Design and modelling of a novel linear electromagnetic vibration energy harvester

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Abstract. This paper presents the design and evaluation of a novel permanent magnet (PM) energy harvesting system for scavenging electrical energy from ambient vibrations. A two-phase tubular linear PM vibration energy harvester consisting of a mover attached with permanent magnets and a slotted stator with built-in two-phase electromagnetic coils is proposed to convert vibrational kinetic energy into electrical energy. Aiming at maximizing the efficiency of vibration-to-electrical energy conversion under designated vibration and limited space requirement, a systematic research, including innovative device design, theoretical modelling and analysis, and finite element evaluation on the PM vibration energy harvester will be presented in this paper. In addition, the methodology of winding the two-phase coils in slotted stator is explicated in order to fully utilize the harvested electrical energy. A two-phase rectifier circuit is developed to convert the alternative voltage generated by the PM harvester into DC voltage that can be used directly by the external resistive load. Simulation results indicate that the proposed linear PM vibration energy harvesting system is able to generate about 100 watt DC electrical power under the vibration with the velocity of 0.4 m/s and the output electrical power is proportional to the levels of vibration excitations.

Keywords: Energy harvesting, Vibration, Electromagnetic, Magnetic circuit, Theoretical model
1. Introduction

Energy harvesting, also known as energy scavenging or power harvesting, is the process of converting non-electrical energy (e.g. solar, thermal, wind, and kinetic energy) into electrical energy that can be used to drive electronics with low-power requirement, such as wireless autonomous device or sensors. Specifically, the conversion of vibrational energy into electrical energy is called vibration energy harvesting. Recently, there have been substantial raise of interests in scavenging electrical energy from ambient vibrations [1-4].

As stated by Williams and Yates [5], three basic methods, i.e. piezoelectric effect, electrostatic generation, and electromagnetic induction, can be used to harvest electrical energy from surrounding vibrations. While each of these methods can scavenge a useful amount of energy from ambient vibrations, the piezoelectric harvester based on piezoelectric effect has been studied by many researchers around the world due to its ability to directly convert applied strain energy into usable electrical energy and ease of application [6,7]. However, piezoelectric harvesters have relatively low energy conversion efficiency and the piezoelectric materials, as the critical component, are normally stiff and brittle [8], which requires careful mechanical design of a buffer area to isolate the piezoelectric material and vibration sources. In addition, high stiffness results in high natural frequency ranges of piezoelectric harvesters, which make them difficult or even inapplicable for the implementations in low-frequency vibration environments such as in bridge and building applications [9]. For the electrostatic energy harvesting system, an external voltage source is inevitably needed in order to charge the capacitor for converting vibrational energy into electrical energy, which will increase the complexity of the system [10]. In contrast, the vibration energy harvester based on electromagnetic induction has high energy conversion efficiency and are very suitable for low-frequency ambient vibrations [11]. Moreover, it does not require any separate voltage source as electrostatic conversion does.

Electromagnetic devices have been used to generate electricity since the early 1930s, not long after Faraday’s fundamental breakthrough in electromagnetic induction. A typical electromagnetic device is composed of a PM system, which is used to generate magnetic field, and a coil system to be used for the induction. When there is a relative motion between the PM system and the coil system, either linear or rotational motion, an electromotive force or electrical energy will be induced in coil system. In generally, there are two types of
electromagnetic harvesters, i.e. rotating and linear electromagnetic harvesters. Rotating electromagnetic harvesters are usually used to convert unidirectional motions into electricity via a linear-to-rotary mechanism, such as commercial used hydroelectric generators, wind-driven generators and steam-driven generators. On the other hand, linear electromagnetic harvesters are very suitable for harmonic motions, such as ocean waves and mechanical vibrations. Because linear electromagnetic harvesters do not require linear-to-rotary mechanism, their energy harvesting efficiency is normal higher than rotating harvesters under the same conditions. Therefore, the linear electromagnetic harvester is a natural candidate to be utilized for vibration energy harvesting.

To date, many linear electromagnetic harvesters or prototypes have been developed to scavenge electrical energy from vibrational environments and power micro-to-medium scale electronics, such as MEMS sensors [4,10] and wireless sensors [12,13]. Williams and Yates [13,14] have presented a general design methodology for linear vibration energy harvesters and proposed a millimetre scale linear electromagnetic harvester to power sensors for bridge health monitoring. Konotchick [15] proposed a linear electromagnetic harvester to convert vibrational energy into electrical energy. The device is simple and easy of application. However, the coil in this harvester is winded on a non-magnetic plastic tubular cylinder, and the gap between the PM and coil (also called effective air-gap) equals to the thickness of the plastic tubular cylinder plus the air-gap between the PM and the cylinder. Therefore, the effective air-gap between PM and coil is very big in this design, which leads to huge magnetic reluctance and decreases the vibration-to-electrical energy conversion efficiency under a same vibration condition. Saha et al. [16] developed an AA-sized tube-type linear electromagnetic harvester for powering sensors or electronics worn by human body, which has the ability of converting human body motion into electrical energy. It uses an “air spring” system rather than the mechanical spring to decrease the friction loss. However, their design has considerable flux leakage and cannot adequately utilize the magnetic flux to generate electrical power resulting in low energy conversion efficiency. Tu and Yeh [17] proposed various designs of linear electromagnetic harvesters in order to scavenge energy from different vibrational sources. These harvesters can be easily manufactured and are compact and efficient to be used for recharging power sources in daily electronic device such as cellular telephones, portable CD players, etc. However, the harvesters designed by Tu and Yeh [17] also possess the
same problems as flux leakage, big actual air-gaps and not being able to fully utilize the magnetic field
generated by the permanent magnets.

Afore-mentioned researches are mainly focus on the development of small-scale electromagnetic harvesters.
Recently, there are also few researches have been done on the large-scale electromagnetic energy harvesting
system with capability to provide large power. Rhinefrank et al. [18] presented a PM tubular linear
electromagnetic harvester to scavenge energy from ocean wave motions. The ocean wave harvester uses
several novel designs to reduce the flux leakage and has potential to fully utilize the flux for electrical power
generation. However, the effective air-gap of the ocean wave harvester is still considerable large and the
conversion efficiency is low. Prudell et al. [19] proposed another ocean wave harvester to convert ocean wave
motions into electrical energy. However, permanent magnets used in this wave harvester are magnetized in
radial direction, and the effective magnetic field for generating inductive voltages is far smaller than axially
magnetized permanent magnets, which leads to lower energy conversion efficiency. Sapinski [20] proposed a
linear electromagnetic harvester to convert structural vibrations into electrical energy and to power the semi-active MR fluid damper. The harvester developed by Sapinski [20] is simple and easy to manufacture.
However, it uses only single magnetic pole-coil interaction and cannot fully use the magnetic field generated
by the PMs resulting in lower energy conversion efficiency.

To sum up, the success in designing a linear electromagnetic harvester with high energy conversion efficiency
lies in efficient utilization of the magnetic field and minimization of the magnetic reluctance as much as
possible. For this purpose, this paper presents a novel linear PM harvester with high efficiency of vibration-to-
electrical energy conversion to scavenge energy from ambient vibrations. The presented PM harvester uses
slotted stator structure, and coils are wounded in the slots which can greatly reduce the size of the effective air-
gap and the magnetic reluctance. At the same time, two-phase coils were chosen in this paper in order to fully
utilize the magnetic field and maximize the vibration-to-electricity efficiency. Therefore it is able to harvest
more energy under same vibration conditions. Comprehensive investigations on the designing and modelling
of the two-phase linear PM harvester are conducted, including an innovative structure design, magnetic circuit
analysis, theoretical modelling on the energy harvesting ability and finite element analysis of the novel device.
Specifically, a lumped equivalent magnetic model, which has been proven to be effective and efficient in
predicting the magnetic field distribution in tubular permanent-magnet machines [18, 21-23], was used in this paper to analyse the magnetic field in the designed vibration energy harvester. Built on the theoretical analysis, this paper reveals an innovative methodology for making two-phase coils in the slotted stator in order to fully utilize the electric powers generated in each stator coils. Series of simulations are undertaken to evaluate the capability of energy harvesting under ambient vibrations. The results indicated that the proposed two-phase tubular linear PM harvester could generate about 100 watt DC electrical power under the vibration with velocity of 0.4 m/s, and the output electrical power is proportional to the levels of vibration excitations.

2. Design of the tubular linear PM harvester

2.1 Novel device design

In this paper, a novel two-phase tubular linear PM vibration energy harvester is proposed to scavenge energy from ambient vibrations, and the structure of this novel PM harvester is illustrated in Fig. 1. The main structural parameters are given in Table 1. The PM harvester mainly consists of a mover system, a stator system and a flux-leakage shield. The mover system is made up of an assembly of 7 alternating permanent magnets, interspersed with 8 magnetic pole steels mounted on a non-magnetic shaft. There are 21 slots with innovative shape inside the stator and 22 magnetic coils are wrapped between the slots. Furthermore, there is a flux-leakage shield covering the stator to prevent the magnetic flux leakage of the device. Two fitting screws are used to fix the flux-leakage shield with the stator. In practical applications, the mover system shuttles horizontally under external vibrations in the stator and thus will cause the change of the magnetic flux linkage in the coils in stator system, which will produce voltage induction in the coils.

To obtain high efficiency in the energy conversion, it is desirable to fully utilize the magnetic flux inside the device and reduce the magnetic reluctance to lowest. To this end, several innovative designs are used to reduce the magnetic reluctance, prevent the magnetic flux leakage and enhance the magnetic flux in the harvester. These innovative designs include: 1) the stator is made as slotted structure in order to provide a return path for the flux and reduce the thickness of the air-gap leads to the considerable reduce of the magnetic reluctance; 2) the PM used in this design is magnetized in axial direction, and seven PMs are stacked in pairs with opposite magnetic polarization direction such that opposing magneto-motive force will drive the flux through the pole steels and reduce the magnetic reluctance; 3) the material of PMs is selected as Neodymium-
Iron-Boron (NdFeB-N50) which have the highest energy production approaching 50 KJ/m³ and very high coercive force; 4) the stator and magnetic poles are made up of high-permeability steel, i.e. pure iron DT4E, in order to reduce the magnetic reluctance; 5) the mover shaft and connection adapters are made up of non-magnet material, i.e. stainless steel, to prevent flux leakage; 6) a flux-leakage shield, made up of non-magnet aluminium, is used to cover the stator and prevent the magnetic flux leakage.

2.2 Magnetic circuit model analysis
In structural design, magnetic field generation is associated with components such as electromagnetic coils, ferromagnetic cores and air gaps, which can be treated as magnetic circuit analysis. Magnetic analysis is of great importance to the successful design of PM energy harvester because a good magnetic circuit design will guarantee the majority of the magnetic flux passes through the stator to participate in the process of generating inductive voltages and thus have the optimal electrical power conversion. Fig. 2 shows the inner structure and parametric dimensions of the tubular linear PM vibration energy harvester. The shaft is made of stainless steel, a non-magnetic material, combined with the high-permeability magnetic poles, so the magnetic flux from the permanent magnets are forced to pass through the poles, air gap and stator core, forming an enclosed magnetic path. For a single magnetic pole pitch, the equivalent magnetic circuit model can be illustrated in Fig. 3, where \( F_m \) is the magneto-motive force (magnetic circuit source) of the permanent magnet; \( \Phi_g \) is the average magnetic flux in the air-gap; \( R_{pole} \), \( R_{gap} \) and \( R_{stator} \) are the reluctances of poles, air-gap and stator, respectively; \( R_{leak} \) is the magnetic reluctance representing the leakage flux path in the air-gap and the shaft.

Similarly as current in an electric circuit, the average magnetic flux, \( \Phi_g \), in the magnetic circuit will follow the path with the least reluctance. Because the poles and stator are made of high-permeable pure iron DT4E, the magnetic reluctance of stator and poles is very small and can be neglected when compared with the magnetic reluctance in the air-gap. Also, because the mover shaft is made of non-magnetic stainless steel and there is a flux-leakage shield to prevent the magnetic flux leakage, the reluctance \( R_{leak} \) can be omitted in this structure.

Therefore, according to the fundamental principle of [24], the average magnetic flux in the air-gap can be expressed as follows:

\[
\Phi_g = \frac{F_m}{R_m + 2R_{gap}} = \frac{B_{rem} N_{m} A_{g} h_{m}}{h_{m} A_{g} + 2d_{rem} A_{m} R_{c} l}
\]  

(1)
where $B_{\text{rem}}$ is the PM residual flux density; $K_C$ is the Carter coefficient, which is used to compensate the slots effects [25]; $A_g$ is the lateral area of the air-gap; $h_m$ and $A_m$ are the width and the cross section area of the PM respectively.

Other parameters, such as $F_m, R_m, R_{\text{gap}}, A_m, A_g,$ and $K_C$ in equation (1), can be obtained from the following equations:

$$F_m = \frac{B_{\text{rem}}}{\mu_{\text{rem}}\mu_0} = H_C h_m$$  \hspace{1cm} (2)

$$R_m = \frac{h_m}{\mu_{\text{rem}}\mu_0 A_m}$$  \hspace{1cm} (3)

$$R_{\text{gap}} = \frac{K_C t}{\mu_0 A_g}$$  \hspace{1cm} (4)

$$A_m = \frac{\pi(D_m^2 - D^2)}{4}$$  \hspace{1cm} (5)

$$A_g = \frac{\pi(D_m + D_s)(r + h_p)}{4}$$  \hspace{1cm} (6)

$$K_C = \frac{t}{\tau r_w \left( \frac{t}{2 \tan^{-1}(\frac{1}{\varphi})} - \frac{1}{\varphi} \ln(1 + \varphi^2) \right)}$$  \hspace{1cm} (7)

$$\varphi = \frac{w_s}{2t}$$  \hspace{1cm} (8)

where $H_C$ is the PM coercive force; $\mu_{\text{rem}}$ is the recoil permeability of the PM; $\mu_0$ is the permeability of vacuum; $t$ is the thickness of the annular air-gap; $h_p$ is the width of the magnetic pole steel; $D_m, D_s,$ and $D$ are the outer diameters of the PM, stator and shaft, respectively; $\tau$ is the length of the pole pitch.

Based on equation (1), the average magnetic flux density in the air-gap can be calculated by:

$$B_g = \frac{\phi_g}{A_g} = \frac{B_{\text{rem}} A_m h_m}{h_m A_g + 2\mu_{\text{rem}} A_m K_C t}$$  \hspace{1cm} (9)

As shown in Fig. 2, the stator slot structure is used in this linear PM vibration energy harvester. Therefore, in order to guarantee the capacity of harvesting vibration energy, all the magnetic flux generated by the PM should pass the stator slots and form an enclosed close magnetic path to generate electrical energy. Based on this principle, the stator back-iron width $t_2$ and slot width $w_2$, as illustrated in Fig. 2, can be calculated by:

$$t_2 = \frac{4\phi_g}{\pi \theta_{\text{max}} (D_{si} + D_{so} + 2h_l)}$$  \hspace{1cm} (10)
\[ w_2 = \frac{2\phi_g}{\pi N_{tm} B_{\text{max}} (D_{st1} + 2(b_2 + h_3))} \]  

(11)

where \( B_{\text{max}} \) is the maximum flux density of the DT4E; \( N_{tm} \) is the number of slots per magnetic pole.

2.3 Optimization of the harvester

The designed linear PM harvester utilizes electromagnetic induction between the PMs and the coils to convert vibration energy into electrical energy when the stator moves through the magnetic field generated by the PMs. To achieve optimal conversion efficiency with given external and internal space requirements, it is very important to optimize the dimensions of the PMs to obtain a maximum magnetic flux in harvester. For a single pole pair with a certain pole pitch \( \tau = 20 \text{ mm} \), as shown in figure 2, the average magnetic flux in the air-gap for different structural parameters can be plotted in Fig. 4 to Fig. 6.

Fig. 4 shows that the flux in the air-gap is reverse proportional to the thickness of the air-gap and a little decrease of the air-gap will cause the dramatic increase of the flux in the air-gap. Also, for the given \( D, D_{m} \) and \( t \), the maximum flux in the air-gap occurs at the condition of \( h_{m}/\tau \) equal to 0.5. Fig. 4 indicates the vital role of the air-gap in the designing of the PM harvester. Small air-gap can effectively increase the flux in the air-gap and thus the electrical output of the PM harvester. However, small air-gap will induce a bigger cogging force that results from the attraction to the permanent magnets by the stator back-iron and the slots. Therefore, a trade-off between a big air-gap flux and a small cogging force in the process of designing the PM harvester should be carefully considered. Fig. 5 shows that the air-gap flux is proportional to the outer diameter of the permanent magnet, and figure 6 shows that the air-gap flux also increases with the decrease of the diameter of the connected shaft within a given space condition. Same as Fig. 4, Fig. 5 and Fig. 6 indicate that there is an optimal value of \( h_{m}/\tau \) (equal to 0.5), on which the air-gap flux trends to maximum regardless of the values of \( D, D_{m} \) and \( t \). For a given space, the volume of the permanent magnet increases with the increase of the outer diameter of the permanent magnet \( (D_{m}) \) and the decrease of the diameter of the connected shaft \( (D) \). Therefore, Fig. 5 and Fig. 6 indicate that the magnetic flux in the air-gap with increase with the permanent magnet volume ratio within a given space condition, and it is useful to increase the volume of the permanent magnet under the condition of satisfying the space and design requirement.
The fundamental component of the Fourier series expansion of the air-gap flux density is another important item [21], since it is proportional to the flux linkage, and the electrical output under given exciting vibration. Based on the device design, it is given by:

\[ B_{gf} = \frac{4}{\pi} B_g \sin \left( \frac{\pi \tau - h_m}{\tau} \right) \]  

(12)

Fig. 7 shows the variation of normalized \( B_g \) and \( B_{gf} \) for different \( h_m/\tau \) values. It can be seen that, without considering the magnetic saturation, \( B_g/B_{ren} \) always increases with \( h_m/\tau \). Although the flux density \( B_g \) still increases with \( h_m/\tau \), \( B_{gf} \) has a maximum saturation with respect to \( h_m/\tau \), as shown in Fig. 7, and the maximum \( B_{gf} \) occurs when \( h_m/\tau \) is around 0.7. Also, based on the results from Fig. 4 to Fig. 6, it can be seen that the flux in the air-gap, \( \Phi_g \), is still large when \( h_m/\tau \) is equal to 0.7. Therefore, the value of \( h_m/\tau \) is selected as 0.7 in order to get a large \( B_{gf} \) and guarantee the kinetic-to-electrical conversion. Based on the results above, the optimised structural parameters of the linear PM harvester are given in Table 2.

2.4. Finite element analysis

A finite element (FE) model of the optimized PM linear harvester was built in ANSYS 13.0 for a magnetic field analysis to verify the design. PLANE13 element type was used for the FE analysis in ANSYS 13.0. The FE analysis provides results on the flux linkage that was used in the theoretical analysis on electrical power output. The FE model also provides a ‘sanity check’ for the magnetic circuit and analytical calculations.

Fig. 8 shows the FE model of the designed PM harvester for a given position of the mover. The FE model includes the permanent magnets, the permeable magnet pole steel, the air-gap between the magnets and the slotted teeth. Generally, it is assumed that there is no flux leakage at the perimeter of the model in an ideally closed magnetic circuit, and the magnetic flux is assumed to flow parallel to the surface. The assumption of no flux leakage is reasonable in this design because the air gap is very small, 2 mm, and several innovative designs were used to prevent the flux leakage.

Fig. 9 shows the flux lines in the linear PM harvester, and it can be seen that almost all of flux lines are across the pole steel, air-gap, and stator to form closed paths. Fig. 10 shows the comparison on magnetic field distribution in air-gap of two adjacent magnetic pole areas from FE analysis and analytical results. It can be seen that the two distributions are not very close. This is caused by the slotting effect. However, two curves
follow same trend and the areas of the two regions bounded by the x-axis and the two curves are approximately equal to each other. Therefore, FM analysis indicates that the structural design and magnetic analysis is efficient and the PM harvester can generate adequate magnetic field to harvest electrical energy.

3. Theoretical analysis on harvested electrical power

3.1 Output inductive voltages

As the magnetic mover system moves due to the external vibration, the stator coils experience a change in flux linkage and therefore induced voltage is generated in the coils. Electrical power is extracted from the harvester when the coils connect with a resistive load. With the pole pitch fixed at 20 mm, the flux linkage of a coil is dependent on the relative motion between the mover and stator. At low frequencies (a few Hz), it is normally assumed that the coils inductance is negligible [23]. Therefore, under open circuit condition, the inducted voltage of a coil, \( j \), can be obtained from the following equation:

\[
E_j = \frac{d\lambda_j}{dt} = \frac{d\lambda_j}{dz} \frac{dz}{dt} = -N_j \Phi_{g} \sin \left( \frac{\pi}{\tau} z + \theta \right) \frac{dz}{dt}
\]  

where \( \lambda_j \) is the flux linkage in coil \( j \) due to the permanent magnet; \( \Phi_{g} \) is the peak flux in the air-gap; \( N_j \) is the number of turns of coil \( j \); \( \theta \) is the phase angle; \( z \) and \( \frac{dz}{dt} \) are the displacement and velocity of the mover relative to the stator, respectively.

Because the pole pitch \( \tau = 2\tau_c \), where \( \tau_c \) is the coil slot pitch, the phase shift in the next coil is 90°. The number of magnets used in this design is 7, so 14 of the 22 coils will be objected to electromagnetic induction and hence induced voltage. Note that the phase angle \( \theta_1 \) of the first coil is equal to 0°, and the voltages in the first four working coils can be calculated as follows:

\[
E_1 = -N_1 \Phi_{g} \frac{\pi}{\tau} \sin \left( \frac{\pi}{\tau} z \right) \frac{dz}{dt} = -N \Phi_{g} \frac{\pi}{\tau} \sin \left( \frac{\pi}{\tau} z \right) \frac{dz}{dt}
\]  

\[
E_2 = -N_2 \Phi_{g} \frac{\pi}{\tau} \sin \left( \frac{\pi}{\tau} z + \frac{\pi}{2} \right) \frac{dz}{dt} = -N \Phi_{g} \frac{\pi}{\tau} \cos \left( \frac{\pi}{\tau} z \right) \frac{dz}{dt}
\]  

\[
E_3 = -N_3 \Phi_{g} \frac{\pi}{\tau} \sin \left( \frac{\pi}{\tau} z + \frac{3\pi}{2} \right) \frac{dz}{dt} = +N \Phi_{g} \frac{\pi}{\tau} \sin \left( \frac{\pi}{\tau} z \right) \frac{dz}{dt}
\]  

\[
E_4 = -N_4 \Phi_{g} \frac{\pi}{\tau} \sin \left( \frac{\pi}{\tau} z + \frac{5\pi}{2} \right) \frac{dz}{dt} = +N \Phi_{g} \frac{\pi}{\tau} \cos \left( \frac{\pi}{\tau} z \right) \frac{dz}{dt}
\]
Therefore, the voltage outputs of the all 14 working coils can be summarized as:

\[ E_{2i+1} = (-1)^i E_1, \quad E_{2i+2} = (-1)^i E_2 \quad i = 1, 2, 3, \ldots, 6 \tag{18} \]

In equations (14) to (17), \( N \) is the number of turns of any stator coils and is given by:

\[ N = \frac{4k\pi (r_0 - w_d)}{\pi w_d} \tag{19} \]

where \( w_d \) is the diameter of the polyurethane-enamelled copper wire used for winding the coils; \( \kappa \) is the fill factor of the coils, which is defined by relating the area of the polyurethane-enamelled copper wire, \( A_{\text{wire}} \), to the cross-sectional area of the coil, \( A_{\text{coil}} \), where \( A_{\text{wire}} = \kappa A_{\text{coil}} / N \). The copper fill factor depends on tightness of winding, insulation thickness, and winding shape. In general, most coils are scramble wound and have a typical fill factor from 0.5 to 0.75 [26], and in this paper \( \kappa = 0.65 \) is selected.

### 3.2 Coils winding design

From equations (14) to (18), it can be found that there is 90° phase difference between the induced voltages from coil-1 and coil-2, \( E_1 \) and \( E_2 \). Following the same trend, there are -90° and 90° phase differences between the coils before and after any particular coil. To avoid the elimination between the voltage outputs from adjacent coils, two polyurethane-enamelled copper wires, wire-A and wire-B, wrap simultaneously to make the stator coils, and the winding direction of one coil is opposite with the adjacent two coils, as the positive and negative symbol shown in Fig. 11.

When the coils are winded by using the novel method shown in Fig. 11, the output voltages of all the working coils of the PM harvester can be rewritten as:

\[
\begin{align*}
E_1 &= E_{2i+1} = -N \frac{\pi}{r} \sin \left( \frac{\pi z}{r} \right) \frac{dx}{dt} \\
E_2 &= E_{2i+2} = -N \frac{\pi}{r} \cos \left( \frac{\pi z}{r} \right) \frac{dx}{dt}, \quad i = 1, 2, 3, \ldots, 6
\end{align*}
\tag{20}
\]

Here, it is assume that \( E_1, E_3, \ldots, E_{13} \) are harvested in wire-A coils, and \( E_2, E_4, \ldots, E_{14} \) are generated in wire-B coils. Then the output voltage of wire-A and wire-B can be presented as:

\[
\begin{align*}
E_A &= 7E_1 = -7N \frac{\pi}{r} \sin \left( \frac{\pi z}{r} \right) \frac{dx}{dt} \\
E_B &= 7E_2 = -7N \frac{\pi}{r} \cos \left( \frac{\pi z}{r} \right) \frac{dx}{dt}
\end{align*}
\tag{21}\]
From equation (21), it can be seen that there are totally two voltage outputs with phase difference of 90°, that is why it is called as two-phase PM harvester. Assumed that wire-A coils and wire-B coils are phase-A and phase-B, respectively. By substituting equation (19) into equation (21), the output voltage of phase-A and phase-B can be rewritten as follows:

\[ E_A = -\frac{2\beta h G e}{\tau w_d^2} \sin \left( \frac{\pi z}{\tau} \right) \frac{dz}{dt} \]

\[ E_B = -\frac{2\beta h G e}{\tau w_d^2} \cos \left( \frac{\pi z}{\tau} \right) \frac{dz}{dt} \]  

(22)

### 3.3 On-load output electrical power estimation

In practice, phase-A coils and phase-B coils are connected in series to power the external load. \( R_c \) is the total resistance of the stator coils. When the PM harvester is connected to an external load, the output electrical power \( P \) can be expressed as:

\[ P = P_A + P_B = \frac{E_A^2 + E_B^2}{R_c + R_{\text{load}}} = \frac{784\beta^2 h_G^2 h_1^2 (\tau_1 - w_2)^2}{\tau^2 w_d^4 (R_c + R_{\text{load}})} \left( \frac{dz}{dt} \right)^2 \]  

(23)

where \( P_A \) and \( P_B \) are the output electrical power of phase-A coils and phase-B coils, respectively; \( R_{\text{load}} \) is the resistance of the resistive load.

From equation (23), when the resistance of \( R_{\text{load}} \) equal to \( R_c \), the output electrical power of the PM harvester reaches the maximum. In this paper, \( R_c \) can be determined as follows:

\[ R_c = \rho \frac{352 e h_1 (\tau_1 - w_2) (D_{so} - 2t_2 - h_1)}{\pi w_d} \]  

(24)

where \( \rho \) is the conductivity of the electrical wire; \( h_1, D_{so}, w_2, t_2 \) and \( \tau_1 \) are the structural parameters of the stator, as shown in figure 2.

Based on equation (23) and (24), the maximum harvested electrical power of the linear PM harvester can be expressed as:

\[ P_{\text{max}} = \frac{49\pi k h_1 \beta^2 (\tau_1 - w_2)}{44 \rho \tau^2 (D_{so} - 2t_2 - h_1)} \left( \frac{dz}{dt} \right)^2 \]  

(25)

Equations (23) and (25) describe the relationship between the harvested power and external vibration excitation. The generated power applied to the external resistive load is proportional to the square of the
excitation velocity. The equations also show the importance of the magnetic flux in the air-gap, $\Phi_g$. A doubling of $\Phi_g$ results in a quadratic increase in the maximal harvested power. Therefore, it is very important in section 2 of this paper to adopt several innovative designs to enhance the magnetic flux in the air-gap and prevent the magnetic leakage.

Also, for a fixed dimension of the linear PM harvester, comparing equations (22) with equation (25), it can be found that the generated voltage of the coils is inversely proportional to the square of the diameter of the electrical wire $w_d$, while the maximal harvested power is not affected by the selection of the diameter of the electrical wire. However, for easy post-processing of the generated power, it is desirable to have high voltage output from coils. Therefore, the electrical wire with small diameter is preferred. Details of the polyurethane-enamelled copper wire used in this paper are listed in Table 3. Based on the structure of the harvester and the properties of the wire, the total electrical resistance of the coils is 49 $\Omega$ and the self-inductance of the coils is 0.043 H. The self-inductance is very small, so it is reasonable to neglect the effect of the self-inductance of the coils under low frequency vibrations.

3.4 Thrust force estimation

One of the major concerns with permanent-magnet machines is the thrust force that results from the interaction between the inductive current and the permanent magnet field. The total thrust force on the coils of the PM harvester is position- and relative-velocity dependent. Based on the energy conservation law, the kinetic power used to drive the harvester is equal to the electrical power generated by the harvester. Therefore, the electromagnetic thrust force can be calculated by:

$$f_n = \frac{p}{v}$$

where $v$ is the relative velocity between the coils and the permanent magnets.

Substituting equation (23) into equation (26) yields:

$$f_n = \frac{p}{v} = \frac{794 \Phi_g^2 2k^2 h^2(t_1-w_2)^2 \, dz}{\tau^2 w_d^3(R_e+R_{load})}$$

Equation (27) indicates that the thrust force of the harvester determined by the relative velocity, magnetic field, external load and the structure of the harvester. Fig. 12 shows the comparison of thrust force waveforms, predicted by both analytical and FE solution, at a relative velocity of 0.4 m/s. It can be seen that the value of
thrust force changes with the relative position between the coils and the permanent magnets.

4. Evaluation on electrical power output

4.1 Open circuit voltages of the PM harvester

A series of investigations on the open circuit voltages of the phase-A and phase-B coils are explored firstly by using MATLAB software. In the analysis, sinusoidal signals are used to simulate the ambient vibrations. Limited to the space constrain of the PM harvester, the amplitude of the sine wave cannot exceed 30 mm. Fig. 13(a) shows the comparison of open circuit voltages of phase-A and phase-B coils under an excitation with frequency of 1 Hz and amplitude of 20 mm. It can be seen from the figure that $E_A$ and $E_B$ are AC voltages, changing its direction and value with the time. In addition, and there is a phase difference between $E_A$ and $E_B$ and the difference is around 90° angle degree, as stated in section 3. Also, there is a slight phase difference between the waveform of $E_A$ and $E_B$. Based on the equation (22), it can be found that the difference between the two-phase output voltages waveforms is caused by the interaction of multiple trigonometric operation and derivative operation.

Fig. 13(b) shows the comparison of $E_A$ and $E_B$ under an excitation with frequency of 3 Hz and amplitude of 20 mm. Comparing Fig. 13(a) with Fig. 13(b), it can be found that, under the same vibration amplitude conditions, amplitudes of $E_A$ and $E_B$ under 3 Hz excitation are about 3 times of the amplitudes of $E_A$ and $E_B$ under 1 Hz excitation, which means that the output voltage of the PM harvester is proportional to the velocity of exciting vibration and it agrees with the results of equation (22).

4.2 Two-phase rectified circuit

As shown in Fig. 13, open circuit voltages of the linear PM harvester are AC voltages and cannot used directly by normal used electronics. Also, there is 90° phase difference between the output voltage of phase-A coils, $E_A$, and phase-B coils, $E_B$. In order to fully utilize the output voltages, a two-phase rectified circuit is designed to convert the AC voltages, $E_A$ and $E_B$, into DC voltage so as to eliminate the phase difference which is normally associated with AC voltage.

Fig. 14 shows the output voltage simulation diagram of the linear PM harvester with a designed two-phase rectified circuit in PSPICE software. In Fig. 14, V1 and V2 represent open circuit voltages $E_A$ and $E_B$, respectively; R1 and R2 are resistances of phase-A and phase-B coils respectively; R3 represents the external resistive load; C1 is the smoothing capacitance.
In the analysis, the input signals of V1 and V2 use the MATLAB simulation results from the open circuit voltages analysis, indicated by \( E_A \) and \( E_B \), as shown in Fig. 13. The external resistive load (i.e., R3 in Fig. 14) in the PSPICE simulation is set to 49Ω, the internal resistance of the harvester, in order to induce a maximum electrical power. Note that, as shown in equation (22), the output voltages of the harvester are determined by its structure and the vibrational driving velocity. Therefore, under a given ambient vibration, open circuit voltages (\( E_A \) and \( E_B \)) of the proposed harvester remain constant and does not change with the external loads. Fig. 15 gives the simulation results of the output voltage from the PM harvester after the rectification by the two-phase rectified circuit. Fig. 15(a) shows the output voltage on external load through two-phase rectified circuit under sinusoidal excitation with frequency of 1 Hz and amplitude of 20 mm. As shown in the figure, after about 8 seconds, the output voltage reaches saturation voltage around 22 V and the ripple in the stable state is caused by the charge and discharge effect of the smoothing capacitance. Fig. 15(b) shows output voltage from the two-phase rectified circuit under sinusoidal excitation with frequency of 3 Hz and amplitude of 20 mm. Similar as the results in Fig. 15(a), the output voltage will reach saturation around 66 V after few seconds. However, the ripple in Fig. 15(b) is smaller than the ripple in Fig. 15(a), which is caused by the fact that the smoothing capacitance needs smaller time to charge and discharge under higher frequency. Fig. 15 indicates that the designed two-phase rectified circuit can convert the AC voltages from the PM harvester into DC voltage that can be used directly by external load. Also, compared Fig. 15(a) with Fig. 15(b), it can be found that the output voltage of the PM harvester after rectification is also proportional to the velocity of the exciting vibration.

4.3 Electrical power harvesting capacity

To evaluate the capacity of the PM harvester, the harvested electrical powers are summarized in this part. Fig. 16 shows the DC voltage outputs on resistive load under different sinusoidal vibrations. From Fig. 16, it can be seen that the DC voltage outputs of the linear PM harvester are proportional to the frequency and amplitude of the input vibration. Fig. 17 shows the DC electrical outputs of the harvester under different sinusoidal excitations. As shown in Fig. 17, the DC electrical outputs will increase with the frequency and amplitude of the sinusoidal inputs.
In order to reveal clearly the relationship between the harvested electrical energy and the strength of the input vibrations, Fig. 18 shows the relation between the voltage outputs and the RMS value of vibration velocity. As shown in the figure, the DC voltage outputs on resistive load are approximately proportional to the excitation velocities. The relation between the harvested electrical power and the RMS value of exciting velocity is shown in Fig. 19. It can be found that the harvested electrical power of the PM harvester increases with the RMS value of exciting velocity. And the trend is not linear, with a great increase in the incremental power with increasing velocity. Also, Fig. 19 shows that the value of harvested electrical power can reach up to 100 W when the exciting velocity exceeds 0.4 m/s, which is sufficient for most electronics.

4.4 Power harvesting capacity comparison

The main aim of this paper is to improve the energy harvesting efficiency of PM vibration energy harvesters by using a novel two-phase linear PM energy harvester. Therefore, a comparison of the PM harvester proposed in this paper with other typical PM harvesters presented in the state of art is necessary. Table 4 shows a comparison of the proposed PM harvester with different designs in the literatures.

Equations (1) and equation (25) indicate that the electrical power generated by PM harvesters is proportional to the volume and the square of the exciting velocity. Therefore, it is fair to employ the power density as a calibration item to evaluate their abilities to generate electrical power under a nominal velocity of 0.4m/s. From Table 4, it can be seen that the PM harvester proposed by Prudell et al. [18] could generate the maximum electrical power among five selected designs, about 1 kW. However, the ocean wave PM harvester proposed by Prudell et al. [18] has the biggest volume, around 3.4 m$^3$, and the energy was harvested under the ocean wave motions with linear velocity of 0.76 m/s. Therefore, its power density under nominal velocity, i.e. 0.4 m/s, is 83.1 $\mu$W/cm$^3$. Table 4 clearly shows that the novel linear PM harvester proposed in this research, with an amazing power density of 90.3 mW/cm$^3$, has better power density than other devices. In other words, it shows that the novel linear PM harvester proposed in this paper has higher vibration-to-electrical energy conversion efficiency and can generate more electrical power under the same volume and vibration conditions.

5. Summary

This paper presented systematic investigations, including innovative designing, magnetic circuit modeling, theoretical analysis on output power and FE analysis of a novel two-phase linear PM energy harvester for
ambient vibration energy applications. The proposed two-phase linear PM harvester has minimal magnetic reluctance and magnetic leakage, and higher vibration-to-electricity conversion efficiency. The average flux in the air-gap has been expressed as a function of the parameter of the harvester, and several general design criteria have been found. Among them, the optimum ratio of the PM thickness to the pole has been set to 0.7 in order to maximize the magnetic flux density in the air-gap. The methodology of winding the two-phase coils in the stator slots were developed based on the theoretical analysis of the output electrical power in order to fully use the magnetic field generated by the PMs. The simulation results indicate that the proposed PM harvester has higher energy harvesting efficiency and the magnitude of the harvested electrical power can reach to 100 W under vibrations with 0.4 m/s velocity.

Acknowledgements

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References


Table 1. PM harvester parameters

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<th>Value</th>
<th>Units</th>
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<td>Power</td>
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<tr>
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<td>Stator inner diameter</td>
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<td>Magnet max energy product</td>
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Table 2. Structural parameters of the linear PM harvester

<table>
<thead>
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<th>Parameter</th>
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<td>$D$</td>
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<td>Diameter</td>
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Table 4. A comparison of different PM vibration energy harvesters

<table>
<thead>
<tr>
<th>Design</th>
<th>Vibration conditions</th>
<th>Power</th>
<th>Volume</th>
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<th>Power density @0.4 m/s</th>
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<tbody>
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<td>2.5 mW</td>
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<td>16.3 mW/cm³</td>
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<td>Prudell et al. [19]</td>
<td>Ocean wave motion 0.76 m/s</td>
<td>1 kW</td>
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<td>0.3 mW/cm³</td>
<td>83.1 µW/cm³</td>
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<td>Sapinski [20]</td>
<td>Relative velocity 0.16 m/s</td>
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<td>1099 cm³</td>
<td>728.2 µW/cm³</td>
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<tr>
<td>This design</td>
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<td>1108 cm³</td>
<td>90.3 mW/cm³</td>
<td>90.3 mW/cm³</td>
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</tbody>
</table>
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