynamic simulation with two 6 degrees-of-freedom robots shows the effectiveness of the proposed idea.

I. INTRODUCTION

The interests and demands on a bimanual robotic system have been consistently increasing in recent years. Especially, in the industrial robotic area, the bimanual robotic system is utilized to various applications owing to its potential for enhancing productivity and flexibility in the manufacturing process [1], [2].

One of the applications to the factory automation is a bimanual machining, which is an operation using coordinated manipulators as the machine tool; one arm is manipulated with a workpiece flexibly and the other arm is manipulated with a machining tool. Compared with a single-arm machining system, the bimanual machining system can increase the workspace, machining volume and the efficiency of production.

To conduct the machining on the bimanual robotic system, however, involves a difficulty that stems from the kinematic redundancy\(^1\) of the system, since the two coordinated robots can generate an infinite number of machining configurations.

To resolve this problem, a relative Jacobian technique can be considered [3], [4]. As the kinematic formulation based on the relative Jacobian provides a perspective to view the two-arms as a single arm with kinematic redundancy, the redundancy in the bimanual system can be easily exploited. There has been some studies which utilize the redundancy to avoid joint-limit and obstacle [3], to minimize joint-torque [5], and maximize structural compliance [1] for bimanual machining or assembly.

Nevertheless, it is still an open problem how to choose one solution among infinitely many possible solutions, so we take note that it is instructive to observe how similar problems are solved by humans [6].

In [7], Guiard has observed the motion of human’s two-arms and presented principles for the asymmetric division of labor in human bimanual action, which means that our hands have different roles and perform distinctly different tasks. More specifically, left hand (non-preferred hand) sets the spatial frame of reference for the right hand (preferred hand), and performs coarse movement; and the right hand makes relative motion to the left hand, and performs fine movement.

In this paper, the redundancy of the bimanual system is resolved via optimizing a cost function representing an aspect of human arms movement based on the Guiard’s principles. In order to implement coarse motion of one arm and fine motion of the other arm, we propose a cost function which employs task-compatibility indices [8] for the aforementioned goals. Furthermore, the redundancy resolution is considered at the acceleration-level for the sake of dynamic control involving position control as well as force or impedance control [9], [10].

Cost functions to model human arm movements have been proposed in the previous studies [11]–[13], however they focused on a single arm manipulation. Although the authors in [6], [14]–[16] have studied the coordination of bimanual robots in the sense of human behavior, the redundancies of bimanual system have not been fully exploited. Thus we believe that it is worthy to extend the human inspired approach to the redundancy resolution of bimanual robotic systems.

The proposed approach is verified by a dynamic simulation of a machining task with two 6 degrees-of-freedom (DOFs) arms.

This paper is organized as the following. In section II, the relative Jacobian technique and the task-compatibility index are introduced as a background. In section III, a cost function based on human-inspired approach is proposed for a bimanual machining, and a redundancy resolution of the bimanual system is provided at the acceleration-level. Section IV offers the procedure and result of numerical verifications. In Section V, the conclusion is drawn.

II. PRELIMINARIES

In this section, provided below is a brief introduction of the relative Jacobian which is useful to describe the relative motion between two arms for bimanual machining [1] or assembly [3], [4]. We also briefly introduce the task-compatibility index [8] which is utilized to formulate a cost
function in this paper.

A. Relative Jacobian

As shown in Fig. 1, when the two arms perform a bimanual machining, one arm holding a workpiece serves as a basis and the other holding a tool has relative motions toward it; the former arm is referred to a reference robot, whereas the latter arm is referred to a tool robot.

Coordinate frames and vectors used in this paper then are defined as follows: \( R \) and \( T \) denote task frames of the reference robot and the tool robot, respectively; \( O_R \) and \( O_T \) the base frames of the reference robot and the tool robot, respectively; \( x_r \) the relative position/orientation vector referenced to frame \( R \); \( q_R \) and \( q_T \) the joint angle vector of the reference robot and the tool robot, respectively.

The relative Jacobian provides the first-order differential kinematics relating the relative velocity between two end-effectors, \( x_r \), to the joint-space velocities of the two arms as follows:

\[
\dot{x}_r = J_r \dot{q}_r \tag{1}
\]

where \( J_r \in \mathbb{R}^{n_r \times (n_R + n_T)} \) denotes the relative Jacobian matrix, with \( n_R, n_T \) and \( n_r \) being the DOFs of the reference robot, the tool robot and the task, respectively, \( x_r \in \mathbb{R}^{n_r} \) is a position vector consisting of both relative position and relative orientation as illustrated in Fig. 1, and \( q = [q_R^T \ q_T^T]^T \in \mathbb{R}^{(n_R + n_T)} \) is the vector consisting of joint angles of both arms. Refer to [3], [4], [17] for the calculation of the relative Jacobian.

Differentiating (1) leads to the second-order differential kinematics of the bimanual robot system as follows:

\[
J_r \ddot{q}_r = \ddot{x}_r - \dot{J}_r \dot{q}_r \tag{2}
\]

Equations (1) and (2) display that the two arms can be treated as a single redundant arm operating in the task-space referenced in \( R \), if the DOFs of the whole system are greater than the DOFs required for the task, i.e. \( (n_R + n_T) > n_r \).

B. Task-compatibility index

The task-compatibility index is proposed to measure the level of agreement between the optimal direction and the actual motion/force of the manipulator with respect to task requirements [8]. In the perspective to view the manipulator as a mechanical transformer with joint velocity and force as input and velocity and force in the task frame as output, the index employs two transmission ratios—velocity transmission ratio and force transmission ratio. One can express the task-compatibility index \( c \) as follows:

\[
c = \sum_{i=1}^{l} w_i [u_i^T (J J^T) u_i] + \sum_{i=1}^{l} w_i [u_i^T (J J^T)^{-1} u_i] \]

\((\text{force transmission ratio})^{-2} \) \((\text{velocity transmission ratio})^{-2} \)

(3)

where \( u_i \) denotes a unit vector in the direction of interest, \( J \) denotes Jacobian matrix of a robot, \( w_i \) denotes a weighting factor, and \( l \) denotes DOFs of the target task.

We can see that the task-compatibility index shown in (3) is a weighted sum of the squares of the reciprocals of transmission ratios. Hence, large value of the index means small velocity and force transmission ratios which enable accurate force and motion control, while small value of the index means large velocity and force transmission ratios which enable quick adaption of the motion and resistance against large disturbance force, i.e. coarse motion [8], [18].

In summary, the physical meaning of the task-compatibility index is as follows: maximizing the index indicates the arm posture is suitable for fine manipulation tasks, where accurate control of small velocity and force is required; and minimizing the index indicates the arm posture is suitable for coarse manipulation tasks, where exertion of large velocity and force, i.e. a quick adaption of the motion and a resistance against large disturbance forces, is required.

III. FORMULATION OF COST FUNCTION INSPIRED BY HUMAN ARMS MOVEMENT

In this section, a cost function is formulated with consideration of the human’s behavior for the bimanual machining. The kinematic redundancies of the bimanual system imposed by the relative Jacobian technique is resolved as a constrained optimization problem with the proposed cost function. For the sake of dynamic control, an acceleration-level redundancy resolution is considered with a gradient projection method (GPM).

In [7], as we aforementioned, Guiard explains when humans conduct a bimanual task, roles of two arms are distinguished in the scale(or resolution) of motion as follows: left (non preferred) hand performs coarse motion; and right (preferred) hand performs fine motion.

Bringing this concept to the robotic manipulation, one can interpret the bimanual machining as follows: the reference robot performs coarse manipulation supporting a workpiece, as a basis of the task; and the tool robot performs fine manipulation controlling force and motion of machining tools relative to the workpiece.

For implementing the coarse and fine manipulation for each robot arms, the task-compatibility index is employed to formulate the cost function reflecting human’s behavior,
since the characteristics of bimanual motion for the machining fits well to the sense of the task-compatibility as addressed in Section II-B.

As illustrated in Fig. 2, by the inspection of the bimanual machining task, one can find that there are two decoupled directions: non-contact direction where is position-controlled and contact-direction where is force-controlled. For the non-contact direction, the reference robot can perform a coarse motion for quick adaption of the motion, whereas the tool robot can perform a fine motion for accurate machining; for the contact direction, the reference robot can resist large force disturbances due to the machining force, whereas the tool robot can perform fine force control. Therefore, the task-compatibility index should be minimized for the reference robot, and maximized for the tool robot, respectively.

Note that it is easy and intuitive for users that the bimanual machining task is described by the relative position between the task-frames at the end-effector of two robots, i.e. $R$ and $T$, rather than expressed at base frames of the two robots, i.e. $O_R$ and $O_T$. Therefore the unit task vectors, $u_R$ and $u_T$, are defined at respective task-frames $R$ and $T$, attached at the end-effector of each manipulator in order to utilize the task-compatibility index with ease.

Then, coordinate-transformed task-compatibility indices for two robots can be expressed as follows. For the reference robot,

$$c_R(q_R) = \sum_{i=1}^{l} w_i \left[ u_{T,R,i}^T (\bar{J}_R \bar{J}_R^T) u_{R,i} \right] + \sum_{i=1}^{l} w_i \left[ u_{T,R,i}^T (\bar{J}_R \bar{J}_R^T)^{-1} u_{R,i} \right]$$

(4)

with

$$\bar{J}_R = \left[ \begin{array}{cc} R_{O_R} & 0 \\ 0 & R_{O_R} \end{array} \right] J_R,$$

(5)

and for the tool robot,

$$c_T(q_T) = \sum_{i=1}^{l} w_i \left[ u_{T,T,i}^T (\bar{J}_T \bar{J}_T^T) u_{T,i} \right] + \sum_{i=1}^{l} w_i \left[ u_{T,T,i}^T (\bar{J}_T \bar{J}_T^T)^{-1} u_{T,i} \right]$$

(6)

with

$$\bar{J}_T = \left[ \begin{array}{cc} T_{O_T} & 0 \\ 0 & T_{O_T} \end{array} \right] J_T,$$

(7)

where $u_{R,i}$ and $u_{T,i}$ denote the unit vectors referenced to $R$ and $T$, respectively, $\bar{J}_R$ and $\bar{J}_T$ denote Jacobian matrices of the reference robot and the tool robot, respectively, and $R_{A}$ denote a rotation matrix from frame $A$ to $B$.

With the consideration of the optimization by using GPM [19], the following cost function is defined as an augmented form of the gradient of task-compatibility indices:

$$\nabla H(q) = \left[ \begin{array}{c} \frac{\partial c_R(q_R)}{\partial q_R} \\ \frac{\partial c_T(q_T)}{\partial q_T} \end{array} \right] \Delta = \left[ \begin{array}{c} \nabla c_R \\ \nabla c_T \end{array} \right]$$

(8)

From (2), one can derive the acceleration-level redundancy resolution via optimization as follows:

$$\ddot{q} = J_t^+(\dot{x}_c - \bar{J}_c \dot{q}) + (I - J_t^+ J_t) \eta,$$

(9)

where $J_t^+ \in \mathbb{R}^{(n_R+n_T) \times m}$ denotes a Moore-Penrose pseudo-inverse [20] of $J_t$, $(I - J_t^+ J_t)$ denotes the orthogonal projection matrix in the null space of relative Jacobian, and $\eta \in \mathbb{R}^{(n_R+n_T)}$ denote a joint-acceleration optimization term.

From (8) and (9), $\eta$ is then designed to minimize the task-compatibility index for coarse manipulation of the reference robot and to maximize the index for fine manipulation of the tool robot as follows:

$$\eta = \left[ -k_R \nabla c_R \\ k_T \nabla c_T \right] - k_D \ddot{q},$$

(10)

where $k_R, k_T > 0$ denote step-size scalars for GPM, and $k_D > 0$ denote a scalar gain motivated by the desire of damping [21]. Note that second term of the right-hand side of (10) is important to avoid the instability phenomena in the self-motion which may occur when the redundancy is resolved at the acceleration-level [9].

IV. NUMERICAL VERIFICATION

The effectiveness of the redundancy resolution by using the proposed cost function is validated by numerical simulations for two 6 DOFs industrial robot, i.e. total 12 DOFs bimanual system. A dynamic simulator “Simstudio 2” developed by SimLab Co. is used in this simulation.

A. Settings

A bimanual machining task is conducted by two arms as follows. One arm, i.e. tool robot, is perpendicularly carving on a moving workpiece along a circle trajectory, and the workpiece is being manipulated by the other arm, i.e. reference robot, as depicted in Fig. 3.

Of the 12 DOFs provided by the two robots, 6 DOFs are allocated for the machining task, and the remaining 6 degrees of redundancy for self-motion optimized by the proposed cost function. Therefore, the variables used in the acceleration-level redundancy resolution (9) for the bimanual system take the following dimensions: $x_c \in \mathbb{R}^6$, and $\eta \in \mathbb{R}^{12}$.
The joint-acceleration optimization term by using the proposed cost function, (10), is determined with suitably tuned gains as follows:

$$\eta = \begin{bmatrix} -20.0 \nabla c_R \\ 10.0 \nabla c_T \end{bmatrix} - 25.0 \dot{q},$$ (11)

where the $c_R$ and $c_T$ denote the coordinate-transformed compatibility indices of reference robot and tool robot, respectively.

Note that since we are mainly interested in the contact direction where the machining force is exerted in this case, the compatibility indices in the cost function use only the force transmission ratio, that is, the weighting factor of the compatibility indices in the cost function use only the direction where the machining force is exerted in this case, the compatibility indices of reference robot and tool robot, respectively.

The bimanual robot system is dynamically controlled by using the impedance control to deal with the contact between two arms; the sampling period is 1 ms and \( \dot{q}, \dot{q}, \dot{x}, \) and \( \dot{J}_r \) in (9) are calculated by numerical differentiation.

### B. Simulation result

Figure 4 shows simulation results as a sequence of snapshots. As aforementioned, the bimanual system performs to carve a circle trajectory on the workpiece with 6 degrees of freedom for total 12 DOFs system, and the self-motion of the remaining 6 degrees-of-redundancy is determined by optimizing the proposed cost function.

The result with the proposed cost function is indicated in Fig. 4 (a), and the result without the cost function is indicated in Fig. 4 (b), respectively. In the latter case, the joint-acceleration optimization term \( \eta \) in (9) is forced to set to zero. It means that the null-space projection for the self-motion with the proposed cost function is deactivated as \( \ddot{q} = \dot{J}_r^T (\ddot{x} - \dot{J}_r \dot{q}) \), that is, the resolved acceleration results in a minimum-norm acceleration. We can notice that bimanual systems show different configurations for the result with or without consideration of the proposed cost function.

For a further investigation of the difference, the task-compatibility indices of the cost function throughout the task are presented in Fig. 5 (the values are saved to data files in real-time during the simulation.) When the cost function is activated, the task-compatibility index of the reference robot is minimized as shown in Fig. 5 (a), and that of the tool robot is maximized as shown in Fig. 5 (b). It implies that the posture of the reference robot is suitable for supporting the workpiece against a large force due to the contact with the tool and the posture of the tool robot is suitable for conducting a fine machining task as we expect in the accordance with the human bimanual action referred to [7].

It is fair to mention the downside of the proposed cost function is that it requires efficient matrix computation for the real-time control, since two task-compatibility indices for two arms need to be computed in parallel. We have solved this problem by using Newmat 11 [22], a C++ based-matrix library, which is known as efficient and fast computation, so real-time simulation with 1 ms sampling period could be conducted.

### V. Conclusion

We have proposed a cost function with a human-inspired approach for the bimanual robotic system to perform a machining. By the numerical simulation, we have demonstrated that the proposed cost function can bestow coarse motion to one arm and fine motion to the other arm, which is analogous to human’s bimanual movement.

For a future study, we prepare experiments using the proposed method, and multiple performance criteria together with the proposed cost function will be also considered. And we are also preparing some comparative studies to existing cost functions.

### REFERENCES


Fig. 4. A sequence of snapshots, for every 4 sec, performing a bimanual machining on the workpiece along a circle trajectory: (a) with the proposed cost function; and (b) without the cost function.

Fig. 5. Task-compatibility indices: (a) for the reference robot and (b) for the tool robot.


