Design of thinned planar array using genetic algorithm and hadamard matrix arrangement

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Abstract— Directive antenna array with reduced side lobe level can be obtained using thinned array. This paper describes the design of planar thinned array using Hadamard matrix-like arrangement. Here ‘on’ and ‘off’ states of the antenna elements are arranged based on Hadamard Matrix and inter-element spacing in x and y-directions are varied and then genetic algorithm is used to optimize lowest side lobe level (SLL) of the thinned array. The result for this thinned array for 4X4 planar array is compared with the fully populated antenna array. Very low side lobe level is achieved using this combined genetic algorithm and Hadamard matrix arrangement.

Keywords- Hadamard matrix; thinned array; genetic algorithm; side lobe level;

I. INTRODUCTION

To increase the overall gain of the antenna system and to control the radiation pattern (like, beam tilting, beam shaping etc.) of the array, antenna array is used. With main beam and nulls, the appearance of side lobe in array radiation pattern is a physical phenomenon. Side lobes consume power and causes interference in communication towards the undesired directions and therefore side lobe levels must be lowest in a practical antenna array [1, 2]. Many array synthesis methods are available to reduce side lobe levels keeping desired gain of the array constant [1-3]. If in an array, all the antenna elements are excited (or switched on), the array is called fully populated array. In a thinned array antenna some of the antenna elements are kept switched off and in this way narrowest beam can be produced with lowest side lobe level, without degrading the performance of the array. In the ‘off’ state the antenna element is either open circuited or terminated by a matched load [4, 5]. There are different methods of reducing side lobe level of an antenna array, some of which are, statistically tapered thinned arrays [6], thinning based on empirical or analytical formula [7], and thinning using optimization techniques. Some of the popular optimization techniques, used for thinned array are genetic algorithm [4, 8, 9], particle swarm optimization [10-12], ant colony optimization [13]. Design of thinned array antenna also done based on difference sets including Hadamard difference set [14-17].

In this paper, thinned planar arrays are designed using Hadamard matrix. The ‘on’ and ‘off’ states in the array arrangement correspond to ‘1’ and ‘-1’ of the Hadamard matrix. The inter-element spacing in both the directions of the array are varied. Genetic algorithm is used to optimize lowest side lobe level of the planar array. Numerical results show that lowest side love can be obtained using this technique. The results are compared with the fully populated array. Genetic algorithm is one of the global optimization methods which became a subject of discussion during last decade. The genetic algorithm is an example of a search procedure that uses random selection for optimization of a function by means of the parameters space coding [18] and has been proven successful for robust searches in complex spaces and this method can handle the common characteristics of electromagnetic [19]. In this work, MATLAB genetic algorithm is used for thinned array antenna design.

II. HADAMARD MATRIX

Hadamard matrix of order n is an n×n matrix H with entries from (±1) such that

\[ HH^T = n I_n \]  \hspace{1cm} (1)

Here, \( I_n \) is the n×n identity matrix and \( H^T \) is the transpose of matrix H. Hadamard matrix is defined as [20]

\[ H = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \]  \hspace{1cm} (2)

Hadamard matrix of order 4 is defined as

\[ H_4 = \begin{bmatrix} H & H \\ H & -H \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \]  \hspace{1cm} (3)
Some important properties of Hadamard matrix are i) Any two rows or two columns are orthogonal. ii) Every pair of rows or every pair of columns differs in exactly n/2 places.

III. PLANAR ANTENNA ARRAY

In a planar rectangular array antenna M × N elements are grouped in a configuration of M rows, having an inter-element distance d_x between the rows and N columns having an inter-element distance d_y between the columns, as shown in Fig. 1.

\[ r_{mn} = (m - 1)d_x u_x + (n - 1)d_y u_y. \]  \hspace{1cm} (4)

The planar array antenna therefore may be regarded as either a linear array antenna directed along the y-axis, having an inter-element distance d_x or a linear array antenna directed along the x-axis, having an inter-element distance d_y. The position of an element (m,n) in the array, \( r_{mn} \), where m is a counter into the x-direction (m = 1, 2, ..., M) and n is a counter into the y-direction (l = 1, 2, ..., N), is given by

\[ A_{m} = \frac{1}{M} \sum_{m=1}^{M} \sum_{n=1}^{N} \sin\left(\frac{M - \Psi_x}{2}\right) \cdot \sin\left(\frac{N \Psi_y}{2}\right). \]  \hspace{1cm} (5)

The pattern of a rectangular array is the product of the array factors of the linear arrays in the x and y directions. The normalized array factor is obtained as

\[ A_{F_{a}(\theta, \phi)} = \begin{bmatrix} \sin \left(\frac{M \Psi_x}{2}\right) \\ \sin \left(\frac{N \Psi_y}{2}\right) \end{bmatrix} \]  \hspace{1cm} (6)

Where \( \Psi_x = k d_x + \beta_x \) and \( \Psi_y = k d_y + \beta_y \).

In this paper normalized array factor, given by (6), is used for thinned array design using Hadamard matrix arrangement of antenna elements and this is the cost function for thinned array optimization for lowest side lobe level using genetic algorithm.

IV. COMPUTED RESULTS

In computation, for all the cases normalized array factor (6) is used to obtain main lobe and side lobes. In fully populated array, all the antenna elements are assumed to be excited (‘on’ state) whereas in thinned array some of the elements are switched off according to Hadamard matrix arrangement.

The ‘on’ and ‘off’ positions of antenna elements in a 4X4 planar thinned array, according to Hadamard matrix arrangement of (3), is shown in Fig. 2. Here ‘1’ corresponds to ‘on’ state and ‘-1’ corresponds to ‘off’ state.

The inter-element spacing along x-axis \( d_x \) and y-axis \( d_y \) are varied from 0.50λ to 0.75λ, and then genetic algorithm is used to optimize the inter-element spacing to achieve lowest side lobe level.
Best optimized result for the array arrangement of Fig. 2 is compared in Fig. 3 with the result of fully populated array of same size. The lowest side lobe level of -75dB is obtained at $d_x=0.75\lambda$, $d_y=0.56\lambda$ at frequency of 34.88GHz.

![Figure 3](image)

**Figure 3.** Comparison of 4X4 thinned planar array based on Hadamard matrix and fully filled array

Sometimes depending on inter-element spacing of the array side lobes appear which are comparable with the peak of the main beam and these are known as grating lobes [1]. These grating lobes are undesired and must be avoided in array design. In this optimization also sometimes grating lobes appeared. Optimized results for Hadamard matrix 4X4 thinned array are tabulated in table-I. Serial number 8 in the table-I is the best result for lowest SLL where minimum SLL of -75dB is obtained when $d_x = 0.75\lambda$ and $d_y = 0.56\lambda$. In this case, fully filled array has SLL of -45dB. The cases when grating lobe appears are also shown in the table-I.

**TABLE I. Side Lobe Levels for Different Inter-element Spacing**

<table>
<thead>
<tr>
<th>Sl. No.</th>
<th>Inter-element spacing in x-axis ($d_x$)</th>
<th>Inter-element spacing in y-axis ($d_y$)</th>
<th>SLL obtained for 4x4 Hadamard Matrix Planar Array (dB)</th>
<th>SLL obtained for fully filled 4x4 Planar Array (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.50 $\lambda$</td>
<td>0.50 $\lambda$</td>
<td>-50</td>
<td>-30</td>
</tr>
<tr>
<td>2</td>
<td>0.50 $\lambda$</td>
<td>0.52 $\lambda$</td>
<td>Grating lobe</td>
<td>Grating lobe</td>
</tr>
<tr>
<td>3</td>
<td>0.50 $\lambda$</td>
<td>0.57 $\lambda$</td>
<td>-45</td>
<td>-50</td>
</tr>
<tr>
<td>4</td>
<td>0.56 $\lambda$</td>
<td>0.50 $\lambda$</td>
<td>-56.5</td>
<td>-30</td>
</tr>
<tr>
<td>5</td>
<td>0.65 $\lambda$</td>
<td>0.65 $\lambda$</td>
<td>Grating lobe</td>
<td>Grating lobe</td>
</tr>
<tr>
<td>6</td>
<td>0.63 $\lambda$</td>
<td>0.55 $\lambda$</td>
<td>-66</td>
<td>-41</td>
</tr>
<tr>
<td>7</td>
<td>0.70 $\lambda$</td>
<td>0.50 $\lambda$</td>
<td>-70</td>
<td>-42</td>
</tr>
<tr>
<td>8</td>
<td>0.75 $\lambda$</td>
<td>0.56 $\lambda$</td>
<td>-75</td>
<td>-45</td>
</tr>
<tr>
<td>9</td>
<td>0.75 $\lambda$</td>
<td>0.75 $\lambda$</td>
<td>Grating lobe</td>
<td>-15</td>
</tr>
</tbody>
</table>

Appearance of grating lobes in an optimized 4X4 thinned planar array based on Hadamard matrix, when $d_x = 0.75\lambda$ and $d_y = 0.75\lambda$, is shown in Fig. 4.

![Figure 4](image)

**Figure 4.** Appearance of grating lobes in 4X4 thinned planar array based on Hadamard matrix

4X4 thinned Hadamard matrix antenna arrays at different frequencies are optimized and lowest side lobe levels are compared with those of fully filled array in Fig. 5. In this case inter-element spacing in all the frequencies are different but array arrangement is fixed according to Fig. 2.
V. Conclusion

Optimization of thinned planar array antenna based on Hadamard matrix arrangement and genetic algorithm is easier than optimization, using only genetic algorithm. In optimization, using only genetic algorithm, it is necessary to optimize the ‘on’ and ‘off’ states of the array also, for which lowest SLL will be obtained, in addition to optimization for inter-element spacing. Genetic algorithm optimization can not produce SLL less than the present combined method. In fact, result (not shown in this paper) using only genetic algorithm, shows SLL which is far above the present lowest value of SLL.

REFERENCES