Intermediate Observer-Based Robust Distributed Fault Estimation for Nonlinear Multiagent Systems With Directed Graphs

Jian Han, Xiuhua Liu, Xianwen Gao, and Xinjiang Wei

***Abstract*—This article focuses on the problem of robust distributed fault estimation for nonlinear multiagent systems with actuator faults and sensor faults. The communication topology of the multiagent systems is assumed to be directed. A novel intermediate observer design method is proposed to estimate the system states, actuator faults, and sensor faults. For the observer constructed in one agent, the output estimation errors of itself and its neighbors are considered, simultaneously. The observer matching condition is not needed in the observer design process. Based on Schur decomposition, the observer parameter calculation method is presented in terms of solution to one linear matrix inequality, which is with the same order as it is for the single agent system. Thus, the calculated amount remains unchanged even when the number of agents increases, since the inequality dimension is independent of the agent number. At last, simulation results are provided to illustrate the effectiveness of the proposed technique.**

***Index Terms*—Actuator and sensor faults, directed graphs, distributed fault estimation, intermediate observer, multiagent systems.**

I. Introduction

With the development of industry, the industry systems are becoming much more complex. One of the powerful tools to describe the complex systems is multiagent systems. Recently, multiagent systems have become a hot topic of research, and excellent results about this type of systems have been obtained, such as [1]–[4]. So far, multiagent systems have been applied in many fields, such as smart distribution grids [5], wireless sensor networks [6], and so on.

It is worth pointing out that the probability of system failure will naturally increase with the increase of the agent number. Compared with the traditional centralized system or decentralized control system, the effect of fault in multiagent systems is much more serious, since the fault in one agent can affect the other agents by the information transfer. Hence, many researchers have paid much attention to fault diagnosis for multiagent systems. In [7], the unknown input observer has been designed for the multiagent systems. In these results, the fault in one agent could be detected by its neighbors. For heterogeneous multiagent systems, an observer-based fault detection and isolation (FDI) method has been proposed in [8]. In [9], the fault detection (FD) filter has been constructed to detect the fault. In [10], FD observer has been designed, in which the H_∞ performance and H_- performance have been considered simultaneously. In addition, a passive fault-tolerant control strategy has been proposed in [10] to ensure that the multiagent systems achieved state consensus. In [11], a mixed H_∞/H_2 optimization approach has been proposed to handle the distributed cooperative FD problem for multiagent systems.

It should be noted that most of them focused on the problem of distributed FDI, and very few of them considered the problem of fault estimation. In fact, fault estimation can accomplish the goal of FD–isolation and identification [12]. The information of fault, such as the size and the shape, can be achieved by the fault estimation technique. And the information may play an important role in fault-tolerant control. So far, some excellent fault estimation approaches have been reported. In [13], the descriptor observers have been designed to estimate the sensor faults. For the systems with actuator and sensor faults, a proportional integral observer design method has been reported in [14]. In [15] and [16], fault reconstruction schemes have been proposed for the nonlinear system with uncertainty, where sliding mode observer-based methods have been proposed. In [17], the reduced-order observer has been designed to estimate the faults in switched systems. It should be noted that in [15]–[17], the observer matching condition was needed. This condition might not be satisfied in many practical systems. In [18] and [19], adaptive technique has been utilized to estimate the varying

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fault signals, while the strictly positive real assumption was needed. Compared with the observer matching condition, the strictly positive real assumption is more conservative. To overcome these problems, a novel fault estimation method, which was named as the intermediate observer-based fault estimation method, was first proposed in [20], in which both the observer matching condition and the strictly positive real assumption were not needed. It should be pointed out that fault-tolerant control is a major purpose of fault estimation, and it is very important in practice. For some practical nonlinear systems, excellent fault-tolerant control methods have been reported, such as [21] and [22], and the references of them.

Unfortunately, most of the existing results only dealt with fault estimation for centralized system, and few results considered distributed fault estimation. In [23], sliding-mode observers have been considered to estimate the actuator faults in multiagent systems. In [24] and [25], distributed intermediate observers have been designed. In the results obtained in [23]–[25], the communication topologies of the multiagent systems were assumed to be undirected, which can be treated as special cases of directed graph. For multiagent systems with directed graphs, the full-order and reduced-order fault estimation observers have been designed in [26] and [27], respectively. In [28], a novel adjustable parameter-based distributed fault estimation observer has been designed to estimate the actuator faults. In order to calculate the observer parameter matrices, the linear matrix inequality (LMI) needed to be solved in [26]–[28], and the order of the LMI was dependent on the number of the agents. Thus, the calculated amount would increase when the number of agents was bigger.

In addition, in [23] and [26], the observer matching condition was needed. This assumption has been removed by designing intermediate observer in [24] and [25]. However, in the existing intermediate observer design process, the observer performance depended on a positive real number, which was selected artificially. Thus, the improper selection of the number might lead to undesirable estimation performance. Furthermore, as mentioned in the pioneer results [29], the multiple faults are common. In multiagent systems, because of the information transfer of each agent, the fault in one component may spread to other components of the whole systems. Thus, the multiple faults may lead to more serious effects. To the best of the authors' knowledge, there are few results about fault estimation for multiagent systems with actuator and sensor faults, simultaneously. This background provides the motivation for the present article. For the nonlinear multiagent systems with directed graph, a set of intermediate observers are designed to estimate the system states and the faults. The actuator fault and sensor fault are considered, simultaneously. The main contributions are summarized as follows.

1) In this article, a novel intermediate observer design method is proposed. This observer can be used for the systems in which the observer matching condition is not satisfied. Compared with the traditional intermediate observer design method, the output estimation error feedback terms, which include the centralized and distributed output estimation errors, are introduced. This may extend the application range of the observer and enhance the estimation performance.
2) Based on Schur decomposition, only one LMI is needed in order to calculate the observer parameter matrices, and the dimension of the LMI is equal to that of a single agent system. This implies that the calculated amount does not change when the number of agents increases.
3) This article considers the case that nonlinear multiagent systems are with directed graph. The proposed method can be used for the case that the communication topology is an undirected graph, since the undirected graph can be treated as a special case of the directed graph.

II. Systems Preliminaries

In this section, the graph theory basics will be introduced, and the problem formulation will be presented.

A. Graph Theory Basics

For a directed graph $\mathcal{G}(\mathcal{V}, \mathcal{E}, \mathcal{A})$, $\mathcal{V} = \{V_1, V_2, \cdots, V_N\}$ represents the vertex set, where V_i is the ith node, $i = 1, 2, \ldots, N$; $\mathcal{E} = \{(V_i, V_j) : V_i, V_j \in \mathcal{V}\} \subset \mathcal{V} \times \mathcal{V}$ represents the edge set; $\mathcal{A} = [a_{ij}] \in R^{N\times N}$ represents the adjacency matrix of the graph $\mathcal{G}$, where $a_{ij} = 1$ for $(V_i, V_j) \in \mathcal{E}$, and $a_{ij} = 0$ for other cases. The node V_j is a neighbor of node V_i if and only if $(V_j, V_i) \in \mathcal{E}$. Matrix $\mathcal{L} = [l_{ij}] \in R^{N\times N}$ denotes the Laplacian matrix of graph $\mathcal{G}$, where $l_{ii} = \sum_{j=1, j\neq i}^{N} a_{ij}$, $(i = 1, \ldots, N)$ and $l_{ij} = -a_{ij}$, $(i \neq j)$. It should be noted that more details about graph theory, such as the definitions of connected graph, path, and spanning tree, can be found in references [3], and the interested readers can refer to them.

B. Problem Formulation

Consider the nonlinear multiagent systems with the following dynamic of agent i:

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t) + B_h h(x_i(t)) + B_a f_{ai}(t) + B_d d_i(t) \tag{1}$$

$$y_i(t) = Cx_i(t) + D_s f_{si}(t) \tag{2}$$

where $i \in \{1, 2, \ldots, N\}$, $x_i(t) \in R^n$, $u_i(t) \in R^{n_u}$, $y_i(t) \in R^{n_y}$ are the system state vector, control input vector, and output vector of the ith agent, respectively. $h(x_i(t)) \in R^{n_h}$ represents the nonlinear dynamic of agent i. $f_{ai}(t) \in R^{n_{f_a}}$ and $f_{si}(t) \in R^{n_{f_s}}$ are the actuator fault vector and the sensor fault vector, respectively. $d_i(t) \in R^{n_d}$ is the external disturbance vector and it is assumed that $d_i(t) \in L[0, \infty)$. $A, B, B_h, B_a, B_d, C, D_s$ are known matrices with appropriate dimensions. It is supposed that C is of full row rank, B_a and D_s are of full column rank. Assume that the nonlinear function $h(x_i(t))$ is Lipschitz with respect to $x_i(t)$, i.e., $||h(v_1(t) - h(v_2(t))|| \leq \delta_h ||v_1(t) - v_2(t)||$, where $v_1(t), v_2(t) \in R^n$, δ_h is a Lipschitz constant.

It is assumed that both $f_{ai}(t)$ and $f_{si}(t)$ are differentiable. In fact, similar assumptions are common in many fault estimation results, such as [20], [23]–[28]. It should be pointed out that some pioneer results have considered the case that the fault is nondifferentiable, readers who are interested in it can refer to [29] and [30].

The main objective of this article is: for each agent i, utilizing the output information of itself and its neighbor agents, design a fault estimation observer to estimate the states $x_i(t)$, faults $f_{ai}(t)$ and $f_{si}(t)$, simultaneously.

The following assumption and lemma are useful:

Assumption 1: The graph $\mathcal{G}$ is directed and contains a spanning tree.

Lemma 1 (Young's Inequality): For any vectors $a \in R^n$ and $b \in R^n$, we have

$$a^T b \leq \frac{1}{p}\alpha^p ||a||^p + \frac{1}{q}\alpha^{-q}||b||^q$$

where $\alpha > 0$, $p > 0$, $q > 0$, and $pq = p + q$.

As pointed out in [20], the observer matching condition, that is, $rank(CB_a) = n_{f_a}$, plays an important role in many observer design results. And this condition is needed in many observer design methods [23] and [26]. However, this condition is not needed in this article. In Section IV, Example 2 is provided to verify it.

Remark 1: In this article, we consider the directed graph rather than the undirected graph. For undirected graph, $(V_j, V_i) \in \mathcal{E}$ means $(V_i, V_j) \in \mathcal{E}$. Thus, the Laplacian matrix of an undirected graph is a symmetric matrix [24]. While, the Laplacian matrix of a directed graph can be a symmetric matrix or a nonsymmetric matrix. From this point of view, the directed graph is more general than the undirected graph. In fact, the undirected graph can be treated as a special case of the directed graph [26] and [28], which implies that the proposed method can be used for the case that the communication topology is an undirected graph. Moreover, we only need that the graph contains a spanning tree, rather than the graph is strongly connected. In Section IV, Example 2 will indicate that the proposed method can be used for the case that the directed graph is not strongly connected.

III. Main Results

In this section, the fault estimation problem will be addressed.

A. State Transformation

Let $\bar{x}_i^T(t) = [x_i^T(t)\ f_{si}^T(t)]^T$, then the dynamic (1)–(2) can be rewritten as

$$K_0\dot{\bar{x}}_i(t) = A_1\bar{x}_i(t) + B_1u_i(t) + B_{h1}h(T_0\bar{x}_i(t)) + B_{a1}f_{ai}(t) + T_1D_sf_{si}(t) + B_{d1}d_i(t) \quad (3)$$

$$y_i(t) = C_1\bar{x}_i(t) = C_0\bar{x}_i(t) + D_sf_{si}(t) \quad (4)$$

where

$$K_0 = \begin{bmatrix} I_n & O \\ O & O_{n_y\times n_{f_s}} \end{bmatrix},\quad A_1 = \begin{bmatrix} A & O \\ O & -D_s \end{bmatrix},$$

$$B_1 = \begin{bmatrix} B \\ O_{n_y\times n_u} \end{bmatrix},\quad B_{h1} = \begin{bmatrix} B_h \\ O_{n_y\times n_h} \end{bmatrix},$$

$$B_{a1} = \begin{bmatrix} B_a \\ O_{n_y\times n_{fa}} \end{bmatrix},\quad B_{d1} = \begin{bmatrix} B_d \\ O_{n_y\times n_w} \end{bmatrix},$$

$$T_1 = \begin{bmatrix} O_{n\times n_y} \\ I_{n_y} \end{bmatrix},\quad T_0 = [I_n\ O_{n\times f_s}],$$

$$C_1 = [C\ \ D_s],\quad C_0 = [C\ \ O_{n_y\times n_{f_s}}].$$

From (4), it is obvious that $D_sf_{si}(t) = y_i(t) - C_0\bar{x}_i(t)$. Substituting it into (3), we have

$$K_0\dot{\bar{x}}_i(t) = (A_1 - T_1C_0)\bar{x}_i(t) + B_1u_i(t) + B_{h1}h(T_0\bar{x}_i(t)) + B_{a1}f_{ai}(t) + T_1y_i(t) + B_{d1}d_i(t). \quad (5)$$

Let $E = [\begin{smallmatrix} O_{n\times n_y} \\ E_0 \end{smallmatrix}]$, where $E_0 \in R^{n_y\times n_y}$ is a nonsingular matrix. Add $EC_1\dot{\bar{x}}_i(t)$ to both sides of (5), then

$$K_1\dot{\bar{x}}_i(t) = (A_1 - T_1C_0)\bar{x}_i(t) + B_1u_i(t) + B_{h1}h(T_0\bar{x}_i(t)) + B_{a1}f_{ai}(t) + T_1y_i(t) + B_{d1}d_i(t) + EC_1\dot{\bar{x}}_i(t)$$

where $K_1 = K_0 + EC_1 = [\begin{smallmatrix} I_n & O \\ E_0C & E_0D_s \end{smallmatrix}]$. It can be proved that K_1 is of full column rank, since E_0 is a nonsingular matrix and D_s is of full column rank. Let $K_1^- = [\begin{smallmatrix} I_n & O \\ -D_s^-C & D_s^-E_0^{-1} \end{smallmatrix}]$, where D_s^- is a left inverse matrix of D_s, that is, $D_s^-D_s = I$. Thus, $K_1^-K_1 = I$.

Then we can obtain the following system:

$$\dot{\bar{x}}_i(t) = A_2\bar{x}_i(t) + B_2u_i(t) + B_{h2}h(T_0\bar{x}_i(t)) + B_{a2}f_{ai}(t) + T_2y_i(t) + B_{d2}d_i(t) + E_2C_1\dot{\bar{x}}_i(t) \quad (6)$$

where $A_2 = K_1^-(A_1 - T_1C_0)$, $B_2 = K_1^-B_1$, $B_{h2} = K_1^-B_{h1}$, $B_{a2} = K_1^-B_{a1}$, $T_2 = K_1^-T_1$, $B_{d2} = K_1^-B_{d1}$, $E_2 = K_1^-E$.

Combining (4) and (6), we have

$$\dot{\bar{x}}_i(t) = A_2\bar{x}_i(t) + B_2u_i(t) + B_{h2}h(T_0\bar{x}_i(t)) + B_{a2}f_{ai}(t) + T_2y_i(t) + E_2\dot{y}_i(t) + B_\omega\omega_i(t) \quad (7)$$

where $\omega_i(t) = [d_i^T(t)\ \dot{f}_{ai}^T(t)]^T$, $B_\omega = [B_{d2}\ O_{(n+n_{f_s})\times n_{f_a}}]$.

Next, the observer will be designed to estimate $\bar{x}_i(t)$ and $f_{ai}(t)$.

B. Observer Design

In this subsection, the observer will be designed in each agent i.

Note that $\dot{y}_i(t)$ exists in the dynamic (7). Thus, the intermediate variables definition in [20], [24], and [25] cannot be utilized directly. In this article, the intermediate variable $\zeta_i(t)$ is defined as follows:

$$\zeta_i(t) = f_{ai}(t) - S(\bar{x}_i(t) - E_2y_i(t)) \quad (8)$$

where S is a designed parameter and will be defined later.

According to (7), we can obtain that

$$\dot{\zeta}_i(t) = \dot{f}_{ai}(t) - S[A_2\bar{x}_i(t) + B_2u_i(t) + B_{h2}h(T_0\bar{x}_i(t)) + B_{a2}f_{ai}(t) + T_2y_i(t) + B_\omega\omega_i(t)]. \quad (9)$$

From (4), (7), and (9), the intermediate observer for the ith agent can be constructed as follows:

$$\dot{\hat{z}}_i(t) = A_2\hat{z}_i(t) + B_2u_i(t) + B_{h2}h(T_0\hat{\bar{x}}_i(t)) + B_{a2}\hat{f}_{ai}(t) + (A_2E_2 + T_2)y_i(t) + \rho_1L_1\xi_{1i}(t) + \rho_2L_2\xi_{2i}(t) \quad (10)$$

$$\begin{aligned}\dot{\hat{\zeta}}_i(t) = & -SB_{a2}\hat{\zeta}_i(t) - S[A_2\hat{\bar{x}}_i(t) + B_2u_i(t) \\ & + B_{h2}h(T_0\hat{\bar{x}}_i(t)) + B_{a2}S\hat{z}_i(t) + T_2y_i(t)] \\ & + \rho_1 L_3\xi_{1i}(t) + \rho_2 L_4\xi_{2i}(t) \end{aligned} \tag{11}$$

$$\hat{\bar{x}}_i(t) = \hat{z}_i(t) + E_2y_i(t) \tag{12}$$

$$\hat{f}_{ai}(t) = \hat{\zeta}_i(t) + S(\hat{\bar{x}}_i(t) - E_2y_i(t)) \tag{13}$$

$$\hat{y}_i(t) = C_1\hat{\bar{x}}_i(t) \tag{14}$$

where $\hat{z}_i(t) \in R^{n+n_{f_s}}$ and $\hat{\zeta}_i(t) \in R^{n_{f_a}}$ are the observer states, $\hat{\bar{x}}_i(t)$, $\hat{\zeta}_i(t)$, $\hat{f}_{ai}(t)$, $\hat{y}_i(t)$ are the estimations of $\bar{x}_i(t)$, $\zeta_i(t)$, $f_{ai}(t)$, and $y_i(t)$, respectively. In addition, ξ_{1i} and ξ_{1i} are defined as

$$\xi_{1i}(t) = y_i(t) - \hat{y}_i(t)$$

$$\xi_{2i}(t) = \sum_{j=1}^{N} a_{ij}\left((y_i(t) - \hat{y}_i(t)) - (y_j(t) - \hat{y}_j(t))\right)$$

which represent the centralized output estimation error and distributed output estimation error, respectively. $L_1 \in R^{(n+n_{f_s})\times n_y}$, $L_2 \in R^{(n+n_{f_s})\times n_y}$, $L_3 \in R^{n_{f_a}\times n_y}$, $L_4 \in R^{n_{f_a}\times n_y}$ are the observer parameter matrices to be designed. Nonnegative constants ρ_1 and ρ_2 are the weighted indexes and satisfy $0 \le \rho_1 \le 1$, $0 \le \rho_2 \le 1$ and $\rho_1 + \rho_2 = 0$. Specifically, ρ_1 is the weight of the centralized output estimation error and ρ_2 is the weight of the distributed output estimation error.

Remark 2: In the observer design process, both the centralized and distributed output estimation errors are considered. Obviously, if $\rho_1 = 1$ and $\rho_2 = 0$, the proposed observer is the traditional decentralized observer; on the other hand, if $\rho_1 = 0$ and $\rho_2 = 1$, the proposed observer is the traditional distributed observer. If a smaller ρ_1 is selected (which implies that ρ_2 is bigger), the influence of the neighbor node increases. Instead, if a bigger ρ_1 is selected (which implies that ρ_2 is smaller), the influence of the neighbor node reduces. In fact, the selection of ρ_1 and ρ_2 provides more design freedom, and the concrete selection of them should be determined by the actual situation.

C. Estimation Error Dynamic Construction

Similar to [20] and [24], in this article, S is defined as

$$S = \mu B_{a2}^T \tag{15}$$

where μ is a chosen constant. Let $\tilde{\bar{x}}_i(t) = \bar{x}_i(t) - \hat{\bar{x}}_i(t)$, from (7), (10), and (12), we have

$$\begin{aligned}\dot{\tilde{\bar{x}}}_i(t) = & (A_2 - \rho_1 L_1C_1)\tilde{\bar{x}}_i(t) + B_{h2}\tilde{h}_i(t) + B_{a2}\tilde{f}_{ai}(t) \\ & + B_\omega\omega_i(t) - \rho_2 L_2\sum_{j=1}^{N} a_{ij}[(y_i(t) - \hat{y}_i(t)) \\ & - (y_j(t) - \hat{y}_j(t))] \end{aligned} \tag{16}$$

where $\tilde{h}_i(t) = h(x_i(t)) - h(\hat{x}_i(t))$ and $\tilde{f}_{ai}(t) = f_{ai}(t) - \hat{f}_{ai}(t)$.

Let $\tilde{\zeta}_i(t) = \zeta_i(t) - \hat{\zeta}_i(t)$. According to (9) and (11), it can be found that

$$\begin{aligned}\dot{\tilde{\zeta}}_i(t) &= \dot{\zeta}_i(t) - \dot{\hat{\zeta}}_i(t) \\ &= \dot{f}_{ai}(t) - SA_2\tilde{\bar{x}}_i(t) - SB_{h2}\tilde{h}_i(t) - SB_{a2}f_{ai}(t) \\ &\quad - SB_\omega\omega_i(t) + SB_{a2}\hat{\zeta}_i(t) + SB_{a2}S\hat{z}_i(t) \\ &\quad - \rho_1 L_3\xi_{1i}(t) - \rho_2 L_4\xi_{2i}(t) \\ &= \dot{f}_{ai}(t) - SA_2\tilde{\bar{x}}_i(t) - SB_{h2}\tilde{h}_i(t) - SB_\omega\omega_i(t) \\ &\quad - \rho_1 L_3\xi_{1i}(t) - \rho_2 L_4\xi_{2i}(t) \\ &\quad - SB_{a2}[f_{ai}(t) - \hat{\zeta}_i(t) - S\hat{z}_i(t)]. \end{aligned}$$

According to (8) and (12), we have $f_{ai}(t) = \zeta_i(t) + S(\bar{x}_i(t) - E_2y_i(t))$ and $\hat{z}_i(t) = \hat{\bar{x}}_i(t) - E_2y_i(t)$. Substituting them into above formula, we have

$$\begin{aligned}\dot{\tilde{\zeta}}_i(t) &= \dot{f}_{ai}(t) - SA_2\tilde{\bar{x}}_i(t) - SB_{h2}\tilde{h}_i(t) - SB_\omega\omega_i(t) \\ &\quad - \rho_1 L_3\xi_{1i}(t) - \rho_2 L_4\xi_{2i}(t) - SB_{a2}[\tilde{\zeta}_i(t) + S\tilde{\bar{x}}_i(t)] \\ &= \dot{f}_{ai}(t) - (SA_2 + SB_{a2}S)\tilde{\bar{x}}_i(t) - SB_{h2}\tilde{h}_i(t) \\ &\quad - SB_\omega\omega_i(t) - \rho_1 L_3\xi_{1i}(t) - \rho_2 L_4\xi_{2i}(t) - SB_{a2}\tilde{\zeta}_i(t). \end{aligned}$$

Based on (4) and (14), it can be found that $\xi_{1i}(t) = y_i(t) - \hat{y}_i(t) = C_1\tilde{\bar{x}}_i(t)$, combining (15) and $\xi_{2i}(t) = \sum_{j=1}^{N} a_{ij}((y_i(t) - \hat{y}_i(t)) - (y_j(t) - \hat{y}_j(t)))$, then we have

$$\begin{aligned}\dot{\tilde{\zeta}}_i(t) = & -\mu B_{a2}^T B_{a2}\tilde{\zeta}_i(t) - \mu B_{a2}^T B_{h2}\tilde{h}_i(t) + D_\omega\omega_i(t) \\ & - (\mu B_{a2}^T(A_2 + \mu B_{a2}B_{a2}^T) + \rho_1 L_3C_1)\tilde{\bar{x}}_i(t) \\ & - \rho_2 L_4\sum_{j=1}^{N} a_{ij}\left[(y_i(t) - \hat{y}_i(t)) - (y_j(t) - \hat{y}_j(t))\right] \end{aligned} \tag{17}$$

where $D_\omega = -\mu B_{a2}^T B_\omega + [O_{n_{f_a}\times n_w}\ I_{n_{f_a}}]$ and $\omega_i(t) = [d_i^T(t)\ \dot{f}_{ai}^T(t)]^T$.

From (8), (13), and (15), we have that

$$\tilde{f}_{ai}(t) = \tilde{\zeta}_i(t) + S\tilde{\bar{x}}_i(t) = \tilde{\zeta}(t) + \mu B_{a2}^T\tilde{\bar{x}}_i(t).$$

Thus, (16) can be rewritten as

$$\begin{aligned}\dot{\tilde{\bar{x}}}_i(t) = & (A_2 + \mu B_{a2}B_{a2}^T - \rho_1 L_1C_1)\tilde{\bar{x}}_i(t) + B_{a2}\tilde{\zeta}_i(t) \\ & + B_{h2}\tilde{h}_i(t) + B_\omega\omega_i(t) - \rho_2 L_2\sum_{j=1}^{N} a_{ij} \\ & \times \left[(y_i(t) - \hat{y}_i(t)) - (y_j(t) - \hat{y}_j(t))\right]. \end{aligned} \tag{18}$$

Letting $e_i(t) = [\tilde{\bar{x}}_i^T(t)\ \tilde{\zeta}_i^T(t)]^T$, combine (17) and (18), then the following dynamic can be achieved:

$$\begin{aligned}\dot{e}_i(t) = & (A_3 - \rho_1\bar{L}_1C_3)e_i(t) + B_{h3}\tilde{h}_i(t) + \bar{B}_\omega\omega_i(t) \\ & - \rho_2\bar{L}_2\sum_{j=1}^{N} a_{ij}\left((y_i(t) - \hat{y}_i(t)) - (y_j(t) - \hat{y}_j(t))\right) \end{aligned} \tag{19}$$

where

$$A_3 = \begin{bmatrix} A_2 + \mu B_{a2} B_{a2}^T & B_{a2} \\ -\mu B_{a2}^T (A_2 + \mu B_{a2} B_{a2}^T) & -\mu B_{a2}^T B_{a2} \end{bmatrix},$$

$$\bar{L}_1 = \begin{bmatrix} L_1 \\ L_3 \end{bmatrix}, \ \bar{L}_2 = \begin{bmatrix} L_2 \\ L_4 \end{bmatrix},$$

$$C_3 = \begin{bmatrix} C_1 & O_{n_y \times n_{fa}} \end{bmatrix},$$

$$B_{h3} = \begin{bmatrix} B_{h2} \\ -\mu B_{a2}^T B_{h2} \end{bmatrix}, \ \bar{B}_\omega = \begin{bmatrix} B_\omega \\ D_\omega \end{bmatrix}.$$

In (19), we have

$$A_3 - \rho_1 \bar{L}_1 C_3 = \begin{bmatrix} A_2 + \mu B_{a2} B_{a2}^T - \rho_1 L_1 C_1 & B_{a2} \\ -\mu B_{a2}^T (A_2 + \mu B_{a2} B_{a2}^T) - \rho_1 L_3 C_1 & -\mu B_{a2}^T B_{a2} \end{bmatrix}.$$

If we do not substitute S into (19), it can be found that the (2,2) element of the matrix $A_3 - \rho_1 \bar{L}_1 C_3$ is $-SB_{a2}$. Note that $B_{a2} = K_1^- B_{a1} = [\begin{smallmatrix} B_a \\ -D_s^- C B_a \end{smallmatrix}]$. Since B_a is of full column rank, we have B_{a2} is of full column rank. As it is pointed out in (15), the parameter S is selected as $S = \mu B_{a2}^T$. Thus, we have that $-SB_{a2} = -\mu B_{a2}^T B_{a2} < 0$, which may be helpful to ensure that all eigenvalues of $A_3 - \rho_1 \bar{L}_1 C_3$ are with negative real part. In addition, it can be found that μ can adjust the convergence speed of the estimation error system. Generally speaking, a bigger μ may lead to a faster convergence speed.

Thus, we can obtain the global error dynamic as follows:

$$\dot{e}(t) = I_N \otimes (A_3 - \rho_1 \bar{L}_1 C_3) e(t) - \mathcal{L} \otimes (\rho_2 \bar{L}_2 C_3) e(t) + I_N \otimes B_{h3} \tilde{h}(t) + I_N \otimes \bar{B}_\omega \omega(t) \tag{20}$$

where

$$e(t) = [e_1^T(t), e_2^T(t), \ldots, e_N^T(t)]^T,$$
$$\tilde{h}(t) = [\tilde{h}_1^T(t), \tilde{h}_2^T(t), \ldots, \tilde{h}_N^T(t)]^T,$$
$$\omega(t) = [\omega_1^T(t), \omega_2^T(t), \ldots, \omega_N^T(t)]^T.$$

D. Stability Analysis

Next, we will consider the stability of error dynamic (20).

Theorem 1: For given constants ρ_1, ρ_2, μ, and γ, the global error dynamic (20) is asymptotically stable with a H_∞ performance level γ, if there is symmetric positive definite matrix $P \in R^{(n+n_{fs}+n_{fa}) \times (n+n_{fs}+n_{fa})}$, matrix $Q \in R^{(n+n_{fs}+n_{fa}) \times n_y}$, positive constants κ, ε, such that the following LMI holds:

$$\begin{bmatrix} \Psi - \lambda_{\min} \rho_2 \kappa C_3^T C_3 & P\bar{B}_\omega & PB_{h3} \\ * & -\gamma^2 I & O \\ * & * & -\varepsilon I \end{bmatrix} < 0 \tag{21}$$

where $\lambda_{\min}$ represents the minimum eigenvalue of $\mathcal{L}^T + \mathcal{L}$,

$$\Psi = (PA_3 - \rho_1 Q_1 C_3) + (PA_3 - \rho_1 Q_1 C_3)^T + \varepsilon \delta_h^2 \bar{I}^T \bar{I} + I,$$

and $\bar{I} = [I_n \, O_{n \times n_{fs}} \, O_{n \times n_{fa}}]$. The observer gain matrices are $\bar{L}_1 = P^{-1} Q$, $\bar{L}_2 = \kappa P^{-1} C_3^T$.

Proof: Select Lyapunov function as

$$V(t) = e^T(t)(I_N \otimes P) e(t).$$

Consider the case that $\omega(t) = 0$. For this case, the differential of $V(t)$ along (20) can be achieved as

$$\begin{aligned} \dot{V}(t) &= e^T(t)[I_N \otimes (PA_3 - \rho_1 Q_1 C_3) \\ &\quad + I_N \otimes (PA_3 - \rho_1 Q_1 C_3)^T] e(t) \\ &\quad - e^T(t)[(\mathcal{L}^T + \mathcal{L}) \otimes (\rho_2 \kappa C_3^T C_3)] e(t) \\ &\quad + 2e^T(t)(I_N \otimes (PB_{h3})) \tilde{h}(t) \end{aligned} \tag{22}$$

where $Q_1 = P\bar{L}_1$, $\bar{L}_2 = \kappa P^{-1} C_3^T$.

Since $h(x_i(t))$ is Lipschitz with respect to $x_i(t)$, we have that

$$||\tilde{h}_i^T(t)|| = ||h(x_i(t)) - h(\hat{x}_i(t))|| \le \delta_h ||\tilde{x}_i(t)||$$

where $\tilde{x}_i(t) = x_i(t) - \hat{x}_i(t)$, δ_h is a Lipschitz constant.

Note that $\tilde{h}(t) = [\tilde{h}_1^T(t), \tilde{h}_2^T(t), \ldots, \tilde{h}_N^T(t)]^T$. Thus, it is not difficult to obtain that

$$||\tilde{h}(t)||^2 = \sum_{i=1}^N ||\tilde{h}_i(t)||^2 \le \sum_{i=1}^N \delta_h^2 ||\tilde{x}_i(t)||^2 = \delta_h^2 ||\tilde{x}(t)||^2$$

where $\tilde{x}(t) = [\tilde{x}_1^T(t), \tilde{x}_2^T(t), \ldots, \tilde{x}_N^T(t)]^T$.

That is

$$\begin{aligned} \tilde{h}^T(t) \tilde{h}(t) &\le \delta_h^2 \tilde{x}^T(t) \tilde{x}(t) \\ &= \delta_h^2 e^T(t)(I_N \otimes \bar{I})^T (I_N \otimes \bar{I}) e(t) \\ &= \delta_h^2 e^T(t)(I_N \otimes (\bar{I}^T \bar{I})) e(t) \end{aligned}$$

where $\bar{I} = [I_n \, O_{n \times n_{fs}} \, O_{n \times n_{fa}}]$.

According to Lemma 1, we have

$$\begin{aligned} & 2e^T(t)(I_N \otimes (PB_{h3})) \tilde{h}(t) \\ &= \frac{1}{\varepsilon} e^T(t)(I_N \otimes (PB_{h3} B_{h3}^T P)) e(t) + \varepsilon \tilde{h}^T(t) \tilde{h}(t) \\ &\le \frac{1}{\varepsilon} e^T(t)(I_N \otimes (PB_{h3} B_{h3}^T P)) e(t) \\ &\quad + \varepsilon \delta_h^2 e^T(t)(I_N \otimes (\bar{I}^T \bar{I})) e(t) \\ &= e^T(t) \left(I_N \otimes \left(\frac{1}{\varepsilon} PB_{h3} B_{h3}^T P + \varepsilon \delta_h^2 \bar{I}^T \bar{I} \right) \right) e(t). \end{aligned} \tag{23}$$

Substitute (23) into (22), it can be found that

$$\begin{aligned} \dot{V}(t) &\le e^T(t)[I_N \otimes (\Phi + \frac{1}{\varepsilon} PB_{h3} B_{h3}^T P) \\ &\quad - (\mathcal{L}^T + \mathcal{L}) \otimes (\rho_2 \kappa C_3^T C_3)] e(t) \end{aligned} \tag{24}$$

where

$$\Phi = (PA_3 - \rho_1 Q_1 C_3) + (PA_3 - \rho_1 Q_1 C_3)^T + \varepsilon \delta_h^2 \bar{I}^T \bar{I}$$

Obviously, if

$$I_N \otimes \left(\Phi + \frac{1}{\varepsilon} PB_{h3} B_{h3}^T P \right) - (\mathcal{L}^T + \mathcal{L}) \otimes (\rho_2 \kappa C_3^T C_3) < 0 \tag{25}$$

we have $\dot{V}(t) < 0$, which implies that the global error dynamic (20) is asymptotically stable for the case that $\omega(t) = 0$.

Note that $\mathcal{L}^T + \mathcal{L}$ is a symmetric matrix. Based on the matrix Schur decomposition theory, there is a matrix M such that

$$M^T(\mathcal{L}^T + \mathcal{L})M = \Lambda \tag{26}$$

where $\Lambda = \text{diag}\{\lambda_1, \lambda_2, \ldots, \lambda_N\}$, λ_i is the ith eigenvalue of $\mathcal{L}^T + \mathcal{L}$, $M \in R^{N\times N}$ is an orthogonal matrix, i.e., $M^T M = I$.

Premultiplying $M^T \otimes I$ and postmultiplying $M \otimes I$ in (25), simultaneously, the following inequality can be obtained:

$$I_N \otimes \left(\Phi + \frac{1}{\varepsilon}PB_{h3}B_{h3}^T P\right) - \Lambda \otimes (\rho_2\kappa C_3^T C_3) < 0. \tag{27}$$

Since Λ is a diagonal matrix, then (27) holds if and only if for $i = 1, 2, \ldots, N$, the following inequality holds:

$$\Phi + \frac{1}{\varepsilon}PB_{h3}B_{h3}^T P - \lambda_i\rho_2\kappa C_3^T C_3 < 0. \tag{28}$$

According to Schur complement, (28) is equivalent to

$$\begin{bmatrix} \Phi - \lambda_i\rho_2\kappa C_3^T C_3 & PB_{h3} \\ * & -\varepsilon I \end{bmatrix} < 0. \tag{29}$$

Note that $\rho_2 \geq 0$ and $\kappa > 0$, we have that $\rho_2\kappa C_3^T C_3 \geq 0$. Obviously

$$\Phi - \lambda_i\rho_2\kappa C_3^T C_3 \leq \Phi - \lambda_{\min}\rho_2\kappa C_3^T C_3 \tag{30}$$

where $\lambda_{\min}$ is the minimum eigenvalue of $\mathcal{L}^T + \mathcal{L}$.

Thus, if the following LMI holds:

$$\begin{bmatrix} \Phi - \lambda_{\min}\rho_2\kappa C_3^T C_3 & PB_{h3} \\ * & -\varepsilon I \end{bmatrix} < 0 \tag{31}$$

we have that LMI (29) is satisfied.

Note that if (21) is feasible, we have that (31) holds, which implies $\dot{V}(t) < 0$. Hence, if (21) is feasible, the global error dynamic (20) is asymptotically stable for the case that $\omega(t) = 0$.

Next, let us consider the case that $\omega(t) \neq 0$. For this case, the differential of $V(t)$ along (20) is

$$\begin{aligned} \dot{V}(t) = e^T(t)[&I_N \otimes (PA_3 - \rho_1 Q_1 C_3) \\ &+ I_N \otimes (PA_3 - \rho_1 Q_1 C_3)^T]e(t) \\ &- e^T(t)[(\mathcal{L}^T + \mathcal{L}) \otimes (\rho_2\kappa C_3^T C_3)]e(t) \\ &+ 2e^T(t)(I_N \otimes (PB_{h3}))\tilde{h}(t) \\ &+ 2e^T(t)(I_N \otimes (P\bar{B}_\omega))\omega(t). \end{aligned} \tag{32}$$

Similar to the case that $\omega(t) = 0$, substitute (23) into (32), we have

$$\begin{aligned} \dot{V}(t) \leq e^T(t)[&I_N \otimes \left(\Phi + \frac{1}{\varepsilon}PB_{h3}B_{h3}^T P\right) \\ &- (\mathcal{L}^T + \mathcal{L}) \otimes (\rho_2\kappa C_3^T C_3)]e(t) \\ &+ 2e^T(t)(I_N \otimes (P\bar{B}_\omega))\omega(t). \end{aligned} \tag{33}$$

Let

$$\mathcal{J}(t) = \dot{V}(t) + e^T(t)e(t) - \gamma^2\omega^T(t)\omega(t). \tag{34}$$

Combine (33) and (34), it is obvious that

$$\mathcal{J}(t) \leq \begin{bmatrix} e(t) \\ \omega(t) \end{bmatrix}^T \begin{bmatrix} I_N \otimes \bar{\Phi} - (\mathcal{L}^T + \mathcal{L}) \otimes (\rho_2\kappa C_3^T C_3) & I_N \otimes (P\bar{B}_\omega) \\ * & -\gamma^2 I_N \otimes I_{n_w+n_{fa}} \end{bmatrix} \begin{bmatrix} e(t) \\ \omega(t) \end{bmatrix}$$

where $\bar{\Phi} = \Phi + \frac{1}{\varepsilon}PB_{h3}B_{h3}^T P + I$.

Obviously, if the following inequality holds:

$$\begin{bmatrix} I_N \otimes \bar{\Phi} - (\mathcal{L}^T + \mathcal{L}) \otimes (\rho_2\kappa C_3^T C_3) & I_N \otimes (P\bar{B}_\omega) \\ * & -\gamma^2 I_N \otimes I_{n_w+n_{fa}} \end{bmatrix} < 0 \tag{35}$$

we have $\mathcal{J}(t) < 0$. Under zero initial condition, $\mathcal{J}(t) < 0$ means that

$$V(t) + \int_0^t [e^T(s)e(s) - \gamma^2\omega^T(s)\omega(s)]ds < 0. \tag{36}$$

Since $V(t) = e^T(t)(I_N \otimes P)e(t)$ and P is a positive definite matrix, (36) implies

$$\int_0^t e^T(s)e(s)ds < \gamma^2 \int_0^t \omega^T(s)\omega(s)ds. \tag{37}$$

In a word, the inequality (35) is a sufficient condition of (37).

Premultiplying $\begin{bmatrix} M^T\otimes I_{n+n_{fa}+n_{fs}} & O \\ O & M^T\otimes I_{n_w+n_{fa}} \end{bmatrix}$ and postmultiplying $\begin{bmatrix} M\otimes I_{n+n_{fa}+n_{fs}} & O \\ O & M\otimes I_{n_w+n_{fa}} \end{bmatrix}$ in (35), simultaneously, we have

$$\begin{bmatrix} I_N \otimes \bar{\Phi} - \Lambda \otimes (\rho_2\kappa C_3^T C_3) & I_N \otimes (P\bar{B}_\omega) \\ * & -\gamma^2 I_N \otimes I_{n_w+n_{fa}} \end{bmatrix} < 0 \tag{38}$$

where the definition of M can be found in (26).

Based on Schur complement, (38) is equivalent to

$$\begin{bmatrix} I_N \otimes \Psi - \Lambda \otimes (\rho_2\kappa C_3^T C_3) & I_N \otimes (P\bar{B}_\omega) & I_N \otimes PB_{h3} \\ * & -\gamma^2 I_N \otimes I & O \\ * & * & -\varepsilon I_N \otimes I \end{bmatrix} < 0 \tag{39}$$

where $\Psi = \Phi + I$, Φ is defined in (24).

Note that $\Lambda = \text{diag}\{\lambda_1, \lambda_2, \ldots, \lambda_N\}$ is a diagonal matrix, where λ_i is the ith eigenvalue of $\mathcal{L}^T + \mathcal{L}$. Thus, (39) holds if and only if for $i = 1, 2, \ldots, N$, the following inequalities are satisfied

$$\begin{bmatrix} \Psi - \lambda_i\rho_2\kappa C_3^T C_3 & P\bar{B}_\omega & PB_{h3} \\ * & -\gamma^2 I & O \\ * & * & -\varepsilon I \end{bmatrix} < 0. \tag{40}$$

Similar to the case $\omega(t) = 0$, inequality (21) is feasible, we have that (40) holds, which implies (37) is satisfied, i.e., the error dynamic (20) is asymptotically stable with the H_∞ performance level γ.

Thus, the proof is completed. ■

Remark 3: From Theorem 1, it is obvious that if the actuator fault $f_{ai}(t)$ is a constant vector and the disturbance $d_i(t) = 0$, the accurate estimations of the system state $x_i(t)$, the actuator fault $f_{ai}(t)$ and sensor fault $f_{si}(t)$ can be obtained simultaneously, since the error dynamic (20) is asymptotically stable for this case. It can be found that the estimation results are not affected by the sensor fault. In [20], where the intermediate observer method was first proposed, if the sensor fault was not a constant vector, the accurate fault estimation could not be obtained, even if there was no actuator fault and disturbance. In Theorem 1, the Lyapunov function is selected as $V(t) = e^T(t)(I_N \otimes P)e(t)$, where P is a symmetric positive definite matrix. This type of Lyapunov function is common in the existing results, such as [24] and [26]. While, the matrix P in [24] and [26] was a block diagonal matrix. Note that this restriction is removed in this article. However, it should be pointed out that the Lyapunov matrix is $I_N \otimes P$, which is a block diagonal matrix, and it may induce conservatism. How to design a more general Lyapunov matrix is really worth considering, and it will be considered in our further work.

Remark 4: The traditional intermediate observer design method was proposed in [20], and was used for the robust distributed fault estimation problem in [24] and [25]. It is worth pointing out that in the traditional intermediate observer design method, the observer parameter matrix is chosen as $S = \mu B_a^T$, where B_a is the actuator fault distribution matrices, μ is a constant which is selected beforehand. Thus, the structure of the observer parameter matrix is limited. To overcome this problem, a novel intermediate observer design method is proposed in this article, in which the output estimation error feedback terms are introduced. This increases the possibility of the feasibility of the LMI, which is used for calculating the observer parameter matrices. In addition, the observer estimation performance can be improved by adding the output estimation error feedback terms.

Remark 5: In [23] and [24], fault estimation was considered for the multiagent systems with undirected graph. Since the Laplacian matrix of the undirected graph is symmetrical, then the method used in [23] and [24] cannot be used for this article. In this article, we consider the directed graph. It is worth pointing out that our method can be extended to the case of undirected communication graph, since the undirected graph can be treated as a special case of the directed graph.

Remark 6: It is worth pointing out the problem of fault estimation for multiagent systems with directed graph has been considered in pioneer references [26] and [28]. In these results, the distributed fast adaptive fault estimation algorithm and distributed fault estimation observer were proposed to estimate the system state and the actuator fault. However, the sensor fault was not considered in [26] and [28]. In addition, in these methods, in order to calculate the observer parameter matrices, a higher order LMI should be solved. Specifically, the orders of the LMI were $N(n + n_{f_a})$ and $N(n + 4n_{f_a} + n_{w_a})$, respectively, where N was the number of the agents, n and n_{f_a} represented the number of the system state and actuator fault in one agent. This implies that the calculated amount would increase when the number of agents was bigger. Compared with these results, this article proposes a novel fault estimation method to estimate the system state, actuator fault and sensor fault, simultaneously. And only one LMI, whose order is $n + 2n_{f_a} + n_w + n_h$, needs to be solved to calculate the observer parameter matrices. Thus, the calculated amount of our method does not change when the number of agents increases, since the inequality dimension is independent of the agent number.

IV. Simulation Study

In this section, two examples will be considered to show the effectiveness of the proposed method.

Example 1: Consider the multiagent systems comprising four agents. The adjacency matrix and Laplacian matrix are listed as follows:

$$\mathcal{A} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \; \mathcal{L} = \begin{bmatrix} 1 & 0 & 0 & -1 \\ -1 & 1 & 0 & 0 \\ -1 & -1 & 2 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}.$$

For $i = 1, 2, 3, 4$, the dynamic of the ith agent is

$$\dot{x}_i(t) = \begin{bmatrix} -1 & 1 & 0 \\ 1 & -2 & 0 \\ 1 & 0 & -1 \end{bmatrix} x_i(t) + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} u_i(t) + \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \cos(x_{i1}) + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} f_{ai}(t) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} d_i(t)$$

$$y_i(t) = \begin{bmatrix} 1 & 2 & -1 \\ -1 & 0 & 0 \\ 1 & 1 & 2 \end{bmatrix} x_i(t) + \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} f_{si}(t).$$

For $j = 1, 2, 3$, $x_{ij}(t) \in R$ represents the j element of $x_i(t)$, i.e., $x_i(t) = [x_{i1}(t), x_{i2}(t), x_{i3}(t)]^T$.

It is assumed that agent 1 and agent 2 occur actuator faults, agent 1 and agent 3 occur sensor faults, and agent 4 is fault free. That is, $f_{s2}(t) = f_{s4}(t) = 0$, $f_{a3}(t) = f_{a4}(t) = 0$. Specifically, we assume that $f_{a1} = 0$ for $0 \le t < 10$, and $f_{a1} = \sin(0.5t - 1)$ for $10 \le t < 25$, and $f_{a1} = 2$ for $25 \le t \le 40$; $f_{a2} = 0$ for $0 \le t < 10$, and $f_{a2} = 1 - e^{-3t+30}$ for $10 \le t \le 40$; $f_{s1} = 0$ for $0 \le t < 10$, and $f_{s1} = 0.1t - 1$ for $10 \le t < 25$, and $f_{s1} = 0$ for $25 \le t \le 30$, and $f_{s1} = \sin(t - 2)$ for $30 \le t \le 40$; $f_{s3} = 0$ for $0 \le t < 10$, and $f_{s1} = \sin(2t - 1)$ for $10 \le t < 25$, and $f_{s1} = \frac{1}{1+2^{-t+30}}$ for $25 \le t \le 40$.

Based on the method proposed in Section III-A, we choose the matrix $E_0 = I$. The weighted indexes of the centralized output estimation error and the distributed output estimation error are selected as $\rho_1 = 0.2$, $\rho_2 = 0.8$, respectively. The constant in (15) is selected as $\mu = 3$. According to Theorem 1, the observer parameter matrices can be achieved as follows:

$$L_1 = \begin{bmatrix} 4.7658 & 14.0981 & 14.1576 \\ 2.3986 & 5.2538 & 5.4716 \\ 6.6216 & 18.6295 & 22.0861 \\ -4.3914 & -16.5389 & -17.2908 \end{bmatrix}$$

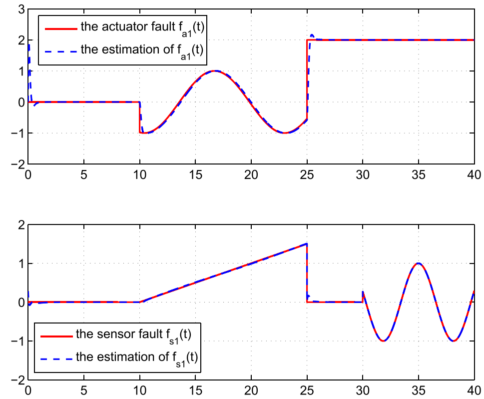

Fig. 1. Actuator fault and sensor fault in agent 1 (the solid lines), and the estimations of them (the dashed lines).

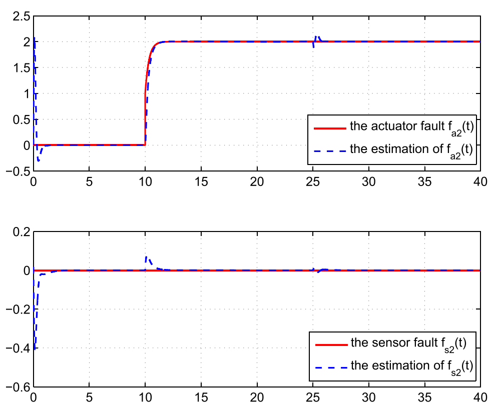

Fig. 2. Actuator fault and sensor fault in agent 2 (the solid lines), and the estimations of them (the dashed lines).

$$L_2 = \begin{bmatrix} 0.0823 & -0.8437 & 2.2303 \\ 2.6828 & 0.4241 & 1.1654 \\ -2.9414 & 0.8216 & 5.5846 \\ 1.5940 & -2.4532 & -0.5003 \end{bmatrix}$$

$$L_3 = \begin{bmatrix} -5.1942 & -70.5286 & -93.9870 \end{bmatrix}$$

$$L_4 = \begin{bmatrix} -3.9986 & -3.5456 & -3.0926 \end{bmatrix}.$$

The simulation results are shown in Figs. 1–4, where the observer initial values are selected as zero vectors with appropriate dimensions. In Figs. 1–4, the solid lines represent faults in each agent, and the dashed lines represent the estimations of them. From Figs. 1–4, it can be found that the observers can estimate the actuator fault and the sensor fault simultaneously. Note that the estimation curves of sensor faults in agent 2, 4 and actuator faults in agent 3, 4 are around zero, which indicates that there are no sensor faults in agent 2, 4 and no actuator faults in agent 3, 4. In order to illustrate the advantage of the proposed intermediate observer, the comparison is carried out to compare the performance of our method with that of the fault estimation method proposed in [14]. The comparison results are shown in Figs. 5 and 6, where the dashed lines represent the fault estimation errors obtained by our method, and the solid lines are the results obtained by the previously proposed method [14]. From these figures, it can be found that our observer has better behavior, especially for the estimation of actuator fault.

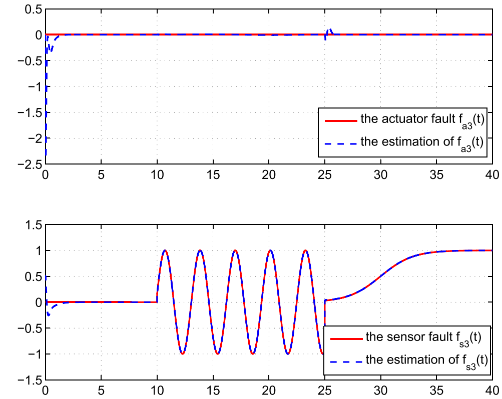

Fig. 3. Actuator fault and sensor fault in agent 3 (the solid lines), and the estimations of them (the dashed lines).

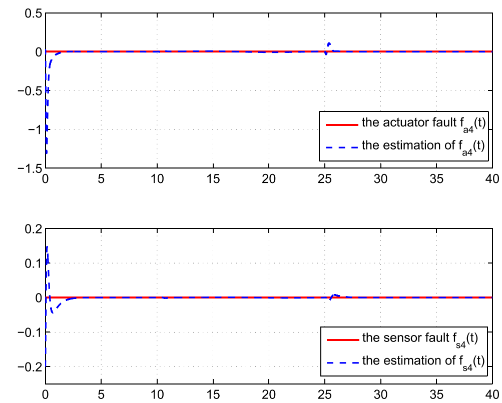

Fig. 4. Actuator fault and sensor fault in agent 4 (the solid lines), and the estimations of them (the dashed lines).

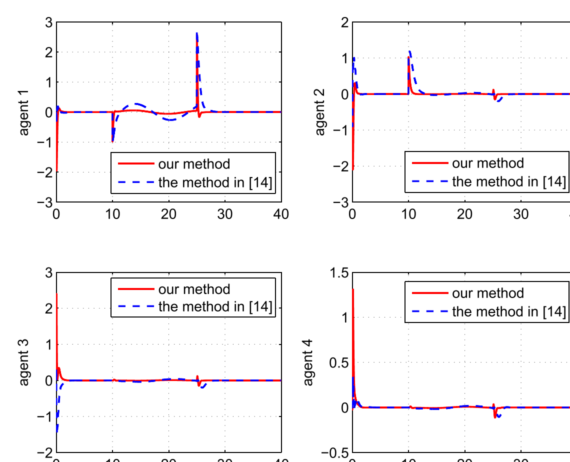

Fig. 5. Actuator fault estimation errors of the multiagent systems.

Example 2: In this example, consider a network of four one-link flexible joint manipulator systems. The adjacency matrix

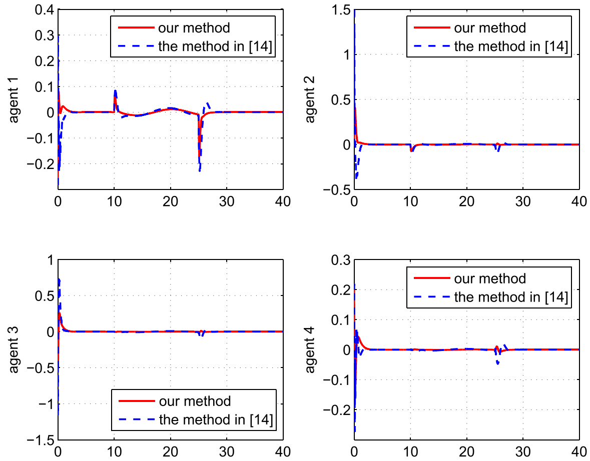

Fig. 6. Sensor fault estimation errors of the multiagent systems.

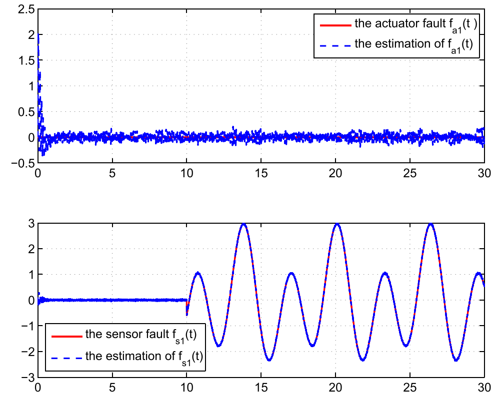

Fig. 7. Actuator fault and sensor fault in agent 1 (the solid lines), and the estimations of them (the dashed lines).

and Laplacian matrix are listed as follows:

$$\mathcal{A} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \mathcal{L} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ -1 & -1 & 2 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}.$$

It can be found that this graph contains a spanning tree, but it is not strongly connected.

Similar to [18], each system dynamic is written as

$$\dot{\theta}_m = \omega_m \quad \dot{\omega}_m = \frac{k}{J_m}(\theta_1 - \omega_m) - \frac{b}{J_m}\omega_m + \frac{K_\tau}{J_m}u$$
$$\dot{\theta}_1 = \omega_1 \quad \dot{\omega}_1 = -\frac{k}{J_1}(\theta_1 - \omega_m) - \frac{mgb}{J_1}\sin(\theta_1)$$

which can be described by the model with form as (1)–(2). The parameter matrices A, B, B_h, C are selected as them in [18], that is

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -48.6 & -1.25 & 48.6 & 0 \\ 0 & 0 & 0 & 10 \\ 1.95 & 0 & -1.951 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 21.6 \\ 0 \\ 0 \end{bmatrix},$$

$$B_h = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -0.333 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$

Assume that agent 2 and agent 4 occur actuator faults, agent 1 and agent 4 occur sensor faults, and agent 3 is fault free. That is, $f_{a1}(t) = f_{a3}(t) = 0$, $f_{s2}(t) = f_{s3}(t) = 0$. Specifically, we assume $f_{a2}(t) = 0$ for $0 \leq t < 10$, and $f_{a2}(t) = 1 - e^{-3t+30}$ for $10 \leq t \leq 30$; $f_{a4}(t) = 0$ for $0 \leq t < 10$, and $f_{a4}(t) = 2$ for $10 \leq t \leq 30$; $f_{s1}(t) = 0$ for $0 \leq t < 10$, and $f_{s1}(t) = 2\sin(2t-1) + \cos(t-1)$ for $10 \leq t \leq 30$; $f_{s4}(t) = 0$ for $0 \leq t < 10$, and $f_{s4}(t) = \sin(2t-1)$ for $10 \leq t \leq 30$.

The distribution matrices of the faults and disturbance are chosen as $B_a = [0\,0\,0\,1]^T$, $D_s = [0\ -1\ 2]^T$, $B_d = [0\,0\,0\,1]^T$. It should be pointed out that $CB_a = [0\,0\,0\,0]^T$, that is, $\text{rank}(CB_a) = 0$. Based on the meaning of the observer matching condition(i.e., $\text{rank}(CB_a) = n_{f_a}$), it can be found that the observer matching condition is not satisfied in this example. Thus, the methods proposed in [23] and [26] cannot be used for this example.

Based on the method proposed in Section III, we choose the matrix $E_0 = I$. The weighted indexes of the centralized output estimation error and the distributed output estimation error are selected as $\rho_1 = 0.3$, $\rho_2 = 0.7$, respectively. The constant in (15) is selected as $\mu = 5$. According to Theorem 1, we can calculate the observer parameter matrices as follows:

$$L_1 = \begin{bmatrix} 4.2094 & 2.4824 & 13.0619 \\ -143.3707 & -106.4742 & -120.4204 \\ -17.8859 & 27.7080 & 196.9692 \\ -9.9190 & 57.2653 & 255.5579 \\ -65.1760 & -101.4689 & -189.6980 \end{bmatrix}$$

$$L_2 = \begin{bmatrix} 1.0036 & -0.0027 & -0.0055 \\ -0.1885 & 1.1811 & 0.3622 \\ 0.0874 & 0.2194 & 1.4389 \\ 0.0798 & 0.6294 & 1.2587 \\ -0.0929 & -1.9714 & 1.0571 \end{bmatrix}$$

$$L_3 = \begin{bmatrix} -19.9724 & -11.9460 & -138.8656 \end{bmatrix}$$

$$L_4 = \begin{bmatrix} -0.0580 & -0.5361 & -1.0721 \end{bmatrix}.$$

To close to the practical conditions, we assume that the measurement noise exists in each agent system. In addition, the observer initial values are selected as zero vectors with appropriate dimensions. The simulation results of agent 1 and agent 4 are shown in Figs. 7 and 8, where the solid lines represent faults in each agent, and the dashed lines represent the estimations of them. The estimation results of agent 2 and agent 4 are omitted due to the limited space. Note that the estimation curve of actuator fault in agent 1 is around zero, since there is no actuator fault in agent 1. From these figures, it is clear that the actuator fault and the sensor fault can be estimated by the proposed observer.

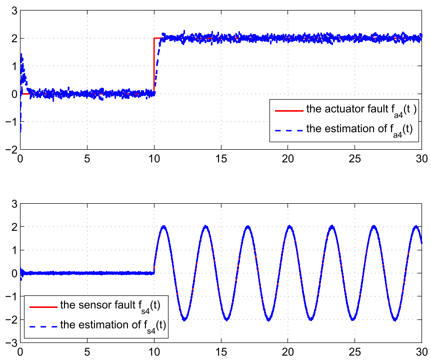

Fig. 8. Actuator fault and sensor fault in agent 4 (the solid lines), and the estimations of them (the dashed lines).

V. Conclusion

In this article, a novel intermediate observer design method was proposed to estimate the system states and faults in the multiagent systems with directed graphs. Both the actuator faults and sensor faults were considered. In the observer design process, the centralized and distributed output estimation errors were introduced, simultaneously, and the observer matching condition and strictly positive real assumption were not needed. Based on Schur decomposition, the observer parameter calculation method was presented in terms of solution to one LMI, and the dimension of the LMI is independent of the number of agents. Thus, the calculated amount remains unchanged even when the number of agents increases. At the end of the article, simulation results were provided to illustrate the effectiveness of the proposed technique. In this article, we assumed that each agent could receive the output information of its neighbors. In the future, we will consider the case that each agent can only receive the estimation states instead of the measured outputs from its neighbors. In addition, based on the fault estimation results, the problem of consensus control for the multiagent systems with faults and actuator saturation will be considered in our future work.

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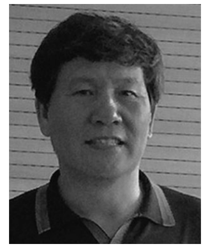

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