Finite-Density Black Holes in a Quantum Gravity Framework

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Introduction

This document outlines a theoretical framework aimed at addressing the singularity problem in black holes by incorporating quantum corrections through a scalar field [1–3]. The modifications to the Einstein field equations are derived from an action principle to ensure consistency and avoid double counting of scalar field contributions [4, 5].

To resolve these singularities and unify general relativity with quantum mechanics, various approaches have been proposed, including loop quantum gravity [6, 7].

This work has been developed in collaboration with McCade Smith, a brilliant mathematician from Columbia University who helped resolve the doubling of the scalar field.

1 Action Integral

We start with the action:

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{16\pi G} \left(R - 2\Lambda(\phi) \right) - \frac{1}{2} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi - V(\phi) + \mathcal{L}_{\text{matter}} \right] \tag{1}$$

[5, 8]

2 Modified Einstein Field Equations

Varying the action with respect to the metric $g^{\mu\nu}$ yields:

$$G_{\mu\nu} + \Lambda(\phi) g_{\mu\nu} = 8\pi G \left(T^{(\phi)}_{\mu\nu} + T^{\text{matter}}_{\mu\nu} \right) \tag{2}$$

[9]

where:

$$T^{(\phi)}_{\mu\nu} = \nabla_\mu \phi \nabla_\nu \phi - g_{\mu\nu} \left(\frac{1}{2} \nabla^\lambda \phi \nabla_\lambda \phi + V(\phi) \right) \tag{3}$$

3 Scalar Field Equation of Motion

Varying the action with respect to ϕ gives:

$$\Box \phi - \frac{dV}{d\phi} + \frac{1}{8\pi G} \frac{d\Lambda}{d\phi} = 0 \tag{4}$$

[5, 7]

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4 Cosmological Constant as a Function of ϕ

We consider:

$$\Lambda(\phi) = \Lambda_0 + f(\phi) \tag{5}$$

[10, 11]

5 Total Stress-Energy Tensor

$$T_{\mu\nu} = T_{\mu\nu}^{(\phi)} + T_{\mu\nu}^{\text{matter}} \tag{6}$$

[4, 5]

6 Objectives

- Choose appropriate forms for $V(\phi)$ and $f(\phi)$ to achieve finite-density black hole solutions.
- Solve the modified field equations for static, spherically symmetric spacetimes.
- Extend the model to rotating black holes.
- Ensure the modified equations respect energy-momentum conservation.
- Explore potential observational implications.

Conclusion

Any insights or suggestions on the above aspects would be greatly appreciated.

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