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Near-complete teleportation of two-mode four-component entangled coherent states

Jie-Qiao Liao and Le-Man Kuang[1]

Department of Physics, Hunan Normal University, Changsha 410081, People's Republic of China

E-mail: lmkuang@hunnu.edu.cn

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Abstract

In this paper, we propose an optical scheme to almost completely teleport a two-mode four-component entangled coherent state in terms of optical devices such as nonlinear Kerr media, beam splitters, phase shifters and photon detectors. Different from those previous schemes in which exact photon number discrimination is needed, in our scheme one only needs to make 'yes' and 'no' measurements upon the photon numbers in related modes. This scheme can also be understood as an entanglement swapping protocol.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

Quantum teleportation, first proposed by Bennett and coworkers [1], has attracted much attention of both theorists and experimenters in the last decade. It is a disembodied transport of quantum states from a sender (Alice) to a receiver (Bob) through a classical communication channel requiring a prepared quantum entangled channel. The original proposals for quantum teleportation [1, 2] focused on teleporting quantum states of a system with a finite-dimensional (discrete variable) state space, such as the two polarizations of a photon or the discrete levels of an atom. Discrete-variable teleportation has been demonstrated experimentally in optical systems [3, 4], ion trap systems [5] and liquid-state nuclear magnetic resonance systems [6].

In recent years, quantum teleportation has been extended to continuous-variable (CV) cases corresponding to quantum states of infinite-dimensional systems [7–20] such as optical fields or the motion of massive particles like trapped ions. Particularly due to the great progress achieved in the last two decades for CV quantum optical systems, CV quantum states can be well generated, manipulated and detected [21]. And comparing to discrete-variable states coded on the degrees of freedom of single photon polarization or photon number, CV states are robust against the loss of photons. Following the theoretical proposal of [8], CV

[1] Author to whom any correspondence should be addressed.

teleportation has been realized for coherent states of a light field [9] by using entangled two-mode squeezed optical beams produced by parametric down-conversion in a sub-threshold optical parametric oscillator. Although coherent states are continuous and nonorthogonal states, they are very close to classical states. A real challenge for quantum teleportation is to teleport truly nonclassical states like quantum superposition states, squeezed states and entangled states.

Quantum teleportation of entangled coherent states have been widely studied in the past few years [22–26]. Wang [22] proposed a teleportation scheme of two-component entangled coherent states $x|\alpha, \alpha\rangle \pm y|-\alpha, -\alpha\rangle$ in terms of only linear optical elements. In a previous paper [23], we presented a near perfect teleportation protocol of the same quantum states. In our scheme we need only to make 'yes' and 'no' measurements upon the photon numbers in related modes. Consequently, it can increase the experimental feasibility. In fact, the teleportation of two-mode entangled coherent states of type of $x|\alpha, \alpha\rangle \pm y|-\alpha, -\alpha\rangle$ essentially can be realized through teleporting a single-mode superposed coherent state. This is because $x|\alpha, \alpha\rangle \pm y|-\alpha, -\alpha\rangle$ can be disentangled by a equipped beam splitter as $\hat{B}(x|\alpha, \alpha\rangle \pm y|-\alpha, -\alpha\rangle) = (x|\sqrt{2}\alpha\rangle \pm y|-\sqrt{2}\alpha\rangle)|0\rangle$. Using this disentangling property, teleportation of the two-mode entangled coherent states can be achieved by the following three steps. Firstly, we disentangle the entangled coherent states to superposed coherent states as shown above. Secondly, we implement teleportation of the superposed coherent states. Finally, we recover the entangled coherent states from the superposed coherent states through mixing them with a vacuum state in an equipped beam splitter. In this paper, we want to propose a teleportation scheme of another type of two-mode entangled coherent states, i.e., four-component entangled states, which cannot be disentangled into direct product states of two single-mode quantum states. Our scheme below cannot be completed through teleporting a single-mode superposed coherent state. The rest of this paper is organized as follows. In section 2, we present the scheme for quantum teleportation of two-mode four-component entangled coherent states. We conclude the paper with some remarks and discussions in section 3.

2. Teleportation of an entangled coherent state

We now consider quantum teleportation of the following two-mode four-component entangled coherent state,

$$|\phi\rangle_{12} = \mathcal{N}(a_1|\alpha, \alpha\rangle + a_2|\alpha, -\alpha\rangle + a_3|-\alpha, \alpha\rangle + a_4|-\alpha, -\alpha\rangle)_{12}, \tag{1}$$

where the normalization constant $\mathcal{N}$ is given by

$$\begin{aligned}\mathcal{N}^{-2} = [&|a_1|^2 + |a_2|^2 + |a_3|^2 + |a_4|^2 + 2\mathrm{e}^{-2\alpha^2}\,\mathrm{Re}[a_1^*a_3 + a_2^*a_4 + a_1^*a_2 + a_3^*a_4] \\ &+ 2\mathrm{e}^{-4\alpha^2}\,\mathrm{Re}[a_1^*a_4 + a_2^*a_3]].\end{aligned} \tag{2}$$

And $|\alpha\rangle$ is the usual Glauber coherent state defined by

$$|\alpha\rangle = \mathrm{e}^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}}|n\rangle. \tag{3}$$

Throughout this paper, for simplicity, we assume that the coherent amplitude α is real.

We present a schematic diagram for teleportation of the two-mode four-component state given by equation (1) in figure 1. From figure 1, it can be seen that, for realizing this teleportation, we need ten modes of light, four of them are ancillary modes. In addition, two cross Kerr media, four beam splitters and eight photon detectors are needed to realize the present scheme.

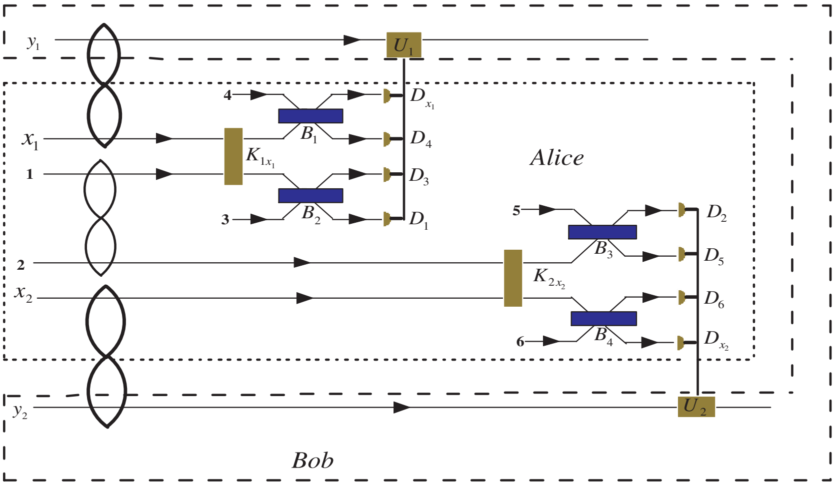

Figure 1. The schematic diagram for teleportation of the arbitrary two-mode quantum state given in equation (1).

We want to teleport the state from modes 1 and 2 on Alice's side to modes y_1 and y_2 on Bob's side. We assume that there are two prepared entangled channel states $|\lambda\rangle_{x_1y_1}$ and $|\lambda\rangle_{x_2y_2}$ between Alice and Bob. The forms of the two channel states are given by

$$|\lambda\rangle_{x_iy_i} = \tfrac{1}{2}[|\alpha\rangle(|\alpha\rangle + |-\alpha\rangle) + |-\alpha\rangle(|\alpha\rangle - |-\alpha\rangle)]_{x_iy_i}, \tag{4}$$

for $i = 1, 2$. Note that modes x_1 and x_2 belong to Alice.

The above entangled channel states are very interesting since the two states $|\pm\alpha\rangle$ of mode x_i are non-orthogonal while the two states $|\alpha\rangle \pm |-\alpha\rangle$ of the other mode y_i are orthogonal. It is well known that coherent states are nonorthogonal, and the fidelity between two coherent states is minus exponential to the square of the distance between the positions of the two states in phase space. So two coherent states having large distance in phase space can be considered approximately as orthogonal. Then we can consider states $|\pm\alpha\rangle$ as a pair of logic basis of a quasi-qubit. From this viewpoint, the states given by equation (4) are similar to two-qubit cluster states [27]. It should be pointed out that these channel states can be generated using cross Kerr interaction.

In what follows, we shall describe the present scheme in detail. First of all, Alice mixes modes 1 and x_1 in cross Kerr medium $\hat{K}_{1x_1}$. The Hamiltonian of a cross-Kerr interaction involving modes a and b can be written as [28]

$$\hat{H} = -\chi\hat{a}^\dagger\hat{a}\hat{b}^\dagger\hat{b}, \tag{5}$$

which leads to the following unitary evolution operator

$$\hat{K}_{ab}(\tau) = \mathrm{e}^{\mathrm{i}\tau\hat{a}^\dagger\hat{a}\hat{b}^\dagger\hat{b}}, \tag{6}$$

where $\tau = \chi t$ with t is the evolution time, $\hat{a}^\dagger(\hat{b}^\dagger)$ and $\hat{a}(\hat{b})$ are the creation and annihilation operators for modes $a(b)$, respectively. It is easy to see that when $\chi t = \pi$ the action of the cross Kerr unitary evolution operator on two modes with the coherent-state input $|\alpha\rangle_a|\beta\rangle_b$ is given by

$$\hat{K}_{ab}(\pi)|\alpha\rangle_a|\beta\rangle_b = \tfrac{1}{2}[|\alpha\rangle_a(|\beta\rangle_b + |-\beta\rangle_b) + |-\alpha\rangle_a(|\beta\rangle_b - |-\beta\rangle_b)]. \tag{7}$$

From the above equation, it is straightforward to see that the channel states (4) is just the above state with $\beta = \alpha$.

After passing the cross-Kerr interaction $\hat{K}_{1x_1}(\pi)$, the initial state $|\phi\rangle_{12}|\lambda\rangle_{x_1y_1}$ of modes $1, 2, x_1$ and y_1 becomes

$$\begin{aligned}|\Psi\rangle_{12x_1y_1} &= \hat{K}_{1x_1}|\phi\rangle_{12}|\lambda\rangle_{x_1y_1}\\ &= \tfrac{1}{2}\mathcal{N}\big\{|\alpha\rangle_1|\alpha\rangle_{x_1}\big[(a_1|\alpha\rangle + a_2|-\alpha\rangle)_2|\alpha\rangle_{y_1} + (a_3|\alpha\rangle + a_4|-\alpha\rangle)_2|-\alpha\rangle_{y_1}\big]\\ &\quad + |\alpha\rangle_1|-\alpha\rangle_{x_1}\big[(a_1|\alpha\rangle + a_2|-\alpha\rangle)_2|\alpha\rangle_{y_1} - (a_3|\alpha\rangle + a_4|-\alpha\rangle)_2|-\alpha\rangle_{y_1}\big]\\ &\quad + |-\alpha\rangle_1|\alpha\rangle_{x_1}\big[(a_3|\alpha\rangle + a_4|-\alpha\rangle)_2|\alpha\rangle_{y_1} + (a_1|\alpha\rangle + a_2|-\alpha\rangle)_2|-\alpha\rangle_{y_1}\big]\\ &\quad + |-\alpha\rangle_1|-\alpha\rangle_{x_1}\big[(a_3|\alpha\rangle + a_4|-\alpha\rangle)_2|\alpha\rangle_{y_1} - (a_1|\alpha\rangle + a_2|-\alpha\rangle)_2|-\alpha\rangle_{y_1}\big]\big\}. \end{aligned} \quad (8)$$

From the above equation, we can see that, if we measure the states of modes 1 and x_1, the information coded in the coefficients before the four basis states $|\alpha\rangle_1|\alpha\rangle_2$, $|\alpha\rangle_1|-\alpha\rangle_2$, $|-\alpha\rangle_1|\alpha\rangle_2$ and $|-\alpha\rangle_1|-\alpha\rangle_2$ of modes 1 and 2 can be transferred to modes y_1 and 2, and the states of modes y_1 and 2 will depend on the states of modes 1 and x_1 which will be measured by Alice. For realizing this measurement, Alice introduces two ancillary modes 3 and 4 which are initially prepared in state $|\alpha, \alpha\rangle_{34}$, and then mixes modes x_1 and 4 in beam splitter $\hat{B}_1$, modes 1 and 3 in beam splitter $\hat{B}_2$, respectively. Note that the beam splitters we used are equipped by a usual 50/50 one and a pair of $\pi/2$ phase shifters. The usual 50/50 beam splitter transform on modes i and j is described by $\hat{\mathcal{B}}_{ij} = \exp\big[\mathrm{i}(\pi/4)\big(\hat{a}_i^\dagger\hat{a}_j + \hat{a}_j^\dagger\hat{a}_i\big)\big]$. When a usual beam splitter is equipped together with a pair of $\pi/2$ phase shifters described by the unitary operator $\hat{P}_j = \exp\big(-\mathrm{i}\pi\hat{a}_j^\dagger\hat{a}_j/2\big)$ with j being mode label, then the total unitary operator of the equipped beam splitter can be written as

$$\hat{B}_{ij} = \hat{P}_j\hat{\mathcal{B}}_{ij}\hat{P}_j, \quad (9)$$

which transforms the state $|\alpha, \beta\rangle_{ij}$ as

$$\hat{B}_{i,j}|\alpha\rangle_i|\beta\rangle_j = |(\alpha+\beta)/\sqrt{2}\rangle_i|(\alpha-\beta)/\sqrt{2}\rangle_j. \quad (10)$$

After passing the two equipped beam splitters $\hat{B}_1$ and $\hat{B}_2$, the output state becomes

$$\begin{aligned}|\Psi'\rangle_{12x_1y_134} &= \hat{B}_1\hat{B}_2\hat{K}_{1x_1}|\phi\rangle_{12}|\lambda\rangle_{x_1y_1}|\alpha\rangle_3|\alpha\rangle_4\\ &= \tfrac{1}{2}\mathcal{N}\big\{|\sqrt{2}\alpha\rangle_1|0\rangle_3|\sqrt{2}\alpha\rangle_{x_1}|0\rangle_4\big[(a_1|\alpha\rangle + a_2|-\alpha\rangle)_2|\alpha\rangle_{y_1}\\ &\quad + (a_3|\alpha\rangle + a_4|-\alpha\rangle)_2|-\alpha\rangle_{y_1}\big] + |\sqrt{2}\alpha\rangle_1|0\rangle_3|0\rangle_{x_1}|-\sqrt{2}\alpha\rangle_4\big[(a_1|\alpha\rangle\\ &\quad + a_2|-\alpha\rangle)_2|\alpha\rangle_{y_1} - (a_3|\alpha\rangle + a_4|-\alpha\rangle)_2|-\alpha\rangle_{y_1}\big]\\ &\quad + |0\rangle_1|-\sqrt{2}\alpha\rangle_3|\sqrt{2}\alpha\rangle_{x_1}|0\rangle_4\big[(a_3|\alpha\rangle + a_4|-\alpha\rangle)_2|\alpha\rangle_{y_1}\\ &\quad + (a_1|\alpha\rangle + a_2|-\alpha\rangle)_2|-\alpha\rangle_{y_1}\big] + |0\rangle_1|-\sqrt{2}\alpha\rangle_3|0\rangle_{x_1}|-\sqrt{2}\alpha\rangle_4\big[(a_3|\alpha\rangle\\ &\quad + a_4|-\alpha\rangle)_2|\alpha\rangle_{y_1} - (a_1|\alpha\rangle + a_2|-\alpha\rangle)_2|-\alpha\rangle_{y_1}\big]\big\}. \end{aligned} \quad (11)$$

It is well known that coherent states are superposition of number states. From the above state (11), it is easy to see that we can discriminate the four states $|\alpha\rangle_1|\alpha\rangle_{x_1}$, $|\alpha\rangle_1|-\alpha\rangle_{x_1}$, $|-\alpha\rangle_1|\alpha\rangle_{x_1}$ and $|-\alpha\rangle_1|-\alpha\rangle_{x_1}$ from the following responses of the detectors D_1, D_3, D_4 and D_{x_1} whose photon number detected denoted by n_1, n_3, n_4 and n_{x_1},

$$n_1 > 0, \qquad n_{x_1} > 0, \qquad n_3 = n_4 = 0, \quad (12)$$

$$n_1 > 0, \qquad n_4 > 0, \qquad n_3 = n_{x_1} = 0, \quad (13)$$

$$n_3 > 0, \qquad n_{x_1} > 0, \qquad n_1 = n_4 = 0, \quad (14)$$

$$n_1 > 0, \qquad n_{x_1} > 0, \qquad n_3 = n_4 = 0. \quad (15)$$

Here we have neglected other groups of responses of the detectors D_1, D_3, D_4 and D_{x_1}, because we cannot discriminate the four states shown above for these cases. Alice records the above measurement outcomes and then tell them to Bob via a classical communication channel.

For the case of equation (12), modes 2 and y_1 collapse to state

$$|\varphi\rangle_{2y_1} = \mathcal{N}\big[(a_1|\alpha\rangle + a_2|-\alpha\rangle)_2|\alpha\rangle_{y_1} + (a_3|\alpha\rangle + a_4|-\alpha\rangle)_2|-\alpha\rangle_{y_1}\big], \tag{16}$$

with the probability being given by

$$\begin{aligned} P_1 &= \sum_{n,m=1}^{\infty} \big|{}_1\langle n|_3\langle 0|_{x_1}\langle m|_4\langle 0|\Psi'\rangle_{12x_1y_134}\big|^2 \\ &= \frac{1}{4}\big(1 - \mathrm{e}^{-2\alpha^2}\big)^2 \end{aligned} \tag{17}$$

Corresponding to the case of equation (13), the state of modes 2 and y_1 is

$$|\varphi'\rangle_{2y_1} = \mathcal{M}\big[(a_1|\alpha\rangle + a_2|-\alpha\rangle)_2|\alpha\rangle_{y_1} - (a_3|\alpha\rangle + a_4|-\alpha\rangle)_2|-\alpha\rangle_{y_1}\big], \tag{18}$$

with the normalization constant

$$\begin{aligned} \mathcal{M}^{-2} = {}&[|a_1|^2 + |a_2|^2 + |a_3|^2 + |a_4|^2 + 2\mathrm{e}^{-2\alpha^2}\,\mathrm{Re}[a_1^*a_2 + a_3^*a_4 - a_1^*a_3 - a_2^*a_4] \\ &- 2\mathrm{e}^{-4\alpha^2}\,\mathrm{Re}[a_4^*a_1 + a_3^*a_2]]. \end{aligned} \tag{19}$$

It is easy to obtain the corresponding probability as follows:

$$\begin{aligned} P_2 &= \sum_{n,m=1}^{\infty} \big|{}_1\langle n|_3\langle 0|_{x_1}\langle 0|_4\langle m|\Psi'\rangle_{12x_1y_134}\big|^2 \\ &= \frac{\mathcal{N}^2}{4\mathcal{M}^2}\big(1 - \mathrm{e}^{-2\alpha^2}\big)^2. \end{aligned} \tag{20}$$

For the above two cases (12) and (13), Bob needs to do nothing. However, corresponding to the outcomes of cases (14) and (15), what Bob should do is to transform respectively the states of modes 2 and y_1 to states (16) and (18) by a phase shifter $\hat{P}_{y_1} = \exp(\mathrm{i}\pi\hat{a}^\dagger\hat{a})$ whose action on coherent states is $\hat{P}_{y_1}|\pm\alpha\rangle_{y_1} = |\mp\alpha\rangle_{y_1}$. And the probabilities corresponding to cases of equations (14) and (15) can be obtained as $P_3 = P_1$ and $P_4 = P_2$ in that $\hat{P}_{y_1}$ is a unitary operator. Note that the above phase shifter with parameter π acts effectively as σ_x operator on coherent inputs of the form $|\pm\alpha\rangle$ a two-dimensional quasi-qubit space.

After finishing the above operation, the states of modes 2 and y_1 are either of the states (16) and (18). And then Alice mixes modes 2 and x_2 in the cross-Kerr interaction $\hat{K}_{2x_2}$. First of all, we consider the case of state (16) of modes 2 and y_1. After passing the cross-Kerr interaction $\hat{K}_{2x_2}$, the initial state $|\varphi\rangle_{2y_1}|\lambda\rangle_{x_2y_2}$ of modes 2, y_1, x_2 and y_2 becomes

$$\begin{aligned} |\Phi\rangle_{2y_1x_2y_2} &= \hat{K}_{2x_2}|\varphi\rangle_{2y_1}|\lambda\rangle_{x_2y_2} \\ &= \tfrac{1}{2}\mathcal{N}\big\{|\alpha\rangle_2|\alpha\rangle_{x_2}\big[(a_1|\alpha\rangle + a_3|-\alpha\rangle)_{y_1}|\alpha\rangle_{y_2} + (a_2|\alpha\rangle + a_4|-\alpha\rangle)_{y_1}|-\alpha\rangle_{y_2}\big] \\ &\quad + |\alpha\rangle_2|-\alpha\rangle_{x_2}\big[(a_1|\alpha\rangle + a_3|-\alpha\rangle)_{y_1}|\alpha\rangle_{y_2} - (a_2|\alpha\rangle + a_4|-\alpha\rangle)_{y_1}|-\alpha\rangle_{y_2}\big] \\ &\quad + |-\alpha\rangle_2|\alpha\rangle_{x_2}\big[(a_2|\alpha\rangle + a_4|-\alpha\rangle)_{y_1}|\alpha\rangle_{y_2} + (a_1|\alpha\rangle + a_3|-\alpha\rangle)_{y_1}|-\alpha\rangle_{y_2}\big] \\ &\quad + |-\alpha\rangle_2|-\alpha\rangle_{x_2}\big[(a_2|\alpha\rangle + a_4|-\alpha\rangle)_{y_1}|\alpha\rangle_{y_2} - (a_1|\alpha\rangle + a_3|-\alpha\rangle)_{y_1}|-\alpha\rangle_{y_2}\big]\big\}. \end{aligned} \tag{21}$$

From the above equation we can see that, depending on both the discrimination of the states of modes 2 and x_2 and the appropriate operations on mode y_2, we can realize near perfect quantum teleportation of quantum states given by equation (1). Using the same method shown above, we introduce another two modes 5 and 6 which initially prepared in state $|\alpha, \alpha\rangle_{56}$ and mix modes 2 and 5, x_2 and 6 in the two equipped beam splitters B_3 and B_4, respectively. And then we obtain the following state:

$$\begin{aligned}|\Phi'\rangle_{2y_1x_2y_256} &= \hat{B}_3\hat{B}_4\hat{K}_{2x_2}|\varphi\rangle_{2y_1}|\lambda\rangle_{x_2y_2}|\alpha\rangle_5|\alpha\rangle_6\\
&= \tfrac{1}{2}\mathcal{N}\big\{|\sqrt{2}\alpha\rangle_2|0\rangle_5|\sqrt{2}\alpha\rangle_{x_2}|0\rangle_6\big[(a_1|\alpha\rangle+a_3|-\alpha\rangle)_{y_1}|\alpha\rangle_{y_2}\\
&\quad+(a_2|\alpha\rangle+a_4|-\alpha\rangle)_{y_1}|-\alpha\rangle_{y_2}\big]+|\sqrt{2}\alpha\rangle_2|0\rangle_5|0\rangle_{x_2}|-\sqrt{2}\alpha\rangle_6\big[(a_1|\alpha\rangle\\
&\quad+a_3|-\alpha\rangle)_{y_1}|\alpha\rangle_{y_2}-(a_2|\alpha\rangle+a_4|-\alpha\rangle)_{y_1}|-\alpha\rangle_{y_2}\big]\\
&\quad+|0\rangle_2|-\sqrt{2}\alpha\rangle_5|\sqrt{2}\alpha\rangle_{x_2}|0\rangle_6\big[(a_2|\alpha\rangle+a_4|-\alpha\rangle)_{y_1}|\alpha\rangle_{y_2}+(a_1|\alpha\rangle\\
&\quad+a_3|-\alpha\rangle)_{y_1}|-\alpha\rangle_{y_2}\big]+|0\rangle_2|-\sqrt{2}\alpha\rangle_5|0\rangle_{x_2}|-\sqrt{2}\alpha\rangle_6\big[(a_2|\alpha\rangle\\
&\quad+a_4|-\alpha\rangle)_{y_1}|\alpha\rangle_{y_2}-(a_1|\alpha\rangle+a_3|-\alpha\rangle)_{y_1}|-\alpha\rangle_{y_2}\big]\big\}. \end{aligned} \tag{22}$$

It is straightforward to see that the following records of the photon detectors correspond to perfect or near perfect teleportation via appropriate transform,

$$n_2>0, \qquad n_{x_2}>0, \qquad n_5=n_6=0, \tag{23}$$

$$n_2>0, \qquad n_6>0, \qquad n_5=n_{x_2}=0, \tag{24}$$

$$n_5>0, \qquad n_{x_2}>0, \qquad n_2=n_6=0, \tag{25}$$

$$n_5>0, \qquad n_6>0, \qquad n_2=n_{x_2}=0, \tag{26}$$

where we have denoted the detected photon number of detectors D_2, D_{x_2}, D_5 and D_6 as n_2, n_{x_2}, n_5 and n_6, respectively. For the case of outcome given by equation (23), the state given by equation (1) is teleported perfectly from modes 1 and 2 to modes y_1 and y_2. It happens with the following probability:

$$\begin{aligned}P_5 &= \sum_{n,m=1}^{\infty}\left|{}_2\langle n|_5\langle 0|_{x_2}\langle m|_6\langle 0|\Phi'\rangle_{2y_1x_2y_256}\right|^2\\
&= \frac{1}{4}\left(1-\mathrm{e}^{-2\alpha^2}\right)^2.\end{aligned} \tag{27}$$

If the response of detectors D_2, D_5, D_6 and D_{x_2} is given by case (24), however, we can see that in order to realize a near-perfect teleportation of the entangled coherent states one has to require the following state transform:

$$\begin{aligned}|\mu\rangle_{y_1y_2} &= \mathcal{K}\big[(a_1|\alpha\rangle+a_3|-\alpha\rangle)_{y_1}|\alpha\rangle_{y_2}-(a_2|\alpha\rangle+a_4|-\alpha\rangle)_{y_1}|-\alpha\rangle_{y_2}\big],\\
&\Rightarrow |\phi\rangle_{y_1y_2}=\mathcal{N}\big[(a_1|\alpha\rangle+a_3|-\alpha\rangle)_{y_1}|\alpha\rangle_{y_2}+(a_2|\alpha\rangle+a_4|-\alpha\rangle)_{y_1}|-\alpha\rangle_{y_2}\big].\end{aligned} \tag{28}$$

Here the normalization constant $\mathcal{K}$ is introduced,

$$\begin{aligned}\mathcal{K}^{-2} &= [|a_1|^2+|a_2|^2+|a_3|^2+|a_4|^2+2\mathrm{e}^{-2\alpha^2}\,\mathrm{Re}[a_1^*a_3+a_2^*a_4-a_1^*a_2-a_3^*a_4]\\
&\quad-2\mathrm{e}^{-4\alpha^2}\,\mathrm{Re}[a_1^*a_4+a_2^*a_3]]\end{aligned} \tag{29}$$

and we can calculate the corresponding probability of case (24) as

$$\begin{aligned}P_6 &= \sum_{n,m=1}^{\infty}\left|{}_2\langle n|_5\langle 0|_{x_2}\langle 0|_6\langle m|\Phi'\rangle_{2y_1x_2y_256}\right|^2\\
&= \frac{\mathcal{N}^2}{4\mathcal{K}^2}\left(1-\mathrm{e}^{-2\alpha^2}\right)^2.\end{aligned} \tag{30}$$

We can further see that equation (28) implies the following transform between coherent states of mode y_2,

$$|-\alpha\rangle \to -|-\alpha\rangle, \qquad |\alpha\rangle \to |\alpha\rangle, \tag{31}$$

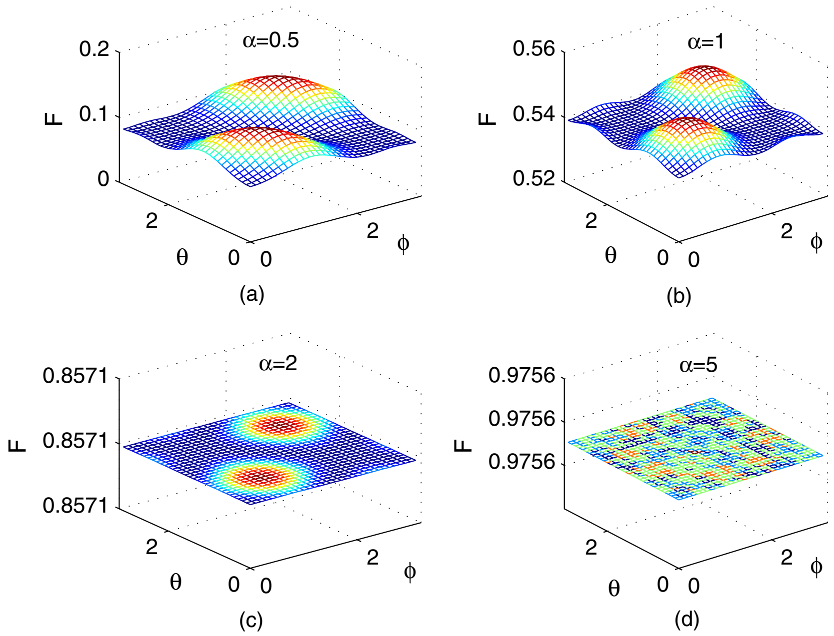

Figure 2. The fidelity of the displacement transform as a function of parameters θ and ϕ with respect to different values of α as shown.

which is generally not a unitary transform except for the limit case of $|\alpha| \to \infty$. In order to do this, we use the displacement operator with the parameter value $\mathrm{i}\pi/(4\alpha)$ to approximately realize the transform given by equation (31).

$$\begin{aligned}|\eta\rangle_{y_1y_2} &= \hat{D}_{y_2}\left(\frac{\mathrm{i}\pi}{4\alpha}\right)\mathcal{K}\big[(a_1|\alpha\rangle + a_3|-\alpha\rangle)_{y_1}|\alpha\rangle_{y_2} - (a_2|\alpha\rangle + a_4|-\alpha\rangle)_{y_1}|-\alpha\rangle_{y_2}\big]\\ &= \mathrm{e}^{\mathrm{i}\frac{\pi}{4}}\mathcal{K}\left[(a_1|\alpha\rangle + a_3|-\alpha\rangle)_{y_1}\left|\frac{\mathrm{i}\pi}{4\alpha}+\alpha\right\rangle_{y_2} + \mathrm{i}(a_2|\alpha\rangle + a_4|-\alpha\rangle)_{y_1}\left|\frac{\mathrm{i}\pi}{4\alpha}-\alpha\right\rangle_{y_2}\right]. \qquad (32)\end{aligned}$$

where the displacement operator $\hat{D}(\mathrm{i}\pi/4\alpha)$ can be effectively performed using a beam splitter with the transmission coefficient T close to unity and a high-intensity coherent field [29].

For seeing the quality of this transform, we calculate the fidelity of the state (32) with respect to the target state (1) as follows:

$$\begin{aligned}F &= |\langle\phi|\eta\rangle|^2\\ &= |\mathcal{N}|^2|\mathcal{K}|^2\exp\left(-\frac{\pi^2}{16\alpha^2}\right)\big||a_1|^2 + |a_2|^2 + |a_3|^2 + |a_4|^2\\ &\quad + 2\mathrm{e}^{-2\alpha^2}(\mathrm{Re}[a_1^*a_3 + a_2^*a_4] + \mathrm{Im}[a_1a_2^* + a_3a_4^*]) + 2\mathrm{e}^{-4\alpha^2}\,\mathrm{Im}[a_2^*a_3 + a_4^*a_1]\big|^2 \qquad (33)\end{aligned}$$

Without loss of generality, we let $a_1 = \cos\theta$, $a_2 = \sin\theta$, $a_3 = \cos\phi$ and $a_4 = \sin\phi$, and in figure 2 we plot the fidelity of the above displacement transform as a function of parameters θ and ϕ with respect to $\alpha = 0.5, 1, 2$ and 5, respectively. Note that for a two-mode four-component quantum state, two introduced parameters θ and φ are not enough to describe it; however our motivation is only to investigate the fidelity of the displacement transform given above and the average fidelity which will be given later, hence from this standpoint the above suggestion is reasonable.

From figure 2 we can see that this fidelity increases with the increase of coherent amplitude α. We find that the fidelity approaches to 1 for $\alpha \geqslant 5$. And the fidelity has some dependence of the superposed coefficients in the small α regime, it becomes stable with the increase of the coherent amplitude α.

Moreover, corresponding to cases (25) and (26), the states of modes y_1 and y_2 can be transformed to modes corresponding to cases (23) and (24) by acting a π phase shifter on mode y_2, respectively. And the probabilities of cases (25) and (26) are also the same as the probabilities of cases (23) and (24).

Now, we return to equation (18) again. Using the same method we can obtain the following state corresponding to equation (21):

$$\begin{aligned}|\Phi\rangle_{2y_1x_2y_2} &= \hat{K}_{2x_2}|\varphi'\rangle_{2y_1}|\lambda\rangle_{x_2y_2} \\ &= \tfrac{1}{2}\mathcal{M}\big\{|\alpha\rangle_2|\alpha\rangle_{x_2}\big[(a_1|\alpha\rangle + a_3|-\alpha\rangle)_{y_1}|\alpha\rangle_{y_2} - (a_2|\alpha\rangle + a_4|-\alpha\rangle)_{y_1}|-\alpha\rangle_{y_2}\big] \\ &\quad + |\alpha\rangle_2|-\alpha\rangle_{x_2}\big[(a_1|\alpha\rangle + a_3|-\alpha\rangle)_{y_1}|\alpha\rangle_{y_2} + (a_2|\alpha\rangle + a_4|-\alpha\rangle)_{y_1}|-\alpha\rangle_{y_2}\big] \\ &\quad - |-\alpha\rangle_2|\alpha\rangle_{x_2}\big[(a_2|\alpha\rangle + a_4|-\alpha\rangle)_{y_1}|\alpha\rangle_{y_2} - (a_1|\alpha\rangle + a_3|-\alpha\rangle)_{y_1}|-\alpha\rangle_{y_2}\big] \\ &\quad - |-\alpha\rangle_2|-\alpha\rangle_{x_2}\big[(a_2|\alpha\rangle + a_4|-\alpha\rangle)_{y_1}|\alpha\rangle_{y_2} + (a_1|\alpha\rangle + a_3|-\alpha\rangle)_{y_1}|-\alpha\rangle_{y_2}\big]\big\}, \end{aligned} \tag{34}$$

which indicates that by acting a π phase shifter on mode x_2, we can transform the above equation to a new one which is different from equation (21) only at the sign before the last two terms. From the point of view of performing the teleportation protocol, this sign difference shown above will make no difference. So the following process is the same as what we used for equation (16) previously.

Based on the above discussions, we can summarize the process for teleportation of the two-mode four-component entangled coherent state by dividing it into two steps. The first step implements teleportation of the state (1) from modes 1 and 2 to modes y_1 and 2, this process has four possible outcomes corresponding to cases given by equations (12), (13), (14) and (15). On one hand, for the cases given by equations (12) and (14), the state of modes 1 and 2 is perfectly teleported to modes 1 and y_1 with the probability P_1. Then the second step realizes teleportation of the state from modes y_1 and 2 to modes y_1 and y_2. This process has also four cases given by equations (23), (24), (25) and (26). Cases given by equations (23) and (25) correspond to perfect teleportation with the probability P_5. However, cases given by equations (24) and (26) correspond to approximate teleportation with the probability P_6, and the fidelity between the approximate state (32) and the state (1) is given by equation (33). On the other hand, for the cases given by equations (13) and (15), the state (18) of modes 2 and y_1 is recovered. Though the state (18) is different from state (16), the second step realizing teleportation of the state from modes y_1 and 2 to modes y_1 and y_2 has the same effect as that corresponding to cases given by equations (12) and (14). In short, for the given teleported state (1) held by the sender Alice, the receiver Bob may have several output states corresponding to different projective measurements, different states may have different fidelities. In order to evaluate the quality of the whole process, we introduce the average fidelity which is defined through summating the product of the fidelity of each output state and corresponding probability.

$$\begin{aligned} F_{\mathrm{av}} &= 4(P_1 + P_2)(P_5 + P_6F) \\ &= \frac{1}{4}\left(1 + \frac{\mathcal{N}^2}{\mathcal{M}^2}\right)\left(1 - \mathrm{e}^{-2\alpha^2}\right)^4\left(1 + \frac{\mathcal{N}^2}{\mathcal{K}^2}F\right). \end{aligned} \tag{35}$$

Here $\mathcal{KM}$ and $\mathcal{N}$ are normalization constants given by equations (29), (19) and (2), respectively. And F is the fidelity given by equation (33). We plot the average fidelity F_{av} as a function of parameters θ and ϕ with respect to $\alpha = 0.5$, 1, 2 and 5 in figure 3.

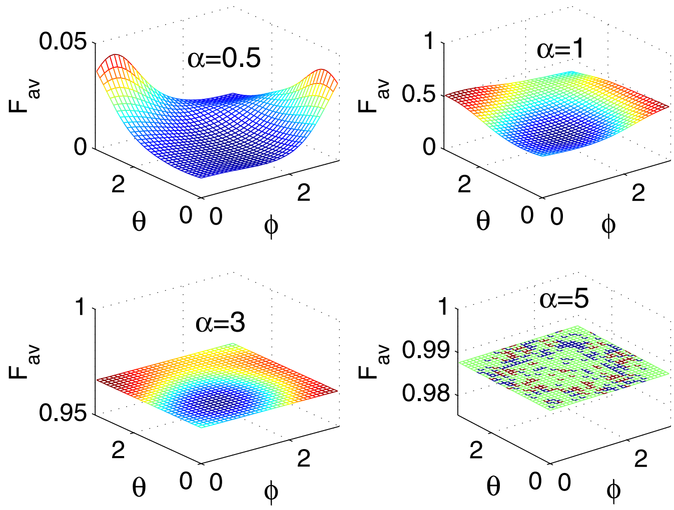

Figure 3. The average fidelity of the whole teleportation process as a function of parameters θ and ϕ with respect to different values of α as shown.

From figure 3 we can see that the average fidelity increases with the increase of coherent amplitude α, it approaches to unity for $\alpha \geqslant 5$. And for small coherent amplitude α, the fidelity is dependent on the superposed coefficients, with the increase of α, the fidelity becomes independent of the superposed coefficients. So our scheme has well stability in the large α regime. This is because states $|\pm\alpha\rangle$ with large α are approximately orthogonal, and then we can discriminate them with a probability approaching 1. In addition, with the increase of α, the displacement transform $\hat{D}_{y_2}$ corresponds approximately to an effective σ_z operation in the quasi-qubit space with basis states $|\pm\alpha\rangle$.

On the other hand, because the main motivation of this scheme is to teleport entanglement, we should also discuss the quality of entanglement transfer in this teleportation protocol. For the cases of perfect teleportation, the state is perfectly teleported, consequently, the entanglement is completely transferred. For the cases of approximate teleportation, however, the state (32) is only approximately recovered by Bob, so the entanglement transfer corresponding to approximate teleportation is not complete. In fact, we can evaluate the quality of entanglement transfer for this approximate teleportation process by calculating the ratio of the degree of entanglement of output state (32) to that of the input state (1). In what follows, we will firstly calculate the degree of entanglement of the states (1) and (32) in terms of concurrence [30].

For investigating the concurrence of the state (1), we introduce the Schrödinger cat states as follows:

$$\begin{aligned} |\alpha_+\rangle &= N_+(|\alpha\rangle + |-\alpha\rangle), & N_+^{-2} &= 2\left(1 + \mathrm{e}^{-2|\alpha|^2}\right); \\ |\alpha_-\rangle &= N_-(|\alpha\rangle - |-\alpha\rangle), & N_-^{-2} &= 2\left(1 - \mathrm{e}^{-2|\alpha|^2}\right). \end{aligned} \tag{36}$$

It is well known that the above two Schrödinger cat states are orthogonal to each other. Hence, we can use them to express the state (1) as follows:

$$\begin{aligned} |\phi\rangle = \frac{\mathcal{N}}{4}\bigg[&\frac{(a_1 + a_2 + a_3 + a_4)}{N_+^2}|\alpha_+\rangle|\alpha_+\rangle + \frac{(a_1 - a_2 + a_3 - a_4)}{N_+N_-}|\alpha_+\rangle|\alpha_-\rangle \\ &+ \frac{(a_1 + a_2 - a_3 - a_4)}{N_-N_+}|\alpha_-\rangle|\alpha_+\rangle + \frac{(a_1 - a_2 - a_3 + a_4)}{N_-^2}|\alpha_-\rangle|\alpha_-\rangle\bigg]. \end{aligned} \tag{37}$$

Similarly, for the state (32), we use the Schrödinger cat states defined in equation (36) for mode y_1 and introduce the following orthogonal basis for mode y_2,

$$|0\rangle = \left|\frac{\mathrm{i}\pi}{4\alpha} + \alpha\right\rangle, \qquad |1\rangle = \frac{1}{\sqrt{1-|p|^2}}\left(\left|\frac{\mathrm{i}\pi}{4\alpha} - \alpha\right\rangle - p\left|\frac{\mathrm{i}\pi}{4\alpha} + \alpha\right\rangle\right),$$
$$p = \left\langle\frac{\mathrm{i}\pi}{4\alpha} + \alpha\,\middle|\,\frac{\mathrm{i}\pi}{4\alpha} - \alpha\right\rangle = \mathrm{i}\,\mathrm{e}^{-2\alpha^2}. \tag{38}$$

So the state (32) can be written as

$$|\eta\rangle = \frac{\mathcal{K}}{2}\left\{\frac{[a_1 + a_3 + \mathrm{i}(a_2 + a_4)p]}{N_+}|\alpha_+\rangle|0\rangle + \frac{[a_1 - a_3 + \mathrm{i}(a_2 - a_4)p]}{N_-}|\alpha_-\rangle|0\rangle \right.$$
$$\left. + \frac{\mathrm{i}(a_2 + a_4)\sqrt{1-|p|^2}}{N_+}|\alpha_+\rangle|1\rangle + \frac{\mathrm{i}(a_2 - a_4)\sqrt{1-|p|^2}}{N_-}|\alpha_-\rangle|1\rangle\right\}. \tag{39}$$

It is clear that both states (37) and (39) are two-qubit pure states. It has been shown that for a pure state

$$|\psi\rangle = \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle, \tag{40}$$

the degree of entanglement can be measured by the concurrence with the following expression [31],

$$C = 2|\alpha\delta - \beta\gamma|. \tag{41}$$

Using the above formula, we obtain the concurrence of states (37) and (39),

$$C_1 = \frac{\mathcal{N}^2}{2N_+^2 N_-^2}|a_1 a_4 - a_2 a_3|,$$
$$C_2 = \frac{\mathcal{K}^2}{\mathcal{N}^2}C_1, \tag{42}$$

where $\mathcal{N}$, $\mathcal{K}$, N_+ and N_- have been given before. Equation (42) indicates that for the cases of approximate teleportation, the ratio of the concurrence of the input state to that of the output state is $\mathcal{K}^2/\mathcal{N}^2$. And similar to the arguments given for defining the average fidelity before, we can also define the quality of entanglement transfer for the whole process by substituting the ratio of concurrence C_2/C_1 for the fidelity F in equation (35),

$$Q_{\mathrm{et}} = 4(P_1 + P_2)\left(P_5 + P_6\frac{C_2}{C_1}\right)$$
$$= \frac{1}{2}\left(1 + \frac{\mathcal{N}^2}{\mathcal{M}^2}\right)\left(1 - \mathrm{e}^{-2\alpha^2}\right)^4. \tag{43}$$

It is clear to see that the quality of entanglement transfer Q_{et} is better than the average fidelity. In figure 4, we plot the quality of entanglement transfer Q_{et} as a function of parameters θ and ϕ with respect to different coherent amplitude α as shown in figure 4.

From figure 4, we can see that the quality of entanglement transfer of the present protocol increases with the coherent amplitude α. The quality of entanglement transfer $Q_{\mathrm{et}} \simeq 1$ for $\alpha > 2$.

Depending on the above discussions, we can see that the whole teleportation process can also be understood as a protocol of entanglement swapping. We assume that the entangled state $|\lambda\rangle_{x_1 y_1}$ of modes x_1 and y_1 is held respectively by Alice and Bob while the entangled state $|\lambda\rangle_{x_2 y_2}$ of modes x_2 and y_2 belongs respectively to Alice and Claire. And the mission of entanglement swapping is to entangle the two modes y_1 and y_2 which maybe never interact each other by doing joint measurement on modes x_1 and x_2. Based on the above discussions, we can see that entanglement swapping protocol can also be implemented.

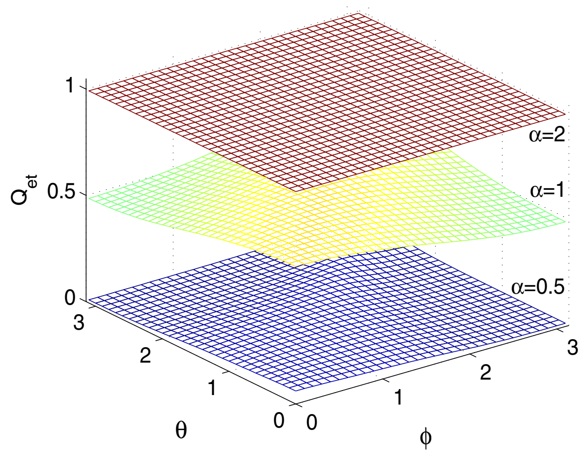

Figure 4. The quality of entanglement transfer $Q_{\rm et}$ as a function of parameters θ and ϕ with respect to the coherent amplitude $\alpha = 0.5$, 1 and 2.

3. Concluding remarks

In conclusion, we have proposed an optical scheme for the quantum teleportation of a two-mode four-component entangled coherent state by using optical elements such as nonlinear Kerr media, beam splitters, phase shifters and photon detectors. We have calculated the average fidelity of our protocol. It has been shown that the average fidelity is generally dependent on the coherent amplitude and the superposed coefficients of the teleported states. Although for small coherent amplitude α, the average fidelity for the whole teleportation process is low and depends on the transposed coefficients, with the increase of α, the average fidelity increases and becomes independent of the superposed coefficients. This average fidelity approaches to unity when $\alpha \geqslant 5$. On the other hand, we have also investigated the quality of entanglement transfer of the present scheme, we found that the quality of entanglement transfer increased with the increase of the coherent amplitude α, and it approached to unity for $\alpha \geqslant 2$. Hence, this is a near-complete scheme. In addition, different from some previous optical schemes of teleportation of entangled coherent states which is only superposition of two logic basis $\{|\alpha, \alpha\rangle, |-\alpha, -\alpha\rangle\}$ of two modes of light, our scheme considered the teleportation of two-mode four-component entangled coherent states. In addition, in those previous schemes, one has to make the exact photon-number measurements to distinguish between even and odd photons. However, in our protocol what we need is only the *yes* or *no* measurements of the photon numbers of the related modes. We also have suggested this scheme can be understood as an entanglement swapping processing.

Finally, we discuss the feasibility of the present scheme. In our teleportation protocol, except linear optical elements, we need the nonlinear one, the cross-Kerr medium. It is a greater challenge to experimentally produce large Kerr nonlinearities. Although sufficiently large Kerr nonlinearities have been difficult to produce, significant progress is being made in this area. In particular, recent progress on atomic quantum coherence [32–36] indicates that it is possible to prepare the Kerr medium with the giant Kerr nonlinearities through using the electromagnetically induced transparency (EIT) technology. Paternostro and coworkers [32] proposed a EIT scheme which can enhance the cross-Kerr effect in a dense atomic medium in the EIT regime. In particular, it has been proved that the interaction of two travelling fields of light in an atomic medium is able to show giant Kerr nonlinearities by means of the so-called cross-phase-modulation. Measured values of the χ^3 parameter are up to six orders of magnitude larger than usual [33]. Recently, Wang *et al* [34] have shown that it is possible to obtain large cross-phase modulation between slow copropagating weak pulse in ^{87}Rb via double EIT. Therefore, our protocol is at the reach of current experiment. We would emphasize that our proposed protocol can potentially be applied to quantum information processing based on CV. Actually, there have appeared tendencies to encode information in quantum states with CV, since such an encoding allows the information to be manipulated much more efficiently than with traditional discrete variable states.

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