OPTIMAL UNKNOWN INPUT DISTRIBUTION MATRIX SELECTION IN ROBUST FAULT DIAGNOSIS

Ron J. Patton, Jie Chen

Department of Electronics, University of York

YORK, YO1 5DD, UK

Keywords: Failure (fault) detection; robustness; optimal disturbance de-coupling; uncertain systems; unknown input observer

Abstract: Uncertainties in dynamic systems are an inevitable consequence of non-linearity and complexity, and obscure the performance of fault diagnosis. In order to achieve robust and reliable fault diagnosis, the unknown input (disturbance) de-coupling principle has been employed in recent research. In this paper, a method of computing the unknown input distribution matrix is proposed as a powerful alternative method to either re-identification of plant parameters arising from different operating points or to the use of non-linear residual generation. The determination of a suitable unknown input distribution matrix to achieve disturbance de-coupling is described as an optimization problem which is solved here via a Singular Value Decomposition. An example of robust fault detection applied to a jet engine system is included as an illustration.

INTRODUCTION

Safety is an increasingly important issue in modern process plant and aerospace systems. To ensure reliable operation, a fault monitoring and diagnosis scheme needs to be available. The fault diagnosis (mainly including fault detection & isolation (FDI) tasks) approaches of today are either based on hardware redundancy, realized by additional physical equipment or on analytical redundancy (AR), mostly implemented as computer software on process supervision computers. The AR approach has gained increasing consideration world-wide (Willsky, 1976; Patton, Frank & Clark, 1989; Frank, 1990; Patton & Chen, 1991a). Research is still under way into the development of more reliable and robust methods for achieving effective FDI. Certainly, the reliability of a FDI scheme must be higher than the monitored system and, for some cases (eg. for uncertain systems) this is difficult to achieve. The main problem obstructing the progress and improvement in reliability of FDI schemes is the robustness with respect to uncertainty which arises for example, due to process noise, turbulence, parameter variations and modelling error. All AR approaches to FDI employ a model of the monitored system. If the model is accurate and the characteristics of all the disturbances are known, FDI is very straightforward and robust solutions are trivial. Uncertainties are inevitably present and may interfere seriously with the FDI performance. AR will be a practically viable alternative to the use of hardware & software replication in safety-critical systems if the robustness with respect to uncertainty can be demonstrated in a simple & certifiable manner.

To design robust FDI schemes, we need the description of uncertainties acting upon
the system during typical process operations. Furthermore, it is necessary to find a
description of these uncertainties which can be handled in a straightforward and
systematic manner. A typical uncertainty description makes use of the concept of
“unknown inputs” acting upon a nominal linear model of the system. All uncertainties of
the system are summarised as unknown inputs (disturbances) acting on the system and
their effect can be considered as bounded or unbounded and structured or unstructured.
Based on this description, one can use the “unknown input observer (UIO)” to estimate
the state (e.g. Yang & Richard, 1988). Watanabe & Himmelblau (1982) and Frank &
Wünnenberg (1989) used the UIO approach to robust FDI. In this scheme, the unknown
input does not affect the residual so that robust FDI is achievable. Patton et al (1987,
1989, 1991b) have shown that an approach to solving this problem using assignment of
suitable eigenvectors and eigenvalues (eigenstructure) as a way of providing robustness
through disturbance de-coupling. By assigning the suitable eigenstructure to an observer,
the residual signal can be completely de-coupled from the disturbance. In this way, robust
FDI is achievable. Gertler (1991) has shown that the two approaches can be used to give
identical designs.

The most important contributions to robust FDI make direct use of the disturbance
de-coupling principle. An observer can be designed to be robust in the sense of
disturbance de-coupling; the robustness will ensure that the residual is insensitive to
disturbances and modelling errors. For all disturbance de-coupling methods, a necessary
assumption is that the unknown input distribution matrix must be known, but within the
framework of international research on this subject, the approaches to obtain this
distribution matrix have been lacking. This shortage obstructs the application of the
disturbance de-coupling approach for robust FDI in real engineering systems. Here, a
method of computing this matrix is proposed. In order to achieve the de-coupling
condition, an optimal approximation to the distribution matrix is used. For complex
non-linear systems, the operating point changes according to working conditions, and
different operating points correspond to different unknown input distribution matrices.
The paper gives a method that uses an optimal matrix to represent the changing structure
of the disturbances over the practical plant operation. A 17th order jet engine system
model is used to illustrate the method proposed in this paper.

**PROBLEM STATEMENT**

Consider a real system subject to parameter variations, disturbances, etc:

\[
\begin{align*}
\dot{x}(t) &= \begin{bmatrix} \dot{x}_0 \\ \dot{x}_u \end{bmatrix} = \begin{bmatrix} A_0 + \Delta A & A_{e0} \\ A_{u1} & A_{u2} \end{bmatrix} \begin{bmatrix} x_0(t) \\ x_u(t) \end{bmatrix} + \begin{bmatrix} B_0 + \Delta B \\ B_u \end{bmatrix} u(t) + \begin{bmatrix} G_0 \\ G_u \end{bmatrix} \mu(t) \\
y(t) &= [C_0 0] \begin{bmatrix} x_0(t) \\ x_u(t) \end{bmatrix} + D_0 u(t) + f_s(t)
\end{align*}
\]

where \( x \in \mathbb{R}^N \) is the state vector (\( N \) the real order of the plant dynamics), \( x_0 \in \mathbb{R}^n \) is
a partial state vector corresponding to dominant part of the system, \( u \in \mathbb{R}^r \) input vector,
\( y \in \mathbb{R}^m \) output vector, \( \mu \in \mathbb{R}^g \) noise vector. \( A_0, B_0, C_0 \) and \( D_0 \) matrices form the
linearised nominal model to be used in the observer, $A_{c0}, A_{u1}, \Delta A, A_{u2}, \Delta B$ and $B_u$ represent the parameter variations and plant uncertainty, whilst $G_{0}$ and $G_{u}$ are the system (process) noise distribution matrices, $f_s(t)$ is a sensor fault vector. A typical application of this partitioned state-space structure arises when comparing a reduced order model with the full-scale system, for example, in an observer used for FDI. For this case, the nominal model $(A_0, B_0, C_0, D_0)$ is the reduced order model and the remaining parameters define the coupling with unmodelled system dynamics. It is assumed that the $n$ reduced order state variables correspond to $n$ of the $N$ state variables of the full-scale system. An observer based on the nominal linear model is used to generate the residual for FDI:

\[
\hat{x}(k + 1) = (A_0 - KC_0)\hat{x}(k) + (B_0 - KD_0)u(k) + Ky(k)
\]

\[
\hat{y}(k) = C_0\hat{x}(k) + D_0u(k)
\]

\[
r(k) = Wc_\gamma(k) = W(y(k) - \hat{y}(k))
\]

where $K$ is the observer gain matrix, $W \in \mathbb{R}^{p \times m}$ is a weighting matrix and $r(t)$ is the residual vector. When this observer is applied to the actual system, the state estimation error ($\varepsilon(t) = x(t) - \hat{x}(t)$) and the residual dynamics are:

\[
\dot{\varepsilon}(t) = (A_0 - KC_0)\varepsilon(t) + [\Delta A \quad \Delta B \quad A_{c0} \quad G_0] E_d(t) - Kf_s(t)
\]

\[
r(t) = H\varepsilon(t) + Wf_s(t)
\]

where $H = WC \in \mathbb{R}^{p \times n}$, $d(t) \in \mathbb{R}^{n_1}$ is an unknown disturbance input vector and $E \in \mathbb{R}^{n \times n_1}$ is, in general, an unknown distribution matrix. The uncertainty term $Ed(t)$ drives the residual away from its zero steady-state. The response of the residual is:

\[
r(s) = [W - WC(sI - A + KC)^{-1}K]f_s(s) + WC[sI - (A - KC)]^{-1}Ed(s)
\]

It can be difficult to distinguish the effects of faults from the effects of disturbances. The effects of disturbances obscure the performance of FDI and act as a source of false alarms which must be minimized. The residual generator must be robust with respect to disturbances. The ideal solution is to make the residual itself decoupled from disturbances.

**BRIEF DETAILS OF ROBUST FAULT DETECTION BY USING EIGENSTRUCTURE ASSIGNMENT**
In order to make the residual $r(t)$ de-coupled from disturbances, it is necessary to null the entries in the transfer function matrix between residuals and disturbances. i.e.

$$G_{rd}(s) = H[sI - (A_0 - KC_0)]^{-1}E = 0$$

Once the matrix $E$ is known, the robust design problem is to choose the matrices $K$ and $W$ to satisfy Eq. (9). Patton et al (1987, 1988, 1991b) have shown that this equation can be solved by assigning the eigenstructure of the observer. This is a direct alternative to the Kronecker Canonical Form (KCF) approach of Frank & Wünnenberg (1989). Eigenstructure assignment provides more insight and is simpler to use than the KCF method, and is thus gaining in popularity. This problem can be solved by either assigning left or right eigenvectors to an observer according to the following methods:

**Method 1:** If $WCE = HE = 0$, and all rows of the matrix $H = WC$ are left eigenvectors of $(A_0 - KC_0)$ corresponding to any eigenvalues, Eq.(9) holds true.

**Method 2:** If $WCE = HE = 0$, and all columns of the matrix $E$ are right eigenvectors of $(A_0 - KC_0)$ corresponding to any eigenvalues, Eq.(9) holds true.

Further details of the approach can be found in Patton & Chen (1991b). Both methods involve a necessary condition:

$$HE = 0$$

As the residual is completely de-coupled from disturbances, robust FDI is achievable. For this approach, a primary assumption is that the unknown input distribution matrix $E$ must be known a priori, this is an example of structured uncertainty. For a wider range of problems, the uncertainty is unstructured, i.e., the $E$ matrix is not known. For this case, can the term $Ed(t)$ characterize the uncertainty? How can we determine the term $Ed(t)$ and the structure of $E$, even approximately? These questions must be answered, otherwise the application domain of the disturbance de-coupling approach for robust FDI will be very limited. In the following part of this paper, we will discuss how to obtain this matrix (albeit approximate) for real uncertain systems. Note that, in the UIO approach for robust FDI, an expression similar to Eq. (10) is used (Frank & Wünnenberg, 1989), i.e., the determination of the optimal disturbance distribution matrix $E$ is a common problem for all disturbance de-coupling approaches including the KCF approach of Frank & Wünnenberg (1989).

**OPTIMIZATION OF UNKNOWN INPUT DISTRIBUTION MATRIX**

We consider here a large scale time-varying non-linear system. In the design of a robust residual generator (based on the use of an observer), a dominant low order time-invariant linear model is used. The modelling error is summarized in $Ed(t)$. Firstly, we consider the case that the full order system model is known a priori. For this case, according to the Eq. (6), we can compute the matrix $E$ as:
\[ E = [\Delta A \mid \Delta B \mid A_{c0} \mid G_0] \in \mathbb{R}^{n \times n_1} \]

In general, this is a non-square matrix, its column number \( n_1 \) is larger than the row number \( n \). The key step in achieving robustness (disturbance de-coupling) is to find a \( p \times n \) matrix \( H \) to satisfy Eq. (10). If \( \text{rank}(E) \leq n - p \), (10) has solutions and exact de-coupling is possible. If, however, \( \text{rank}(E) > n - p \), (10) has no solutions and exact de-coupling is impossible. The approximate de-coupling must be taken. The procedure will be to first compute a matrix \( E^* \) that is as close as possible to \( E \), and \( \text{rank}(E^*) \leq n - p \), i.e. to determine the matrices \( E^* \) and \( H \) so that:

\[
\begin{align*}
HE^* &= 0 \\
\|E - E^*\|_F^2 &= \# \\
\end{align*}
\]

The constraint condition (12) is equivalent to requiring that \( \text{rank}(E^*) \leq n - p \). If only \( p \) linearly independent rows of \( H \) are required, the constraint condition can be changed to \( \text{rank}(E^*) = n - p \). Here \( \| \cdot \|_F \) denotes the Frobenius norm, defined as the root of the sum of squares of the entries of the associated matrix. The matrix \( E^* \) is thus chosen so that the sum of the squared distances between the columns of \( E \) and \( E^* \) is minimized, subject to the constraint that \( E^* \) contains only \( n - p \) linearly independent columns. The optimization problem just posed can be easily solved by the Singular value Decomposition (SVD) of \( E \):

\[
E = S[\text{diag}\{\sigma_1, \cdots, \sigma_n\}, \ 0]T
\]

where \( S \) and \( T \) are orthogonal matrices, \( \sigma_1 \leq \sigma_2 \leq \cdots \leq \sigma_n \) are the Singular Values of \( E \). As shown in Lou et al (1986), the matrix \( E^* \) minimizes (13) is given by:

\[
E^* = S[\text{diag}\{0, \cdots, 0, \sigma_{p+1}, \cdots, \sigma_n\}, \ 0]T
\]

and an orthonormal solution of the matrix \( H \) is:

\[
H = [s_1 \ s_2 \ \cdots \ s_p]^T
\]

where \( s_1, \cdots, s_p \) are first \( p \) columns of \( S \). Certainly, the matrix \( H \) in (16) and \( E^* \) in (15) together satisfy the Eq. (12).

We now give an alternative statement of the optimization problem: an ideal matrix \( H \) should make \( H e_i = 0 \) for all \( i = 1, 2, \cdots, n_1 \), where \( e_i \) is the \( i \)th column of the matrix \( E \). This is not always possible. Hence, it makes sense is to choose a matrix \( H \) that is “as orthogonal as possible” to all \( e_i \) \( (i = 1, 2, \cdots, n_1) \), i.e. to make each of \( H e_i \) \( (i = 1, 2, \cdots, n_1) \) as close to zero as possible. The optimization criterion can be defined as:

\[
J = \sum_{i=1}^{n_1} \|He_i\|_F^2.
\]

The optimal solution for \( H \) follows by minimizing \( J \), subject to \( HH^T = I \). Lou et al (1986) show that the choice of \( H \) given in (16) also minimizes \( J \), yielding the minimum value as \( J = \sum_{i=1}^{p} \sigma_i^2 \). The new statement of the optimization problem provides some very useful insight as \( J^* \) can be used as a robustness measure which is clearly relative to the independent row number \( p \) of the matrix \( H \).

Typically some components of unknown input vector \( d \) are larger than others. Furthermore, certain components of the unknown input vector have more effect on the
residual. Hence we must pay varied attention to the different components of the disturbance in the optimization procedure. For example, if the jth component of the disturbance is significantly larger than the ith component, the term Hej will be more important than the term Hei. The criterion J must be replaced by:

$$J = \sum_{i=1}^{n_1} \alpha_i \|He_i\|_F^2$$

where $$\alpha_i(i = 1, 2, \cdots, n_1)$$ are positive weighting factors. The relative magnitudes of the $$\alpha_i$$ correspond to relative magnitudes of components of the disturbance weighting. By rewriting the weighting optimization criterion as:

$$J = \sum_{i=1}^{n_1} \|H(\sqrt{\alpha_i} e_i)\|_F^2$$

the problem can be solved using the procedure described above, but with $$\alpha_i$$ replaced by $$\sqrt{\alpha_i} e_i$$ and with E replaced by $$E' = [\sqrt{\alpha_1} e_1, \sqrt{\alpha_2} e_2, \cdots, \sqrt{\alpha_n} e_n]$$.

The operating point of the system varies according to different plant conditions, and different operating points correspond to different unknown input matrices, $$E_i(i = 1, 2, \cdots, M)$$. It is attractive to be able design a single FDI scheme for a whole range (or a set) of operating points. The success of the single FDI design depends on its robustness properties. In order to make the disturbance de-coupling hold for all operating points, we must make:

$$HE_i = 0, \quad \text{for} \quad i = 1, 2, \cdots, M$$

or

$$H[E_1 \ E_2 \ \cdots \ E_M] = HP = 0$$

If rank(P) $\leq$ n $-$ p, (18) has solutions and the exact de-coupling at all operating points is achievable. If rank(P) $>$ n $-$ p, approximate de-coupling must be used. This is equivalent to the solution of Eq. (10) and can be solved by defining an optimization problem:

$$\min \|P - P^*\|_F^2$$

$$\exists H \neq 0 \quad HP^* = 0$$

Consider the case when the full-order system model is not available. An identification procedure is used to obtain the nominal model $$\{A_0, B_0, C_0, D_0\}$$ with the estimation error $$\{\Delta A, \Delta B, \Delta C, \Delta D\}$$. Normally, $$\Delta A$$ and $$\Delta B$$ are unknown but bounded:

$$A_1 \leq \Delta A \leq A_2$$

$$B_1 \leq \Delta B \leq B_2$$

where $$A_1, A_2, B_1$$ and $$B_2$$ are known and $$\Delta A \leq A_2$$ denotes that each element of $$\Delta A$$ is not larger than the corresponding element of $$A_2$$. This typifies an unstructured and bounded uncertainty. Consider $$\Delta A$$ and $$\Delta B$$ in a finite set of possibilities, say $$\\{\Delta A_i, \Delta B_i\}(i = 1, 2, \cdots, M)$$ within the interval $$A_1 \leq \Delta A \leq A_2$$ and $$B_1 \leq \Delta B \leq B_2$$. This might involve choosing representative points, reflecting desired weighting on the likelihood or importance of particular sets of parameters. In this situation, a set of unknown input distribution matrices is obtained:

$$E_i = [\Delta A_i, \Delta B_i] \quad i = 1, 2, \cdots, M$$

In order to make the disturbance de-coupling valid for a wide range of model parameter variations, an optimal matrix $$E^*$$ should be made to be near all
\(E_i(i = 1, 2, \cdots, M)\) as closely as possible. The optimization problem is thus defined as:

\[
\min_{\{s.t. \exists H \neq 0 \text{ for } HE^* = 0\}} \|E^* - [E_1 \ E_2 \ \cdots \ E_M]\|_F^2
\]

\(E^*\) is then used to design disturbance de-coupling robust residual generators. As \(E^*\) is close to all \(E_i\), the approximate de-coupling is achieved over whole range of parameter variations.

**AN EXAMPLE**

A thermodynamic simulation model of a jet engine is utilised as an example to illustrate the approach taken. The linearized 17th model is used in the paper. The state variables include pressures, air and gas mass flow rates, shaft speeds, absolute temperatures and static pressure. The nominal operating point is set at 70% of the demanded high spool speed (\(N_{H}\)). The measurements are \(N_L, N_H, T_7, P_6, T_{29}\) (\(N\) denotes compressor shaft speeds, the \(P\) variables denote pressures, whilst \(T\) represents a temperature). The control inputs are: the main engine fuel flow rate and the exhaust nozzle area. For practical reasons and convenience of design, a 5th order model is used to approximate the 17th order model. The model reduction and other errors are represented by the disturbance term \(E_d(t)\). The 5th order model matrices are:

\[
A_0 = \begin{bmatrix}
-0.078 & 0.294 & -0.022 & 0.021 & -0.029 \\
0.007 & -0.028 & 0.002 & -0.002 & 0.003 \\
-1.325 & 5.326 & -0.526 & 0.221 & -0.477 \\
1.081 & -4.445 & 0.377 & -0.463 & 0.403 \\
2.152 & -8.639 & 0.781 & -0.575 & 0.782
\end{bmatrix} \times 10^3
\]

\[
B_0 = \begin{bmatrix}
-0.0072 & 0.0030 \\
0.0035 & 0.0003 \\
1.2185 & -0.0329 \\
1.3225 & 0.0201 \\
-0.0823 & 0.0244
\end{bmatrix}
\]

\[
C_0 = I_{5 \times 5} \quad D_0 = 0_{5 \times 2}
\]

As shown in Section 3, the robust residual generation design procedure based on eigenstructure assignment involves: (1) Computing the matrix \(H\) such that \(HE = 0\) (when the matrix \(E\) has been given), (2) Assignment of the suitable eigenstructure of the observer. The emphasis here is the derivation of the matrix \(E\), hence we are not concerned with the eigenstructure assignment details. We simplify the observer design in order to emphasize the optimal selection of the disturbance distribution matrix. As \(C_0\) is an identity matrix, the observer eigenstructure has full design freedom. As \(C_0\) and \(A_0\) are both full rank matrices, if we choose all eigenvalues to be the same (e.g. \(-100\)), then any row vector is an assignable left eigenvector of the observer. This means that we do not
need to re-assign the observer eigenstructure when using a different matrix $H$. For this case the gain matrix $K = -\left(100I_{5\times5} + A_0\right)$ is fixed in the following discussion.

The matrix $E$ corresponds to structured uncertainty arising from the application of the 5th order observer to the 17th order plant is derived as:

$$E = [E_1 \ E_2 \ E_3 \ E_4] \times 10^3$$

$$E_1 =$$

$$\begin{bmatrix}
0.076 & -0.294 & 0.022 & -0.021 & 0.029 \\
-0.008 & 0.026 & -0.001 & 0.002 & -0.003 \\
1.309 & -5.024 & 0.305 & -0.333 & 0.478 \\
-1.031 & 4.152 & -0.255 & 0.274 & -0.403 \\
-2.146 & 8.637 & -0.787 & 0.611 & -0.842
\end{bmatrix}$$

$$E_2 =$$

$$\begin{bmatrix}
0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.093 & 0.005 & 0.003 \\
0.0 & 0.0 & -0.073 & -0.015 & -0.008 \\
0.0 & 0.0 & 0.0 & -0.001 & -0.002
\end{bmatrix}$$

$$E_3 =$$

$$\begin{bmatrix}
0.0 & 0.004 & -0.003 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.003 & 0.0 & 0.0 & 0.004 & 0.0 \\
-0.001 & 0.0 & 0.0 & -0.013 & 0.0 \\
-0.001 & -0.001 & -0.009 & -0.003 & 0.0
\end{bmatrix}$$

$$E_4 =$$

$$\begin{bmatrix}
0.0 & -0.0013 & 0.0 & 0.0 & 0.0 \\
0.0 & -0.0002 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0269 & 0.0 & 0.0 & 0.0 \\
0.0 & -0.0804 & 0.0 & 0.0 & 0.0 \\
-0.0169 & 0.0025 & 0.0126 & -0.0091 & 0.0
\end{bmatrix}$$

Rank($E$) = 5 = n, and hence Eq. (10) has no solution, the optimization procedure must be employed. The singular values of $E$ are $\{1, 5, 60, 198, 11268\}$, and the matrices $S$ and $T$ are omitted for brevity. Different values of $p$ (the number of independent rows of $H$) give rise to different optimal matrix $E^*$ and the corresponding matrix $H$. Here, we use the simulation to compare the fault detection robustness performance. Fig.1 shows the output estimation error norm which is very large, and cannot be used to detect the fault reliably. Fig. 2 shows the residuals corresponding to different designs. Compared with the output estimation error, the residual is very small, i.e., disturbance de-coupling is
achieved. The diagram also shows that the magnitude of the fault-free residual increases when the independent row number \( p \) increases and the optimal robustness is obtained when \( p = 1 \). In this case: 

\[
E^* = S[\text{diag}(0.5, 0.60, 0.198, 0.11268)_{0 \times 14}]^T \text{ and } H = [0.0813, 0.9966, 0.0099, 0.0025, 0.0043].
\]

Fig. 3 shows the improvement in robustness of the weighting optimization design. The choice of the weighting factor \( \omega_i \) is based on the relative magnitudes of components of the disturbance. The solution is: 

\[
H = [0.1038, 0.9945, 0.0106, 0.0037, 0.0049].
\]

In order to evaluate the power of the robust FDI design, a small fault is added to the exhaust gas temperature measurement (T7); this simulates the effect of an *incipient fault*, here the effect of which is too small to be noticed in the measurements. Fig. 4(a) shows the faulty output of the measurement (T7); the fault is very small compared with the output, and consequently, is not detectable in the measurement. Fig. 4(b) shows the absolute value of the corresponding residual. The fault can be detected easily using the residual.

An ideal FDI scheme should provide robust de-coupling over all operating points of interest. Of course, the disturbance distribution matrix \( E \) will vary corresponding to changes in the operating point. Here, we consider the jet engine operating points set at 70%\( N_H \) and 90%\( N_H \). The procedure given in Section 4 is used to obtain an optimal solution for \( H \) and \( E^* \): 

\[
H = [-0.0578, -0.9982, -0.0132, -0.0048, -0.0044].
\]

This design is also verified by using simulation. Fig. 5 shows the absolute values of the residuals when the jet engine is run at a different operating point. Note that the robustness is clearly demonstrated.

**CONCLUDING DISCUSSION**

One crucial limitation of the disturbance de-coupling approach to robust FDI arises as the distribution(s) of uncertainties acting upon the observer is (are) never known exactly, in practice. As this information is not available compromises have to be met. This paper has described a method to obtain the distribution matrix of disturbances by making use of relationships between the modelled large-scale plant and the reduced observer model to achieve near disturbance de-coupling over the entire operating range. One of the conditions to achieve de-coupling is to find a weighting matrix which is orthogonal to the distribution matrix. In most practical situations, the distribution matrix is a full row rank matrix whose orthogonal complement does not exist. A low rank matrix has been chosen to approximate the original matrix in the Frobenius norm sense, whilst preserving its orthogonal complement. A real 17th order jet engine system has been used to illustrate the method. The simulation shows that approximate de-coupling is achievable, even when the operating points of the non-linear engine system are changed over a typical speed range; this illustrates the strong robustness of this FDI method.

**ACKNOWLEDGMENT**

The authors acknowledge funding support (GR/G2586.3) from the UK Science & Engineering Research Council.
REFERENCES


Captions

Fig.1 Norm of the output estimation error

Fig.2 Residual for different designs

(a) Absolute value of the residual when $p = 1$
(b) Norm of the residual when $p = 2$

Fig.3 Absolute value of the residuals for weighted (dashed line) and no weighted (solid line) design

Fig.4 Faulty output and residual when a fault occurs in the measurement ($T_7$)

(a) Faulty output
(b) Absolute value of the residual

Fig.5 Absolute values of the residuals

(a) Jet engine working at $70\%N_H$
(b) Jet engine working at $90\%N_H$
output estimation error

time (seconds)
faulty pressure measurement

ΔP₆ (KPa)

time (seconds)