Random Walks and Sustained Competitive Advantage

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Strategy is concerned with sustained interfirm profitability differences. Observations of such sustained differences are often attributed to unobserved systematic a priori differences in firm characteristics. This paper shows that sustained interfirm profitability differences may be very likely even if there are no a priori differences among firms. As a result of the phenomenon of long leads in random walks, even a random resource accumulation process is likely to produce persistent resource heterogeneity and sustained interfirm profitability differences. A Cournot model in which costs follow a random walk shows that such a process could produce evidence of substantial persistence of profitability. The results suggest that persistent profitability does not necessarily provide strong evidence for systematic a priori differences among firms. Nevertheless, since the phenomenon of long leads is highly unrepresentative of intuitive notions of random sequences, such evidence may still be persuasive.

Key words: sustained competitive advantage; resource-based view; random walks; luck

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1. Introduction

Consider two fictional computer firms, Firm 1 and Firm 2, which compete for the same consumers. Since these consumers care primarily about the speed of the computer they buy, the profit that a firm makes in a year depends on the speed of the computer the firm is able to produce in that year. The firm that is able to produce the fastest computer in a given year makes a profit of $10 million in that year. The firm that produces the slowest computer in a given year makes a profit of $5 million in that year. If the computers produced are equally fast each firm makes a profit of $7.5 million in that year.

Although each firm tries to increase the speed of its computers as much as possible, the actual increase is random. Specifically, in each year the speed of the computers produced by Firm 1 increases with a random number between 0 and 100 megahertz. Similarly, in each year the speed of the computers produced by Firm 2 increases with a random number between 0 and 100 megahertz.

Suppose that we observe the firms during a 10-year period from 1991 to 2000. Furthermore, suppose that in 1990 the computers produced by Firm 1 and Firm 2 were equally fast: They both had a speed of 200 megahertz. Which of the following outcomes do you believe is most likely?

(A) The average profit of Firm 1 over the 10 years from 1991 to 2000 is equal to $6 million.

(B) The average profit of Firm 1 over the 10 years from 1991 to 2000 is equal to $7.5 million.

(C) The average profit of Firm 1 over the 10 years from 1991 to 2000 is equal to $9 million.

(D) The average profit of Firm 1 over the 10 years from 1991 to 2000 is equal to $10 million.

If you answered (B) you belong to a majority. Most people think that (B) is most representative of a random process with identically distributed random changes in speed. However, it is notoriously difficult to make subjective judgments about probability (Kahneman and Tversky 1972). The correct, but potentially counterintuitive, answer in this case is (D)—that Firm 1 has an average profit of $10 million. In fact, of all possible outcomes, the most likely outcome is that one firm has an average profit of $10 million and the other one an average profit of $5 million. The least likely outcome is that both firms have an average profit of $7.5 million. Thus, in this case, sustained interfirm profitability differences are the most likely outcome of a random process. This paper deals with the sources and consequences of this phenomenon. It demonstrates how a purely random process of resource accumulation is very likely to produce sustained interfirm profitability differences.

1 Surveys distributed to 200 students and faculty members show that 70% to 90% tend to answer (B). Only 10% to 20% give (D) as the answer. In fact, when asked about the least likely outcome, a majority tend to answer (D).
Explanations of sustained interfirm profitability differences are central in strategy research. Observations of such differences are often attributed to persistent differences in firm characteristics and resources (Rumelt 1984, Barney 1991, Peteraf 1993). Persistent resource heterogeneity, in turn, is typically explained by a priori heterogeneity in initial stocks or in expected flows of resources (Rumelt 1984, Peteraf 1993, Teece et al. 1997). In this scenario firms have different resources, since they started out with different resources or differed in their ability to develop resources. In addition, it has been noted that chance events (Barney 1986), especially combined with path dependency in the resource accumulation process, may generate persistent heterogeneity (Arthur 1989). Yet, as this paper demonstrates, persistent resource heterogeneity is actually the expected outcome of even a random resource accumulation process.

More specifically, following Levinthal (1991), this paper examines the consequences of a random walk model of resource accumulation. In this process resource stocks are modeled as the sum of random resource flows. The flow of resources is assumed to be random, and all firms are assumed to have the same probability of an increase of a given amount. Thus, no firm is, ex ante, more likely than others to increase the level or quality of its resources. Furthermore, all firms are assumed to start with the same initial level of resources. Building upon a famous result on long leads in random walks (Feller 1968), this paper demonstrates that such a process will typically generate sustained differences in resources. As a result, in situations where profitability is an increasing function of the level or the quality of resources, the most likely outcome is not that the average profits of firms equalize over time. Rather, the most likely outcome is that there is a group of firms with consistently high profits and a group of firms with consistently low profits. Using a Cournot model in which costs follow a random walk, I demonstrate that this process can generate estimates of the persistence of profitability consistent with empirical observations (Mueller 1986, 1990).

The possibility that a stochastic model of resource accumulation could account for persistent resource heterogeneity and sustained interfirm profitability differences does not demonstrate that success is random. It does suggest, however, that observations of persistent resource heterogeneity and sustained interfirm profitability differences do not provide much evidence for heterogeneity in initial resource stocks or in expected resource flows. Observing a firm with high performance for 10 consecutive years, most people may become convinced that the firm must have been different from the start. However, the model presented in this paper suggests that chance, rather than systematic a priori differences among firms, can generate a consistent performance record. Although the idea that persistent performance differences may be due to chance is not new (Rumelt 1984, Barney 1986, Arthur 1989, Stinchcombe 2000), this paper extends the scope of such chance explanations by proposing a simple and counterintuitive alternative way in which chance events can result in sustained interfirm profitability differences. Formulated differently, this paper suggests an alternative null hypothesis about sustained interfirm profitability differences.

The possibility of such a null hypothesis suggests that, empirically, it may be difficult to distinguish between random and systematic processes underlying profit series. In particular, given intuitive notions of random sequences (Kahneman and Tversky 1972), it seems unlikely that laymen would attribute the pattern of persistent profitability generated by this model to chance. Rather, persistent profitability is likely to be attributed to prior systematic differences among firms. In contexts where long leads are present, such underestimation of the role of chance implies that learning about, and evaluations of, strategies may be strongly influenced by noise. Spurious theories may be developed to account for essentially random phenomena.

2. Explanations of Persistent Resource Heterogeneity

Studies of firm accounting profitability have demonstrated substantial differences in average profitability within industries (Cubbin and Geroski 1987, Rumelt 1991, Waring 1996, McGahan and Porter 1997, McGahan 1999). Although such differences tend to be eliminated over time as profitability regresses to the mean (Ghemawat 1991, Waring 1996, Fama and French 2000), studies of the autoregressive process of accounting profitability show that there is not complete convergence (Mueller 1986, 1990; Waring 1996). Firms with above-average profitability tend to remain above average. In addition, persistence of profitability varies among industries—in some industries and in some periods there is substantial persistence of profitability (Waring 1996). Finally, studies have documented that there are some firms that obtain high profitability relative to the industry average during long periods (Mueller 1986, Waring 1996). More systematic statistical evidence comes from the study of Wiggins and Ruefli (2002), who used a nonparametric method for identifying outliers. Using 25 years of data, they demonstrated that there are some firms, although few, that obtain high profitability relative to their industry mean for periods of 10 to 25 years.

Although the phenomenon is perhaps rare, explanations of sustained above-average returns have been
central in strategic management; several interpretations have been offered. Recent discussions have emphasized the importance of persistent resource heterogeneity (Demsetz 1973; Lippman and Rumelt 1982; Rumelt 1984; Barney 1986, 1991; Peteraf 1993). In this interpretation, above-average returns are seen as rents to scarce firm-specific resources. Firms with superior resources are able to earn economic rents while firms without superior resources are able to earn only a competitive rate of return (Rumelt 1984, Peteraf 1993). Sustained interfirm profitability differences are thus explained by persistent resource heterogeneity. It follows that explanations of persistent resource heterogeneity have become central within this perspective.

Following Dierickx and Cool (1989), explanations of resource heterogeneity can be characterized by the assumptions they make about stocks and flows in the resource accumulation process. In Dierickx and Cool’s bathtub metaphor, firm $i$’s stocks of resources in period $t$, $R_{i,t}$, depend on resources in period $t-1$, $R_{i,t-1}$, and flows during period $t$, $f_{i,t}$. Formally, $R_{i,t} = R_{i,t-1} + f_{i,t}$.

Early explanations of persistent resource heterogeneity assumed initial heterogeneity in stocks of resources and focused on the mechanisms that sustain such heterogeneity. In the simplest version of this theme firms are supposed to be endowed with different stocks of resources. In situations where there are strong isolating mechanisms and flows are negligible, any initial heterogeneity is likely to persist over time (Rumelt 1984, Peteraf 1993). If we denote firm $i$’s stocks of resources at time $t$ by $R_{i,t}$, the underlying process can be described as follows:

$$R_{i,t} = R_{i,t-1},$$  \hspace{1cm} (1)

In this process, stocks of resources are entirely determined by initial stocks. As a result, heterogeneity in initial stocks will result in persistent resource heterogeneity.

A later version of the same theme puts the emphasis on heterogeneity in expected flows rather than stocks. In this version, initial resource endowments may, in fact, be similar, but expected flows of resources are assumed to differ among firms. Thus, it is assumed that some firms are better able to accumulate and develop resources; they have superior “dynamic capabilities” (Teece et al. 1997, Zollo and Winter 2002). If we denote the expected value of firm $i$’s flows by $\theta_i$, the underlying process can be described as follows:

$$R_{i,t} = R_{i,t-1} + \theta_i + \epsilon_{i,t},$$  \hspace{1cm} (2)

where $\epsilon_{i,t}, t = 1, 2, \ldots, n$, are independently and identically distributed random variables with zero mean. In this process, stocks of resources at a given time are a function of past levels of resources and the size of flows of resources. Furthermore, the expected size of flows is assumed to differ among firms. Such heterogeneity in expected flows, will, after some time, produce persistent heterogeneity in stocks.

Both of these explanations assume some form of a priori heterogeneity in either initial stocks or in expected flows. The implication of this assumption is that if the process was, hypothetically, run again, the same firms would tend to end up on top. It is possible, however, to imagine a different process in which chance would play an important role. In this process, there is homogeneity in initial stocks and no ex ante differences in expected flows. However, the process of resource accumulation is assumed to be path dependent (Arthur 1989, Noda and Collis 2001). Specifically, expected flows are assumed to be an increasing function of past stocks of resources; there are “asset mass efficiencies” (Dierickx and Cool 1989). The underlying process can be described as follows:

$$R_{i,t} = R_{i,t-1} + g_i(R_{i,t-1}) + \epsilon_{i,t},$$  \hspace{1cm} (3)

where $g_i(\cdot)$ is an increasing function and $\epsilon_{i,t}$, $t = 1, 2, \ldots, n$, are independently and identically distributed random variables with zero mean. In this process, stocks of resources at a given time are a function of past levels of resources and the size of flows. However, the size of flows is assumed to be an increasing function of past levels of resources. Thus, a firm that happens to gain an advantage is more likely to accumulate large resources in the future. As a result of this path dependency, small random events may produce persistent heterogeneity in stocks of resources even if all firms start with the same initial stocks of resources. This implies that if the process was run again, it is unlikely that the same firm would end up at top. Most likely, the firm with the most resources would be a different firm.

While persistent resource heterogeneity is usually attributed to one of the above three processes, this paper shows that there is, in fact, a much simpler process that also produces persistent resource heterogeneity. This process assumes homogeneity in both initial stocks and in expected flows. Specifically, it assumes that the distribution of flows is the same for all firms and for all periods. Thus, there are no increasing returns in the resource accumulation process. The underlying process is simply:

$$R_{i,t} = R_{i,t-1} + \epsilon_{i,t},$$  \hspace{1cm} (4)

where $\epsilon_{i,t}, t = 1, 2, \ldots, n$, are independently and identically distributed random variables with zero mean. As in Levinthal (1991), the accumulation process is thus assumed to be a random walk.
Similar to the first process, this process assumes that heterogeneity in stocks is the result of variation in prior stocks. In fact, in this process the prior period resource stocks provides a base for the entire subsequent history. However, while the first process assumes that resource stocks are entirely determined by an initial random draw, this process assumes that resource stocks can be modeled as the sum of several independent random draws. Thus, the existing resource stocks are the cumulative result of resource flows during previous periods (Levinthal 1991). As such it provides a simple model of the cumulative character of capability and resource development (Nelson and Winter 1982, Dosi 1988, Teece et al. 1997). In particular, it is consistent with a view of a capability development as the result of a local and cumulative process in which, "What a firm can hope to do technologically in the future is narrowly constrained by what it has been capable of doing in the past" (Dosi 1988, p. 1130). In addition, it recognizes the strong uncertainty present in most search processes (Levinthah and March 1981, Dosi 1988, Levinthal 1991). In this stochastic model, however, initial stocks of resources and the distribution of the increments are assumed to be identical for all firms. In other words, no firm has a first-mover advantage and no firm is assumed to be better able to accumulate and develop resources.

These assumptions may seem to be unrealistic. In most markets firms enter with different resources and capabilities. Furthermore, there can be large interfirm differences in the development of productivity, research, and functional capabilities. Given this, it may seem pointless to examine a process that ignores these features. However, as Levinthal (1991) emphasizes, the explanatory power of a process that includes these features can only be assessed by comparing it to a process in which these features are absent. The situation is analogous if one wants to examine the hypothesis that a series of numbers is the result of a biased die. The hypothesis of a biased die is tested by computing the probability of the series given the null hypothesis of an unbiased die. The above process introduces a null hypothesis about persistent resource heterogeneity. In contrast, if persistent resource heterogeneity is very unlikely given this null hypothesis, observations of persistent resource heterogeneity and sustained interfirm profitability differences indicate heterogeneity in either initial stocks or in expected flows. In contrast, if persistent resource heterogeneity is very likely given this null hypothesis, observations of persistent resource heterogeneity and sustained interfirm profitability differences do not provide much evidence for heterogeneity in either initial stocks or in expected flows.

### 3. Random Walks and Persistent Resource Heterogeneity

To examine the implication of the above null hypothesis, consider an industry with two firms where the resource accumulation process of firm $i$ follows a random walk. Specifically, suppose that the stocks of resources for firm $i$ in period $t$ is $R_{i,t} = R_{i,t-1} + \epsilon_{i,t}$, where $\epsilon_{i,t}$, $t = 1, 2, \ldots, n$, are independently and identically normally distributed random variables with zero mean and finite variance. We assume that the initial level of resources is the same for both firms. Suppose that we would observe these two firms over a period of $n$ years. Let $k$ denote the number of years when the stocks of resources of Firm 1 is higher than the stocks of resources of Firm 2, i.e., when $R_{1,t} > R_{2,t}$. How likely is it that $k$ is close to zero or to $n$, implying that one firm has a higher stock of resources most of the time?

Although such episodes of persistent differences in resources may seem unlikely (Feller 1968, Kahneman and Tversky 1972), a famous result in probability theory regarding long leads in random walks (Feller 1968) implies that they are to be expected. Specifically, the most likely event is that $k$ is equal to zero or to $n$, while the least likely event is that $k$ is equal to $n/2$.

To see this, note that the distribution of $k$ is identical to the distribution of the number of periods in which $R_{1,t} - R_{2,t}$ is positive. Since $R_{1,t} - R_{2,t}$ follows a symmetric random walk, however, this distribution is well known. For any symmetric random walk, $X_t = X_{t-1} + \epsilon_t$, satisfying the condition that $P(X_t \geq 0) = P(X_t < 0) = 0.5$, it can be demonstrated (Feller 1971, p. 419) that the probability that the random walk spends $k$ ($0 \leq k \leq n$) periods on the positive side and $n-k$ periods on the negative side equals:

$$
P_{k,n-k} = \binom{2n}{k} \left( \frac{2n-2k}{n-k} \right) 2^{-2n}.
$$

The distribution is plotted in Figure 1 for $n = 10$. As illustrated, rather than being concentrated on $k = 5$, which one would have expected if the random walk spent about half the time on the positive and half the time on the negative side, the distribution is bimodal, with most weight on the extreme values of $k = 0$ and $k = 10$. More generally, we have that for any $n$ the central term, i.e., $P_{n/2,n/2}$, is always the smallest and the probabilities $P_{n,0}$ and $P_{0,n}$ are always the largest. Thus, the random walk tends to stay at one side for a large proportion of time. Using an approximation of the distribution in (5), which holds exactly as $n \to \infty$, it is possible to show that the expected value of this
they are very close, even for asymptotically, calculations using the distribution in (5) show that and the median is about 0.85. Although these values only hold on the positive side. Let one firm will lead all the time is very high. Typically lead to a different distribution where the probability that a random walk as the difference between the stocks of resources of two firms implies that there is a 50% chance that one firm will have a higher stock of resources during at least 85% of the observed period. For comparison, it is interesting to note that the same distribution could also emerge through other processes. For example, suppose that initial stocks were independently normally distributed with zero mean and a standard deviation of 1.5. Furthermore, suppose that the resources of each firm in each period are equal to initial stocks plus an independently and normally distributed noise term with zero mean and a standard deviation of 1. Simulations show that this process produces an almost identical distribution of leads. The proportion is about 0.82 and that the median is 0.85. Thus, on average, a random walk tends to spend 82% of the time on one side and 50% of all random walks tend to spend more than 85% of the time on one side. The interpretation of a random walk as the difference between the stocks of resources of two firms implies that there is a 50% chance that one firm will have a higher stock of resources during at least 85% of the observed period. For comparison, it is interesting to note that the same distribution could also emerge through other processes. For example, suppose that initial stocks were independently normally distributed with zero mean and a standard deviation of 1.5. Furthermore, suppose that the resources of each firm in each period are equal to initial stocks plus an independently and normally distributed noise term with zero mean and a standard deviation of 1. Simulations show that this process produces an almost identical distribution of leads.

Let $Y = k/n$ be the proportion of time a random walk spends on the positive side. Let $f(y)$ denote the density function of $Y$. It can be demonstrated that

$$f(y) \xrightarrow{n \to \infty} \frac{1}{\pi \sqrt{y(1-y)}}$$

as $n \to \infty$ (Karlin and Taylor 1981, p. 474). Let $Z$ denote the maximum of $Y$ and $1 - Y$, i.e., $Z$ is the largest proportion of time that the random walk spends on one side. Since we have that $P(Z > z) = 2(1 - F(z))$, the cumulative distribution function of $Z$ is $G(z) = 2F(z) - 1$ and the density is $g(z) = 2f(z)$. The expected value of $Z$ is thus

$$E[Z] = 2\int_0^1 \frac{z}{\pi \sqrt{z(1-z)}} \, dz = 2[\sqrt{\pi} - 2^2 + \frac{1}{2} \arcsin(2z - 1)]_{z=1}^{z=1} \approx 0.82,$$

and the median is about 0.85. Although these values only hold asymptotically, calculations using the distribution in (5) show that they are very close, even for $n = 10$. A process with randomly distributed expected flows, however, typically leads to a different distribution where the probability that one firm will lead all the time is very high.

To understand these results it should be noted that they follow from the assumed cumulative character of the resource accumulation. Specifically, resource accumulation was assumed to follow a random walk. This assumes that resources obtained in previous periods remain relevant and that the expected flow of resources is identical for all firms and all periods. Under these assumptions the prior period resource stock provides a base for the entire subsequent history. As a result, a firm that has obtained a high stock of resources is likely to remain among the firms with the largest stocks of resources. Conversely, it is quite unlikely that this firm would end up among the firms with the smallest stocks of resources.

The importance of the cumulative character of random walks can be illustrated by examining a more general process where resources obtained in past periods are less relevant than resources obtained in recent periods. In the above model of resource accumulation as a random walk, resources obtained in previous periods were assumed to be retained and relevant in all later periods. However, due to technological change (Tushman and Anderson 1986) and failures of retention (Argote 1999), resources obtained in recent periods may be more relevant than resources obtained in distant periods. To model this, suppose the stocks of resources for firm $i$ follow a modified random walk, $R_{i,t} = bR_{i,t-1} + \epsilon_{i,t}$, where $\epsilon_{i,t}, t = 1, 2, \ldots, n$, are independently and identically distributed standard normal variables and $b$ is a parameter measuring the importance of the past flow of resources. As $b$ decreases, resources obtained in previous periods become less important, and with $b = 0$, resources obtained in previous periods have no effect on present resource stocks. Figure 2 plots the distribution of the number of times $R_{1,t} - R_{2,t}$ is positive during the

![Figure 1](image1.png)

**Note.** Based on Equation (5).

![Figure 2](image2.png)

**Figure 1** Probability That a Symmetric Random Walk Spends $k$ Periods on the Positive Side and $n-k$ Periods on the Negative Side When $n=10$

**Figure 2** Probability That the Difference Between Two Modified Random Walks, $R_{i,t+1} = bR_{i,t} + \epsilon_{i,t}$, Is Positive During $k$ of 10 Periods

**Note.** Each line is based on 100,000 simulations, where $\epsilon_{i,t} \sim N(0, 1)$. 

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first 10 periods for various values of $b$. Clearly, as $b$ decreases, the distribution becomes unimodal and concentrated around 5. Obviously, when $b = 0$ the distribution is the binomial distribution with $n = 10$ and $p = 0.5$. The implication of this change in distribution is that the expected maximum proportion of time that one firm leads is reduced. For example, while the expected maximum proportion is 0.83\(^4\) when $b = 1$, it is 0.73 when $b = 0.7$, and 0.62 when $b = 0$.\(^5\) Thus, as resource accumulation becomes less cumulative, and prior stocks become less significant for future development, it is less likely that one firm will lead most of the time. Similar results hold in a model in which accumulated resources may become a liability, perhaps due to competency traps where prior learning constrains the ability to cope with environmental changes (Levinthal and March 1981, Barnett and Hansen 1996, Ingram and Baum 1997). A simple way of modeling such a process is to assume that if an environmental change occurs, then the stocks of resources change sign. Thus, previous leaders become laggards. Simulations show that if $b = 1$ and the probability of an environmental change is very small, the distribution of the proportion of time that Firm 1 leads will still be bimodal, with most weights on the extremes. However, as the probability of an environmental change increases, the distribution of leads quickly becomes unimodal and concentrated around 0.5. As a result, it is less likely that one firm will lead most of the time. More generally, these results imply that persistent resource heterogeneity is most likely to occur in industries and in periods in which past resources are retained and continue to be valuable. In this case, persistent resource heterogeneity is the expected outcome of a random resource accumulation process.

Although it is difficult to obtain analytical results, the general logic of the phenomenon of long leads obviously also applies to models with many firms. If resources follow a random walk, it is still true that firms with a high stock of resources in comparison with other firms are likely to continue to be among the firms with the highest stock of resources. To illustrate this, consider a model with 100 firms where the resources of each firm $j$ starts at zero and evolves according to a modified random walk $R_{i,t} = bR_{i,t-1,j} + e_{i,j,t}$ where $b$ is a positive fraction and $e_{i,j,t}$, $t = 1, 2, \ldots, n$ are independently and identically distributed normal random variables with mean zero and variance 1. Simulations show that when $b = 1$ the distribution of the proportion of time the stock of resources of firm $i$ is among the 50% largest has a shape very similar to the distribution of the proportion of time a symmetric random walk is positive.\(^6\)

Thus, a firm is most likely to be among the 50% best or among the 50% worst most of the time, and is least likely to be among the best half of the time. Moreover, simulations show that the expected maximum proportion of time that a firm spends among the 50% best or among the 50% worst in the first 10 periods is 0.83, identical to the two-firm model. Thus, during the first 10 periods a firm spends, on average, 83% of the time either among the 50% best or among the 50% worst. Again, decreasing $b$ reduces this proportion. Thus, it is 0.73 when $b = 0.7$ and 0.62 when $b = 0$ (based on 10,000 simulations). Further illustration of the persistence of relative position of stocks of resources can be obtained by using a more fine-grained classification of relative position. Consider, for example, the first to the fifth quintiles. Simulations with $b = 1$ show that, on average, a firm tends to spend 58% of the first 10 periods in one quintile (based on 10,000 simulations). Again, this is reduced if resource accumulation is less cumulative. Thus, if $b = 0.7$ the proportion is 0.47 and 0.38 if $b = 0$.

The stability of relative positions when $b$ is high implies that it is quite likely that some firms will be among the best firms during almost all periods. Indeed, simulations with 100 firms show that if $b = 1$ the probability that there will be a firm which is among the 10 best firms during all of the first 10 periods is 0.54, as opposed to 0.02 when $b = 0.7$, and almost 0 when $b = 0$ (based on 10,000 simulations). Similar results apply to the probability of observing a firm that is among the 10 worst during all periods. The implication is that if resource accumulation is cumulative it is quite likely that some firms will be among the best and some firms will be among the worst all the time. In other words, persistent resource heterogeneity is likely to be observed.

These results have been derived using a very specific model of resource accumulation, as a partial sum of independent and identically distributed increments, which is unlikely to be a good model of resource accumulation in many contexts. The results can, however, be extended to several other contexts and processes. First, the model of a random walk can easily be generalized to situations in which the competitive advantages of firms depend on the evolution of several different types of resources rather than on only one. Since any linear combination of normally distributed variables, not necessarily independent, is normally distributed, the above results would be identical if the competitive advantage of

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\(^4\) Using Equation (5). It is 0.82 if the asymptotic formula is used.

\(^5\) Based on 20,000 simulations. Note that the lowest possible value is always 0.5.

\(^6\) The results are similar to the case with only two firms since the situation is essentially similar: Both cases concern the occupation time of a random walk above or below its expected value.
firm was a linear function of several different types of resources. Second, the phenomenon of long leads also emerges for stochastic processes other than the random walk. The essential property underlying the phenomenon of long leads is the fact that a continuation of a lead is more likely than a change in lead. In addition to random walks, several other stochastic processes satisfy this requirement. Consider, for example, Muth's stochastic model of the learning curve (Muth 1986). In this model the firm samples a pool of alternatives in each period, and the cost in period \( t \) is the minimum of the sampled alternatives. Formally, the cost in period \( t \) is \( \min(X_1, \ldots, X_i) \), where \( X_i \) are independently identically distributed random variables. Simulations with 100 firms, using a beta distribution for \( X_i \), show that the distribution of the proportion of time the cost of firm \( i \) is among the 50% largest is, again, bimodal with most weight on 0 and 1, and with least weight on 0.5. Alternatively, consider a process of problemistic search in which the probability that resources change is higher if the resource stock is low in comparison with other firms (Greve 1998). Formally, suppose that in each period a normally distributed zero mean increment occurs to the resources of firm \( i \) with probability \( 1/(1 + e^{d_{i,t}}) \), where \( d_{i,t} \) is the difference between the resources of firm \( i \) and the average resources of all firms. Simulations with 100 firms show that the proportion of time that a firm spends among the firms with 50% largest resources is again bimodal with most weight on the extremes.

4. Long Leads and Sustained Interfirm Profitability Differences

To the extent that profitability differences can be explained by resource differences (Barney 1991), the phenomenon of long leads suggests that sustained interfirm profitability differences could be explained by a random resource accumulation process. Specifically, if the level or quality of resources follows a cumulative process with randomly and identically distributed increments, and firms are unable to imitate the resources of other firms, the most likely outcome is not that the average profits of firms will converge over time—rather, the expected outcome is that there will be a group of firms with consistently high profits and a group of firms with consistently low profits.

For an initial illustration of this outcome, consider the situation featured in the introduction. Two firms compete for customers, and a critical resource, such as the speed of the computers produced, follows a random walk. The firm that is able to produce the faster computer in a given year makes a profit of $10 million, while the other firm makes a profit of $5 million. Due to the phenomenon of long leads, it is very likely that one firm has the faster computers, and thus, the higher profit, during most of the observed period. For example, if the firms are observed during 10 years, i.e., \( n = 10 \), it follows from Equation (5) that the probability that either Firm 1 or Firm 2 will have the highest product quality during all 10 years, and thus an average profit of $10 million, is 35%. This also implies that the distribution of average profits will not be concentrated at $7.5 million. Rather, the distribution of average profits will be bimodal, with $5 million and $10 million as the most likely outcomes.

The implication of such a bimodal distribution is that it is very likely that the average profit of the two firms will differ substantially. Using the approximation introduced in the previous section, we have that the expected absolute difference in average profitability is 3.18 and the median is 3.54. Observations of such large differences in average profitability might suggest that it would be unlikely that the difference was generated by chance. And indeed, a two-tailed \( t \)-test of the difference between average profits shows most often that the difference is significant at a significance level of 5%. However, since the data are generated by dependent rather than independent draws, such a test would be inappropriate. A proper test, using the distribution generated by a random walk as the null hypothesis, would instead show that even a difference in average profitability of $5 million based on 10 years is not significant. Indeed, it would require approximately 510 periods before a difference in average profitability of $5 million would lead to rejection of the null hypothesis of a random walk.

Obviously, the mapping between resource stocks and profitability levels is unlikely to have the “tournament-like” quality of this example. However, to the extent that resource stocks are related to competitive advantage (Barney 1991), profitability is likely to be an increasing function of the stocks of resources. If so, the general proposition that a random resource accumulation process is likely to generate sustained interfirm profitability differences follows. Specifically, consider an industry with \( n \) firms in which profitability of firm \( i \) in period \( t \), \( \pi_{i,t} \), is a strictly increasing function of the stocks of the resources of firm \( i \) in period \( t \), \( R_{i,t} \). Formally, \( \pi_{1,t} > \pi_{2,t} \) if and only if \( R_{1,t} > R_{2,t} \). In this case the profitability rankings of firms will be subject to long leads exactly as outlined in the previous section. For example, if \( b = 1 \) and \( n = 100 \), a firm spends, on average, 83% of the time either among the 50% most profitable or among the 50% least profitable firms during the first 10 periods. Thus, there will be persistent differences in profitability rankings.

\footnote{Following the notation in §3, we have that the expected absolute difference in average profitability is \( 10 \cdot E[Z] − 5 = 3.18 \). The median can be calculated using a transformation of the distribution of \( Z \).}
To examine in more detail the connection between this model of random resource accumulation and existing empirical studies of the persistence of profitability (Mueller 1986, Waring 1996), the mapping between resource stocks and profitability levels must be specified. Following previous work on efficiency differentials as explanations of profitability differences we make use of a Cournot model with heterogeneous firms that differ in their marginal costs (Clark et al. 1984, Schmalensee 1987).

Specifically, we examine an industry with a fixed number of \( n \) firms. In each period each firm decides the quantity it will produce, \( q_i \), taking into account the reactions of other firms and the effect on price. The cost of firm \( i \) in period \( t \) is assumed to follow a modified random walk with a negative drift, \( c_{i,t} = c_0 - z_{i,t} \), where \( c_0 \) is the initial cost level, common for all firms, and where the accumulated cost reduction, \( z_{i,t} \), is assumed to follow an autoregressive process: \( z_{i,t} = b z_{i,t-1} + e_{i,t} \), where \( e_{i,t} \) are independently and identically distributed random variables with a uniform distribution on \((-k, 0)\), and \( b \in [0, 1] \) is a parameter regulating the retention of previous cost reductions.\(^8\) The inverse demand function, assumed to be common knowledge, is specified as

\[
p_t = \alpha - \beta \left( \sum_{i=1}^{n} q_i \right),
\]

where \( p_t \) is the market price, and \( \alpha \) and \( \beta \) are positive constants. Profits for firm \( i \) at time \( t \) is \( \pi_{i,t} = (p_t - c_{i,t}) q_i \). Finally, to apply the Nash equilibrium solution concept, it is assumed that the costs of all firms in period \( t \) are common knowledge at the beginning of period \( t \).

Given this specification, the first-order condition for profit maximization for firm \( i \) can be expressed as

\[
\alpha - \beta \left( \sum_{i=1}^{n} q_i \right) - \beta q_i - c_i = 0
\]

(period indices are suppressed). Summing the first-order conditions, we have that total supply is

\[
\sum_{i=1}^{n} q_i = \frac{n \alpha - \sum_{i=1}^{n} c_i}{\beta n + \beta}
\]

and that the quantity supplied by firm \( i \), given positive profits, is

\[
q_i = \frac{\alpha - \sum_{i=1}^{n} q_i - c_i}{\beta}.
\]

On the basis of these numbers it is possible to calculate the profit for firm \( i \) as

\[
\pi_i = \left( \alpha - \beta \sum_{i=1}^{n} q_i - c_i \right) q_i^g.
\]

Although in this model all firms have identical initial costs and identical expected cost declines, if \( b \) is close to 1 it typically generates substantial and persistent differences in average profitability. Consider, first, the distribution of average profitability. To make comparisons with empirical data possible I calculate normalized profitability ratios, \( \bar{\pi}_{i,t} = (\pi_{i,t} - \bar{\pi}_t) / \bar{\pi}_t \), where \( \bar{\pi}_{i,t} \) is the profitability of firm \( i \) in period \( t \), and \( \bar{\pi}_t \) is the average profitability of all firms in period \( t \) (cf. Mueller 1986). Figure 3 plots the distribution of the average profitability ratio during periods 1 to 10 for a model with 20 firms and for different values of \( b \) (in this simulation \( \alpha = 450, \beta = 1, k = 3, \) and \( c_0 = 100 \)). As illustrated, if \( b = 1 \), there is a relatively wide distribution of profitability ratios. Thus, in this case it is quite likely that a firm obtains an average profitability, based on 10 periods, which is substantially below or above the industry average.\(^{10}\) Observations of an industry consistent with this model would thus show substantial differences in average profitability. On the one hand, it can be noted that the distribution of the average profit ratio, as well as the distribution of average profitability, is positively skewed (the skewness coefficient is 0.31), consistent with empirical data for most industries (Foster 1986, p. 109). On the other hand, if \( b \) is 0.7, the distribution of the average profit ratio is more concentrated around 0. The distribution is also less skewed, with a skewness coefficient of 0.10.

Consider, next, the persistence of such differences in average profitability. For a simple illustration of the degree of persistence we divided the sample of 20 firms into two groups of firms based on their profitability in period 10. Keeping the firms in the group in which they started, the group average profitability ratio is tracked through period 20 (Ghemawat 1991, Waring 1996). Figure 4 shows the result for various levels of \( b \). As illustrated, if \( b \) is equal to 1 there is a very high degree of persistence, as observed in some industries such as the U.S. automobile industry during the 1970s (Waring 1996). For lower values of \( b \) there is more rapid convergence to the mean. These

\(^8\) The distribution of the cost reduction was chosen to avoid the possibility of negative costs. Simulations show that the precise distribution has a negligible effect, however.

\(^{10}\) Obviously, the dispersion in profitability ratios could be changed by incorporating fixed costs, initial capital outlays, and depreciation in the model. Thus, no clear inferences can be drawn from the absolute levels of the dispersion in profit ratios.
results can be compared to the empirical observations in Waring (1996, Figure 1), who finds that the average return on asset (ROA) differential (defined as ROA minus the industry’s mean ROA) for the firms among the 50% most profitable in 1970 had been reduced by 50% in 1976. Simulations using an equivalent definition of return differentials, where assets were assumed to be constant and identical for all firms, show that a value of $b$ around 0.9 would replicate these results most closely. It should be noted, however, that even a process with $b = 1$ can produce substantial regression to the mean if environmental changes can turn prior cost reductions into a liability. To examine the persistence of profitability in more detail I estimate autoregressive time-series model using the data generated by the simulations. Following Mueller (1986), I estimate the autoregressive equation $\hat{\pi}_{t+1} = \theta_{t} + \lambda_{t} \hat{\pi}_{t} + \epsilon_{t+1}$ for each firm in the industry using ordinary least-square regression. Combining the estimate of permanent rents, $\hat{\theta}_{t}$, with the estimate of short-run persistence, $\hat{\lambda}_{t}$, an estimate of the long-run projected profitability ratio can be derived as $\pi^{*} = \hat{\theta}_{t}/(1 - \hat{\lambda}_{t})$, provided that $\hat{\lambda}_{t} \in (-1, 1)$. Table 1 summarizes the average of these estimates for various values of $b$ and various intervals. It also shows the average estimates for firms with the 50% highest and 50% lowest profitability in the initial period (i.e., period 10), as well as the average initial profit ratio for these groups ($\pi_{0}$).

As illustrated in Table 1, the projected long-term profit ratios of firms with different initial profitability are likely to differ substantially if $b$ is equal to 1. Thus, if cost reductions were consistent with such a model, studies using the above autoregressive equation are likely to find evidence of substantial persistence of profitability, observed in some industries (Waring 1996). If $b$ is below 1, however, the projected profit ratios are unlikely to differ by much, indicating substantial regression to the mean, observed in many industries (Mueller 1986, Waring 1996). As noted above, even a process with $b = 1$ can produce substantial regression to the mean if environmental changes can turn prior cost reductions into a liability. It can also be noted that the values of the estimates of $\theta_{t}$ and $\lambda_{t}$ in Table 1 are not far from empirical estimates. For example, an estimate of $\lambda_{t}$ around 0.5 is common in the empirical literature (Mueller 1986,

\[ \hat{\theta}_{t} = \theta_{t} + \epsilon_{t} \]

\[ \hat{\lambda}_{t} = \lambda_{t} + \epsilon_{t} \]

\[ \pi^{*} = \hat{\theta}_{t}/(1 - \hat{\lambda}_{t}) \]

\[ \pi_{0} = \theta_{0} \]

\[ \epsilon_{t} \sim N(0, \sigma^{2}) \]

\[ \theta \sim N(\mu, \tau^{2}) \]

\[ \lambda \sim N(\mu, \tau^{2}) \]

\[ \pi = \theta/(1 - \lambda) \]

\[ \pi_{0} = \theta \]

\[ \epsilon = \sigma \]

\[ \pi^{*} = \theta/(1 - \lambda) \]

\[ \pi_{0} = \theta \]

\[ \epsilon = \sigma \]

\[ \pi^{*} = \theta/(1 - \lambda) \]

\[ \pi_{0} = \theta \]

\[ \epsilon = \sigma \]

\[ \pi^{*} = \theta/(1 - \lambda) \]

\[ \pi_{0} = \theta \]

\[ \epsilon = \sigma \]

\[ \pi^{*} = \theta/(1 - \lambda) \]

\[ \pi_{0} = \theta \]

\[ \epsilon = \sigma \]

\[ \pi^{*} = \theta/(1 - \lambda) \]

\[ \pi_{0} = \theta \]

\[ \epsilon = \sigma \]

\[ \pi^{*} = \theta/(1 - \lambda) \]

\[ \pi_{0} = \theta \]

\[ \epsilon = \sigma \]

\[ \pi^{*} = \theta/(1 - \lambda) \]

\[ \pi_{0} = \theta \]

\[ \epsilon = \sigma \]

\[ \pi^{*} = \theta/(1 - \lambda) \]

\[ \pi_{0} = \theta \]

\[ \epsilon = \sigma \]

\[ \pi^{*} = \theta/(1 - \lambda) \]

\[ \pi_{0} = \theta \]

\[ \epsilon = \sigma \]

\[ \pi^{*} = \theta/(1 - \lambda) \]

\[ \pi_{0} = \theta \]

\[ \epsilon = \sigma \]

\[ \pi^{*} = \theta/(1 - \lambda) \]

\[ \pi_{0} = \theta \]

\[ \epsilon = \sigma \]

\[ \pi^{*} = \theta/(1 - \lambda) \]

\[ \pi_{0} = \theta \]

\[ \epsilon = \sigma \]

\[ \pi^{*} = \theta/(1 - \lambda) \]

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\[ \pi^{*} = \theta/(1 - \lambda) \]

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\[ \epsilon = \sigma \]

\[ \pi^{*} = \theta/(1 - \lambda) \]

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\[ \epsilon = \sigma \]

\[ \pi^{*} = \theta/(1 - \lambda) \]

\[ \pi_{0} = \theta \]

\[ \epsilon = \sigma \]

\[ \pi^{*} = \theta/(1 - \lambda) \]

\[ \pi_{0} = \theta \]

\[ \epsilon = \sigma \]

\[ \pi^{*} = \theta/(1 - \lambda) \]

\[ \pi_{0} = \theta \]

\[ \epsilon = \sigma \]

\[ \pi^{*} = \theta/(1 - \lambda) \]

\[ \pi_{0} = \theta \]

\[ \epsilon = \sigma \]

\[ \pi^{*} = \theta/(1 - \lambda) \]

\[ \pi_{0} = \theta \]

\[ \epsilon = \sigma \]

\[ \pi^{*} = \theta/(1 - \lambda) \]

\[ \pi_{0} = \theta \]

\[ \epsilon = \sigma \]

\[ \pi^{*} = \theta/(1 - \lambda) \]

\[ \pi_{0} = \theta \]

\[ \epsilon = \sigma \]

\[ \pi^{*} = \theta/(1 - \lambda) \]

\[ \pi_{0} = \theta \]

\[ \epsilon = \sigma \]

\[ \pi^{*} = \theta/(1 - \lambda) \]

\[ \pi_{0} = \theta \]

\[ \epsilon = \sigma \]

\[ \pi^{*} = \theta/(1 - \lambda) \]

\[ \pi_{0} = \theta \]

\[ \epsilon = \sigma \]

\[ \pi^{*} = \theta/(1 - \lambda) \]

\[ \pi_{0} = \theta \]

\[ \epsilon = \sigma \]

\[ \pi^{*} = \theta/(1 - \lambda) \]

\[ \pi_{0} = \theta \]

\[ \epsilon = \sigma \]

\[ \pi^{*} = \theta/(1 - \lambda) \]

\[ \pi_{0} = \theta \]

\[ \epsilon = \sigma \]

\[ \pi^{*} = \theta/(1 - \lambda) \]

\[ \pi_{0} = \theta \]

\[ \epsilon = \sigma \]

\[ \pi^{*} = \theta/(1 - \lambda) \]

\[ \pi_{0} = \theta \]

\[ \epsilon = \sigma \]

\[ \pi^{*} = \theta/(1 - \lambda) \]

\[ \pi_{0} = \theta \]

\[ \epsilon = \sigma \]

\[ \pi^{*} = \theta/(1 - \lambda) \]

\[ \pi_{0} = \theta \]

\[ \epsilon = \sigma \]

\[ \pi^{*} = \theta/(1 - \lambda) \]

\[ \pi_{0} = \theta \]

\[ \epsilon = \sigma \]

\[ \pi^{*} = \theta/(1 - \lambda) \]

\[ \pi_{0} = \theta \]

\[ \epsilon = \sigma \]

\[ \pi^{*} = \theta/(1 - \lambda) \]

\[ \pi_{0} = \theta \]

\[ \epsilon = \sigma \]

\[ \pi^{*} = \theta/(1 - \lambda) \]

\[ \pi_{0} = \theta \]

\[ \epsilon = \sigma \]
Table 1  Estimates of the Autoregressive Equation $\tilde{\pi}_{t,1} = \theta + \lambda \tilde{\pi}_{t,1} + \epsilon_{t,1}$, as a Function of $b$ and the Number of Periods Observed

<table>
<thead>
<tr>
<th>Periods 10–20</th>
<th>$b = 1$</th>
<th>$b = 0.9$</th>
<th>$b = 0.8$</th>
<th>$b = 0.7$</th>
</tr>
</thead>
<tbody>
<tr>
<td>All observations</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\theta$</td>
<td>$-0.0008$</td>
<td>$-0.0001$</td>
<td>$-0.0003$</td>
<td>$-0.0002$</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>$0.5535$</td>
<td>$0.5139$</td>
<td>$0.4573$</td>
<td>$0.3949$</td>
</tr>
<tr>
<td>$\pi^*$</td>
<td>$0.0084$</td>
<td>$0.0126$</td>
<td>$0.0084$</td>
<td>$0.0049$</td>
</tr>
<tr>
<td>50% best in period 10</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\theta$</td>
<td>$0.0975$</td>
<td>$0.0418$</td>
<td>$0.0196$</td>
<td>$0.0111$</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>$0.5639$</td>
<td>$0.5115$</td>
<td>$0.4555$</td>
<td>$0.3939$</td>
</tr>
<tr>
<td>$\pi_0$</td>
<td>$0.2371$</td>
<td>$0.1658$</td>
<td>$0.1299$</td>
<td>$0.1118$</td>
</tr>
<tr>
<td>$\pi^*$</td>
<td>$0.1882$</td>
<td>$0.0328$</td>
<td>$0.0021$</td>
<td>$-0.0013$</td>
</tr>
<tr>
<td>50% worst in period 10</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\theta$</td>
<td>$-0.0791$</td>
<td>$-0.0420$</td>
<td>$-0.0202$</td>
<td>$-0.0116$</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>$0.5630$</td>
<td>$0.5161$</td>
<td>$0.4591$</td>
<td>$0.3960$</td>
</tr>
<tr>
<td>$\pi_0$</td>
<td>$-0.2371$</td>
<td>$-0.1658$</td>
<td>$-0.1299$</td>
<td>$-0.1118$</td>
</tr>
<tr>
<td>$\pi^*$</td>
<td>$-0.1714$</td>
<td>$-0.0777$</td>
<td>$0.0147$</td>
<td>$0.0111$</td>
</tr>
<tr>
<td>Periods 10–30</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All observations</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\theta$</td>
<td>$-0.0012$</td>
<td>$-0.0004$</td>
<td>$-0.0003$</td>
<td>$-0.0003$</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>$0.7487$</td>
<td>$0.6918$</td>
<td>$0.6207$</td>
<td>$0.5404$</td>
</tr>
<tr>
<td>$\pi^*$</td>
<td>$0.00453$</td>
<td>$0.0068$</td>
<td>$0.0030$</td>
<td>$0.0010$</td>
</tr>
<tr>
<td>50% best in period 10</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\theta$</td>
<td>$0.0536$</td>
<td>$0.0152$</td>
<td>$0.0057$</td>
<td>$0.0030$</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>$0.7494$</td>
<td>$0.6906$</td>
<td>$0.6192$</td>
<td>$0.5394$</td>
</tr>
<tr>
<td>$\pi_0$</td>
<td>$0.2374$</td>
<td>$0.1661$</td>
<td>$0.1303$</td>
<td>$0.1117$</td>
</tr>
<tr>
<td>$\pi^*$</td>
<td>$0.1920$</td>
<td>$0.0224$</td>
<td>$0.0054$</td>
<td>$0.0026$</td>
</tr>
<tr>
<td>50% worst in period 10</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\theta$</td>
<td>$-0.0560$</td>
<td>$-0.0159$</td>
<td>$-0.0064$</td>
<td>$-0.0035$</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>$0.7482$</td>
<td>$0.6929$</td>
<td>$0.6211$</td>
<td>$0.5414$</td>
</tr>
<tr>
<td>$\pi_0$</td>
<td>$-0.2374$</td>
<td>$-0.1661$</td>
<td>$-0.1303$</td>
<td>$-0.1117$</td>
</tr>
<tr>
<td>$\pi^*$</td>
<td>$-0.1830$</td>
<td>$-0.0088$</td>
<td>$0.0005$</td>
<td>$-0.0006$</td>
</tr>
</tbody>
</table>

Note: Each entry shows average values based on 20,000 simulations, where $\alpha = 450$, $\beta = 1$, $k = 3$, $n = 20$, $c_0 = 100$.  

As illustrated in Table 1, however, the estimates are sensitive to the number of periods examined. As the number of periods examined increases, the absolute value of the estimate of $\lambda$ increases and that of $\theta$ decreases. However, estimates more in line with empirical observations can be obtained if noise is included in the model. For example, if a noise term with a standard deviation of 35 is added to yearly profitability (which has an average of about 340 during periods 10–30), the estimate of $\lambda$ is approximately 0.5, even if the number of periods examined is 20 and $b = 1$. Finally, although these simulation results have been derived using specific parameter values, simulations using alternative parameter values for $\alpha$, $\beta$, $c_0$, $k$, and $n$, as well as alternative specifications of the distribution of the reduction, show that the results are robust. In particular, if $b = 1$, simulations generate substantial persistence of profitability. The magnitude of such differences in profitability, however, obviously depends on the values of parameters such as $k$ that regulate the variance of costs, and $\alpha$ that regulate the sensitivity of profitability differences to differences in costs.15  

Overall, the above simulations suggest that a model of random resource accumulation is capable of replicating several regularities in a time-series of profitability. By varying a single parameter, the model is capable of accommodating the wide range of estimates of the persistence of profitability observed empirically. In particular, if $b$ is equal to or close to 1, the model is capable of generating the substantial persistence of profitability observed in some industries and during some periods (Waring 1996, Wiggins and Ruefli 2002). Obviously, such persistence could also be produced by alternative processes. For example, a model with heterogeneity in initial costs, a model with heterogeneity in expected cost reductions, or a path-dependent model of cost reductions in which the expected cost reduction was an increasing function of the size of past cost reductions would also produce substantial persistence of profitability.16 The main point to be stressed here is that substantial persistence of profitability can also emerge in a model that neither assumes heterogeneity in initial stocks or expected flows nor assumes path dependency in flows.

5. Discussion  
The idea that profitability, and even sustained above-average profitability, may be due to chance is not new. It is well known that whenever isolating mechanisms are strong, a single lucky break can generate persistent above-average returns (Mancke 1974, Rumelt 1984, Barney 1986, Stinchcombe 2000). Moreover, it is well known that path-dependent processes can make market outcomes sensitive to chance events (Arthur 1989). Thus, in markets with increasing returns, small chance events may lead to the dominance of a single actor (Arthur 1989). Similarly, in stochastic growth models where expected absolute sales growth is proportional to size, random growth processes typically result in highly skewed size distributions (Gibrat 1931, Simon 1955, Steindl 1965). Finally, it has often been noted that even if firm profitability in each year were independently and identically distributed, in a large population of firms we are very likely to find some firm with sustained above-average profitability (e.g., Alchian 1950, Barney 1997).

15 Thus, if $k$ is small, the variance in costs and the variance in profitability will be small. Similarly, if $\alpha$ is large, profitability will be less sensitive to differences in costs. As a result, the variance in profitability will be lower.

16 It should be noted, however, that in the latter two models the difference between firms with low and high profitability tends to increase over time. As illustrated in Figure 4, this does not hold for the random walk model.
This paper extends the scope of such chance explanations by proposing a simple yet powerful alternative way in which chance events can result in sustained interfirm profitability differences. In contrast to models where heterogeneity in resource stocks is the result of a single lucky draw protected by strong isolating mechanisms, which may be a strong assumption, this process assumes that resource stocks is the cumulative result of many independent random draws. And in contrast to models of path-dependent processes, this process need not assume that the expected flow of resources is an increasing function of the existing stock of resources. Nevertheless, the expected outcome of such a process is sustained interfirm profitability differences.

Obviously, the fact that a random walk process can produce sustained interfirm profitability differences does not negate the possibility that such differences are the result of heterogeneity in initial stocks or in expected flows. However, it does suggest that observations of persistent resource heterogeneity and sustained interfirm profitability differences do not provide much evidence for heterogeneity in either initial stocks or in expected flows. Even if a firm had consistently high performance during a number of years, this does not provide much evidence that the firm differed in any significant way from the other firms in the industry at the start of the period, and neither does it provide much evidence that this firm was more capable in developing and augmenting its resources or had a more effective strategy. The above model suggests that luck, rather than systematic prior differences among firms, could have generated its consistent performance record. Formulated differently, the above model suggests an alternative null hypothesis about sustained interfirm profitability differences.

From a research perspective, the possibility of such a null hypothesis suggests that, empirically, it may be difficult to distinguish between random processes and systematic prior differences as explanations of patterns in profit series. In particular, it would be difficult to distinguish alternative resource accumulation processes based on estimates from the above autoregressive equation. However, if one is willing to assume that the mapping between resource levels and profitability levels is approximately linear, it might be possible to distinguish between alternative processes by examining the correlation between profitability growth in various periods. Specifically, profitability growth in different periods would be uncorrelated in the random walk model and in a model with heterogeneity in initial stocks, but would be positively correlated in a model with heterogeneity in expected flows or increasing returns. In addition, to distinguish between a random walk model and a model with heterogeneity in initial stocks, one could examine the development of the average profit ratio of the 50% most-profitable firms each year. In a random walk model, with \( b = 1 \), this would increase over time, while in a model with heterogeneity in initial stocks it would remain at the same level. Although the mapping between cost levels and profitability is not strictly linear in the above Cournot model, simulations show that these tests can identify the underlying random walk process. Specifically, if \( b = 1 \), the correlation between profit growth in different periods is approximately 0. In addition, profit ratios for the most profitable firms increase over time. Unfortunately, the last test fails if \( b < 1 \). In this case, profitability ratios remain at the same level. The correlation between profit growth in different periods, however, becomes negative. Nevertheless, this could also happen in a model with heterogeneity in initial stocks if residuals followed an autoregressive process. As an alternative to these tests, specific hypotheses regarding the impact of firm attributes on profit levels or profit growth could be tested, which is the focus of much strategy research. To the extent that specific attributes of firms correlate with profit levels or profit growth, a pure random resource growth process can be rejected, unless, of course, such attributes are themselves the result of the level of resource stocks.

These identification problems obviously also face laymen who try to evaluate the effectiveness of firm strategies on the basis of time-series data of profitability. However, given the difficulties individuals have in the perception of randomness (Wagenaar 1970, Kahneman and Tversky 1972), it seems unlikely that laymen would attribute the profitability patterns generated by a random walk process to chance. As demonstrated by research on the perception of randomness, a belief that random variables are self-correcting seems to characterize most peoples’ intuitive perceptions of randomness. Thus, individuals evaluate series of coin tosses with many alterations between heads and tails as being most likely to be random (Tune 1964, Wagenaar 1970). Kahneman and Tversky (1972) suggest that this tendency results from the use of a representativeness heuristic. Making use of this heuristic, individuals evaluate the probability of an uncertain event by both the degree to which it is similar to the properties of its parent population and the degree to which it reflects salient features of the process by which it is generated. It follows that individuals evaluate a time series as more likely to be random if it is irregular and lacks a systematic tendency. According to this heuristic, however, long leads will be highly unrepresentative of most peoples’ perceptions of a random process and are likely to be rejected as nonrandom (Kahneman and Tversky
1972) as demonstrated by answers, in informal surveys, to the question introducing this paper. Although this heuristic makes some sense if events are independent, it leads individuals astray when events are dependent. When events are dependent, as in random walks, frequent alterations suggest that the underlying process is unlikely to be random, while few alterations indicate that the underlying process may be random.

Such heuristics for evaluating randomness suggest that laymen are likely to underestimate the role of chance in business performance. Persistent differences in profitability will be considered as strong evidence for the existence of systematic prior differences among firms. Given the possibility of long leads, however, such attributions may be misleading. Nevertheless, they are likely to have important consequences for evaluations of and decisions about strategies (Bukszár 1999). Even if there are no systematic differences in the effectiveness of strategies and organizations, there will nevertheless appear to be some. As a result, financial analysts, reporters, consultants, and managers may become convinced of the superiority of industry leaders and the inferiority of laggards. Under such circumstances industry leaders will probably grow more confident in the application of their strategy and their organization (Miller 1993), while laggards will be likely to consider a change in strategy and organization.

Underestimation of the role of chance also makes it difficult for individuals to draw valid lessons from history. Learning requires that the accidents of history be ignored. If individuals are unaware of the phenomenon of long leads, however, they may try to learn from what is essentially a chance phenomenon. Observers may be inclined to examine firms with consistently high performance in order to discover the source of its “extraordinary” performance record. However, in situations where long leads are expected, it would be quite likely that the firm’s performance record be a result of chance fluctuations. Nevertheless, there is always something special in the history of a firm, its strategy, and its organization that could be used to explain its performance record. Thus, even if success is the result of luck, it is possible to construct an “explanation” (Fischhoff 1975, 1982). The explanation may be entirely spurious, however.

6. Conclusion

Do the most profitable firms differ systematically ex ante from less profitable firms, or is profitability mainly the result of historical accidents and chance? Addressing this question is a central challenge in strategic management research (Cockburn et al. 2000, Henderson 2000). While it is difficult to determine the overall importance of chance, the models presented in this paper suggest that distinguishing the effect of chance and systematic ex ante differences might be more difficult than is generally believed. Specifically, if the resource accumulation process is sufficiently cumulative, sustained differences among firms are the expected result, even if resource flows are identically and independently distributed. Using a Cournot model in which costs follow a random walk, this paper demonstrated how such a process can generate evidence of substantial persistence of profitability.

Although illuminating, this paper has several limitations that need to be addressed in future work. First, most results were derived using a simple random walk model of resource accumulation. As illustrated in §3, however, the phenomenon of long leads also applies to other types of stochastic processes. It would be very useful to understand the exact conditions under which long leads emerge. Second, the present paper only examined a specific mapping between resource and profitability levels. Future research is needed to examine the class of mappings that preserve the phenomenon of long leads. Third, the models in this paper need to be extended to include entry and exit. Such turnover is likely to influence the observed pattern of persistence as well as the difficulty of distinguishing systematic and random processes. Fourth, future research is needed to derive more powerful tests that could distinguish between alternative processes. Although it might prove difficult to find robust tests, careful analyses of specific parametric models might be very valuable to suggest how different processes could be distinguished. Finally, the practical implications for evaluations of strategies need to be examined. If sustained above-average performance results from a chance process, how should the effectiveness of strategies be evaluated? Furthermore, what are sensible learning and imitation strategies?

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