Optimal Lot-sizing and Pricing with Markdown for a Newsvendor Problem

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Abstract. This paper deals with the joint decisions on pricing and ordering for a monopolistic retailer who sells perishable goods with a fixed lifetime or demand period. The newsvendor-typed problem is formulated as a two-period inventory system where the first period represents the inventory of fresh or new-arrival items and the second period represents the inventory of items that are older but still usable. Demand may be for either fresh items or for somewhat older items that exhibit physical decay or deterioration. The retailer is allowed to adjust the selling price of the deteriorated items in the second period, which stimulates demand and reduces excess season-end or stale inventory. This paper develops a stochastic dynamic programming model that solves the problem of pre-season decisions on ordering-pricing and a within-season decision on markdown pricing. We also develop a fixed-price model as a benchmark against the dual-price dynamic model. To illustrate the effect of the dual-price policy on expected profit, we conduct a comparative study between the two models. Extension to a generalized multi-period model is also discussed.

Keywords: Inventory, Perishable, Pricing-Ordering, Dynamic Pricing

1. INTRODUCTION

This paper considers a single product with a fixed length of lifetime in a two-period inventory system. The first period represents the inventory of fresh or new arrival items and the second period represents the inventory of items that are older but still usable. New items not sold within a certain period will be markdown in the second period. The older and deteriorated items not sold within a certain period will be salvaged. The new items are replenished from an outside vendor with infinite capacity, stochastic demands occur independently in both periods, and the unfilled demands are lost.

The two-period structure is similar to the price protection model proposed by Lee et al. (2000) and the blood management model given in Goh et al. (1993). Lee et al., focused on channel coordination issues in a bilateral monopoly, dealing with the optimal trade terms such as seller’s price and rebate and buyer’s order quantity. Goh et al. dealt with the issuing policies of new and old items for blood management. In this paper, our interest is in the pricing-ordering decisions for a monopolistic retailer who determines the replenishment quantity, sets the initial price for the new items, and makes a follow-up markdown for the old items so that the expected profit is maximized over the fixed shelf life or demand period.

The fixed lifetime perishability problem falls into the general framework of the classic newsboy model. Goyal and Giri (2001), Nahmias (1982), and Raafat (1991) provided extensive review of the problem. However, most of
the references therein exploit the problem by deriving proper ordering policy under the implicit assumption of first-in-first-out issuing policy. In many real systems such as food and fashion retailing, customers determine the issuing policy. If the valuation of fresh items is higher, stock consumptions in the display shelf will be last-in-first-out. In this paper, the valuations of perishable items with different ages are characterized by different distributions of reservation prices (see Lazear 1986 for details).

Reservation price is defined as the maximum price that customers are willing to pay for the product, i.e., customers buy the product only if their reservation price is higher than or equal to the product’s price. Hence, it tends to have a continuous distribution. In the fashion and food industries, the reservation price distribution shifts to the left over time, i.e., customers in general are willing to pay higher price for the new items and lower price for the aged items. Lazear formulated the obsolescence-prone pricing behavior by a simple two-period dynamic model. Pashigian (1988) and Pashigian and Bowen (1991) investigated the model empirically. The specific version of newsboy problem assumes that the reservation price is random, and management knows its probability distribution. The retailer therefore can adjust price in response to the shift of reservation price distribution during its lifetime.

Due to its effectiveness for short-term control over on-hand stock, price change is increasingly prevalent in practice. The average weekly frequency of price changes at the largest U.S. supermarket chains ranges from 3223 to 4316, yielding an average of 15.66 percent of their products each week (Levy et al., 1997). The supermarket chains tend to achieve more price flexibility by changing prices more often, i.e., multiple times each week. Pashigian (1988) and Pashigian and Bowen (1991) showed that fashion goods increase in the percentage markup and the frequency of sales since 1970. In the less fashion products like automobiles, there still exist within-season price declines (Pashigian et al., 1995). In the airline industry, fare changes are routine and commonly used as a demand-responsive mechanism (see McGill and van Ryzin 1999, Weatherford and Bodily 1992, and Belobaba 1987). For instance, American Airlines made up to 50,000 daily fare changes (Smith et al., 1992). In the context of electronic commerce, on-line retailers adjust their prices more readily than conventional retailers in response to structure changes in supply or demand (Brynjolfsson and Smith 2000).

The reasons of price changes are numerous such as to reflect cost changes or demand fluctuations, to respond to competitor’s price changes, or to comply temporary promotions. This paper focuses on permanent markdown or clearance sales over product lifetime. The price adjustment in the second period is an attempt to compensate for a shifting demand function due to the physical decay or deterioration of the aged items. The advantage of using the proposed strategy, or so-called dual-price or two-fare pricing policy in airlines practice, is that by adjusting the price over product life time it is possible to synthesize a wide range of deteriorating losses and to induce customers to change their buying behaviors.

This paper is organized as follows. Section 2 provides a survey of related research. The underlying problem is formulated as a two-period dynamic programming model in section 3. For comparison purpose, a fixed-price model, i.e., without markdown in the second period, is also introduced in the section. Section 4 illustrates the mathematical behavior of the models and conducts a comparative study between dual-price and fixed-price policies. Extension to a generalized multi-period model and concluding remarks are provided in section 5.

2. LITERATURE REVIEW

Many research works on pricing-ordering/production have been developed in the past decades. Eliaishberg and Steinberg (1993) and Yano and Gilbert (2004) provided a comprehensive survey of early works on this area. For the deterministic demand, Rajan et al. (1992) developed a continuous time pricing and ordering policy for perishable goods. Abad (1996) extended their work by allowing backlogged. Under demand uncertainty circumstance, Smith and Achabal (1998) developed optimal pricing and inventory policies that take into account of price, reduced assortment, and seasonal effects on sales rates. Gilbert (2000) and Deng and Yano (2006) considered the problem of joint decisions on pricing and production schedules for multi-period settings. Bernstein and DeCroix (2004) considered joint decisions on pricing and capacity, instead of pricing and production/inventory decisions, in a multi-tier assembly system. Federgruen and Heching (1999) exploited the pricing-ordering problem for the cases of multi-period as well as infinite sales horizon. A replenishment order may be placed at the beginning of some or all of the periods. In this paper, we consider the pricing-ordering decisions for the newsboy-type problem, i.e., a fixed selling horizon with only one replenishment order at the beginning of the period.

Gallego and van Ryzin (1994) formulated the underlying problem using intensity control mechanism and obtained structural monotonicity results for the optimal dynamic price that is a function of the stock level and the length of the selling horizon. Their work motivates many researchers to develop more general models from a variety of perspectives. Gallego and van Ryzin (1997) generalized the model that allows time-varying demand and multi-product with a network structure. Bitran and Mondshein (1997) developed both continuous time and periodic pricing models for compound Poisson process demand that is a function of the price through the distribution of reservation prices. Chun (2003) studied the variant of the periodic model proposed by Bitran and Mondshein that considers a negative binomial distributed demand function. Zhao and Zheng (2000) exploited the problem by allowing non-homogeneous demand where