Performance of MAC in Wireless Recharging under E-limited Scheduling

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Abstract—Recharging through radio frequency (RF) is a promising approach to enhance battery lifetime of wireless sensor networks (WSNs). In this paper, we design a polling based MAC protocol with round robin scheduling under E-limited service policy. We derive service completion time for E-limited system. In this model, the coordinator sends recharging pulse upon the reception of recharging request(s) from one or many nodes. Normal operation of data communication is postponed during charging process. A probabilistic model for energy depletion within the proposed MAC along with queueing delay model is evaluated as well. Later, we assess the behaviour of time interval between two consecutive recharging events and packet waiting time under varying network size and traffic load.

I. INTRODUCTION

Sensor networks consist of thousands of inexpensive sensors with data sensing, processing and communication components. Sensor nodes usually operate in unattended mode without regular replacement or manual recharging of batteries, communicate with one another over short distances. Due to the limited lifetime of batteries in sensor nodes, periodic recharging of batteries can be accomplished by energy harvesting which requires no or minimum human intervention [15]. Most of the energy harvesting techniques, i.e. windmills, solar system, are best effort basis and can not guarantee continuation of adequate power supply when required most.

On the other hand, RF based recharging through coordinator (base station) is reliable as long as the coordinator’s power source is reliable [17], [18]. Environmental change, in case of RF recharging, has insignificant impact on a node’s battery power reception process. Amount of replenished energy resource in this case is more predictable and depends only on attenuation of wireless signal between coordinator and the node.

In order to achieve both expected level of communications performance and operation without maintenance, it requires a reliably designed and meticulously evaluated MAC protocol. In this paper, we propose a polling based MAC which considers round robin scheduling with E-limited service policy. The protocol allows a node to send recharging request explicitly when its energy drops below a certain threshold value. We use the same RF frequency for alternative operation of data communication and recharging process since the node is having single antenna system which makes a node’s hardware simpler and cheaper [10]. However, sensing unit of the node keeps collecting incoming data even during recharging process. Performance of the network using this protocol is then assessed through probabilistic analysis and a dedicated uplink queueing model focusing on the battery depleting process and monitoring the impact of recharging interval on data communications.

The paper is organised as follows, Section II equips an overview of related work. Section III describes basic operation of MAC for E-limited service system. Section IV models the energy depletion process and derives the probability distribution of the time interval between consecutive recharging events of a node. Section V models vacation period of a node and queueing delay experienced by data packets followed by performance results of the proposed MAC protocol including effective offered load, recharging probability and queueing delay in Section VI. Finally, Section VII concludes the paper and highlights some future research direction.

II. RELATED WORK

A generic model for energy replenishable sensor nodes which include battery replacement or generic recharging was presented in [7]. The work mainly focuses on battery replacement or generic recharging that happens at a certain replacement rate, rather than on energy harvesting from the environment or through RF recharging. Generally, energy harvesting and data communication occur independently but some approaches consider interplay between data communication and energy transfer process as well [12]. Although energy harvesting is an infinite power source theoretically, but it is unreliable [4] due to its dependant on ambient conditions. And hence, energy allocation with the help of redundant energy sources must be planned and optimized with proper attention in order to guarantee uninterrupted network operation. A comparative analysis for different energy harvesting techniques, including practical measurements has been outlined in [3].

Several CSMA based MAC protocols have been proposed to prolong battery lifetime for WSNs [19] [5]. These MACs mainly focus on energy conservation by enabling sleeping mode in a node while other nodes are transmitting data. These types of MACs can not guarantee per node fairness and requires higher latency time. On the other hand, polling based MAC protocol improves real time performance and provides fairness among nodes by implementing different types of scheduling [16]. Several analytical models and simulations...
have been performed both for CSMA and polling based MACs [1][8]. The result shows that polling-based MAC outperforms CSMA based counterparts in case of network performance.

RF recharging may extend the battery lifetime of sensor nodes to "immortal" [11], but normal data operation can be affected to a large extent by the actions related to energy transfer as they extend the vacation period of a node [17]. Vacation period has significant impact in queuing delay as discussed in [20]. In order to reduce mean queue length and data loss MAC based sleep-awake policy is highlighted in [6]. More work is required to figure out the optimal scheduling of energy transfers and the adjustment of different parameters accountable for these interactions.

Earlier, we have proposed a MAC protocol in which recharging is performed through RF frequency [9]. The MAC polls the nodes in round robin fashion and 1-limited service policy was allocated for each node. Network performance for 1-limited service goes down if the traffic arrival rate is not similar among the nodes. In WSN data arrival rate may not be symmetric. In many WSN applications, a node gets frequent smaller packets in a short period and then node experiences no packet arrival for relatively longer period of time. In such scenario, E-limited service policy shows better network performance, i.e. offered load, vacation time and queuing delay, compared to 1-limited service policy [13]. We have extended our work from 1-limited to E-limited service policy. In addition, we have implemented queuing model for network.

III. MAC PROTOCOL

The network consists of \( N \) slave nodes and a coordinator, as shown in Fig. 1. Each of the nodes has sensing unit to produce data that is delivered to the coordinator. We presume that MAC follows round robin scheduling where the coordinator sequentially polls each slave node by sending POLL messages. Polling cycle is the time requirement of the coordinator to visit all slave nodes once as mentioned in the round robin scheduling. The POLL message which is referred as downlink transmission, may or may not contain DATA packets alike Bluetooth. In reply to the POLL message, the destined slave transmission, may or may not contain DATA packets alike scheduling. The POLL message which is referred as downlink transmission, may or may not contain DATA packets alike Bluetooth. The coordinator polls the nodes in round robin fashion and 1-limited service policy was allocated for each node. Network performance for 1-limited service goes down if the traffic arrival rate is not similar among the nodes. In WSN data arrival rate may not be symmetric. In many WSN applications, a node gets frequent smaller packets in a short period and then node experiences no packet arrival for relatively longer period of time. In such scenario, E-limited service policy shows better network performance, i.e. offered load, vacation time and queuing delay, compared to 1-limited service policy [13]. We have extended our work from 1-limited to E-limited service policy. In addition, we have implemented queuing model for the network.

We can redefine polling cycle as the time required for consecutive visits to the same node by the coordinator. In a polling cycle as shown in Fig. 2, each node gets one opportunity to send at most \( M \) packets.

In order to observe the impact of E-limited based round robin MAC scheduling on the recharging model, we first formulate the length of service period of a slave node in each polling cycle. We assume traffic arrival rate follows Poisson distribution and the rate is similar to all nodes. Packet arrival rate to a node is denoted by \( \lambda \). We assume a fixed packet size of \( L \) for this model. Let \( G_p(z) = z^L \) be the PGF for packet length. \( G_p^m(s) = e^{-sL} \) is the Laplace-Stieltjes Transform (LST) of \( G_p(z) \). Our analysis will follow the theory of \( M/G/1 \) queuing systems with vacations.

The coordinator performs as server in the network, while each slave node representing one of the clients which is serviced by the coordinator. The number of packets at the uplink queue of a slave node can be modelled with set of embedded Markov points. The Markov points correspond to the moments when a node’s vacation is terminated due to the arrival of data and moments when a node’s packet is served completely.

Let \( q_i \) be the joint probability that a Markov point in the uplink of a slave node is a vacation termination time and there are \( i = 0, 1, 2 \ldots \) packets at the uplink queue of a slave. Variables \( a_i \) and \( f_i \) represent the probabilities of \( i \) message arrivals during service time of a packet and during each vacation period respectively. Parameter \( \pi_i^m \) denotes \( i \) number of packets in the system after the completion of the \( m^{th} \) packet service, \( m = 1, 2, \ldots, M \). Then, the following equations hold:

\[
\begin{align*}
\pi_i^1 &= \sum_{k=0}^{i+1} q_k a_{i-k+1} \\
\pi_i^m &= \sum_{k=0}^{i+1} \pi_k^{m-1} a_{i-k+1} && m = 2, 3, \ldots, M \\
q_i &= \left( \sum_{m=1}^{M-1} \pi_0^m + q_0 \right) f_i + \sum_{k=0}^{i} \pi_k^M f_{i-k} && i = 0, 1, 2, \ldots
\end{align*}
\]

PGFs for number of packets after each packet service and queue are as:

Fig. 1: Network layout.
During the service period of a node other \((N-2)\) nodes are forced to undertake vacation. LST of vacation time is represented by \(V^*(\lambda - \lambda z)\). Number of packets arrival during a single vacation time is represented by \(V(\lambda - \lambda z)\). PGF of packet arrivals during a vacation can be presented as:

\[
V(\lambda - \lambda z) = \sum_{i=0}^{\infty} a_i z^i
\]

Using equations (1), (2) and (3), we can simplify the PGFs to:

\[
\Pi_m(z) = \sum_{i=0}^{\infty} \pi_m^i z^i \quad m = 1, 2, 3, \ldots, M
\]

\[
Q(z) = \sum_{i=0}^{\infty} q_i z^i
\]

We need to normalize \(Q_\sigma(z)\) as \(Q_\sigma(1) = 1 - \Pi(1)\). The normalized PGF can be represented as \(Q_\sigma'(z)\). The operation of the network can be expressed by \(M(N-1)\) equations. By solving these equations, we can derive probability values in PGF \(Q_\sigma'(z)\).

The uplink transmission is terminated after sending \(M\) packets or less till the queue is empty. PGF for the length of uplink service period including NULL packets is shown as:

\[
S^u(z) = \frac{1}{Q_\sigma(z)} \left[ z + \sum_{i=0}^{\infty} q_i(G_p(z))^M \right.
\]

\[
+ z(G_p(z))^M \sum_{i=1}^{M-1} q_i
\]

\[
+ \sum_{i=1}^{M-1} \sum_{l=1}^{M-k} (z - z(G_p(z))^l)\psi^*_{M-l,i}(G_p(z)) \right]
\]

where

\[
\psi^*_{l,i}(z) = \frac{iz^{l-1}}{l(l-i)!} \cdot \frac{dz}{dy} (zG_p(\lambda - \lambda z))^{2n}
\]

In the downlink, the coordinator sends only POLL message. The slave sends only one packet (DATA or NULL) against each POLL message. Let \(S\) denote the service time for both uplink and downlink. PGF for uplink and downlink service
time together $S(z)$ can be expressed as:

$$S(z) = \frac{1}{Q_p(z)} \left[ z + \sum_{i=M}^{\infty} q_i (zG_p(z))^M \right. \\
+ z(zG_p(z))^M \left. \sum_{i=1}^{M-1} q_i \right] \\
+ \sum_{i=1}^{M-1} q_i \sum_{l=1}^{M-k} (z - z(zG_p(z))^l) \psi_{M-l,i}(zG_p(z)) \right]$$

(8)

Probability of sending only NULL packet in the uplink is:

$$s^u_0 = S^u(0)$$

(9)

PGF for service time for transmitting Data Packets is:

$$S^u(z) = S^u(z) - s^h_0$$

(10)

IV. RECHARGING MODEL

Energy level of a node’s battery is reduced during packet transmission as it requires power consumption to do so. A node needs to be recharged when its energy level falls below a given threshold value $E_\delta$. In such scenario, the slave node containing less than $E_\delta$ energy can send recharging request by enabling some header bits when it sends packet to the coordinator. When the coordinator receives such recharging request, it stops normal operation of data transmission. The next POLL message is destined for all the slaves and the coordinator sends note to all that it is going to send charging pulse immediately by enabling some reserved bits in the POLL header. The coordinator notifies about the power intensity of the charging pulse $P_{\text{crg}}$ along with its duration $T_{\text{crg}}$. Due to the varying distances of slaves nodes from the coordinator, value of received energy for each node $i$ will be different and it will be proportional to the path loss $LP_i$ between the coordinator and the node $i$ as per Friis transmission equation:

$$LP_i = \frac{\lambda^2}{4\pi r_i} \left( \frac{G_i}{G_r} \right)^2 P_{\text{crg}}$$

where $\eta$ is the coefficient of efficiency for RF power conversion, $G_i$ and $G_r$ are antenna gain for the receiver (slave node) and the transmitter (coordinator) respectively, $\lambda$ is the wavelength of the RF signal and $r_i$ is the distance between the receiver and transmitter [2]. Energy level of the node $i$ can be increased by $\Delta E_i = P_{\text{crg}} \cdot T_{\text{crg}} \cdot LP_i$ amount maximally. At the end of recharging operation, normal operation of data transmission is resumed.

To perceive the impact of recharging on E-limited data service system, we need to derive an analytical model coupling with recharging period and queueing delay at the node premise. At the beginning, all slave nodes have equal energy level of $E_{\text{max}}$ which is the maximum capacity of battery sources of all slave nodes. During normal operation, the nodes use energy in sensing, processing, sending and resending (in case of corrupted DATA packets) DATA/NULL packets. A significant amount of energy is used to listen a complete POLL message destined to the node itself and header part of POLL messages designated to other nodes. Energy consumption value depends on traffic volume, bit error rate, number of retransmission $n_{\text{ret}}$, size of the network $N$.

Although, a particular node $i$ sends energy request when its energy falls bellow $E_\delta$ but all the nodes are recharged simultaneously while the coordinator sends charging pulse. After being recharged, a node $i$ will gain in energy level to $\min \left( E_{\text{max}}, \Delta_i \right)$. Due to the different path loss values, the nodes will possess different energy level after the recharging operation.

Initially, energy recharging request can be triggered by any slave nodes as all the nodes have maximum energy value and they have the similar traffic distributions. But in the long run, energy requests will be issued by farthest node regular basis as it receives minimum energy through recharging due to the maximum path loss. As a result its energy will fall towards $E_\delta$ sooner compared to other nodes. The farthest node will generate second, third, $\cdots$ recharging cycles while other nodes closer to the coordinator will have near to the maximum energy level.

Although the nodes have similar traffic distributions, their energy expenditures along with energy levels fluctuate over the polling cycles, due to the randomness of traffic arrival and number of attempts to send erroneous packets. This variation happens despite the assumption of fixed network size, constant reliability. Fig. 3 shows two alternative energy consumption processes of a node starting from maximum energy level $E_{\text{max}}$. Due to random packet losses and difference in sending number of packets in E-limited system, period between two recharging point is a random variable which distribution needs to be analysed. Table I represents units for energy expenditures for various events considered in the model. In this model, we focus to find total number of polling cycles along with energy

### Table I: Elementary energy consumption units.

<table>
<thead>
<tr>
<th>Energy expenditure</th>
<th>label</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data sensing</td>
<td>$E_{\text{ds}}$</td>
</tr>
<tr>
<td>Listening to the POLL packet</td>
<td>$E_{\text{poll}}$</td>
</tr>
<tr>
<td>Listening to the header of POLL packet</td>
<td>$E_{\text{hp}}$</td>
</tr>
<tr>
<td>Data packet transmission</td>
<td>$E_{\text{ds}}$</td>
</tr>
<tr>
<td>Null packet transmission</td>
<td>$E_{\text{null}}$</td>
</tr>
</tbody>
</table>

![Fig. 3: Detail of energy consumptions rate and their variation.](image-url)
TABLE II: Energy consumption units at polling cycle level.

<table>
<thead>
<tr>
<th>Energy expenditure label</th>
<th>Energy expenditure</th>
</tr>
</thead>
<tbody>
<tr>
<td>cycle with no data packet</td>
<td>$E_{null} = E_{pollt} + (m - 2)E_{pa} + E_{null}$</td>
</tr>
<tr>
<td>cycle with first transmission attempt</td>
<td>$E_{tr} = E_{ds} + E_{pollt} + (m - 2)E_{pa}/S_{u}^{z}(z) + E_{dt}$</td>
</tr>
<tr>
<td>cycle with packet re-transmission</td>
<td>$E_{ret} = E_{pollt} + (m - 2)E_{pa}/S_{u}^{z}(z) + E_{ret}$</td>
</tr>
</tbody>
</table>

expenditure for the farthest node starting from a recharging point till its energy level falls below $E_{ds}$. To do so, we need to study the joint probability distribution of consumed energy for E-limited system along with the required time. As the energy consumption for DATA/NULL packet (re)transmission is not exact multiple of sensing energy, we need to consider separate energy units for sensing DATA packets, transmitting DATA and NULL packets. Later, we relax this condition and convert all the energy units to a single unit.

Table I shows three kinds of energy expenditure to transmit DATA, NULL packets and to retransmit corrupted packet. DATA packet retransmission requires less energy value compared to the first attempt to send DATA packet as retransmission does not require sensing energy any more. It requires $E_{null}$ energy to send a NULL packet when a node’s buffer is empty. We need to derive total offered load (effective utilization) of a node $\rho_{tot}$ from the model later. $s_{d}^{z} = 1 - \rho_{tot}$ is the probability that a node’s buffer is empty. The remaining two types of energy expenditures $E_{ret}$ and $E_{tr}$ take places when a node’s buffer has some packets to transmit. Packet transmission energy $E_{tr}$ is larger than packet retransmission energy $E_{ret}$ as $E_{tr} = E_{ret} + E_{ds}$. Sensing energy unit $E_{ds}$ is having the smallest energy among all the energy units considered for the model. Later all the energy units will be replaced as multiples of sensing energy units.

PGF for energy consumption along with required number of time units for successful transmission of packets by a node in a polling cycle has to be calculated. Let $v, w$ be the energy units for sensing data packet, retransmitting a packet respectively and $t$ be the time unit to track how much energy is consumed over time. By replacing $z = (v^{\sum_{i=0}^{\infty} s_{t}^{z} \cdot w \cdot t^{L}})$ in equation (10), PGF for combined energy expenditure and time can be expressed as:

$$E_{data}(w, v, t) = S_{u}^{z} v^{\sum_{i=0}^{\infty} s_{t}^{z} \cdot w \cdot t^{L}}$$

In the proposed model, we should consider energy consumption for NULL packet. To send a NULL packet usually it requires less energy with respect to DATA packet. We assume variable $\phi$ to track energy consumptions for sending NULL packets. In E-limited system only one NULL packet is sent in a polling cycle. Complete PGF for sending DATA and NULL packets can be expressed as:

$$E_{all}(w, v, \phi, t) = E_{data}(w, v, t) + (1 - s_{t}^{u})\phi t$$

Let us consider that the farthest node $Y$ has the energy consumption budget of $\Delta_{Y}$ after it is recharged. By inspecting dynamics of DATA and NULL packet transmission, we conclude that $\Delta_{Y}$ amount of energy consumption is finished between

$$n_{min} = \Delta_{Y}/(n_{ret} + 1)E_{ret} + E_{ds}$$

and

$$n_{max} = \Delta_{Y}/(E_{null})$$

packet transmissions whose value is impacted by traffic arrival rate, transmission error rate and number of retransmission attempts. Minimum number of packet transmission $n_{min}$ happens when a node has packets all the time and it requires $n_{ret}$ of attempts to send each DATA packet successfully. Maximum packet transmission $n_{max}$ occurs when a node has empty queue all the time and the node sends only NULL packet. Probabilities of sending packets between these two boundary limits inclusively has non zero values.

Now, we need model the joint probability distribution of energy consumption along with time duration in order to send all the combinations of DATA and NULL packets between two charging points. The PGF $EEp(w, v, \phi, t)$ considering all possibilities can be written as

$$EEp(w, v, \phi, t) = \sum_{j=n_{min}}^{n_{max}} E_{all}(w, v, \phi, t)^{j}$$

As our model requires to determine total energy expenditure, we require to combine all the energy consumption units. According to tables I and II we define translation ratios between re-transmission and NULL packet transmission energies with sensing energy as:

$$nT = E_{null}/E_{ds}$$

and

$$vT = E_{ret}/E_{ds}$$

Further we need to use these ratios to map $w = v^{T}$ and $\phi = v^{\phi T}$ in expression (13). However since variable $v$ already appears with high powers, it is possible to collect the coefficients, round the powers and combine them in joint energy consumption variable $u$ (which is equivalent to $v$ by dimension but we have considered different variable name for clarity). The algorithm 1 shows merging the variables $w, \phi, v$ into variable $u$ (which has same unit as $v$).

This energy conversion could not be accomplished earlier in equation (12) ( due to the significant inaccuracy of rounding exponents of energy variable to integers). After combining all energy units the new PGF $EEp_{u}(u, t)$ has only one energy unit $u$ which only carries integer multiple of $E_{ds}$ in its exponent. Minimum and maximum exponent value of variable $u$ in $EEp_{u}(u, t)$ is expressed as $min_{exp}$ and $max_{exp}$ respectively. As clarified earlier, recharging will be launched by the furthest node $Y$ from the coordinator. Node $Y$ gains in $\Delta_{Y}$ energy.
Algorithm 1: PGF for combined energy unit and time unit.

Data: $EE_{all}(w, v, \phi, t)$, conversion ratios $nT$ and $rT$
Result: PGF for energy consumption representing in $E_{ds}$ quanta along with number of polling cycles between two consecutive charging points.

1. Find minimal $\min_w$ and maximal $\max_w$ exponent of variable $w$ in $EE_{all}(w, v, \phi, t)$;
2. for $i \leftarrow \min_w$ to $\max_w$ do
3. Derive coefficient $w_{\text{coef}}[i]$ of $w^i$ (polynomial on $v$, $\phi$ and $t$);
4. Find minimal $\min_{\phi}$ and maximal $\max_{\phi}$ exponent of variable $\phi$ in $w_{\text{coef}}[i]$;
5. for $k \leftarrow \min_{\phi}$ to $\max_{\phi}$ do
6. Calculate coefficient $\phi w[i, k]$ of $\phi^k$ in $w_{\text{coef}}[i]$ (polynomial on $v$ and $t$);
7. Find minimal $\min_v$ and maximal $\max_v$, exponent of variable $v$ in $\phi w[i, k]$;
8. for $j \leftarrow \min_v$ to $\max_v$ do
9. Find coefficient $v \phi w[i, k, j]$ of $v^k$ in $\phi w[i, k]$ (polynomial on $t$);
10. calculate combined integer energy consumption coefficient $\exp[i, k, j] = [i \cdot rT + k \cdot zT + j]$;
11. form new element of new polynomial as $v \phi w[i, k, j] \exp[i, k, j]$;
12. Sum third level $\sum_{\max_{k=\min_v}} \sum_{\max_{k=\min_v}}$ of $v \phi w[i, k, j] \exp[i, k, j]$;
13. Sum second level $\sum_{\max_{k=\min_v}} \sum_{\max_{k=\min_v}}$ of $v \phi w[i, k, j] \exp[i, k, j]$;
14. form new PGF as $EE_u(u, t) \leftarrow \sum_{\max_u} \sum_{\max_u} \sum_{\max_u}$:

in recharging process and when its energy level touches $E_\delta$ due to energy consumption new recharging request is sent. We will express energy budget in multiples of sensing energy quanta $n_{ds} = \frac{E_\delta}{E_{ds}}$ so that it matches unit of variable $u$ in PGF $EE_u(u, t)$.

Let $u_{\text{coef}}(i)$ be the coefficient of $w^i$ in $EE_u(u, t)$. In that case, $u_{\text{coef}}(i)$ will be a polynomial function in $t$.

Then polynomial conditioned to the event that energy consumption is exceeded can be derived as:

$$T_r(t) = \sum_{i=\min_u}^{\max_u} u_{\text{coef}}(i)$$

(14)

Since variable $t$ represents time for polling cycles, polynomial $T_r(t)$ is the conditional PGF for polling cycle numbers for which energy budget of a node is exceeded. $T_r(t)$ has to be unconditioned for becoming complete probability distribution for number of polling cycles.

$$T(t) = \frac{T_r(t)}{T_r(1)}$$

(15)

Mean number of polling cycles between two consecutive recharging appeals is therefore:

$$T = T'(1)$$

(16)

A node $i$ will be in outage if it requires to consume power exceeding $\Delta_i$. In that case, outage probability can be calculated as $p_{\text{out}} = \frac{1}{T}$. We can rename this outage probability as recharging probability $p_{\text{rec}} = p_{\text{o}}$ since the node requests for recharge before stepping towards energy outage.

V. VACATION MODEL

A node is forced to undertake vacation (can not perform data sending/receiving operation) when other nodes are transmitting packets or when it receives recharging pulse from the coordinator. In this section, we will formulate probability distribution of this vacation period. MAC time is expressed in basic slots. POLL and NULL packet consume one time slot each where a DATA packet consumes $L$ time slots. $S(z)$ presents a node’s service time for E-limited system in sending packets in the uplink and receiving POLL messages in the downlink. The node receives uplink packet with $\lambda$ arrival rate. Without the impact of vacation, a node’s offered load will be $\rho = \lambda \cdot S$.

But at actual, a node has to undertake vacation. Let $V(z)$ be the PGF of vacation period and $\bar{V}$ be the average vacation time. During vacation period though a node is not allowed to send data but it continues to receive traffic in $\lambda$ rate. This additional traffic arrival during vacation period extends the effective offered load in many folds. New scaled offered load $\rho_{\text{tot}}$ can be derived as:

$$\rho_{\text{tot}} = \rho + \lambda \bar{V}$$

(17)

E-limited polling MAC with round robin scheduling can be modeled with vacations [14]. Total vacation period can be split into two constituents.

1) Cyclical vacation composed of packets transmitting / receiving durations by other nodes in a a polling cycle. This vacation is comprised of service times of other $N - 2$ nodes excluding the target node and the coordinator. PGF of cyclical vacation is:

$$V_{\text{cyc}}(z) = (S(z))^{m-2}$$

(18)

2) Recharging vacation composed of duration when all nodes are forced to stop normal operation of sending packets as they must listen to the charging pulse. PGF for cyclic vacation is represented as:

$$V_{\text{rec}}(z) = P_{\text{out}} z^T_{\text{cr}} + (1 - P_{\text{out}})$$

(19)

PGF for total vacation experienced by a single node calculated as:

$$V(z) = V_{\text{rec}}(z) V_{\text{cyc}}(z)$$

(20)

Mean value and standard deviation of total vacation can be found as:

$$\bar{V} = V'(1)$$

(21)

$$V_{\text{stdev}} = \sqrt{(V''(1) - (V'(1))^2)} + V'(1)$$

(22)
Note that total offered load is dependent on average vacation period and in turn cyclical vacation relies on total offered load and therefore equations (17) and (21) need to be solved in a system.

A. Queuing model

PGF of Queue in equation (5) comprises $\Pi_M(z)$ and $\pi^k_Q$ (where $1 \leq k \leq M - 1$) values. We need to decompose this compound (A PGF function has other PGF function(s) as elements(s)) PGF where each PGF will be simplified and separated from one another. In case of E-limited system, after serving a packet, a node containing non zero DATA packet in the queue may or may not go for vacation depends on how many packet have been served. Beginning of vacation period is a function of number of packets served by the node. We assume E-limited system alike $M/G/1$ system with Bernoulli scheduling where it continues service $p_s = 1 - p_v$ probability where $p_v$ is the vacation probability. In such scenario $q_i$ can be re-written as:

$$q_i = (q_0 + \pi_0) f_i + p_v \sum_{k=1}^{i} \pi_k f_{i-k} \ i = 1, 2, \cdots \ (23a)$$

$$\pi_i = \sum_{k=1}^{i+1} q_k a_{i-k+1} + p_v \sum_{k=1}^{i+1} \pi_k a_{i-k+1} \ i = 0, 1, \cdots \ (23b)$$

$$\sum_{i=0}^{\infty} (q_i + p_i) = 1 \quad (23c)$$

By using equation (23a-c), PGFs for queue and number of packets remaining after service time can be written as:

$$Q(z) = [q_0 + p_s \pi_0 + p_v \Pi(z)] V*(\lambda - \lambda z) \quad (24a)$$

$$\Pi(z) = \frac{[Q(z) + p_s \Pi(z) - (q_0 + p \pi_0)] S_u(z)}{z} \quad (24b)$$

$$Q(z) + \Pi(z) = 1 \quad (24c)$$

From (24a), (24b), we can deduce separate PGFs as:

$$Q(z) = \frac{(q_0 + p_s \pi_0) [z - S_u(\lambda - \lambda z)] V*(\lambda - \lambda z)}{z - [p_s + p_v V*(\lambda - \lambda z)] S_u(\lambda - \lambda z)} \quad (25a)$$

$$\Pi(z) = \frac{(q_0 + p_s \pi_0) [V*(\lambda - \lambda z) - 1]\ S_u(\lambda - \lambda z)}{z - [p_s + p_v V*(\lambda - \lambda z)] S_u(\lambda - \lambda z)} \quad (25b)$$

$$q_0 + p_s \pi_0 = \frac{1 - \rho_{tot} - p_s \lambda V}{1 - \rho_{tot} - \lambda V} \quad (25c)$$

From (25c), $p_v$ value can easily be calculated as other terms are having known values. Considering the bit error, new PGF for queue as:

$$Q \sigma(z) = [(1 - \sigma) + \sigma z] Q(z) \quad (26)$$

$$\frac{Q \sigma(z)}{Q(z)} \text{ and } \frac{\Pi(z)}{\Pi(1)}$$

are normalized PGF functions.

Now, we have to find probability distributions of packet delay from probability distribution of packets left after the packet has been served. System time of packet can be calculated from number of packets arrive during that period. Let $T^*(s)$ be LST of a packet’s staying in the system. Number of packet arrivals can be expressed as

$$\frac{\Pi(z)}{\Pi(1)} = T^*(\lambda - \lambda z) \quad (27)$$

Waiting time includes waiting for all previous packets and unsuccessful transmissions of the target packet. Therefore we can derive probability distribution of waiting time $W^*(s)$ as:

$$\frac{\Pi(z)}{\Pi(1)} = W^*(\lambda - \lambda z) S_u(\lambda - \lambda z) \quad (28)$$

Finally, we can substitute $s = \lambda - \lambda z$ or equivalently $z = 1 - \frac{s}{\lambda}$ to calculate the value of $W^*(s)$. Then probability distribution of the packet delay becomes:

$$W^*(s) = \frac{s(1 - 2\lambda s)}{s - \lambda + \lambda (G^0_z(s))^2} \cdot \frac{1 - V^*(s)}{sV} \cdot \frac{q_0^u}{Q \sigma(1) V^*(s)} \quad (29)$$

Mean value of the queuing delay is obtained as:

$$W = \lambda (T^2 + T^3) \cdot \frac{L}{1 - 2\lambda L} + \frac{V^2}{2V} - V + \frac{Q(1)}{\lambda Q(1)} \quad (30)$$

VI. PERFORMANCE RESULTS

To assess the performance of the proposed MAC protocol, all the slave nodes were located within 10 metre radius of the coordinator. Wireless attenuation follows second degree of distance as per Friis equation (i.e., free space loss is considered). We assume the piconet experiences fixed bit error rate $ER_b = 10^{-5}$. For E-limited service policy, a slave node can send up to $M = 2$ packets. Number of retransmissions for a corrupted packet is set to $n_{ret} = 3$ value. Network size $N$ is varied between 3 to 9 nodes including the coordinator. Uplink packet arrival rate $\lambda$ follows Poisson distribution and the rate value is equal to all nodes. In the experiment, the rate is varied between 0.008 and 0.022 values with 0.002 stepwise increment. In the downlink, the network has only POLL packets. Packet transmission time is constant and equal to one time slot. Charging pulse duration is thousand slot wide. We have considered Maple 16.0V to generate graphs for different parameter values used in our model (mathematical analysis).

In Fig. 4(a) we present total offered load defined in equation (17). Offered load value increases both for growing network size $N$ and higher traffic arrival rate $\lambda$. The later is obvious as offered load is direct proportional to the traffic arrival rate. On the other hand, in case of growing network size, a node gets less proportion of time to send data. The node undertakes longer vacation period for higher network size as mentioned equation (18). Traffic keeps coming during the vacation period. As a result the node’s effective offered load gets increased.

Mean recharging period is shown in Fig.4(b). It may look counter intuitive at the first glance that a node’s average recharging period is decreased with higher network size as
it gets less fraction of active time. Note that, a slave node has to engage in longer vacation for larger network. And the larger network size leads to faster depletion of energy since all the nodes have to listen all POLL messages. Due to this, mean recharging period is inversely related with network size. Recharging period is decreased with higher traffic intensity as it requires more energy to send DATA packet compared to the NULL packet. Recharging probability in Fig.4(c) is the inverse of recharging period and its behaviour is exactly opposite to the recharging period.

Descriptors of vacation time are shown in Fig. 5. Mean vacation time grows gradually with higher traffic intensity. A node consumes energy at a faster rate for increasing traffic intensity. Due to the higher energy consumption recharging probability increases (i.e., more frequent recharging vacation \( V_{rec} \) occurs). On the other hand larger network size \( N \) enlarges cyclic vacation \( V_{cyc} \). As per equation (18), \( V_{cyc} \) is exponential to the network size and longer \( V_{cyc} \) enhances the mean vacation time. Standard deviation of mean vacation time has similar but less precipitous behaviour for packet arrival rate as higher arrival rates decrease NULL packet quantity. Standard deviation also gets increased more rapidly with higher \( N \) since this injects more variability in the transmission process. For this reason coefficient of variation \( V_{stdev}/V \) decreases with higher packet arrival rate but grows with increasing network size. It has been noticed that coefficient of variation is hyper-exponential.

Ensembles of probability distribution of polling cycle periods between two successive recharging pulses is shown in Fig. 6(a) for the network of size \( N = 4 \). Both the boundary limits (‘lower’ and ‘upper’) of distribution space have smaller values for larger traffic intensity. Higher traffic arrival rate increases packet retransmission probability and packet retransmission requires less power consumption. As a result ‘lower boundary’ starts from smaller value range and ‘upper boundary’ ends up with smaller value. Fig. 6(b) is shifted to the left (towards lower value area) for higher \( N = 7 \) network size compared to the Fig. 6(a). For higher network size, nodes consume more power as they need to listen more POLL packets.

Fig.7 shows the mean waiting time. Initially, mean delay time increases slowly with the packet arrival rate \( \lambda \) and network size \( N \). For larger \( N \) and \( \lambda \) values, a steeper increment in mean delay occurs due to the simultaneous impact of high traffic intensity and large vacation period. Note that this waiting period includes the vacation due to the recharging pulse.

**VII. Conclusion**

In this paper, we have proposed and evaluated simple MAC protocol which performs wireless recharging in WSNs. We have shown traffic admission schemes for E-limited system for the proposed MAC. MAC performance is mostly influenced by breaks in activity due to RF recharging. Therefore, we have designed a precise model for recharging period between consecutive charging points. We have analysed the model against varying packet arrival rates and network sizes. We have
Fig. 5: Descriptors of vacation time in piconet with wireless charging.

(a) mean vacation time

(b) standard deviation of vacation time

(c) coefficient of variation of vacation time

Fig. 6: Probability distributions of number of polling cycles between two successive re-chargings for different network size

(a) Ensemble of cycle distributions for $\lambda = 0.008 \cdots 0.020$ when $N = 4$

(b) Ensemble of cycle distributions for $\lambda = 0.008 \cdots 0.020$ when $N = 7$

Fig. 7: Descriptor of waiting time in piconet with wireless charging.

also developed queueing model for nodes in the network and calculate packet waiting time in a queue. Recharging period has significant impact on the distribution of packet waiting time and recharging duration has to be carefully chosen in order to exclude larger access delays in the network. We have noticed that farthest node frequently requests for energy in case of similar traffic distribution to the nodes. To overcome this problem and to provide fairness among nodes, in future, we want to apply zoning concept where traffic arrival will be symmetric for nodes located in the same zone but different for nodes in different zones. In future, we also aim to monitor the network performance by varying network parameters like recharging durations and power intensity values to find the optimal values which render lowest packet waiting times and
minimise per bit power consumption in transferring data.

REFERENCES


