

Editorial

A Future Vision of Mathematics Education Research: Blurring the Boundaries of Research and Practice to Address Teachers' Problems

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We began our editorials in 2017 seeking answers to one complex but important question: How can we improve the impact of research on practice? In our first editorial, we suggested that a first step would be to better define the problem by developing a better understanding of the fundamental reasons for the divide between research and practice (Cai et al., 2017). This sparked subsequent editorials in which we delved deeper into some of the many complicated facets of this issue. In our March (Cai et al., 2017b) editorial, we argued that impact needs to be defined more broadly than it often has been, notably, to include cognitive and noncognitive outcomes in both the near term and longitudinally. This led us to focus our May (Cai et al., 2017a) editorial on the ways that research might have a greater impact on the learning opportunities that help students reach broader learning goals. We argued that it is not enough to identify learning goals—it is also necessary to conduct research that breaks those learning goals into subgoals that can be appropriately sequenced. We highlighted research on learning trajectories as an example of this sort of work but also emphasized the need to work at a grain size that is compatible with teachers' classroom practice. Finally, in our July (Cai et al., 2017c) editorial, we argued that the implementation of learning opportunities in the classroom is an integral element of research that has an impact on practice.

A common thread throughout our editorials has been the importance of the teacher's role in all these aspects of impactful research. We pointed to this in the story we shared in January about Baltimore's public school system reform in which an administrative shift moved away from the existing culture of teachers identifying the problems to be solved to a culture of having those problems defined by researchers and administrators. These newly identified problems were not the ones considered most pressing by teachers, thus decreasing their engagement in solving them. We believe research has a greater impact on practice when teachers play a purposeful role in the research process, whether in defining research problems; in identifying learning goals, subgoals, and learning opportunities; or of course, in implementing learning opportunities in the classroom.

A system that reflects these considerations would require a radically different picture of mathematics education research than the one that exists now. In our January editorial, we asked the following questions: "Can we, as a community, look

ahead 25 years to *JRME*'s diamond jubilee—a hypothetical future in which research is more impactful? What will that future look like?" (Cai et al., 2017, p. 5). Just as Gage (1989) provided a "'historical' sketch" that posited what would ensue in research on teaching over the next 20 years, we now peer into the future of mathematics education research, one that reflects the hypothetical future we asked our community to envision in our first editorial—a future in which mathematics education research has a much greater impact on practice. In this final editorial of 2017, we describe a future vision of mathematics education research that blurs the boundaries of research and practice to address teachers' problems.

We begin by revisiting the story of the fourth-grade teacher, Mr. Lovemath, from our May editorial (Cai et al., 2017a). Recall that, with guidance from Ms. Research, a mathematics education researcher, Mr. Lovemath selected a high-level fraction-ordering task to use with his students (see Figure 1).

However, the students struggled with the task, ultimately prompting Mr. Lovemath to provide a hint that lowered the cognitive demand of the otherwise high-level task. Although the fraction task had many qualities that mathematics education research suggests would make it a powerful learning opportunity for students, Mr. Lovemath's learning goals for his students went unrealized because of a mismatch between the demands of the task and the learning opportunities previously experienced by the students. We now imagine a continuation of this scenario to illustrate our future vision of mathematics education research. We identify a few key factors that, although only possible in a different culture of teaching and research, are not impossible to envision.

Our vision is of a work environment for both Mr. Lovemath and Ms. Research that yields a tight melding of research and practice and that has the potential to produce substantially different outcomes for students. Let us imagine that Mr. Lovemath and Ms. Research have watched the lesson (which was recorded on video). In their subsequent discussion, they consider several questions: Why did the students encounter difficulties in solving this problem? Why did the intended opportunity to learn not materialize? What should Mr. Lovemath do in the next lesson? How could they revise the current lesson for use next time?

In this new research–practice world, Mr. Lovemath consults an online professional knowledge base for mathematics education that contains feedback and

Order these fractions from smallest to largest:

$$\frac{7}{9} \quad \frac{2}{4} \quad \frac{9}{10} \quad \frac{6}{13} \quad \frac{1}{2} \quad \frac{9}{5} \quad \frac{3}{7}$$

Place the fractions in the correct locations on the number line.

Figure 1. Fraction-ordering task.

adaptations that other teachers and researchers have recorded from their experiences helping students achieve the same learning goals, some by using similar tasks. This professional knowledge base is accessible to the mathematics education community writ large; is organized and searchable by learning goals; and includes plans for activities and lessons that have been refined over time and suggested tasks, comments, and findings of teachers and researchers about students' thinking and responses based on repeated trials of similar lessons. Meanwhile, Ms. Research analyzes the task to identify potential learning subgoals that are needed to support students' productive engagement with this task. Then, Mr. Lovemath and Ms. Research discuss how to refine the task and design additional activities that would address those learning subgoals so that these students can take advantage of the intended learning opportunity. They test their refinements to see whether they help the students achieve the learning goals, and then feed their findings back into the professional knowledge base in the form of a knowledge package that includes a more precisely stated set of learning (sub)goals; an improved, annotated lesson and task; a set of improved prompts and expected student responses; assessment items that elicit key information about students' understanding; and data on how students engaged with the task.

Three features of this future scenario warrant further explanation: (a) the collaborative nature of Mr. Lovemath and Ms. Research's partnership, (b) the online shared professional knowledge base accessed by Mr. Lovemath, and (c) a redesigned incentive and accountability system that would make this kind of collaborative work feasible and appealing for both teachers and researchers.

A New Form of Collaboration

In this scenario, Mr. Lovemath and Ms. Research benefited greatly from working together to identify and specify the problem and to design and implement a response in ways that are radically different from the kinds of teacher–researcher partnerships that typically occur now. Although their collaboration shares characteristics with some research–practice and university–school partnerships that others have described (see, e.g., Coburn & Penuel, 2016; Hunter Quartz et al., 2017), it is also qualitatively different in ways we will discuss below and in future editorials. The collaboration between Mr. Lovemath and Ms. Research was not a short-term partnership designed only to improve the learning opportunities for Mr. Lovemath's students. Rather, the collaboration aimed to create learning opportunities that could benefit other teachers facing similar problems around this learning goal. Their collaboration went well beyond simply translating research into practice, and it went beyond Ms. Research suggesting research-based instructional approaches for Mr. Lovemath to follow. In this collaboration, Mr. Lovemath and Ms. Research worked toward the common goal of maximizing (all) students' achievement by playing equally important roles in identifying the problem, analyzing the nature of the problem at a very fine-grained level, gathering data on students' learning, and developing adaptations to address the issues that they identified.

Because this system of collaboration treats Mr. Lovemath and Ms. Research as partners who contribute equally important kinds of expertise, the products of their collaboration (the professional knowledge package that they developed) are likely to have genuine buy-in from Mr. Lovemath. This buy-in means that Mr. Lovemath is more likely to faithfully implement the adaptations that he and Ms. Research developed. This is not surprising. A well-established finding in social psychology suggests that when people are involved in creating a process or product, they are more likely to continue their participation in developing and using it because of their sense of ownership (see, e.g., Lewin & Grabbe, 1945). As a recent example of the significance of this finding, Kramer, Cai, and Merlino (2015) found that “Will to Reform” (p. 22), which included teacher buy-in, was a key factor in schools that successfully implemented reform curricula to improve student learning.

Although Mr. Lovemath and Ms. Research are the major players, this collaboration could also include other teachers, researchers, curriculum writers, and educators who have relevant expertise and who are invested in helping students achieve the same learning goals. That is, with Mr. Lovemath and Ms. Research comprising the central partnership, the collaboration could be expanded to include a broader set of stakeholders. The main requirement for participation is a shared interest in helping students achieve the specified learning goals and a willingness to share one’s expertise to realize this outcome. This form of collaboration is different from typical teacher–researcher collaborations, wherein once a problem is solved, the researcher moves on and the partnership dissolves. We envision this collaboration to be ongoing and a part of a network of many such partnerships, with problems of practice solved gradually in a system of incremental improvement across the profession. Once a set of problems is solved, new problems are taken up by the partnership. Thus, the process continues.

A Shared Online Professional Knowledge Base

A second key component of our future vision is the existence of a rich, searchable, user-friendly, and shared professional knowledge base for mathematics education that is accessible online and consists of useful findings and artifacts that are continuously refined over time, indexed by specific learning goals and subgoals, and that assist teachers and researchers in implementing learning opportunities in their classrooms. Mr. Lovemath and Ms. Research benefited from the resources that were systematically collected in this shared online knowledge base that addressed the specific learning goals and issues encountered by the class. This was possible because the knowledge base comprised contributions from repeated cycles of other teacher–researcher collaborations that had addressed problems of practice around the same learning goals (fraction ordering using multiple strategies). Having done the work of analyzing the learning opportunities that could help students achieve the learning goal and the associated subgoals, designing tasks to create the learning opportunities, and then testing and revising the tasks to address what the students were missing, Mr. Lovemath and Ms. Research then fed their findings back into the professional knowledge base to further enrich the available pool of resources.

Together with the existing resources in the knowledge base, their work helped to refine the learning trajectory around this task and its mathematical concepts.

Several issues arise when contemplating a knowledge base of this type. The form of the knowledge base and its artifacts is itself a subject for research and is likely to continuously evolve. We do not propose to define here what belongs in the knowledge base. Indeed, what information is most useful to collect and store and what grain size the artifacts should address are themselves empirical research questions. We envision that the professional knowledge base would consist of lesson plans, tasks, excerpts, video recordings of lessons with codes and annotations, a repertoire of anticipated student responses, student work including mistakes and how they were addressed, assessment items tapping students' understanding, and formative assessment tools that teachers could use to gain knowledge about students as the lesson plays out. Even with current technology, all these materials can be easily digitized for remote access.

For this knowledge to be useful for others, it needs to be easily accessible. A knowledge base would become unwieldy if, over time, artifacts accrued in a disorganized fashion and outdated knowledge failed to be replaced with updated knowledge, obstructing users from accessing the information that is most relevant to them at any given time. In our May editorial (Cai et al., 2017a), we argued for the importance of defining learning goals at a grain size that fits teachers' practice—in other words, learning goals that are at the lesson and even activity level. The more detailed and specific the learning goal and the associated knowledge, the easier it would be to retrieve on demand and the more likely that teachers would use it. This is because teachers need resources to help students achieve the specific learning goals that exist within a particular lesson (e.g., ordering fractions written in common form using several different strategies). A hierarchically structured and searchable indexing system of learning goals might prove to be the most user-friendly, but again, the most accessible and effective design for the knowledge base would be a subject for empirical research.

Indeed, the knowledge base might itself serve as a springboard for new research questions and approaches to investigating those questions. In other words, not every mathematics education researcher would need to be paired with a "Mr. Lovemath." Some might take advantage of the data gathered and accumulated across classrooms to pose and investigate new research questions about teaching and learning.

New Incentive and Accountability Structures

One might ask how, in our future vision, the collaborative partnerships we have described came into existence. In other words, why would Mr. Lovemath approach Ms. Research to work together on his problem? Why would Ms. Research spend time working with Mr. Lovemath on improving his lesson and contributing to the creation of relevant artifacts for the knowledge base? Morris and Hiebert (2009) referred to the conditions for creating collaborative partnerships like these as "a distant dream" (p. 439). In our view, this new world emerged because incentive and accountability structures for both teachers and researchers shifted in radical ways.

In other words, the collaborative partnerships we have described came into existence because of a change in the culture of expectations.

What incentive and accountability structures could have evolved to explain a radical change in the work environment of both teachers and researchers? In simple terms, our future vision imagines that these structures became singularly focused on the outcomes that readers most value: richer and more robust mathematics learning for students. Teachers were provided incentives and were held accountable for enriching students' learning opportunities. This rhetoric may sound familiar; however, the change in culture was so radical that these phrases came to mean something very different in this new world than they mean today.

First, the incentives for teachers included time during their work week to improve their teaching. Time, although essential, is not enough. A second radical change is that teachers also had support from research partners who were interested in solving the teachers' problems. These partnerships were not unusual in this new world. Teachers and researchers regularly joined forces to study and improve students' learning opportunities.

Third, teachers were held accountable for their students' learning. However, accountability was defined much differently than it is today. Accountability focused on the quality of learning opportunities that teachers created for their students and not just on student test score performance. Moreover, accountability extended to students throughout their district and did not just apply to teachers' own classrooms. Mr. Lovemath's district, as many others, embraced the radical idea that Mr. Lovemath was expected to collaborate with, and contribute to the knowledge base for, all other mathematics teachers in his district who were teaching the same curriculum and helping students to achieve the same learning goals. In return, Mr. Lovemath benefited from other teachers being held accountable for similarly collaborative work.

The fourth radical change was triggered by the fact that Ms. Research began experiencing similar incentive and accountability structures as Mr. Lovemath. The promotion and tenure criteria at her university were rewritten to place more emphasis on her contributions to help students in the local school district learn richer and more challenging mathematics. By the time our new world of research–practice began functioning in the way we described earlier, both Mr. Lovemath and Ms. Research were being evaluated by the learning opportunities provided to, and gains in mathematics learning shown by, the students in Mr. Lovemath's classes and in similar classes across the district.

Holding Mr. Lovemath and Ms. Research accountable for students' learning in the local district changed the dynamics of teacher–researcher partnerships in dramatic ways. We believe, however, that the new incentives played an equally important role. Mathematics education researchers and school mathematics teachers want all students to have increasingly better opportunities to learn mathematics. Seeing students respond to richer opportunities to learn more challenging mathematics was internally rewarding and professionally motivating for both Mr. Lovemath and Ms. Research. Moreover, Mr. Lovemath began engaging in mathematics teaching as a professional endeavor, with opportunities to create and

contribute to something bigger—the professional knowledge base used by others to improve their own practice. His work, with Ms. Research and others, began influencing the way that particular mathematics lessons were taught across the district. He became recognized for his expertise in listening to students, identifying obstacles that they encountered in learning particular concepts, suggesting revisions to lessons, and implementing test versions of these lessons with his students. On par with researchers, Mr. Lovemath participated in improving the profession beyond his own classroom, engaging in stimulating discussions and debates with peers about relationships between teaching and learning and coauthoring artifacts that became part of the knowledge base used by other teachers in his district and beyond.

We imagine Mr. Lovemath and Ms. Research jointly making presentations at the conferences of the National Council of Teachers of Mathematics and the International Congress of Mathematics Education. They share with eager audiences the process of identifying a problem, collaborating to specify the learning goals, designing learning opportunities to service those goals, and making documented progress in improving students' learning. One of the outcomes is a coauthored artifact for the knowledge base. The artifact could be used by teachers everywhere who are trying to help their students who are facing similar difficulties achieve similar learning goals. Other products might include coauthored manuscripts for other teams working to solve related problems of practice. These professional activities are, in this new world, meeting the accountability expectations and providing renewed incentives for both of these engaged educators.

Conclusion

In January 2017, we challenged our readers to envision a radically different future for mathematics education research in which research has a meaningful impact on practice. The future vision we have described in this editorial is the product of a system of individual changes, each depending on the other. Teachers and researchers collaborate on local problems of practice because their joint efforts help their own students learn but are also recognized by other members of the profession and contribute to the improvement of students' mathematics learning across many classrooms. The dissemination of the products of these collaborations depends on the shared professional knowledge base developed by partnerships within schools everywhere. The incentive and accountability structures hold teachers and researchers responsible for improvements in student learning while affording similar professional rewards, both internal and external, to both researchers and teachers. This interdependency is a necessary feature of the world we envision, a world in which research and practice in mathematics education are tightly intertwined, a world in which the boundaries between research and practice become blurred.

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