Optimal Surplus Harmonic Energy Distribution

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***Abstract* – A new optimal programmed PWM technique is described. The voltage total harmonic distortion at the output of an *L-C* filter connected to a PWM inverter is minimized. The proposed technique can be directly applied to constant frequency, constant amplitude sinusoidal power supplies such as uninterruptible power supplies. Without any hardware changes, the new method allows for more than 20 percent smaller THD than the general programmed PWM method. Other indexes that are functions of surplus harmonics, such as acoustic noise, heat losses, torque and speed ripples, can be optimized using the new method. A new model of nonconstraint optimization is developed for the proposed method. It is shown that this model includes both double-level and triple-level PWM waveforms. An energy conservation principle is applied to the model to prove and explain the optimization possibility. An energy conservation based algorithm for obtaining an optimization starting point is demonstrated. A numerical example of triple-level optimal programmed PWM waveforms for single-phase applications is presented.**

I. Introduction

The programmed pulse-width modulation (PWM) technique [1] is a very important and efficient method of eliminating selected harmonics from a PWM waveform spectrum. Theoretically, it can achieve the highest quality of the output waveform among all PWM methods used in voltage-source inverters [2].

Programmed PWM inverters can be used in constant-frequency variable-amplitude applications such as uninterruptible power supplies (UPS's) and in ac drives to optimize various drive performance indexes: efficiency, small torque ripple, accurate speed and position, minimization of device stresses, reduction of EMI, and reduction in acoustic noise. In inverter applications that require a high quality sinusoidal output, e.g., certain UPS's, low-pass *L-C* filters are used. So far, there have been reports on minimization of THD at the output of a programmed PWM inverter before a filter [3]. This paper presents a new method of minimization of the voltage THD at the output of an *L-C* filter. The THD of the output voltage is a popular performance index for PWM inverters. The THD is defined as

$$THD = \frac{100}{V_1}\sqrt{\sum_{n=2}^{\infty} V_n^2}. \qquad (1)$$

where V_n is the amplitude of the n-th harmonic. Fixed-frequency power supplies, such as UPS's, employ an output *L-C* filter to further suppress high-order harmonics. The quality of the UPS output voltage can be predicted from the harmonic content of the inverter output voltage, which is the *L-C* filter input voltage, by considering the so called distortion factor (DF) [3]. DF takes into account how higher order harmonics can be reduced by a second order low-pass filter

$$DF = \frac{100}{V_1}\sqrt{\sum_{n=2}^{\infty}\left(\frac{V_n}{n}\right)^2}. \qquad (2)$$

II. Mathematical Model of Programmed PWM

In typical PWM voltage inverters, the dc bus voltage is chopped N times per half cycle to produce either a double-level $\{-\frac{1}{2},\frac{1}{2}\}$ output (in half-bridge arrangements) or a triple-level $\{-1,0,1\}$ output (in full-bridge arrangements). Owing to the symmetries in the PWM waveforms, only odd harmonics exist. The Fourier coefficients of odd harmonics are given as follows:

- Double-level programmed PWM

$$V_n = \frac{4E}{n\pi}[1-2\cos n\alpha_1+2\cos n\alpha_2+\ldots+(-1)^j 2\cos n\alpha_j + \ldots + (-1)^N 2\cos n\alpha_N] \qquad (3)$$

- Triple-level programmed PWM

$$V_n = \frac{4E}{n\pi}[\cos n\alpha_1 - \cos n\alpha_2 + \ldots + (-1)^{j-1}\cos n\alpha_j + \ldots + (-1)^{N-1}\cos n\alpha_N] \qquad (4)$$

where E is the amplitude of the square wave.

Amplitudes of any M harmonics can be set by solving the M equations obtained from setting (3) or (4) equal to prespecified values. In the general programmed PWM method, the fundamental component is set to a required amplitude and $M-1$ low-order harmonics are set to zero [4]-[8]. Usually, the Newton iteration method is used to solve systems of nonlinear equations (3) or (4). The correct solution must satisfy the condition

$$0 < \alpha_1 < \alpha_2 < \ldots < \alpha_N < \frac{\pi}{2}. \qquad (5)$$

The general programmed PWM method is designed to eliminate as many low-order harmonics as possible. This determines the spectrum of the remaining higher-order harmonics. Implementation of the general programmed PWM requires solution of a system of nonlinear and transcendental equations. Since in any programmed PWM scheme the content of low-order harmonics should be small, some of the nonlinear equations of systems (3) or (4) are used as constraints in optimization of various performance indexes. The resulting computational problems of constrained optimization do not converge easily. Our proposed programmed PWM method makes the minimization of the DF as its target, without imposing which harmonics should be *entirely* eliminated. To assure small low-order harmonic content, the performance index is augmented by components containing low-order harmonics multiplied by penalty factors. A mathematical model of optimization without constraints is obtained which converges easily. In this way, the higher-order harmonics spectrum can be controlled to push more harmonic energy to higher frequencies. Those high order harmonics can be easily suppressed by an *L*-*C* filter. This new UPS harmonic control scheme does not require any hardware additions to a general programmed PWM scheme. (The value of DF decreases more than 20 percent compared with general programmed PWM.)

III. Equivalence of Programmed PWM Models for Optimization

Two classical programmed PWM techniques (3) and (4) have different waveforms and mathematical models. Consider the minimization of DF for each of the control schemes. Since DF is positive, its minimization is equivalent to minimization of the square of DF. It can be seen from (2) that the optimization target functions of different PWM schemes are the same. All are sums of squares of odd surplus harmonics divided by a square of the harmonic order. However, to keep the fundamental equal to one and low order harmonics zero, constraints must be introduced into the optimization model. The constraints can be imposed using (3) or (4) for double-level PWM and triple-level PWM, respectively. Right sides of abovementioned equations represent amplitudes of n-th harmonics. Fundamentals should be set to unity, other low-order harmonics should be equal to zero. Moving the constant term in (3) to the left side one obtains

$$-\frac{1}{2}V_n+\frac{2E}{n\pi}=\frac{4E}{n\pi}[\cos n\alpha_1-\cos n\alpha_2+\ldots+(-1)^{j-1}\cos n\alpha_j$$
$$+\ldots+(-1)^{N-1}\cos n\alpha_N] \quad (6)$$

Comparing equation (6) to equation (4), it can be seen that the right sides are identical and they are equal to constants. Hence, it can be concluded that the minimization of DF in two PWM schemes can be described by one optimization model with different constants in the constraint equations.

IV. Explanation of Optimization Possibility – Energy Conservation

A new concept of energy conservation during the optimization of double-level programmed PWM has been presented in [9]. Because of the quarter-cycle symmetry of programmed PWM, the quarter-cycle PWM energy can be used to represent the total energy property of one period. In the time domain, the energy of a quarter-cycle double-level PWM voltage waveform supplying a resistive load R can be expressed as

$$W=\frac{E^2}{R}\frac{\alpha_1}{2\pi}T+\frac{(-E)^2}{R}\frac{\alpha_2-\alpha_1}{2\pi}T+\ldots+\frac{E^2}{R}\frac{\frac{\pi}{2}-\alpha_N}{2\pi}T \quad (7)$$

where E is the amplitude and T is the period of the square wave. Deduced from (7), the energy of a quarter-cycle waveform can be given as

$$W=\frac{E^2}{R}\frac{T}{4}. \quad (8)$$

The above expression for the energy W does not depend on switching angles. Thus, the value of W will not change when a different set of switching angles is selected. Frequency spectra of time-domain PWM waveforms can be obtained using Fourier series expansion. Energies calculated from time-domain and frequency-domain analysis should satisfy the energy conservation law, that is, the sum of fundamental and surplus harmonic energies should be equal to W.

The expression for the frequency-domain energy is

$$W=\sum_{k=0}^{\infty}I_{2k+1}V_{2k+1}T$$
$$=\frac{V_1}{\sqrt{2}R}\frac{V_1}{\sqrt{2}}T+\frac{V_3}{\sqrt{2}R}\frac{V_3}{\sqrt{2}}T+\frac{V_5}{\sqrt{2}R}\frac{V_5}{\sqrt{2}}T+\ldots \quad (9)$$

where V_1 represents the amplitude of fundamental voltage component, and $V_{2k+1}(k=1,2,...)$ represent the surplus harmonics. By changing the switching angles with a constraint that the fundamental component is kept constant, amplitudes of particular harmonics can be decreased or increased. When a high order harmonic increases, some other harmonics, including lower-order harmonics, will decrease to preserve the energy conservation law. Since high-order harmonics are easier to filter out, by pushing the surplus harmonic energy towards high frequencies, minimization of DF, hence, minimization of THD at the output of an *L*-*C* filter can be achieved. Section VI shows the implementation of this optimization idea using Matlab programming.

V. Development of Optimization Model

A. Constrained Optimization

This section presents an implementation of DF minimization for the triple-level programmed PWM. Although there is no energy conservation property in a triple-level programmed PWM, the equivalence of optimization models for double-level and triple-level schemes validates the DF minimization approach for a triple-level programmed PWM.

To illustrate it, let us consider the triple-level programmed PWM scheme. Assume eleven switching angles ($N=11$) in a quarter cycle of the PWM waveform.

In the proposed DF minimization approach, the amplitude of the fundamental is controlled and nine low-order harmonics are eliminated which leaves one free switching angle for shaping surplus harmonic distribution. The fundamental is set to unity and (4) is used to set the constraints. The optimization target function is

$$G = \frac{1}{V_1}\sqrt{\sum_{n=21}^{\infty}\left(\frac{V_n}{n}\right)^2}. \qquad (10)$$

For practical reasons, it was decided to consider surplus harmonics up to 63rd.

B. Nonconstrained Optimization

A direct application of Matlab optimization toolbox procedures to the model with the optimization target function (10) and constraints (4) results in nonconvergence due to a singular Jacobian matrix. Since in the actual electronic system there are always mismatches and parameter tolerances, low-order harmonics will be small but not entirely eliminated. This gives a rise to an idea of transforming the constraint optimization model (10) and (4) into a nonconstraint one. The nonconstraint optimization is expected to have better convergence properties. The unity amplitude of the fundamental and small amplitudes of low-order harmonics can be required by using appropriate penalty factors in the nonconstraint optimization target function. The target function of the new scheme of optimization without constraints can be written as

$$F = (V_1 - 1)^2 + K_2^2V_3^2 + \ldots + K_{10}^2V_{19}^2 + G^2 \qquad (11)$$

where $K_2, K_3, \ldots, K_{10}$ are penalty factors and G is given by (10). The optimization target function (11) belongs to nonlinear least squares optimization problems. Function *leastsq* in the Matlab optimization toolbox was used to solve this DF minimization problem. The penalty factors were selected as $K_i = \frac{4}{2i-1}$ to put more weight on elimination of lower order harmonics.

C. Starting Point

This subsection shows how the energy conservation concept introduced in Section IV is used to obtain the starting point for DF minimization.

Let us consider the case for $N = 11$. Elimination of ten low-order harmonics with the general programmed PWM method with fundamental equal to unity, results in first significant surplus harmonic crest to be formed by 23th, 25th, 27th, and 29th harmonics (Fig. 2(a)). It follows from the definition of DF (2) that higher order harmonics contribute less to its value. The idea of energy conservation suggests that increase in the amplitude of selected harmonics higher in order than the first crest should decrease amplitudes of harmonics in the crest. Since, in this example, the harmonics 23th, 25th and 27th are a part of the first significant crest, let us select the 29th and 31st as high order harmonics and increase their amplitude. Addition of amplitude requirements for two high-order harmonics means that elimination of two lower order harmonics must be removed from the constraints (4). The highest previously eliminated harmonics, namely 19th and 21st, are made nonconstrained. The new nonlinear system of equations takes on the form

TABLE I
Switching Angles of Three Control Schemes (in Degrees).

Angle	General	Starting point	Optimal
1st	12.0951	12.8405	11.9519
2nd	15.2980	16.3948	15.0000
3rd	24.2877	25.0666	23.0959
4th	30.5558	30.7137	27.6166
5th	36.6808	35.7295	32.2403
6th	45.7335	42.8395	38.0730
7th	49.3718	46.5329	42.1124
8th	60.7622	57.4493	51.7438
9th	62.4524	59.6355	54.3737
10th	75.5559	73.7103	66.8814
11th	75.9914	74.3215	67.8267

$$\begin{aligned}
&\cos\alpha_1 - \cos\alpha_2 + \ldots - \cos\alpha_{10} + \cos\alpha_{11} = \frac{\pi}{4}\\
&\cos 3\alpha_1 - \cos 3\alpha_2 + \ldots - \cos 3\alpha_{10} + \cos 3\alpha_{11} = 0\\
&\ldots\\
&\cos 17\alpha_1 - \cos 17\alpha_2 + \ldots - \cos 17\alpha_{10} + \cos 17\alpha_{11} = 0\\
&\cos 29\alpha_1 - \cos 29\alpha_2 + \ldots - \cos 29\alpha_{10} + \cos 29\alpha_{11} = \frac{29\pi}{4}V_{29}\\
&\cos 31\alpha_1 - \cos 31\alpha_2 + \ldots - \cos 31\alpha_{10} + \cos 31\alpha_{11} = \frac{31\pi}{4}V_{31}.
\end{aligned} \qquad (12)$$

Use the following algorithm to maximize 29th and 31st harmonics:

1) Take V_{29}, V_{31}, and switching angles of general programmed PWM as initial values.
2) Micro-increase amplitude of V_{29} and V_{31}.
3) Calculate new switching angles from (12).
4) Take the result of step 3 as new iteration values.
5) Repeat step 2 until further increase of V_{29} and V_{31} causes convergence problems.

This method is an effective and simple way to gain the starting point of DF minimization.

The calculated switching angles for general programmed PWM ([1], [10]), starting point for the optimal programmed PWM obtained using the algorithm of this subsection, and optimal programmed PWM with the minimum DF are presented in Table I for $N = 11$.

VI. Numerical Examples

The switching angles for general programmed PWM [5] and optimal programmed PWM with the minimum DF were calculated. The frequency spectrum of the general programmed PWM and optimal PWM with the minimum DF is shown in Figs. 1-4 for N = 9, 11, 13, and 15, respectively. Corresponding time domain waveforms for $N = 11$ are presented in Fig. 5. Numeri-

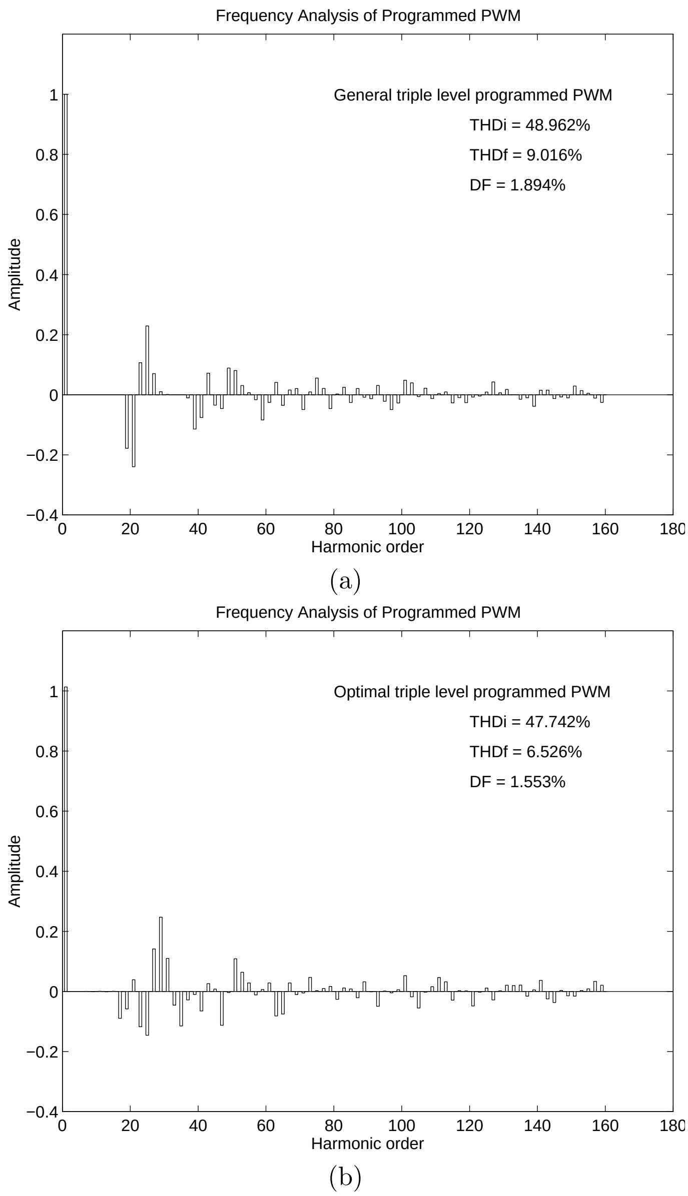


Fig. 1. Frequency spectrum of triple-level programmed PWM with $N = 9$. (a) General. (b) Optimal.

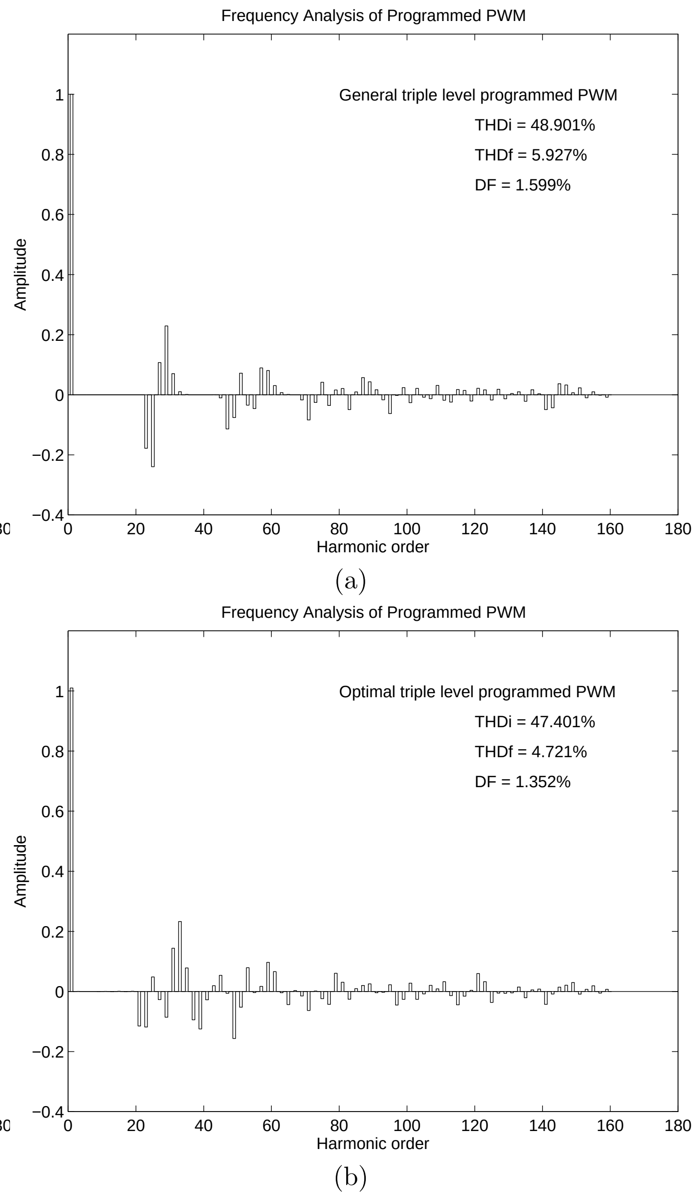


Fig. 2. Frequency spectrum of triple-level programmed PWM with $N = 11$. (a) General. (b) Optimal.

cal values of switching angles in the first quarter-cycle for all the harmonics spectra presented in Figs. 1-4 are given in Table II.

The voltage total harmonic distortion THD_i at the inverter output and the voltage total harmonic distortion THD_f at the output of the L-C filter are also shown in Figs. 1-4. Defining $\omega_r = 1/\sqrt{LC}$ and $p = \omega_r/(2\pi f)$, where f is the frequency of the fundamental, the total harmonic distortion at the filter output is [3]

$$THD_f = \frac{100(p^2-1)}{V_1}\sqrt{\sum_{n=2}^{\infty}\left(\frac{V_n^2}{n^2-p^2}\right)^2}. \qquad (13)$$

An L-C filter with a corner frequency $f_r = \omega_r/(2\pi) =$ 555 Hz was used for calculations.

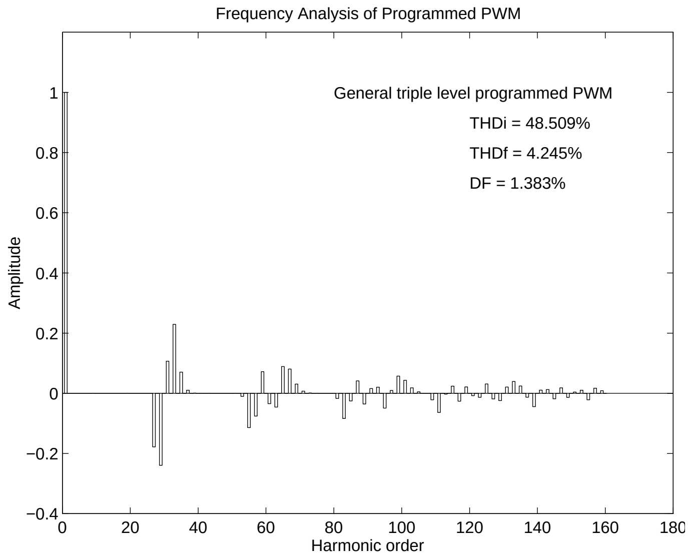
It can be seen in Figs. 1-4 that the obtained minimum values of DF are 18.0%, 15.4%, 13.9%, and 7.6% smaller than DF's for the general programmed PWM for N = 9, 11, 13, and 15, respectively. The corresponding decrease in THD_f's is 27.6%, 20.3%, 17.9%, and 4.2%. It should be noted that the decrease in THD_f depends on the corner frequency of the L-C filter which was kept constant for the abovementioned calculations. For instance, if a filter with a corner frequency f_r = 750 Hz is used, the THD_f decrease for N = 15 would be 6.5% (from 5.54% to 5.18%). Thus, a change in the PWM control scheme allows for load THD decrease from almost 30% to several percent (depending on the number of switchings per quarter-cycle and on the filter size) without any hardware changes.

VII. Conclusions

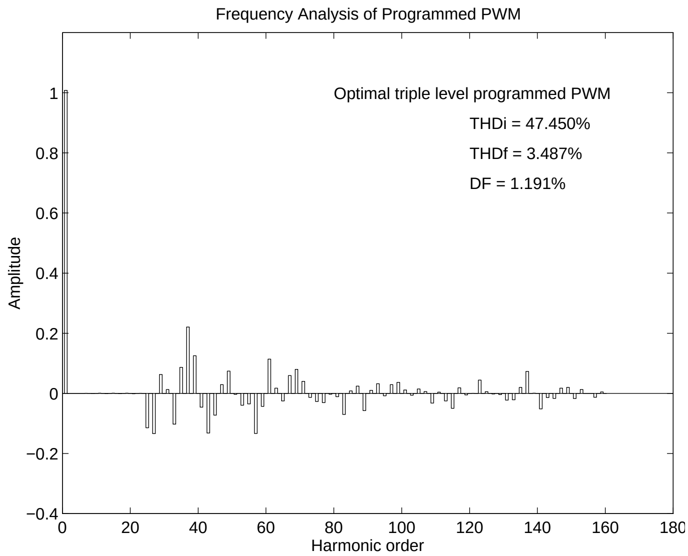
A new optimal programmed PWM technique has been described. The voltage total harmonic distortion at the output of an L-C filter connected to a PWM inverter is minimized. The proposed technique can be

TABLE II
Switching Angles in the First Quarter-Cycle for Harmonic Spectra of Figs. 1-4 (in Degrees).

angle #	Fig. 1(a)	Fig. 1(b)	Fig. 2(a)	Fig. 2(b)	Fig. 3(a)	Fig. 3(b)	Fig. 4(a)	Fig. 4(b)
1	13.98	13.48	12.09	12.23	10.66	10.79	9.11	9.67
2	18.43	17.35	15.30	15.45	13.08	13.24	10.83	11.61
3	28.13	25.47	24.29	23.72	21.38	21.11	18.25	19.10
4	36.77	30.80	30.56	28.29	26.13	24.99	21.64	22.53
5	42.65	35.89	36.68	32.42	32.22	29.27	27.44	27.27
6	54.93	44.78	45.73	37.64	39.15	32.99	32.41	30.36
7	57.71	48.78	49.37	41.67	43.23	36.68	36.71	33.32
8	72.74	62.38	60.76	51.28	52.09	43.63	43.13	38.13
9	73.46	64.00	62.45	54.04	54.48	46.94	46.08	41.55
10			75.56	66.57	64.92	56.51	53.77	48.93
11			75.99	67.59	66.02	58.37	55.59	51.34
12					77.57	69.57	64.31	60.25
13					77.85	70.23	65.25	61.54
14							74.78	71.37
15							75.10	71.84

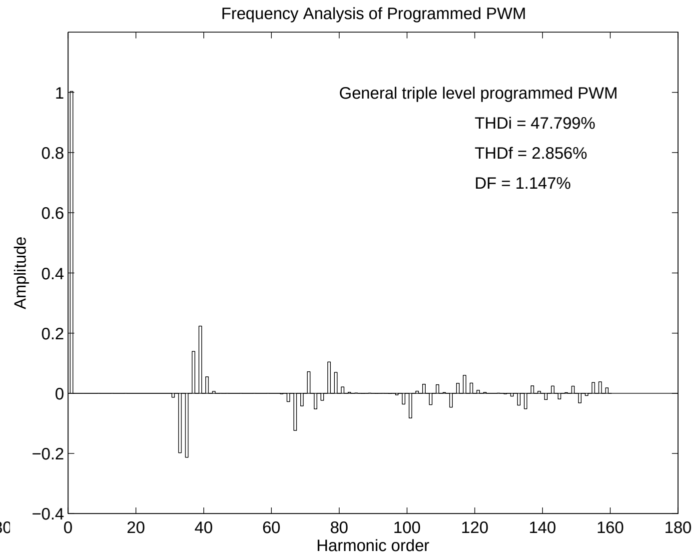


(a)

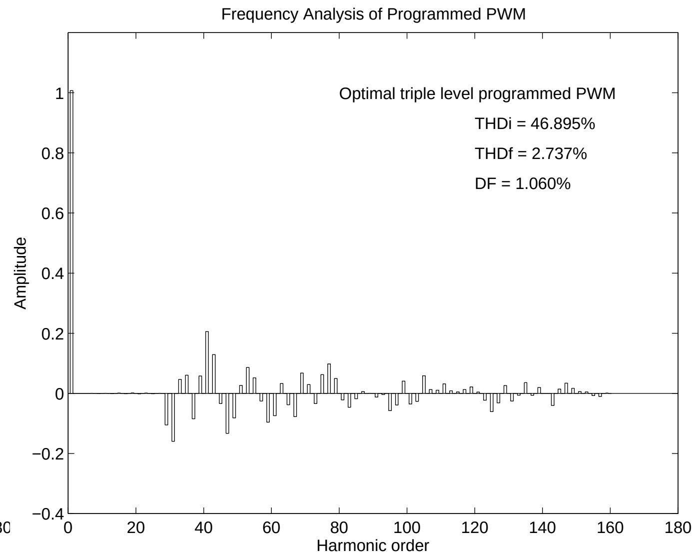


(b)

Fig. 3. Frequency spectrum of triple-level programmed PWM with $N = 13$. (a) General. (b) Optimal.

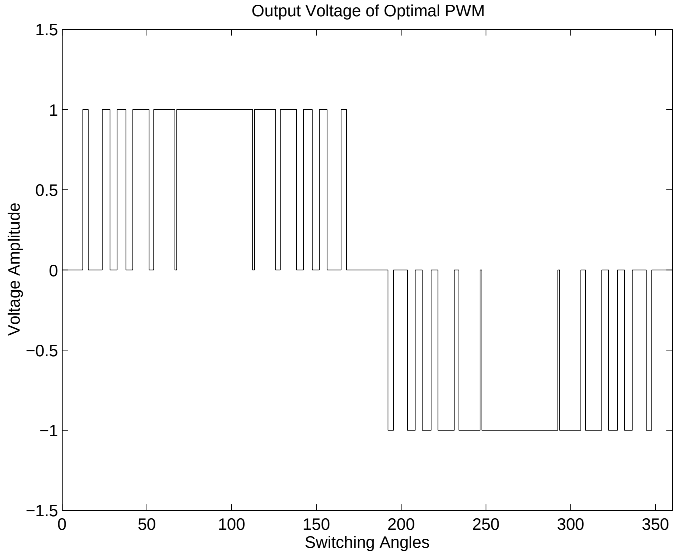


(a)

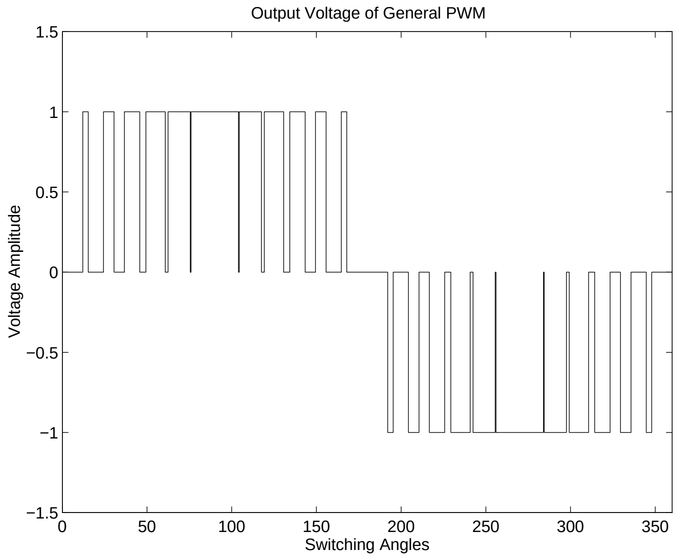


(b)

Fig. 4. Frequency spectrum of triple-level programmed PWM with $N = 15$. (a) General. (b) Optimal.



(a)



(b)

Fig. 5. Frequency spectrum of triple-level programmed PWM with $N = 11$. (a) General. (b) Optimal.

directly applied to constant frequency, variable amplitude sinusoidal power supplies such as uninterruptible power supplies. Without any hardware changes, the new method allows for obtaining more than 20 percent decrease in THD than the general programmed PWM method. The amount of THD decrease depends on the number of switchings per cycle, the required amplitude of the fundamental component, and the size of the filter. The biggest gain in output voltage quality is for small number of switchings which suggest that the method could be used primarily in high-power inverters with slow semiconductor switches. The spectra of the optimized waveforms are more uniform than those obtained from general programmed PWM. Hence, they also exhibit a lower acoustic noise [11], [12].

A new model of nonconstraint optimization has been developed for the new method. It has been shown that this model includes both double-level and triple-level PWM waveforms. An energy conservation principle has been applied to the model to prove and explain the optimization possibility. An energy conservation based algorithm for obtaining an optimization starting point has been demonstrated. The nonconstraint optimization model can be used for minimization of other performance indexes, e.g., reduction of EMI, efficiency, and small torque ripple as well as accurate speed and position in ac drives applications. The presented method is verified by numerical examples of single-phase triple-level optimal programmed PWM waveforms for various numbers of switchings per cycle.

The future work will include hardware implementation of the proposed concept. Theoretical studies on the application of the THD minimization method in closed-loop systems with varying nonlinear load will be performed. The general idea of shaping the frequency spectrum by optimization of a selected performance index can be also used in multilevel inverters and variable frequency applications.

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