Simultaneous Structural-control Optimization of a Coupled Structural-acoustic Enclosure

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ABSTRACT: Simultaneous structural-control optimization of a coupled structural-acoustic enclosure is investigated in this paper. The whole system has a rectangular parallelepiped sound field of which the entire bottom face and the partial top face are covered with two elastic plates while the other boundaries are enclosed with rigid walls. Two point forces applied on the two plates are used as disturbance and control inputs respectively. In the sound field, one sound sensor is placed to measure the sound pressure output. Feedback $H_1$ control is used to generate the control input signal to reduce the noise at the sensor spot induced by the disturbance. The coupled structural-acoustic enclosure is modeled by the finite element method. The element thickness parameters of the top elastic plate along with the controller parameters are adopted as design variables for the simultaneous optimization in which the closed-loop $H_1$ norm is minimized. To solve the simultaneous design problem, a nested approach is used in which the controller syntheses are considered as subprocesses included in the main optimization process dealing with the structural design variables. Moreover, since the matrices are unsymmetrical for structural-acoustic coupling problems, indirect modal analysis is used to reduce the computational cost instead of the direct method. Numerical example shows that the system performance is improved significantly by the simultaneous optimization. The effectiveness and potentiality of the design approach presented in this paper are also demonstrated by the results.

Key Words: simultaneous optimization, structural-acoustic coupling, active noise control

INTRODUCTION

In the traditional design practice of an actively controlled structure, the structure and its control system are designed separately. First, a nominal structure is designed or optimized to meet some open loop performance requirements. Next, a control system is synthesized for the structural model to satisfy some closed loop specifications. Although such a design approach has been employed in many applications due to its simplicity, obviously, it cannot yield the best overall design since the interactions between the two sub systems have not been exploited. As more and more stringent performance requirements are imposed in advanced applications, since the second half of 1980s, a number of studies (Onoda and Haftka, 1987; Kajiwara et al., 1994; Maghami et al., 1996; Zhu et al., 2002a, b) on simultaneous structural-control optimizations have been presented in which a variety of problem formulations and solving methods are used.

In this paper, simultaneous structural-control optimization of an actively controlled coupled structural-acoustic enclosure is investigated. The main purpose of this paper is to develop a procedure which extends the simultaneous structural-control optimization to include the structural-acoustic coupling and demonstrate the potentiality of utilizing the new method to improve the performance of active noise control in coupled enclosures.

Typical examples of coupled structural-acoustic enclosures are automobile compartments or aircraft cabins where the interior noise couples with part or all of the car body or fuselage structure. In this paper, a simple numerical example is considered which has a rectangular parallelepiped sound field enclosed with rigid walls and two elastic plates. Two point forces applied on the two plates are used as disturbance and control inputs respectively. Inside the sound field, one sound sensor is placed to measure the sound pressure output. Feedback $H_\infty$ control is used to generate the control input signal to reduce the noise at the sensor spot induced by the disturbance. The simultaneous optimization is formulated as a nonlinear programming problem in which the closed loop $H_\infty$ norm is minimized with respect to both the structural and control design variables subjective to some constraints. In the example the coupled enclosure is modeled by the finite element method (FEM) (Nefske
et al., 1982; Sung and Nefske, 1984; Fahy, 1985) and the element thickness parameters of one enclosing plate are used as structural design variables.

Since it is not efficient to treat the structural and control design variables equally within the same framework, a nested solving approach is adopted in which the control syntheses are considered as subprocesses included in the main optimization process dealing with the structural variables. Whenever the structure is changed in the main process, a complete control design will be carried out immediately to evaluate the new change. In such an approach, the control optimizations are well separated from the structural optimization so that existing structural and control design techniques can be combined and employed without any difficulty. In this paper, the structural variables are solved in the main optimization by the method of moving asymptotes (MMA) (Svanberg, 1987), while the control parameters are designed in the suboptimizations by state-space $H_\infty$ synthesis (Doyle et al., 1989).

Moreover, as for the structural optimization, the coupled structural-acoustic enclosure needs to be reanalyzed for many times. Due to the structural-acoustic coupling, the “mass” and “stiffness” matrices obtained in the finite element analysis are unsymmetrical. Since the direct modal analysis for unsymmetrical system is computationally very expensive when a large system is considered, in this paper, an indirect modal analysis is used instead of the direct method. It is found that the indirect method can considerably reduce the computational costs for large systems with very little effect on accuracy if an enough number of decoupled modes are used.

The main contents of the paper are organized as below: In “Simultaneous Structural-control Design”, the general formulation and solving approach of the structural-control optimization are presented; In “FEM Analysis of Coupled Structural-Acoustic Enclosures”, the FEM modeling of the coupled enclosure including the direct and indirect methods is discussed; In “Control Design”, the control design used in this research is described briefly; In “Numerical Example”, the numerical example is described; In “Results and Discussion”, the results of the simultaneous optimization and the cause of performance improvement are discussed thoroughly; In “Conclusions”, the conclusions are given.

SIMULTANEOUS STRUCTURAL-CONTROL DESIGN

Generally, simultaneous optimization of a structure-control system can be posed as a nonlinear programming problem in which a certain objective function $f$ is minimized over the structural parameter $p_s$ and control parameter $p_c$.

$$\min_{p_s, p_c} f(p_s, p_c) \quad (1)$$

It is clear that Equation (1) is equivalent to the following one,

$$\min_{p_s} \min_{p_c} f(p_s, p_c) \quad (2)$$

provided that all three minimizations in the two equations are capable of finding their respective global optimal solutions (i.e., the details of the searching strategies, start points, and characteristics of objective functions are not considered). Hence, the simultaneous optimization can be transformed to a structural optimization nested with control optimizations as subprocesses (Zhu et al., 1999).

$$\min_{p_s} \tilde{f}(p_s) \quad (3)$$

where

$$\tilde{f}(p_s) \overset{\Delta}{=} \min_{p_c} f(p_s, p_c) \quad (4)$$

denotes the control optimization using a certain control law. As shown in the above equations, the main optimization process is the structural optimization with respect to the structural parameter $p_s$, while the suboptimization processes nested into the main process is the control designs with respect to the control parameter $p_c$. Whenever $p_s$ is modified in the main process, the subprocess is called and a new controller is designed immediately to find the corresponding $p_c$. From the point of the main process, the explicit (to a certain extent, only) design variable is $p_s$, while $p_c$ is implicit or just by-product introduced when $p_s$ is evaluated. In such a nested approach, since the structural and control design variables are treated separately and the control design is well encapsulated, various existing structural and control design techniques can be combined and employed without any difficulty.

Moreover, some constraints, such as lower and upper bounds on the design parameters $p_s^L$ and $p_s^U$, and other performance requirements $\{g_i\}$, can be imposed on the optimization

$$\min_{p_s} \tilde{f}(p_s) \quad \text{s.t. } \begin{cases} p_s^L \leq p_s \leq p_s^U \\ g_i(p_s) \leq 0 (i = 1, 2, 3, \ldots, n_g) \end{cases} \quad (5)$$
FEM ANALYSIS OF COUPLED STRUCTURAL-ACOUSTIC ENCLOSURES

Direct Modal Analysis

Typical examples of coupled structural-acoustic enclosures are automobile compartments or aircraft cabins where the interior noise couples with part or all of the body or fuselage structure. When the structural-acoustic interactions are strong, instead of separate structural and acoustic analyses, coupled analysis is necessary to obtain the vibration characteristics of the whole system. Finite element method (FEM) is suitable for such applications provided that the sound field is entirely enclosed with elastic structures and rigid walls (Nefske et al., 1982; Sung and Nefske, 1984; Fahy, 1985).

By using FEM, the undamped responses of the coupled structural-acoustic system under the input \( f \) can expressed as

\[
M \ddot{p} + Kp = f
\]  
(6)

where,

\[
p = \begin{bmatrix} \{ p_a \} \\ \{ p_s \} \end{bmatrix}
\]  
(7)

\[
M = \begin{bmatrix} M_{as} & M_{as} \\ 0 & M_{ss} \end{bmatrix}
\]  
(8)

\[
K = \begin{bmatrix} K_{as} & 0 \\ K_{sa} & K_{ss} \end{bmatrix}
\]  
(9)

The subscript \( a \) and \( s \) indicate the acoustic and the structural degrees-of-freedom (DOF) groups respectively. The responses of the whole system are described by vector \( p \) which comprises the sound pressure vector \( p_a \) and the structural displacement vector \( p_s \). The number of DOFs of the sound field, the structure and the whole system are assumed to be \( n_a \), \( n_s \) and \( n \) (\( n=n_a+n_s \)) respectively. For the enclosing elastic structure, \( M_{ss} \) and \( K_{ss} \) are the mass and stiffness matrices, while for the sound field, \( M_{as} \) and \( K_{sa} \) are analogs of mass and stiffness if the sound pressures \( x_a \) are regarded as structural displacements. The symmetrical submatrices \( M_{ss} \), \( K_{ss} \), \( M_{as} \) and \( K_{sa} \) can be derived from the “stand-alone” structure and sound field, therefore their values are independent of the coupled interactions between the air and the structure. The submatrices \( M_{as} \) and \( K_{sa} \) represent the coupling and satisfy the following equation,

\[
M_{as} = -K_{sa}^T
\]  
(10)

Solving the unsymmetrical generalized eigenvalue problem associated with Equation (6), the finite element model of the system response can be restated in terms of system modes:

\[
(K - \lambda_i M) \phi_i = 0,
\]  
\[
\psi_i^T (K - \lambda_i M) = 0
\]  
(11)

where \( \lambda_i \) is the \( i \)th eigenvalue, and \( \phi_i \) and \( \psi_i \) are associated right and left eigenvectors. Since \( M \) and \( K \) are unsymmetrical, the left vectors are not equal to the corresponding right ones. Theoretically, all the \( n \) eigen-solutions have real values. When the system has a large number of DOFs and lower frequency responses are of interest, only the first \( m \) (\( m < n \)) lower order modes are considered, while higher order ones can be neglected:

\[
\Lambda = \text{diag}(\lambda_1, \lambda_2, ..., \lambda_m)
\]  
(12)

\[
\Phi = [\phi_1, \phi_2, ..., \phi_m]
\]  
(13)

\[
\Psi = [\psi_1, \psi_2, ..., \psi_m]
\]  
(14)

When properly normalized, the eigenvectors possess the following orthogonal properties

\[
\Psi^T M \Phi = I
\]  
(15)

\[
\Psi^T K \Phi = \Lambda
\]  
(16)

Using the eigenvector matrix \( \Phi \), physical coordinates \( p \) can be transformed to modal coordinates \( q \)

\[
p = \Phi q
\]  
(17)

Substituting Equation (17) into Equation (6), pre-multiplying by \( \psi_i^T \), applying the orthogonal properties, and adding some modal damping, the system can be expressed by a set of \( m \) single DOF systems

\[
\ddot{q} + \text{diag}(2\xi_i \sqrt{\lambda_i}) \dot{q} + \Lambda q = \Psi^T f
\]  
(18)

where, \( \xi_i \) is the damping ratio for the \( i \)th mode.

Indirect Model Analysis

Due to the unsymmetrical matrices \( M \) and \( K \), the direct modal analysis discussed above is very expensive for large systems. To reduce the cost, the modal analyses of the “stand-alone” structure and sound field can be exploited. Considering the two generalized eigenvalue problems defined by \( (M_{aa}, K_{aa}) \) and \( (M_{ss}, K_{ss}) \), since the matrices are symmetrical, it is easy to obtain the
respective solutions \((\Lambda_a, \Phi_a)\) and \((\Lambda_s, \Phi_s)\), which possess orthogonal properties as

\[
\Phi_a^T M_a \Phi_a = I_a, \quad \Phi_a^T K_a \Phi_a = \Lambda_a \tag{19}
\]

\[
\Phi_s^T M_s \Phi_s = I_s, \quad \Phi_s^T K_s \Phi_s = \Lambda_s \tag{20}
\]

The physical coordinates \(p\) then can be transformed to modal coordinates \(\tilde{p}\) as

\[
\tilde{p} = \Theta \phi
\]

where

\[
\Theta = \begin{bmatrix} \Phi_a & 0 \\ 0 & \Phi_s \end{bmatrix} \tag{22}
\]

It is supposed that \(\Phi_a\) and \(\Phi_s\) respectively have \(m_a\) \((m_a \leq n_a)\) and \(m_s\) \((m_s \leq n_s)\) modes. Provided that \(m_a\) and \(m_s\) are large enough, the transformation will affect the accuracy of the model very little when a lower frequency range is considered, and therefore can be used to reduce the model size for large systems. In the extreme, if all modes of the two decoupled systems are used (i.e., \(m_a = n_a\) and \(m_s = n_s\)), the transformation will not change any behavior of the model. Substituting Equation (21) into Equation (6), premultiplying by \(\Theta^T\), and applying the orthogonal properties shown in Equations (19) and (20), the following reduced equation can be obtained

\[
\tilde{M} \tilde{p} + \tilde{K} \tilde{p} = \tilde{f}
\]

where

\[
\tilde{M} = \begin{bmatrix} I_a & \tilde{M}_{as} \\ 0 & I_s \end{bmatrix} \tag{24}
\]

\[
\tilde{K} = \begin{bmatrix} \Lambda_a & 0 \\ \tilde{K}_{sa} & \Lambda_s \end{bmatrix} \tag{25}
\]

\[
\tilde{M}_{as} = -\tilde{K}^T_{sa} = \Phi_a^T M_a \Phi_s \tag{26}
\]

\[
\tilde{f} = \Theta^T f
\]

(27)

It is clear that Equation (23) has a smaller size than but the same structure as Equation (6). Thus following the procedure discussed in the previous section, the eigenvalues, right and left eigenvectors associated with Equation (23) can be derived by coupled modal analysis

\[
\tilde{\Lambda} = \text{diag}(\tilde{\lambda}_1, \tilde{\lambda}_2, \ldots, \tilde{\lambda}_m) \tag{28}
\]

\[
\tilde{\Phi} = [\tilde{\phi}_1, \tilde{\phi}_2, \ldots, \tilde{\phi}_m] \tag{29}
\]

Finally, after another modal transformation similar to Equation (17),

\[
\tilde{\psi} = [\tilde{\psi}_1, \tilde{\psi}_2, \ldots, \tilde{\psi}_m] \tag{30}
\]

a similar equation to Equation (18) can be obtained

\[
\tilde{q} + \text{diag}(2\tilde{\xi}\sqrt{\tilde{\lambda}_i}) \tilde{q} + \tilde{\Lambda} \tilde{q} = \tilde{\psi}^T \tilde{f}
\]

(32)

The indirect procedure discussed in this section can reduce the computational costs for large system with very little effect on accuracy if an enough number of decoupled modes are used in Equation (21). Furthermore, in our case, the structure will be modified during the optimization while the sound field remains unchanged, thus only one structural problem and one coupled problem for the reduced model need to be solved in one reanalysis if the indirect method is used. Hence, considering the repetitious calculations, the total cost of the indirect method will be much lower than that of the direct one.

**CONTROL DESIGN**

For active interior noise control of coupled structural-acoustic enclosures, even a brief review is beyond the scope of this paper since the subject is huge and still growing rapidly. The fundamental knowledge and recent theoretical and practical developments can be found in many references (Hansen and Snyder, 1997; Tseng et al., 1998). From a large amount of literature on the subject, one popular procedure which employs the feedforward control strategy can be found, in which the quadratic optimization control theory is used to determine the optimum magnitude and phase of the control source (Nelson et al., 1987; Nelson and Elliott, 1992). It permits the use of both acoustic and structural inputs as primary and control sources. Moreover, it allows the use of various local and global quantities as the quadratic objective function, such as the sum of squared acoustic pressures at discrete points, global internal acoustic potential energy (Thomas et al., 1993; Snyder and Hansen, 1994), acoustic energy density at discrete points (Sommerfeldt and Nashif, 1994; Park and Sommerfeldt, 1997) etc.

However, since the investigation into the possibility of improving the control performance by the simultaneous structural-control optimization is emphasized in this research, instead of the popular feedforward control strategy using the global acoustic potential energy as the objective function, a state-space based \(H_\infty\) output
feedback control is used in which the measured outputs are the sound pressures at discrete locations. The choice of such a feedback strategy using local acoustic quantities unavoidably lowers the practical significance of the control strategy itself compared with popular control techniques, but basing the whole thing upon the state-space framework makes the subsequent work (i.e., repetitious control designs after the structural remodeling due to the simultaneous structural-control optimization, and the investigation of the mechanism causing the closed loop performance improvement) more straightforward. However, it should be mentioned here that the use of other control strategies should not cause any obstacles theoretically in the simultaneous optimization if the contributions of structural changes to the overall system performance can be appropriately evaluated.

Assuming the coupled structural-acoustic enclosure system has the disturbance $w$, control input $u$, and measured output $y$, the following state space model can be derived from the results of indirect model analysis

$$\dot{x} = Ax + B_w w + B_u u$$  \hspace{1cm} (33)
$$y = Cx$$  \hspace{1cm} (34)

where,

$$x = \begin{bmatrix} \tilde{q} \\ \tilde{q} \end{bmatrix}$$  \hspace{1cm} (35)

$$A = \begin{bmatrix} 0 & I \\ -\tilde{A} & -\text{diag}(2\tilde{\xi}\sqrt{\lambda}) \end{bmatrix}$$  \hspace{1cm} (36)

$$B_w = \begin{bmatrix} 0 \\ \tilde{\Psi}^T \Theta \tilde{b}_w \end{bmatrix}, \quad B_u = \begin{bmatrix} 0 \\ \tilde{\Psi}^T \Theta \tilde{b}_u \end{bmatrix}$$  \hspace{1cm} (37)

$$C = \begin{bmatrix} c_y \Theta \Phi & 0 \end{bmatrix}$$  \hspace{1cm} (38)

where, $b_w$, $b_u$, and $c_y$ are respectively the location matrices for $w$, $u$, and $y$ which can be related to the total input $f$ and the measured output $y$ of the physical space model shown in Equation (6) as below

$$f = b_w w + b_u u$$  \hspace{1cm} (39)
$$y = c_y p$$  \hspace{1cm} (40)

In Figure 1, the above state-space model is represented by block “$P$” which has inputs $w$ and $u$ and output $y$.

By introducing the feedback controller $K$, sensor noise signal $d$, limitation on control input $u$, and four weighting functions $W_w$, $W_y$, $W_u$, and $W_d$, the closed loop model can be obtained as shown in Figure 2. In this paper, for simplicity, output feedback $H_\infty$ control law is adopted. Therefore, the purpose of the control design is finding the optimal controller $K$ which minimizes the $H_\infty$ norm of the closed loop transfer function from $w$ and $d'$ to $y'$ and $u'$ shown in Figure 2

$$f_1 = \min_K \| T(w',d'; y, u') \|_\infty$$  \hspace{1cm} (41)

**NUMERICAL EXAMPLE**

One example is used in this paper to demonstrate the effectiveness of the approach discussed in the previous sections for the simultaneous structural-control optimization considering structural-acoustic coupling.

It can be seen from Figure 3 that the example has a rectangular parallelepiped sound field of which the three dimensions are $L_x$, $L_y$, and $L_z$. The sound field is enclosed with rigid walls and two elastic plates which are
shown as two gray rectangles in the figure. The bottom face of the sound field is entirely covered with the big plate while the top face is partly covered with the small one. The eight sides of the two plates are supposed to be fully clamped. As shown in the figure, two vertical point forces acting on the top and bottom plates are assumed to be the control input \( u \) and disturbance \( w \) respectively. In the center of the sound field, one sound sensor is placed to measure the sound pressure output. It can be found that the sensor and the two forces lie along the vertical central line of the enclosure.

Figure 4 shows the finite element mesh of the coupled enclosure, from which it can be seen that the sound field and the elastic plates are modeled by eight-node 3D solid and four-node 2D plate elements respectively. Each structural node has six DOFs, while each acoustic node has only one DOF of sound pressure. The mesh is regular and uniform along the three axes of coordinates. Moreover, on the structural–acoustic interfaces, the plate and the sound field have identical mesh.

Table 1 shows several physical parameters used in the example, i.e., the three dimensions of the enclosure, the material properties of the plate, the properties of the air, and the thickness of the plates.

The element thickness parameters of the top plate elements are used as structural design variables while the bottom plate is kept invariable. Since the initial enclosing structure, the sound field, the input and the output are symmetrical about the center \( YZ \) plane of the enclosure, it is assumed that the top plate remains symmetrical in the optimization. As shown in Figure 4 the top plate is discretized into 54 plate elements, therefore considering the symmetry, only 27 design variables need to be determined. In addition, it can be seen from Table 1 that both the bottom plate and the initial top plate are 10 mm thick.

As stated in the previous section, output feedback \( H_\infty \) control law is adopted for simplicity. Again for simplicity, the corresponding weighting functions \( W_w, W_u, W_y, \) and \( W_d \) used in the control design are assumed to be constant scalars of 1, 0.01, 1 and 0.1 respectively. It should be mentioned here that there should be no difficulty if other feedback control law and complicated frequency dependent weighting functions are used, since the control design is independent and well encapsulated in the nested simultaneous optimization. In addition, the coupled modes which have frequencies higher than 500 Hz are omitted in the state-space modeling for control design.

Finally, the simultaneous optimization problem can be expressed as

\[
\begin{align*}
\min_{p_x, p_t} & \quad f_1/F_1 \\
\text{s.t.} & \quad f_2/F_2 \leq 1 \\
& \quad p_t^L \leq p_t \leq p_t^U
\end{align*}
\]

where, \( f_1 \) and \( f_2 \) are the closed loop \( H_\infty \) norm and the top plate weight of the current structure. While the constants \( F_1 \) and \( F_2 \) are the corresponding \( f_1 \) and \( f_2 \) values of the initial structure. The structural design variable \( p_t \) is the vector of thickness ratios of current top elements to the associated initial ones. In addition, lower and upper bounds on \( p_t \), (i.e., \( p_t^L \) and \( p_t^U \)) are supposed to be 0.5 and 1.5 respectively. From the above definition, it is clear that the purpose is to minimize the closed loop \( H_\infty \) norm ratio with respect to the structural and control design variables \( p_x \) and \( p_t \), subject to the bounds of the structural variable \( p_x \) and the top plate weight ratio.

Table 1. Physical properties of the model.

<table>
<thead>
<tr>
<th>Dimensions of Enclosure (mm)</th>
<th>( L_x ) 600</th>
<th>( L_y ) 1200</th>
<th>( L_z ) 400</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material properties of structure</td>
<td>Young’s Modulus (GPa) 200</td>
<td>Poisson’s ratio 0.3</td>
<td>Density (kg/m(^3)) 7.8 ( \times ) 10(^3 )</td>
</tr>
<tr>
<td>Physical properties of air</td>
<td>Sound speed (m/s) 344</td>
<td></td>
<td>Density (kg/m(^3)) 1.115</td>
</tr>
<tr>
<td>Thickness of plates (mm)</td>
<td>Bottom plate, ( Z = 0 ) 10</td>
<td>Top plate (initial value), ( Z = L_z ) 10</td>
<td></td>
</tr>
</tbody>
</table>
The nested approach discussed previously is used to solve the simultaneous design problems defined in Equation (42), in which the structural variables are solved in the main optimization by the method of moving asymptotes (MMA), while the control parameters are designed in the suboptimizations by the $H_\infty$ synthesis. The details of MMA and $H_\infty$ control design are omitted here and can be found in the references (Svanberg, 1987; Doyle et al., 1989).

It is mentioned in the previous section that the use of the feedback strategy considering local acoustic quantities unavoidably lowers the practical significance of the control strategy itself but simplify the whole work including the optimization and the result investigation. Again, for simplicity, the geometry of the example enclosure, including the arrangements of the primary and control sources and the single sound pressure sensor, is symmetric except for the two different elastic enclosing plates. If the configuration of the two plates is also made symmetric, an ideal feedforward controller can generate the control source so that the contributions of the two plates to the sensor signal cancel each other via a phase change of $\pi$ and therefore can exactly drive the sensor signal to zero. By introducing different areas for the two plates, the exact symmetry and the ideal cancellation phenomenon can be eliminated. In addition, by optimizing the thickness distribution of the top plate, it is possible to make the local cancellation at the sensor spot easier for the unsymmetrical plates. Obviously, such configurations further limit the practical application of the example itself, but simplify the following work especially making the evaluation and discussion of the final result derived by the simultaneous optimization easier to carry out. In addition, like the statement made in the previous section concerning the control strategy, it is clear that the use of such a simple and special example itself should not cause any theoretical difficulties when more complicated enclosure geometries and sophisticated sensor or actuator techniques to be used.

In addition, the optimization with respect to the thickness ratios of the top plate elements is very likely to produce a nonuniform plate with discrete thickness values in subareas of the plate. When manufacturing issues are considered, it is difficult to employ such plate structures. In more practical design problems, instead of optimizing the thickness distributions of the plate or shell structures, the attached stiffeners can be adopted as the design objectives.

RESULTS AND DISCUSSIONS

The numerical results are discussed in this part.

The thickness distributions of the initial and optimized top plates are shown in Figures 5 and 6. As mentioned previously, the initial top plate has uniform thickness of 10 mm. For the initial structure, all values of the structural variable $p_s$ and top plate weight ratio $f_2/F_2$ are equal to 1.0 according to their definition. As shown in Figure 6, the optimized thickness distribution is difficult to be described but quite different from the initial one. Most of the design variables are close to their lower or upper bounds after the optimization, and it will be found later that the final top plate weight ratio is equal to 0.876 which means that optimized plate is lighter in weight than the initial one. In addition, the thickness distribution shown in Figure 6 is symmetrical since only the up half is optimized and the down half is symmetrically reproduced.

The changes of the closed loop $H_\infty$ norm and the top plate weight ratio with respect to the iteration number are shown in Figures 7 and 8 respectively. It can be seen that the closed loop $H_\infty$ norm is reduced significantly by the simultaneous optimization. Moreover, the optimized top plate is slightly (compared to the norm reduction)
lighter than the initial one. In addition, as shown in the figures, the $H_\infty$ norm and the weight ratio fluctuate strongly in the early stage of the optimization and levels off to the final values after about 50 and 80 iterations respectively. Although not considered in this research, it is possible to reduce the fluctuations and speed up the convergence by tuning the parameter settings in the optimization algorithm.

The open loop frequency responses of the initial and the optimized enclosures are compared in Figures 9 and 10. It can be found from Figure 9 that the open loop response from the disturbance $w$ to the output $y$ does not change much after the optimization, except that the second peak of the initial system at 97.8 Hz moves to 86.1 Hz which is very close to the first peak at about 85 Hz. While in Figure 10 for the open loop response from the control input $u$ to the output $y$, the situation is totally different. It can be seen that except the peak at about 290 Hz, the optimized system behaves very differently than the initial one does. The two figures imply that the significant change of the top plate thickness distribution will affect the open loop response from $w$ to $y$ slightly but that from $u$ to $y$ greatly. Considering the geometrical arrangement of the whole system including the input and output shown in Figure 3, it is clear that the result is reasonable.

The closed loop responses of the initial and the optimized systems are compared in Figures 11 and 12. From Figure 11 showing the closed loop response from the disturbance $w$ to the output $y$, it can be seen that for both the initial and optimized systems, the sound pressure levels are successfully reduced compared with the open loop responses shown in Figure 9. However, it is obvious that the optimized system has much smaller peak response than the initial one. The responses of the initial closed loop system at 84.6 and 224 Hz are about 20 dB larger than the maximum response of the optimized system at 256 Hz. Moreover, in Figure 12 for the closed loop response from the disturbance $w$ to the control input $u$, it can be seen that the optimized closed loop system needs much smaller peak control effort than the initial one. Considering the peak
responses of control input $u$ at 84.6 and 224 Hz, again more than 20 dB improvements are achieved by the simultaneous optimization. It can be concluded from Figures 11 and 12 that the superiority of the optimized closed loop system over the initial one mainly comes from the change of system characteristics at the frequencies about 85 and 224 Hz.

The cause of the improvement in the closed loop performance can be explained if the open loop responses shown in Figures 9 and 10 are considered again. It can be seen from Figure 9 that for both the initial and optimized systems, the main peaks to be controlled appear around three frequencies of 85, 224 and 290 Hz. However, in Figure 10, it can be found that when the control input $u$ being applied, around 290 Hz both the initial and optimized systems have large peaks of output $y$, but around 85 and 224 Hz, only the optimized system has peaks large enough to control the pressure vibrations induced by the disturbance $w$. Thus for the example considered in this paper, it is the improvement in the controllabilities around 85 and 224 Hz that improves the closed loop response and reduces the control effort.

<table>
<thead>
<tr>
<th>Table 2. Improvement due to optimization.</th>
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<tbody>
<tr>
<td>Initial</td>
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<tr>
<td>-----------------</td>
</tr>
<tr>
<td>Objective function value</td>
</tr>
<tr>
<td>Closedloop $H_\infty$ norm</td>
</tr>
<tr>
<td>Weight ratio of top plate</td>
</tr>
<tr>
<td>Closedloop maximum response $y : w$ (Pa : N)</td>
</tr>
<tr>
<td>$u : w$ (N : N)</td>
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</table>

The values of the objective function, closed loop $H_\infty$ norm, top plate weight ratio, and maximum closed loop responses from disturbance $w$ to output $y$ and control input $u$ are listed in Table 2. It can be seen that with a 12.4% change in top plate weight and the redistribution of top plate material, the simultaneous optimization achieves a reduction of 93% in the closed loop $H_\infty$ norm which leads to 83.5 and 93.2% decreases in maximum responses of the measured output and the control input.

CONCLUSIONS

Simultaneous structural-control optimization of an actively controlled coupled structural-acoustic enclosure is investigated in this paper.

The simultaneous design problem is formulated as a main structural optimization dealing with the structural parameters, into which the control optimizations with respect to the control parameters are nested as sub processes. The control optimizations are encapsulated and executed independently to find the control parameters for any new structures produced during the main optimization. Hence, various methods and tools for structural design and control synthesis can be combined and employed easily.

In the structural optimization, the coupled structural-acoustic enclosure needs to be reanalyzed for many times. Due to the structural-acoustic coupling, the “mass” and “stiffness” matrices obtained in the finite element analysis are unsymmetrical. Since the direct modal analysis for unsymmetrical system is computationally much more expensive than the symmetrical modal analysis when large system is considered, in this paper, indirect modal analysis is used instead of the direct method. It is found that the indirect method can considerably reduce the computational costs for large systems with very little effect on accuracy if a liberal number of decoupled modes are used.

The numerical results of the example considered in this research show that, with a redistribution of the material of the interface structure, the simultaneous optimization achieves an improvement of 93% in the closed loop system performance which includes 83.5 and 93.2% reductions in maximum noise level and
control effort. Furthermore, it is also made clear that the enhancement in controllability around 85 and 224 Hz is the main cause of the closed loop performance improvement.

Although a simple example is considered in this paper, the effectiveness and potentiality of the new design approach are demonstrated by the results.

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