Innovative Applications of O.R.

Newsvendor with multiple options of expediting

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Abstract

We consider a manufacturer facing a single period inventory planning problem with uncertain demand and multiple options of expediting. The demand comes at a certain time in the future. The manufacturer may order the product in advance with a relatively low cost. She can order additional amount by expediting after the demand is realized. There are a number of expediting options, each of which corresponds to a certain delivery lead time and a unit procurement price. The procurement price is decreasing over delivery lead time. The selling price is also decreasing over time. In this paper, we assume that the manufacturer must deliver all products to the customer in a single shipment. The problem can be formulated as a profit maximization problem. We develop structural properties and show how the optimal solution can be identified efficiently. In addition, we compare our model with the classical newsvendor model and obtain a number of managerial insights.

Keywords: Inventory Newsvendor Procurement lead times Expediting Lead-time-dependent pricing

1. Introduction

In the traditional newsvendor model, a manufacturer or retailer procures a single product from her upstream supplier and resells the product to her downstream customers who have one-time uncertain demand. In general, due to the long procurement lead time and customer restriction on delivery lead time, the order of the manufacturer should be placed long ahead of the actual demand being revealed. If the actual demand exceeds the pre-stocked quantity of the manufacturer, unmet demand is lost. In reality, however, if a shortage occurs, some companies have options to place make-up orders by expediting. For example, it is not uncommon that the manufacturer may have other emergency sourcing options or even in-house rush production. The expediting options are quick and responsive, but in general they are much more expensive compared to early procurement. Therefore, the manufacturer should evaluate carefully the trade-off between cost and responsiveness when making inventory and procurement decisions.

In this paper, we study the newsvendor problem with multiple options of expediting under the above environment. A manufacturer (newsvendor) procures a product using normal procedure before the demand is realized. After demand realization, if the demand exceeds the quantity of pre-stock products, the manufacturer can place a make-up order using expediting procedures to fully satisfy the demand. There may be a number of expediting options, each of which corresponds to a certain lead time and a unit price. For example, different expediting options correspond to different transportation modes and a higher price is charged for a shorter procurement lead time. In addition, the price for selling the product depends on the delivery lead time, and a longer delivery lead time leads to a lower unit price, reflecting the fact that late delivery is penalized.

The newsvendor problem is one of the classical problems in the literature on inventory management (Arrow et al., 1951; Silver et al., 1998). There is a rich literature on the newsvendor problem with a wide variety of research issues. We refer readers to Khouja (1999), Qin et al. (2011), Gallego and Moon (1993) and Petruzzi and Dada (1999) for thorough and excellent reviews. In the following we discuss some closely related papers.

A number of research papers have addressed the expediting or multiple sourcing issues, such as Hong and Hayya (1992), Ramasesh et al. (1991) and Tomlin and Wang (2005). For multiple sourcing, a stream of researches focus on supplier selection, risk control and auctions (Ghodsypour and O’Brien, 2001; Treleven and Schweikhart, 1988; Tunca and Wu, 2009), other than inventory management. In the field of inventory management, most researches consider only one expediting option (dual sourcing), in either single-period, multi-period or continuous-time models. Closely related to our research, Khouja (1996) considers a newsboy problem with one emergency supply option where a proportion of customers satisfy their demand from an emergency supply source when there is a shortage. The author solves the problem under the objective of maximizing the expected profit or maximizing the probability of achieving a target profit. Fu et al. (2009) consider expediting in a single-period assemble-to-order (ATO) system with one expediting option. DeYong and Cattani (2012) study a special newsvendor problem where the newsvendor can expedite an
additional order or cancel the previous order based on the updated demand forecast. Examples of researches on multi-period inventory control include Lawson and Porteus (2000), Durán et al. (2004), Huggins and Olsen (2010), Zhou and Chao (2010), Zhu (2012), to name a few. These researches analyze that certain inventory policies, such as the base-stock policy, are optimal or close-to-optimal. Chiang (2010) proposes a single-item continuous-review order expediting inventory policy, which can be considered as an extension of the ordinary (s,Q) policy. The buyer can expedite part of an outstanding order via a fast transportation mode at extra costs when his inventory falls below a certain level. Plambeck and Ward (2007) consider expediting option in a continuous-time ATO system. In addition, a number of recent papers study supply chain models with second production or procurement opportunities, including Cachon and Swinney (2009), Li et al. (2009), Jones et al. (2003) and Jones et al. (2001).

This paper considers the lead-time-dependent procurement cost and product price. Previous researches based on this setting are relatively limited. Fang et al. (2008) consider the case of lead-time-dependent final product price and explore how the manufacturer can use a “Vendor Managed Consignment Inventory” scheme to manage the underlying risk and coordinate independent suppliers’ decisions on the production quantities of their components under demand uncertainty. Hsu et al. (2006) consider an ATO system with lead-time-dependent component and final product price without expediting. Chandra and Grabis (2008) study a single-stage variable lead-time inventory system with lead-time dependent procurement cost. They develop a cost-minimization model without expediting and do not consider the varying product prices. Hsu et al. (2007) address inventory decisions in an assemble-to-order system with lead-time-dependent final product pricing under the full shipment case and propose efficient solution procedures. They assume constant component purchasing prices, and thus do not consider the expediting issue, which is the main focus of our research.

Our model differs from previous work in the following two aspects. First, our paper considers multiple options of expediting, while most of previous researches address only one option. Second, we consider decreasing procurement cost and product price. As will be seen later, the newsvendor problem with these two features reveals some new characteristics and insights. To the best of our knowledge, this has not been addressed in the literature.

The rest of the paper is organized as follows. In Section 2, we discuss our problem and formulate it as a profit maximization model. In Section 3, we develop some structural properties and solve the optimization model. In Section 4, we compare our model with the traditional newsvendor problem and discuss a number of managerial insights. Section 5 concludes the paper and offers some future research directions.

2. The model

We consider the problem under the following assumptions. A manufacturer produces or processes a single product from a supplier and resells the product to customers. Demand comes at a certain time in the future and the manufacturer may order some amount of product using normal procedure in advance. These products will arrive before the demand is revealed. When demand is realized, in case of shortage, the manufacturer will place make-up orders using expediting procedure to satisfy the entire demand. There are a number of expediting options, each of which corresponds to a lead time and a unit price. The unit price is lower with a longer lead time. Hence, the manufacturer wants to determine the quantity of the pre-stocked product to maximize her total expected profit.

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We assume here that the manufacturer must deliver products in a single shipment to satisfy the entire customer demand. The formulation consists of two stages. At stage 1, the manufacturer determines the pre-stocked quantity of the product before the actual demand is revealed. At stage 2 when the demand is realized, given the demand D and the pre-stocked quantity Q, the manufacturer uses the pre-stocked quantity to fully satisfy the demand exactly at time 0 if D ≤ Q, i.e., the demand is not more than the pre-stocked quantity. If D > Q, however, the manufacturer must place make-up orders to procure additional products. When these additional products arrive, the manufacturer will then deliver all products in a single shipment to customer. First, we have the following property.

Proposition 1. For the stage-two problem, if Q < D, i.e., the pre-stocked inventory is less than the demand, there is an optimal procurement and shipment decision such that the manufacturer places a single make-up order at time 0 and makes a single full shipment, where the order lead time and delivery lead time are L’ ∈ {l₁, l₂, …, lₙ}.

Proof. If there are orders placed at any time t > 0, then we can modify this solution by moving all the procurement times to time 0 and keeping other decisions unchanged. It is easy to verify that the new solution will not increase the costs or decrease the revenue. Hence in the following we assume this property is satisfied, i.e., all orders are placed at time 0.

If there is a single order that will arrive at a time lₙ ∈ {l₁, l₂, …, lₙ}, then it is easy to verify that there is an optimal shipment decision that satisfies L’ ≤ lₙ. If there are multiple orders that will arrive at times lₙ, lₙ₊₁, …, lₙ₊m, where 1 ≤ lₙ < l₁ < ⋯ < lₙ₊m ≤ n, we can modify the solution by merging all orders into a single one with the longest lead time lₙ. It is clear that this modification will reduce the total procurement cost without affecting the revenue received from the customer. □

Hereafter we shall consider the procurement and delivery decisions satisfying Proposition 1. Namely, the manufacturer will order the additional products at time 0 and deliver the final products as soon as the ordered products arrive. Therefore, the make-up delivery lead time can be restricted to {l₁, l₂, …, lₙ}. For convenience, define Pᵢ ≡ P(lᵢ), 0 ≤ i ≤ n. Clearly, P₀ ≥ P₁ ≥ ⋯ ≥ Pₙ.

In addition, the unit procurement cost under expediting is also decreasing in expediting lead time and we assume that the the unit
cost for pre-stocked products is no more than the lowest unit cost under expediting, namely,
\[ c_1 > c_2 > \ldots > c_n > c_0. \]

Note that \( c_0 \geq 0 \). This makes sense, as usually the manufacturer has enough time to prepare the pre-stocked products. For example, she may use cheap transportation modes and get price discount for advance order information (Sheopuri et al., 2010).

Based on Proposition 1, we can express the second stage problem as follows. Our objective is to maximize the net revenue, denoted as \( R(Q,D) \).

\[ (F) \quad \max_{Q \geq 0} E_0[\{-c_0Q + R(Q,D)\}]. \]

At stage one, our objective is to maximize the total expected profit under uncertain demand \( D \).

\[ (F') \quad \max_{Q \geq 0} R(Q,D) = \max_{Q \geq 0} \left( \left\{ \frac{P_c D - c_i(D - Q)}{P_c - c_i} + b(Q - D) \right\}^+ \right) \] \[ + b(Q - D)^+. \] (1)

3. Structural properties

In this section, we discuss a number of structural properties of the proposed problem that allow us to simplify problem \((F)\).

Proposition 2. If \( i < j \), where \( i, j \in \{1, 2, \ldots, n\} \), and \( P_1 - c_i > P_1 - c_j \), then regardless of the pre-stocked quantity \( Q \), there is an optimal solution that does not use \( l_i \) as delivery time.

Proof. Suppose there is a solution that uses \( l_i \) as delivery time. Now we change \( l_i \) to \( l_j \) and examine the profit changes. Let \( C_i \) be the total profit when choosing \( l_i \) as delivery time. Let \( C_j \) be the total profit when choosing \( l_j \) as delivery time. Then for arbitrary \( Q \) and \( D \), we have

\[ C_j - C_i = \left( P_j D - c_j(D - Q)^+ - c_j Q + b(Q - D)^+ \right) - \left( P_j D - c_i(D - Q)^+ - c_i Q + b(Q - D)^+ \right) \]

\[ \leq \left( P_j D - c_j D \right) - \left( P_j D - c_i D \right) \leq 0. \]

where the first inequality holds because \( c_j > c_{ij} \) and the second inequality holds by the assumption \( P_j - c_i > P_j - c_j \).

By Proposition 2, we can remove \( l_i \) for any pair \((l_i, l_j)\) such that \( P_j - c_i > P_j - c_j \). After removing all lead times that satisfy the above inequality, we will obtain a smaller set of lead times \( \{l_i\} \) such that \( P_j - c_i \) is increasing in \( i \). Namely, we have

\[ P_1 - c_1 < P_2 - c_2 < \cdots < P_n - c_n. \] (3)

Hereafter, we shall assume the lead times already satisfy this condition. Without loss of generality, we still denote the set of lead times as \( \{l_0, l_1, \ldots, l_n\} \) where \( l_0 \equiv 0 \).

It is intuitively true that for the second stage problem \((F)\), the choice of delivery lead time \( l \) depends on the pre-stocked quantity \( Q \) as well as the realized demand \( D \). The following property states this relationship precisely and is a fundamental insight for us to simplify the problem significantly. Before describing the proposition, we define the following notation:

\[ r_{ij} = \frac{c_i - c_j}{(c_i - c_j) - (P_i - P_j)}, \quad 1 \leq i < j \leq n. \] (4)

It is easy to see that \( r_{ij} \geq 1 \) because \( c_i - c_j > P_i - P_j \) due to inequalities (3).

Clearly, given any two lead times \( L_1 < L_2 \), the choice between the two depends on both \( Q \) and \( D \).

Proposition 3. For the second stage problem \((F)\), for any two lead times \( L_1 \) and \( L_2 \), where \( L_1 < L_2 \), \( 1 \leq i < j \leq n \), if \( D \leq r_{ij} Q \), then there is an optimal solution that does not choose \( l_j \) as delivery lead time.

Proof. To solve problem \((F)\), we need to compare the total profits associated with the two delivery lead times. Given the two lead times \( L_1 \) and \( L_2 \), the profit for each can be expressed respectively as (we ignore the initial ordering cost \( c_0 Q \) as this term is the same for both cases).

\[ P_1 D - c_j(D - Q)^+ + b(Q - D)^+. \] (5)

Clearly, if \( D \leq Q \), then \( L = 0 \). Then we shall choose \( l_1 \) instead of \( l_j \) if \( l_1 \) leads to a higher profit, namely,

\[ P_1 D - c_j(D - Q) \geq P_2 D - c_j(D - Q). \] (6)

or

\[ D \leq \frac{c_j - c_i}{(c_j - c_i) - (P_i - P_j)} Q = r_{ij} Q. \] (7)

which completes the proof. □

Example 1. Suppose \( P(1) = 4 \), \( P(2) = 3 \), \( c(1) = 3 \), \( c(2) = 1 \), \( Q = 10 \), \( D = 11 \), then if we choose \( L_1 \), total net profit is \( P(1) D - (D - Q)^+ - c(1) = 4(11) - 1 = 41. \) If we choose \( L_2 \), then the total net profit is \( P(2) D - (D - Q)^+ c(2) = 3(11) - 1 = 32 \). Therefore, we should choose \( L_1 \), which is also consistent with \( D \leq r_{12} Q \). But if \( Q = 0 \), then we should choose \( L_2 \) since \( D > r_{12} Q \).

The following proposition follows from the above proposition.

Proposition 4. Given \( l_1, l_2, l_n \), \( 0 < i < j < k \leq n \), if \( r_{ij} \geq r_{jk} \) then there is an optimal solution that does not use \( l_j \) as delivery time.

Proof. Consider two cases: (1) \( D \leq r_{ij} Q \) and (2) \( D > r_{ij} Q \).

(1) \( D \leq r_{ij} Q \). From Proposition 3, \( l_i \) is better than \( l_j \). Thus, it is optimal not to use \( l_j \) as delivery time.

(2) \( D > r_{ij} Q \). It is clear that \( D > r_{jk} Q \) since \( r_{ij} \geq r_{jk} \), then \( l_j \) is better than \( l_i \). Again it is optimal not to use \( l_j \) as delivery time.

Therefore, for all \( D \geq 0 \), we will not choose \( l_j \) as delivery time. □

By Proposition 4, we can remove any lead time \( l_j \) such that \( r_j \geq r_{ij} \) \( (0 < i < j < k \leq n) \). This property, along with the earlier discussions in this section, allows us to modify the problem to satisfy the following conditions without affecting its optimal solution.

(a) \( P_i - c_i \) is strictly increasing over \( i \), \( 1 \leq i \leq n \) (by Proposition 2);

\[ P_i - c_i < P_{i+1} - c_{i+1} < \cdots < P_n - c_n. \]

(b) \( r_{ij} \) is strictly increasing over \( i \), \( 1 < i \leq n - 1 \) (by Proposition 4):

\[ r_{i+1} < r_{i+2} < \cdots < r_{n-1,n}. \]

For convenience, we define \( r_1 \equiv r_{ij} \), \( i = 1, \ldots, n - 1 \).

Based on the above discussions, we can summarize the solution for the second stage problem \((F)\). For convenience, we define \( r_n = 0 \) and \( r_n = +\infty \).

Proposition 5. For any pre-stocked quantity \( Q \) and demand realization \( D \), the optimal solution for the second stage problem \((F)\) can be expressed as shown in Table 1:

Based on Proposition 5, we can now express the expected profit function for the first stage problem.

\[ P_t(Q) = E_0[\{-c_0Q + R(Q,D)\}] = -c_0Q + \int_0^Q [P_d x + b(Q - x)] dF(x) \]
Hence the above expression can finally be simplified to the following result.

\[ P_f(Q) = c_0 - c_0 + (P_0 - P_1)Qf(Q) - (c_1 - b)F(Q) + \sum_{i=1}^{n-1} (c_i - c_{i+1})F(r_iQ). \]

In general, the properties of the objective function \( P_f(Q) \) depend on the demand distribution function \( F(x) \). However, based on the structural properties, it is not difficult to obtain optimal solutions to the problem for a number of commonly used demand distributions including Uniform, Gamma, and Normal. To keep our discussions concise, we refer readers to an online technical report of this paper for a more detailed discussion about this problem.

Next, we consider a special case where procurement costs and selling prices are both linear, i.e.,

\[ c(l) = u_0 - u_1l, \]
\[ P(l) = v_0 - v_1l, \]

where \( u_0, u_1, v_0, v_1 \) are constants, and \( u_1, v_1 > 0 \). Then

\[ c_i = u_0 - u_1l_i, \quad i = 1, 2, \ldots, n, \]
\[ P_i = v_0 - v_1l_i, \quad i = 0, 1, \ldots, n. \]

Thus, without loss of generality, we can define analogously to (4)

\[ r_{ij} = \frac{(u_0 - u_1l_i) - (u_0 - u_1l_j)}{(u_0 - u_1l_i) - (u_0 - u_1l_j) - [(v_0 - v_1l_i) - (v_0 - v_1l_j)]}, \]
\[ = \frac{u_1(l_i - l_j) - v_1(l_i - l_j)}{u_1(l_i - l_j)} = \frac{u_1}{u_1 - v_1}. \]

That is, with linear costs, \( r_{ij} = \frac{u_1}{u_1 - v_1} \) will be constants for all \( 1 \leq i < j \leq n \). Based on earlier discussions, it is easy to prove the following result.

**Proposition 6.** For the second stage problem, if \( Q < D \), i.e., the pre-stocked inventory is less than the demand, suppose the procurement cost and the selling price functions are linear as defined above, then (i) if \( u_1 \leq v_1 \), there is an optimal solution that always uses \( l_i \) as delivery time; (ii) if \( u_1 > v_1 \), we consider three subcases: (a) if \( D < \frac{u_1}{u_1 - v_1}Q \), then there is an optimal solution that uses \( l_i \) as delivery time if the additional procurement is necessary; (b) if \( D = \frac{u_1}{u_1 - v_1}Q \), any of \( l_i \) \( (i = 1, 2, \ldots, n) \) can be used as the optimal delivery time; (c) if \( D > \frac{u_1}{u_1 - v_1}Q \), then there is an optimal solution that uses \( l_n \) as the delivery time.

Based on **Proposition 6** and earlier discussions, the problem can be efficiently solved when the costs and prices are linear. **Proposition 6** says that if the procurement cost and selling price functions are both linear, then depending on the relative quantity between the realized demand and the pre-stocked quantity, the manufacturer can choose either the shortest or the longest lead time for expediting. In order to explain how parameters \( u_1 \) and \( v_1 \) affect the optimal delivery time, we divide the total demand \( D \) (when \( D > Q \)) into two parts: the pre-stocked quantity \( Q \) and the second order quantity \( D - Q \). Then we consider two cases. In case (i), \( u_1 \leq v_1 \) implies that \( P_i - c_i \) is decreasing over \( i \). For the pre-stocked quantity \( Q \), it is optimal to be delivered to customers as soon as possible because \( P_i \) is decreasing. For the second order quantity \( D - Q \), the manufacturer should also deliver this part as soon as possible since \( P_i - c_i \) is decreasing. Therefore, the delivery time \( l_i \) is optimal if additional procurement is required. In case (ii), \( u_1 > v_1 \) suggests that \( P_i - c_i \) is increasing over \( i \). Therefore, the pre-stocked quantity \( Q \) should be delivered as soon as possible (\( l_1 \)) while the second order quantity \( D - Q \) should be delivered at time \( l_n \). However, all products must be delivered to the customer in a single shipment here. As a result, there is a tradeoff between \( l_1 \) and \( l_n \) for the manufacturer. This tradeoff in return depends on the relative quantity of \( Q \) and \( D \). If \( D \) is relatively large (this implies the second order quantity \( D - Q \) is relatively large compared to the

### Table 1

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<td>( r_{n-2}Q &lt; D &lt; r_{n-1}Q )</td>
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<td>( n )</td>
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pre-stocked quantity \( Q \), the manufacturer should choose \( l_n \) as the delivery time because the second order quantity has a larger contribution to the total profit. If \( D \) is relatively small, then the manufacturer should choose \( l_1 \) as the delivery time because the pre-stocked quantity has a larger contribution to the total profit. At the critical point when \( D = \frac{P_l}{b+P} Q \), the pre-stocked and the second order quantities are equally important and any delivery time will result in the same profit.

**Remark 1.** In this paper, we assume that the manufacturer must deliver total products in a single full shipment. For the case of partial shipments, where the manufacturer is allowed to deliver products in multiple shipments, it is quite clear that the model is simply a parameter-adjusted newsvendor problem in which the under-stocking cost is reduced from \( P_0 - c_0 \) to \( (P_0 - c_0) - (P^* - c^*) \), where \( P^* - c^* = \max_{1 \leq i \leq n}(P_i - c_i) \). Consequently, under this case, the manufacturer will pre-order less than the classical newsvendor case without expediting options. In addition, the optimal profit in the partial shipments case is higher than that in the classical newsvendor case.

### 4. Comparison and managerial insights

In this section, we compare the full shipment case with the classical newsvendor model. Specifically, we compare the following two models.

- Classical newsvendor. In this case, there is no second order opportunity and hence the demand may not be fully satisfied. We assume that there is no shortage cost for the unfulfilled demand.
- Newsvendor with multiple expediting options and a full shipment. In this case, there is a second order opportunity to fully satisfy the customer demand and a single full shipment is required by the customer.

It can be easily verified that the optimal profit in the partial shipments case is higher than those in the full shipment case and classical newsvendor case. However, whether the full shipment is better than the classical newsvendor is not clear. Although the manufacturer in the full shipment case has a second order opportunity, she has to wait for the make-up procurement, if her pre-stocked product is not enough, to arrive and satisfy the entire demand in a single shipment. A delay in the final delivery means a loss in the revenue from the customer since the final product price is decreasing in the delivery time. Therefore, there is a probability that the profit from full shipment may not be as large as that from classical newsvendor. However, the relationship between the two cases depends on the specific distribution of the demand. The profit from classical newsvendor may be more sensitive to the demand variation because the newsvendor cannot procure for a second time if the demand exceeds his pre-stocked quantity. We will illustrate this relationship in the following numerical analysis. In this analysis, we are interested in the characteristics of optimal order quantities and profits in the two models.

The basic parameters are set as follows. Let \( P_0 = 20, c_0 = 10, b = 5 \). There are three options for expediting: \( P_1 = 19, c_1 = 15; P_2 = 18, c_2 = 12; P_3 = 15, c_3 = 11 \). Assume that the demand follows Normal distribution \( N(\mu, \sigma^2) \), where \( \mu = 100, \sigma = 30 \). Then we can get the optimal order quantities \( Q_0 = 112.9, Q_1 = 113.6 \) and the optimal profits \( \pi_0 = 836.4, \pi_1 = 800.8 \).

Next, we examine how the coefficient of variation (COV) affects the optimal decisions and profits. We fix the demand mean and vary the standard deviation. As shown in Fig. 1, \( Q_0 \) increases in a linear fashion. This is intuitive because \( \frac{\sigma}{\mu} > 0.5 \). However, \( Q_1 \) first increases and then decreases in the standard deviation \( \sigma \). This phenomenon occurs due to the requirement of a full shipment and can be interpreted as follows. Consider an order decision \( Q \) and a demand realization \( D \). If there is an average of the pre-stocked products \( (D < Q) \), then as is the case with classical newsvendor, the manufacturer will satisfy all the demand and salvage the leftover products. However, there is a shortage of the pre-stocked products \( (D > Q) \), then the pre-stocked products may have less value compared to the classical newsvendor, as the manufacturer will have to wait for the make-up order to arrive and then deliver in a single full shipment. Thus, the demand variation may affect the pre-stocked quantity in opposite ways. When the variation is relatively small, the manufacturer may wish to pre-stock more to prevent shortages when the variation increases. When the variation is relatively large, the manufacturer may wish to pre-stock less when the variation increases because the value of the pre-stocked products diminishes with large variation. By similar arguments, we can explain the relationship between the pre-stocked quantities in the classical newsvendor and full shipment cases.

**Fig. 2** shows that the profits of the two cases all decrease in standard deviation, which is consistent with our common understanding that demand variation decreases the expected profit. Furthermore, the profit in the newsvendor case is more sensitive to standard deviation. This is due to the fact that it has only one order opportunity and cannot satisfy the exceeding demand with expediting options. If COV is small, the optimal profit in the newsvendor case is higher than that in the full shipment case. If COV is large, however, the newsvendor case is worse than the full shipment case in terms of profits.

### 5. Conclusions

In this paper, we consider the newsvendor problem with multiple options of expediting. A manufacturer procures a product from a supplier and resells the product to customers who have uncertain
References


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Appendix A. Supplementary material

Supplementary data associated with this article can be found, in the online version, at http://dx.doi.org/10.1016/j.ejor.2012.10.039.

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