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A NUMERICAL STUDY ON FRACTAL DIMENSIONS OF CURRENT STREAMLINES IN TWO-DIMENSIONAL AND THREE-DIMENSIONAL PORE FRACTAL MODELS OF POROUS MEDIA

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Abstract

The fractal dimension of random walker (FDRW) is an important parameter for description of electrical conductivity in porous media. However, it is somewhat empirical in nature to calculate FDRW. In this paper, a simple relation between FDRW and tortuosity fractal dimension (TFD) of current streamlines is derived, and a novel method of computing TFD for different generations of two-dimensional Sierpinski carpet and three-dimensional Sierpinski sponge models is presented through the finite element method, then the FDRW can be accordingly

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predicted; the proposed relation clearly shows that there exists a linear relation between pore fractal dimension (PFD) and TFD, which may have great potential in analysis of transport properties in fractal porous media.

Keywords: Random Walker; Tortuosity; Fractal Dimension; Pore Fractal.

1. INTRODUCTION

There has been great interests in transport properties of porous media in various fields such as reservoir engineering,[1–3] environmental geology,[4,5] composite material processing,[6,7] soil science,[8,9] etc. Understanding the correlation between transport properties and microstructure of porous media is crucial for their widespread use.[10,11] With the current interest in upscaling electrical properties, fractal models are being widely believed as a viable descriptors of reservoir rocks.[12–15]

Fractal theoretical investigations on electrical conductivity of porous media have been presented in the past several decades. Early studies attempted to formulate expressions for electrical conductivity based on the Einstein relation. The results suggested a general relationship between electrical conductivity (σ) and porosity (ϕ) of the form $\sigma \propto \phi^m$, where m is a scaling exponent. Different researchers found different expressions for the exponent m as a function of fractal dimensions.[16–19] Katz and Thompson[18] performed experiments by measuring pore fractal dimension (PFD) (D_f) with length parameter. They expected that the electrical conductivity of rock sample was self-similar for length scales based on the Einstein relation. However, the scaling relationship between σ and ϕ was not covered in their work. Roy and Tarafdar[17] developed electrical conductivity and porosity models for fractal generator block. For a three-dimensional system with deterministic volume, its electrical conductivity scaling law including fractal dimension of random walker (FDRW) (D_w) can be expressed as[19,20]:

$$\sigma \propto \phi^{1+(2-D_w)/(D_f-D_E)}, \tag{1}$$

where D_E is the Euclidean dimension, and $D_E = 2$ and $D_E = 3$ in the two- and three-dimensional spaces, respectively. In Eq. (1), the calculating method of FDRW is somewhat empirical in nature. Furthermore, the predictions of electrical conductivity are usually expressed in terms of observable macroscopic parameters of porous rocks, such as porosity,[21,22] in current methods.

Several micro-structural parameters are important for modeling and simulating transport properties in porous media, such as tortuosity,[23–25] lacunarity,[26,27] pore size distribution,[28] pore surface area,[29] etc. Tortuosity (T) has been recently introduced to describe electrical conductivity,[30,31] which quantifies the tortuous or twisted nature when current passes through a porous medium. The length of the actual path of current is defined as a function of discrete length measuring,[32] $L_T = \lambda^{1-D_T} L_0^{D_T}$, where λ, L_0 and D_T are the scale, straight distance and tortuosity fractal dimension (TFD), respectively. The parameter D_T denotes the complex level of the actual current length ($D_T = 1$ for a straight path).

In this paper, we will derive the interrelation between FDRW and TFD, and introduce an easier method to calculate those fractal dimensions.

2. THE RELATION FOR FRACTAL DIMENSIONS OF RANDOM WALK AND TORTUOSITY

We assume the porous medium can be represented as a pore fractal. Applying fractal theory, Katz and Thompson[18] presented a scaling relation between porosity and fractal pore size range:

$$\phi \propto (\lambda_{\min}/\lambda_{\max})^{D_E-D_f}, \tag{2}$$

where $\lambda_{\min}$ and $\lambda_{\max}$ are minimum and maximum fractal pore size range.

Yu *et al.*[33] derived a scaling for tortuosity and fractal dimension as

$$T \propto \left(\frac{L_0}{\lambda_{\min}}\right)^{D_T-1}. \tag{3}$$

Besides Eq. (3), the assumption that $\lambda_{\max}$ can replace L_0 when minimum pore size is smaller enough than the maximum pore size,[34] is also used in the presented analysis. The electrical conductivity can be expressed with tortuosity and porosity[35]:

$$\sigma \propto \phi/T. \tag{4}$$

Combining Eqs. (2)–(4) yields the scaling law:

$$\sigma \propto \phi^{1+(D_T-1)/(D_E-D_f)} \tag{5}$$

Equation (5) is obtained in terms of fractal nature of porous media, which has the similar form to the classical Archie's equation.[36] From Eq. (5), the electrical conductivity is related to the physical parameter (ϕ) and microstructure characters (D_f and D_T) of porous media. Comparing Eq. (5) to Eq. (1), we have

$$D_w = D_T + 1. \tag{6}$$

Equation (6) provides a method for predicting FDRW of porous media by means of fractal geometry theory. In the following sections, we will investigate the validity of this relationship between FDRW and TFD of porous media by numerical simulations.

3. CALCULATING TORTUOSITY FRACTAL DIMENSION

Tortuosity is one of the important parameters for describing macroscopic current distribution under an applied potential. However, pore geometry decides the position of current flow that is almost impossible to obtain. Coleman and Vassilicos[19] provided a solution to tortuosity of fluid flow problem, in which the tortuosity is considered where the ratio of kinetic energy of tortuous flow to those pores path is straight. Herrick and Kennedy[30] developed a streamline model to calculate "electrical efficiency" in terms of power in two-phase rock/pore system, and the electrical efficiency is expressed as the rock sample power divided by the maximum power of the rock when an electric current is induced to flow axially through a cylindrical rock, and the current model is similar to Coleman and Vassilicos's tortuosity model. Furthermore, a characteristic singular streamline model should completely represent electric current in porous media when the macroscopic current is a specific pore configuration (Fig. 1).

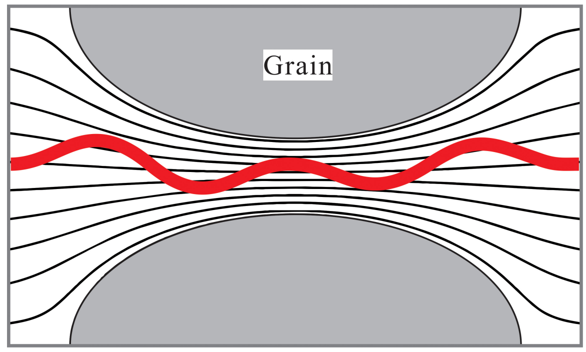

Fig. 1 The tortuosity of a rock depends on the contribution of all conductive components of pore space in rock. A particular singular streamline model (red streamline) effectively represents electric current flow in the rock.

Assuming the geometry of conducting phase of pore is known, the electric field can be expressed as $E = \nabla\psi$, where ψ is the potential, and Ohm's law is $J = \sigma E$, where J is the current density and σ represents each point conductivity in a pore space. These relations give a classic potential equation $\nabla \cdot \sigma\nabla\psi = 0$. The electric charge moves distance d_e in terms of power $P = F \cdot d_e/t$ of electric field in an applied potential force F and time t, and the power will be given[30]

$$\begin{aligned} P &= \int_V \gamma(r')J(r') \cdot E(r')dv' \\ &= \int_V \gamma(r')\sigma(r') \cdot E^2(r')dv', \end{aligned} \tag{7}$$

where r' is a position in rock volume V, and $\gamma(r') = 0, 1$ is a function in which 1 denotes the presence of pore of space. Finite element method (FEM) for the solution of electric field E can be found in literature.[37]

The tortuosity can be considered as the ratio of the kinetic power of equivalent straight streamlines P_s to those of tortuosity streamlines P_t. Therefore, a relationship can be given to tortuosity in a fully-saturated pore space[30] as

$$T = \frac{P_s}{P_t} = \frac{\int_{V_{\text{tube}}} \gamma(r')\sigma_w E^2_{\text{tube}} dv'}{\int_{V_{\text{rock}}} \gamma(r')\sigma_w E^2_{\text{rock}} dv'} \tag{8}$$

where V_{tube} and V_{rock} are pore volume and whole volume of sample, respectively. The numerator of Eq. (8) can be considered as the power in straight streamlines (cross-sectional area A and streamline length L_0) is applied voltage v, the electric field would only be non-zero in the voltage direction (such as x-direction) and given by $\frac{\delta\psi}{\delta x} = \frac{v}{L_0}$. Then the power energy of this straight streamline is $P_s = \frac{\phi A}{L}\sigma_w v^2$, and Eq. (8) can be rewritten as

$$T = \frac{\phi A v^2}{L \int_{V_{\text{rock}}} \gamma(r') E^2_{\text{rock}} dv'}. \tag{9}$$

Equation (9) shows that the tortuosity is only related to electric field distribution in porous media

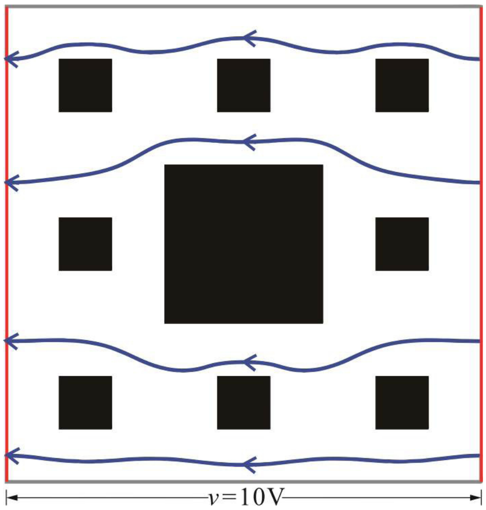

Fig. 2 Schematic of current paths through the 2nd-stage Sierpinski carpet with 10 V electric potential.

(Fig. 2). For two-dimensional space, the tortuosity of the square sample having area A' is

$$T = \frac{\phi v^2}{\int_{A'_{\text{rock}}} \gamma(r') E^2_{\text{rock}} ds'}. \tag{10}$$

The TFD for tortuosity streamline in porous media can be obtained as[38]

$$D_T = 1 + \frac{\ln T}{\ln \frac{L_0}{\lambda_{\text{av}}}}. \tag{11}$$

In Eq. (11), the average pore size of porous media λ_{av} can be expressed as[25]

$$\lambda_{\text{av}} = \frac{D_f}{D_f - 1} \lambda_{\min} \tag{12}$$

and L_0 can be written as[39]

$$L_0 = \sqrt{\frac{\pi}{4} \frac{D_f}{2 - D_f} \frac{1 - \phi}{\phi} \lambda^2_{\max}}. \tag{13}$$

Thus, the ratio L_0/λ_{av} in Eq. (11) can be calculated by

$$\frac{L_0}{\lambda_{\text{av}}} = \frac{D_f - 1}{D_f} \frac{\lambda_{\max}}{\lambda_{\min}} \sqrt{\frac{\pi}{4} \frac{D_f}{2 - D_f} \frac{1 - \phi}{\phi}}, \tag{14}$$

where $\lambda_{\max}/\lambda_{\min}$ is the self-similarity scaling of maximum iterative generator in the same model. According to Eqs. (9)–(11), TFD can be calculated if the electric field is determined in two- and three-dimensional space.

4. NUMERICAL SIMULATION RESULTS AND DISCUSSION

The Sierpinski carpet (Fig. 2) is a two-dimensional pore fractal model with the flow path of simulating a wide range of pore sizes and configurations. This exact self-similar fractal model has long been used as a model substrate for simulating transport problems in complex pore space geometries.[17]

Li and Yu[40] proposed a simple recursive model for tortuosity of flow path in Sierpinski carpet based on the self-similarity of carpet. The structure of the two-dimensional Sierpinski carpet pattern is as follows: a square of size L_0 is divided into $b \times b$ equal smaller squares, as shown in Fig. 2. l of the smaller square are occupied and the remaining $(b^2 - 1)$ is vacant. This gives the generator, or first stage pattern. Higher stages are obtained by repeating n of the division process on each of occupied squares, so the fractal dimension $D_f = \log(b^2 - l)/\log(b)$.

The second stage model of Sierpinski carpet is imported 10 V potential difference between left and right side in Fig. 2, where the blue line represents the electrical streamline induced in right side. The streamline length is different because the current moves to the perpendicular direction of field by passing through the particle. The average length of

Table 1 **The Parameters of the Standard Sierpinski Carpet of 1–5 Stage.**

Structure Parameter	Stage of Sierpinski carpet				
	1st	**2nd**	**3rd**	**4th**	**5th**
D_f	1.892	1.892	1.892	1.892	1.892
ϕ	0.889	0.790	0.702	0.624	0.555
T	1.134	1.265	1.405	1.556	1.716
D_T	1.023	1.041	1.057	1.072	1.086

Table 2 **Numerical Results of D_{wb} and D_T for Different Carpet Shapes from Fig. 3.**

No.	Figure Number	n	D_f	D_{wb}	D_T	$\|\xi\|$
1	(a)	2	1.896	2.160 ± 0.009	1.054	0.010
2	(b)	2	1.896	2.134 ± 0.009	1.053	0.015
3	(c)	2	1.771	2.241 ± 0.009	1.187	0.042
4	(d)	2	1.946	2.086 ± 0.009	1.029	0.002
5	(e)	2	1.946	2.075 ± 0.009	1.028	0.005
6	(f)	2	1.946	2.079 ± 0.009	1.029	0.003
7	(g)	2	1.853	2.131 ± 0.009	1.109	0.038
8	(h)	2	1.853	2.138 ± 0.009	1.114	0.033
9	(i)	2	1.853	2.124 ± 0.009	1.107	0.040
10	(j)	2	1.853	2.130 ± 0.009	1.109	0.038

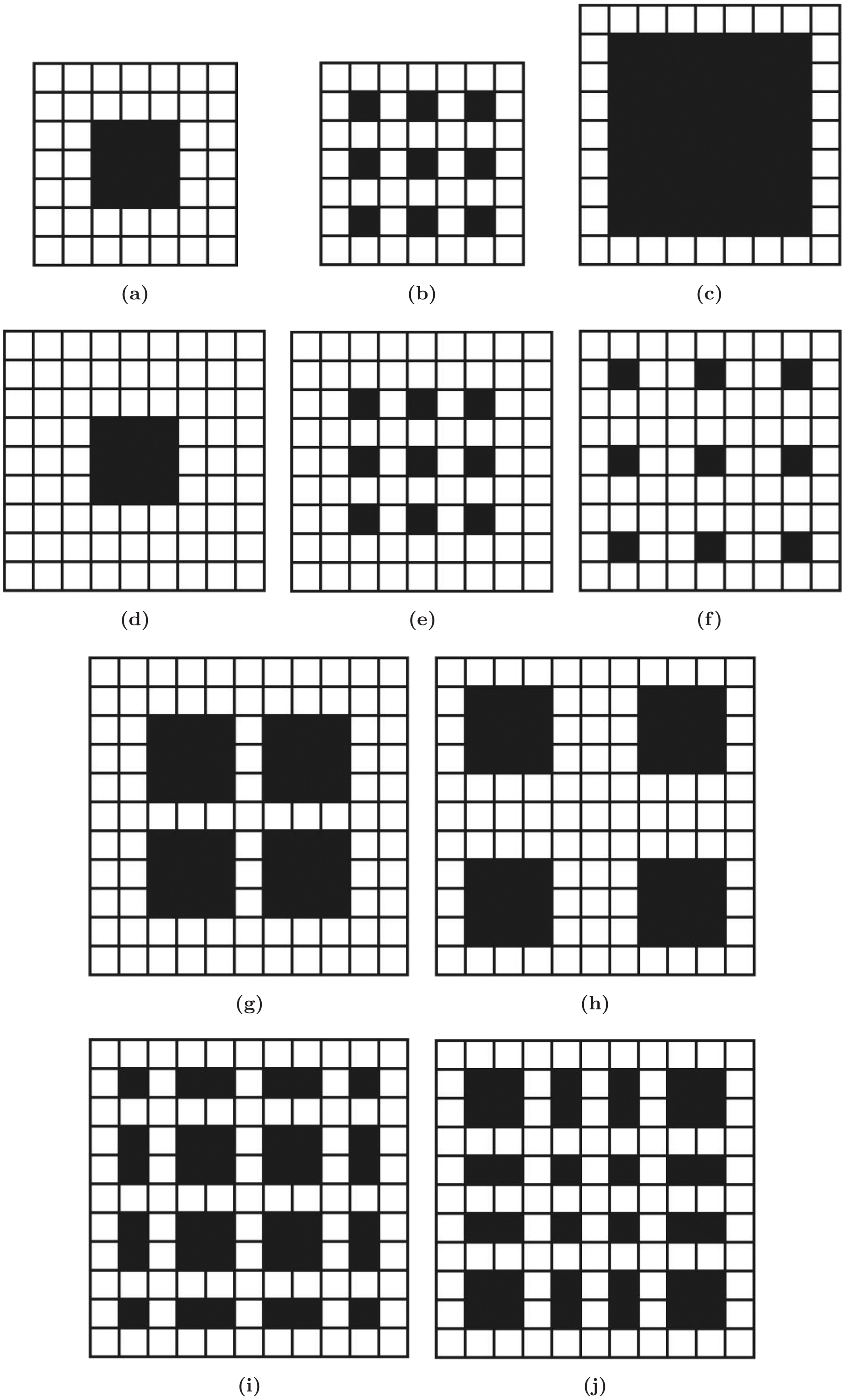

Fig. 3 Some examples of the set of carpets with the same value of PFD. The PFD of four groups (a–b), (c), (d–f), and (g–j).

streamline can be calculated through the computing power of electric field from FEM.

The fractal data for five standard carpet generators are shown in Table 1. It is observed that the tortuosity and TFD of each generator is increasing with the decrease of porosity, which are consistent the current researches.[25,40,41]

Table 2 lists different Sierpinski generators (Fig. 3) with three different dimensions: PFD, FDRW and TFD. The D_{wb} is determined by the blind ant algorithm,[42] the finitely-ramified carpets are calculated via the algorithm for 10,000 random walks of 100,000 walkers that is approximately equal to real FDRW. For TFD, the FEM is used to calculate the electric field distribution of each carpet model in Fig. 3, where the direction of electrical potential is the same as the solution of Fig. 2.

We plotted the relations between FDRW and TFD for the 11 carpets in Fig. 4. From Eq. (6), the PFD is increasing with the increase of D_{wb}, which is a consistent tendency as shown in Fig. 4. This can be explained that the more compact the pores of a carpet are, the less they hinder the random walks. It is obvious that the relation [Eq. (6)] is in better agreement with the numerical calculating results.

Since a random walker on fractal lattices has to move around the pores, the dimension of pore distribution should be related to FDRW. A better approximation of FDRW for infinitely ramified Sierpinski carpets requires a huge amount of simulation of long random walks, whose computation is very expensive and complex.[43] In practice, tortuosity in porous media is reliably obtained through empirical equation[44] or analytical solution[45] than FDRW in which the calculating approach is somewhat empirical in nature. Therefore, TFD is more convenient to calculate electrical conductivity for porous media.

For FDRW of porous media to be calculated, Havlin and Ben-Avraham[46] presented a relation $D_w = D_f + \bar{\xi}$, where $\bar{\xi}$ is the resistance exponent which is defined by scaling of resistance with length. When the front of diffusion is a spherical cut, then $D_w = D_f + 1$. The relation of Havlin and Ben-Avraham can also be written as

$$D_w + D_f = 2D_f + \bar{\xi}. \qquad (15)$$

Combining Eq. (6) gives

$$D_T + D_f = 2D_f - 1 + \bar{\xi}. \qquad (16)$$

The right part of Eq. (16) is replaced by the assumption of $2D_f - 1 + \bar{\xi} = D_E + 1 + \bar{\xi}$ in two-dimensional space ($D_E = 2$), Eq. (16) can be expressed as

$$D_f + D_T = D_E + 1 + \xi, \qquad (17)$$

where ξ is a constant related to PFD and the resistance exponent. We plot the values of $D_f + D_T$ from Table 2 for different carpets in Fig. 5. As shown in Table 2 and Fig. 5, ξ is a varying constant for different fractal carpets and its values are much less. When $\xi = 0$, by inserting Eq. (17) into Eq. (5) we obtain

$$\sigma \propto \phi^2. \qquad (18)$$

Equation (18) is a special case of classical Archie equation with $m = 2$. However, there is not any theory to prove that ξ identically equal to zero in porous media, for $\xi = 0$, which is only used in a limited way.

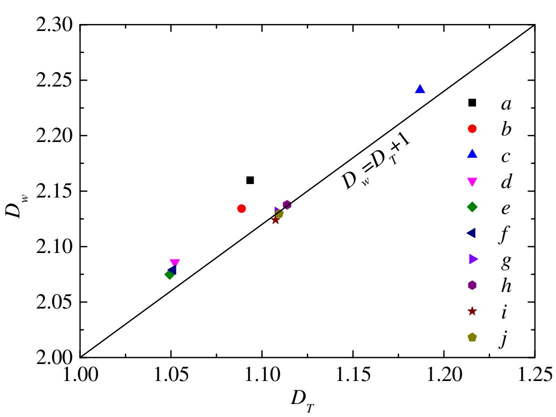

Fig. 4 The FDRW versus TFD for different carpets structure as shown in Table 2. The black line represents $D_w = D_T + 1$.

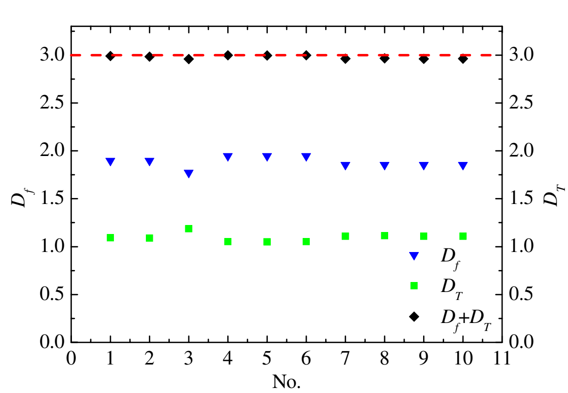

Fig. 5 For the carpets the PFD is plotted over the TFD with data of Table 2. The dashed line's value is 3 which is approximately equal to $D_f + D_T$.

Table 3 The Parameters of the Sierpinski Sponge of 1–3 Stage.

Structure Parameter	Sierpinski Sponge		
	1st	2nd	3rd
D_f	2.966	2.966	2.966
ϕ	0.963	0.927	0.893
T	1.023	1.044	1.060
D_T	1.006	1.011	1.014
$\|\xi\|$	0.028	0.024	0.021

In order to prove the validity of Eq. (17) in three-dimensional space ($D_E = 3$), we calculate TFD for $n = 1, 2, 3$ pore fractal Sierpinski sponge[47] generators with Eqs. (9) and (11) (here $\lambda_{\max}/\lambda_{\min} = 9$) and the results are shown in Table 3. The distribution of the resulting TFD value in Table 3 is similar to Sierpinski carpet in Table 2. The agreement between the predicted relation and numerical results is generally good for Sierpinski sponge, where the prediction of relation is better fitting in $|\xi| \leq 0.1$.

5. CONCLUSIONS

In this paper, we have derived a new analytical scaling relation for fractal dimensions of random walker and tortuosity based on the pore fractal model of porous media. The proposed relation is in full agreement with numerical calculating result of Sierpinski carpet using the FEM. The TFD (D_T) is coupled in models through the process that the current flows in complex pore system. In the presented pore fractal model cases where pore spaces are interconnected, the observed TFD has a linear relationship to PFD (D_f) with $D_f + D_T = D_E + 1 + \xi$. This linear dependence of TFD on PFD would be observed in detailed numerical models of Sierpinski carpet and Sierpinski sponge; subsequently a simple scaling was derived for fractal porous media.

Although these relations of fractal dimensions are presented based on the ideal/exactly self-similar pore fractal geometry structures, we argue that these relations are valid for statistically self-similar fractals (such as random/disordered/natural porous media). The simple linear relations, which perhaps embrace the deep intrinsic property of fractal media, should be given more attention. The relations involved in the prediction of electrical conductivity in experiment will be further discussed in our future work.

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