Comment on ‘Optimal inventory replenishment policy for the EPQ model under trade credit derived without derivatives’

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Abstract: In this article, we complement the shortcoming of the inventory economic production quantity (EPQ) model developed by Huang and Huang (2008), ‘Optimal Inventory Replenishment Policy for the EPQ Model Under Trade Credit Derived Without Derivatives’, International Journal of Systems Science, 39, 539–546), and propose an arithmetic–geometric inequality method to obtain the global optimal solution without taking complex differential calculus or using tedious algebraic manipulations. Finally, we provide an economical interpretation of the theoretical result so that the reader can understand the insight of the result.

Keywords: inventory; trade credits; arithmetic mean; geometric mean; finance; EOQ models; convex optimisation

1. Introduction

Following the publication of Goyal (1985), numerous papers have appeared dealing with various situations of inventory problems under the permissible trade credit. For an up-to-date review of literature, see Chang, Teng and Goyal (2008). The most popular approach for deriving the economic order quantity (EOQ) model is based on differential calculus by taking the first- and second-order derivatives of the average cost per unit. In their recent paper, Huang and Huang (2008) extended Chung and Huang’s economic production quantity (EPQ) model, and obtained the optimal inventory replenishment policy under supplier’s trade credit by using the algebraic method of completing perfect square.

In this article, we first complement the shortcoming in Huang and Huang’s EPQ inventory model (2008) by removing an incorrect and unnecessary case. We then propose an easy-to-use and simple-to-understand arithmetic–geometric inequality method as shown in Teng (2008) to obtain the global optimal solution immediately and explicitly without taking complex differential calculus or using tedious algebraic manipulations. Finally, we provide a simple-to-understand economical interpretation of the theoretical result so that the reader can easily understand the economical meanings and its managerial implications.

2. Notation and assumptions

For simplicity, we use the similar notation as those in Huang and Huang (2008).

- $D$: the demand rate per year
- $P$: the replenishment rate (i.e. production rate) per year, $P \geq D$
- $A$: the ordering (or set-up) cost per order (lot)
- $\rho$: $1 - \frac{\rho}{P} \geq 0$, the fraction of no production
- $c$: the unit purchasing cost
- $s$: the unit selling price, $s \geq c$
- $h$: the unit stock holding cost per item per year excluding interest charges
- $I_e$: the interest earned per dollar per year
- $I_k$: the interest charged per dollar in stocks per year by the supplier
- $M$: the manufacturer’s trade credit period offered by the supplier in years
- $T$: the cycle time in years (a decision variable)
- $TRC(T)$: the annual total relevant cost, which is a function of $T$
- $T^*$: the optimal cycle time of $TRC(T)$

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We are restating the following assumptions of Huang and Huang (2008) for a clearer presentation of our approach:

1. In the maturity stage of a product life cycle, we may assume that demand rate, \( D \), is known and constant.
2. Shortages are not allowed.
3. Replenishment rate, \( P \), is known and constant.
4. Time horizon is infinite.
5. During the trade credit period the account is not settled. The manufacturer deposits the sales revenue, and earns the investment return rate of \( I_e \). At time \( M \) (the end of the supplier’s trade credit), the manufacturer pays off all units sold to the supplier. When \( T > M \), the manufacturer starts paying the interest charged on those unsold items with the interest rate of \( I_k \).

Note that Huang and Huang (2008) assumed that \( I_k \leq I_e \). In this note, we relax this dispensable assumption.

3. Mathematical formulation

The manufacturer buys all parts at time zero and must pay the total cost of the items at time \( M \). Based on the values of \( M \) and \( T \), we have only two possible cases: either \( T \geq M \) or \( T < M \), instead of the three cases considered by Huang and Huang (2008).

Case 1: \( T \geq M \)

In this case, the manufacturer pays off for all the units sold until \( M \), keeps the profits, and starts paying for the interest charges on the items sold after \( M \). The graphical representation of this case is shown in Figure 1. As a result, the interest earned per cycle is \( I_e \) times the area of the triangle \( OMD \) shown in Figure 1. The interest charged per cycle is \( (c/s)I_k \) times the area of the triangle \( BCD \). The interest earned per year is given by

\[
\frac{sI_e}{T} \left[ \frac{DM^2}{2} \right].
\]

The interest charged per year is given by

\[
\frac{cI_k}{T} \left[ \frac{D(T - M)^2}{2} \right].
\]

It may be noted that Huang and Huang (2008) as well as Chung and Huang (2003) incorrectly calculated the interest charged per cycle.

Using Equations (1) and (2), we yield the annual total relevant cost for the manufacturer as

\[
TRC_1(T) = \frac{A}{T} + \frac{hDT\rho}{2} + \frac{cI_k}{T} \left[ \frac{D(T - M)^2}{2} \right] - \frac{sI_e}{T} \left[ \frac{DM^2}{2} \right].
\]

Case 2: \( M > T \)

In this case, the manufacturer receives the total revenue at time \( T \), and is able to pay the supplier the total purchase cost at time \( M \). Hence, the manufacturer faces no interest charge. The interest earned per cycle is \( I_e \) multiplied by the area of the trapezoid on the interval \([0, M]\) as shown in Figure 2. As a result, the interest earned per year is given by

\[
\frac{sI_e}{T} \left[ \frac{DT^2}{2} + DT(M - T) \right].
\]

Therefore, we obtain the annual total relevant cost for the manufacturer as

\[
TRC_2(T) = \frac{A}{T} + \frac{hDT\rho}{2} - \frac{sI_e}{T} \left[ \frac{DT^2}{2} + DT(M - T) \right].
\]
4. Optimal solution by arithmetic-geometric-mean-inequality theorem

We now develop an easy-to-apply method by using the arithmetic-geometric-mean-inequality theorem for obtaining the optimal order quantity that minimises the total relevant cost. It is well known that for any two real positive numbers, say \(a\) and \(b\),

\[
\frac{a + b}{2} \geq \sqrt{ab}. \quad (6)
\]

The equation holds only if \(a = b\). We will use this property to obtain the optimal solution for the EPQ inventory model.

**Case 1:** \(T \geq M\)

From (3), we know that the total relevant cost per unit time is as follows:

\[
TRC_1(T) = \frac{A}{T} + \frac{hDT}{2} + \frac{cl_b}{T} \left[ \frac{D(T - M)^2}{2} - sI_c \left( \frac{DM^2}{2} \right) \right] \\
= \frac{1}{T} \left[ A + \left( \frac{DM^2}{2} \right)(cI_b - sI_c) \right] \\
+ \frac{DT}{2} (h_\rho + cI_b) - cI_b DM. \quad (7)
\]

By using the arithmetic-geometric mean inequality, we can easily obtain that

\[
TRC_1(Q) \geq \sqrt{[2A + DM^2(cI_b - sI_c)][D(\rho h + cI_b)]} \\
- cI_b DM. \quad (8)
\]

Therefore, the optimal replenishment time is

\[
T^*_r = \sqrt{[2A + DM^2(cI_b - sI_c)][D(\rho h + cI_b)]}, \\
\text{if } 2A + DM^2(cI_b - sI_c) \geq 0. \quad (9)
\]

If \(2A + DM^2(cI_b - sI_c) < 0\), then we know that \(T^*_r\) does not exist. Likewise, the optimal lot size \(Q^*_1\) is

\[
Q^*_1 = T^*_r D = \sqrt{D[2A + DM^2(cI_b - sI_c)]/(\rho h + cI_b)}, \\
\text{if } 2A + DM^2(cI_b - sI_c) \geq 0. \quad (10)
\]

Next, we discuss the other case in which \(T < M\).

**Case 2:** \(T < M\)

Simplifying (5), we have

\[
TRC_2(T) = \frac{A}{T} + \frac{hDT}{2} - sI_c \left[ \frac{DT^2}{2} + DT(M - T) \right] \\
= \frac{A}{T} + \frac{DT}{2} (h_\rho + sI_c) - sI_b DM. \quad (11)
\]

Using the arithmetic-geometric mean inequality, we can easily obtain that

\[
TRC_2(Q) \geq \sqrt{2AD(\rho h + sI_c) - sI_b DM}. \quad (12)
\]

Therefore, the optimal replenishment time is

\[
T^*_2 = \sqrt{2A/[D(\rho h + sI_c)]}. \quad (13)
\]

Hence, the optimal lot size \(Q^*_2\) is

\[
Q^*_2 = T^*_2 D = \sqrt{2AD/(\rho h + sI_c)} \quad (14)
\]

Now, we are ready to obtain the theoretical results and their economical interpretations.

5. Theoretical results and economical interpretations

We first restate the theorem and then provide an economical interpretation.

**Theorem 1:** We have the following results:

(A) If \(-2A + DM^2(\rho h + sI_c) < 0\), then \(T^* = T^*_r > M\).

(B) If \(-2A + DM^2(\rho h + sI_c) > 0\), then \(T^* = T^*_r < M\).

(C) If \(-2A + DM^2(\rho h + sI_c) = 0\), then \(T^* = M\).

**Proof:** The proof is similar to that in Huang and Huang (2008). \(\square\)

Note that Theorem 1 here is the proper version of Theorems 3 and 4 in Huang and Huang (2008). A simple economical interpretation of Theorem 1 is as follows. It is clear from the classical EOQ model that the optimal order quantity is obtained when the ordering cost is equal to the inventory cost. Whenever the manufacturer orders items from the supplier, it receives the benefit of \(DM^2sI_c/2\) from the supplier’s trade credit. As a result, the true ordering cost is reduced to \(A - (DM^2sI_c)/2\). On the other hand, we know that the inventory cost (excluding interest charges) for order \(DM\) units is \(DM^2h_\rho/2\). Therefore, if the true ordering cost, \(A - (DM^2sI_c)/2\), is higher than the inventory cost for order \(DM\) units, \(DM^2h_\rho/2\), then the optimal lot size \(Q^* = T^* D\) must be higher than \(DM\) units. Hence, if \(-2A + DM^2(\rho h + sI_c) < 0\), then \(T^* > M\), and vice versa.

6. Conclusions

In this article, we have not only complemented the shortcoming in Huang and Huang’s EPQ model (2008) but also relaxed the dispensable assumption of \(I_c \geq I_r\). By contrast to the quadratic-algebraic method of completing perfect square suggested by Huang and Huang (2008).
Huang (2008), we have proposed an arithmetic-geometric inequality method to obtain the global optimal solution without using differential calculus or algebraic manipulations. Finally, we have provided an economical interpretation of the theoretical result so that the reader can easily understand the economical meanings and its managerial implications of the theoretical results.

The proposed model can be extended in several ways. For instance, we may extend the deterministic demand function to a stochastic demand pattern. Also, we could generalise the model to allow for shortages, quantity discount, and cash discount. Finally, we could consider the problem of simultaneously setting price, quality, and order quantity for a product in which its demand is a function of unit selling price as well as product quality.

Notes on contributors

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