

# GEYSER PUMP SOLAR WATER HEATER SYSTEM MODELING DESIGN OPTIMIZATION

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## ABSTRACT

Solar thermal water heating systems reduce household energy bills by using the free solar radiation provided by the sun to heat water for residential needs. In order to eliminate the need for electricity to run a pump to drive the fluid circulation in these systems, fluid buoyancy effects can be employed to move the fluid from lower elevations to higher elevations. There are several operational challenges with conventional “thermosyphon” systems, such as reversing flow and overheating, which can all be addressed by using a geyser pump mechanism. Although the solar thermal geyser pump water heating system has been on the market, little research has been done on the optimization of the system to improve its efficiency. Engineering Equation Solver (EES) and the Transient System Simulation Tool (TRNSYS) were used to implement a mathematical model for the geyser pump system operation under transient weather conditions. The model allows for parametric studies of the design attributes to investigate optimum efficiency conditions for the thermally driven pump.

## 1. NONMENCLATURE

### Symbols

$A$	Cross-sectional area (m <sup>2</sup> )
$A_1$	Constant
$Bo$	Bond Number
$B_1$	Constant
$Co$	Distribution parameter
COP	Coefficient of Performance
$D$	Diameter of lift tube (m)
$D_o$	Diameter of entrance tube (m)
$D_2$	Reference diameter (m)
$f$	Friction factor

$f'$	Fanning friction factor
$g$	Acceleration of gravity (m/s <sup>2</sup> )
$H$	Height of Generator liquid level (m)
$j$	Superficial velocity (m/s)
$K$	Experimental friction relationship
$Ko$	Correlating fitting parameter
$L$	Length of lift tube (m)
$Lo$	Length of entrance tube (m)
$m$	Constant (different drift flux analysis than in slug/churn transition analysis)
$\dot{m}$	Mass flow rate (kg/s)
$n$	Constant
$N_f$	Viscous effects parameter
$P$	Pressure (bars)
$Q$	Volumetric flow rate (m <sup>3</sup> /s)
$\dot{Q}$	Heat transfer rate (W)
$r$	Correlating fitting parameter
Re	Reynolds number
$S$	Slip between phases of two-phase flow
$T$	Temperature (K)
$V$	Velocity (m/s)
$x$	Quality
$Y$	Mole fractions
Greek characters	
$\epsilon$	Void fraction
$\epsilon_R$	Pipe roughness (m)
$\rho$	Density (kg/m <sup>3</sup> )
$\mu$	Fluid viscosity (kg/m-s)
$\sigma$	Surface tension (N/m)
$\Sigma$	Surface tension number
$\checkmark$	Volumetric Flow Rate (m <sup>3</sup> /s)

## Subscripts

0, 1, 2	States in governing equations
$a$	Ammonia
$BP$	Bubble pump
$G$	Gas
$gj$	Drift
$h$	Homogeneous conditions (in two-phase flow)
$L$	Liquid
$m$	Mixture
$TP$	Two-phase
$v$	Vertical
$w$	Water

## Superscripts

*	non-dimensionalized
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## 2. INTRODUCTION

Renewable energy technologies, such as wind and solar power, are being widely studied by researchers today as many countries are trying to reduce their dependence on non-renewable energy sources (i.e. fossil fuels). Massive use of conventional, non-renewable resources produces greenhouse gases which contribute significantly to climate change. Renewable energy sources, such as solar and wind energy, on the other hand, do not produce greenhouse gases. They are sustainable and free of cost.

Solar thermal water heating (STWH) systems are both cost efficient and energy efficient. The STWH systems are most suitable for places with hot climates and direct sunlight, but can also work quite well in colder climates found throughout the United States. Different designs of the STWH systems have overcome the challenges such as freezing, and overheating. Most STWH systems have a flat plate solar collector tilted towards the south on the roof of the residence. Flat plate solar panels consist of four parts: a transparent cover which allows minimal convection and radiation heat loss, dark color flat plate absorber for maximum heat absorption, pipes which carries heat transfer fluid that remove heat from the absorber, and a heat insulated backing to prevent conduction heat loss. Within this collector, there is a network of black tubing inside which flows water or some other fluid. The fluid in the tube enters the panel relatively cold and is heated as it flows through the panel as the black exterior of the tubes absorbs the heat from the sun. The heat transfer fluid and the materials selected for the tubes are important parameters for the system to withstand drastic temperature differences of freezing and overheating. If the fluid is water, then it goes right into a hot water storage tank. If another working fluid is used, the heat is then transferred from the working fluid to the water in the hot water tank via a heat exchanger in the tank.

Two of the most common STWH system types are passive and active systems. The fluid in a passive system is driven by natural convection, whereas in active systems, the fluid is moved via a pump and thus require another form of energy

input other than solar energy to run the pumps inside the systems. This means that in the case of an electricity outage, an active system may overheat. Thus, passive systems are preferred for economic and reliability purposes, but they have their own set of technical challenges.

One of the most favorable passive systems is a thermosyphon STWH system, shown in Figure 1. A thermosyphon is an open loop system that can only be used for nonfreezing climates. The simple design of the thermosyphon system is economically efficient. Theoretically, in a thermosyphon system, the tank has to be above the solar panel. Using gravity, the cold water from the tank flows downward and enters the tube beneath the panel, and gets heated up inside the tube. During this process, similar to a hot air balloon, the less dense hot water is buoyant and thus floats to the top of the tube and slowly reaches the top of the tank. Then, more cold water would drain down from the tank and get heated up. The process continues on until the water of the tank reaches thermal equilibrium. Although the system seems ideal, there are problems such as overheating and freezing. Furthermore, the temperature differences of the tank from sunrise to sunset will always be positive; therefore, the pressure difference will also be positive. If the pressure of the system is not designed properly, there will be a reverse-thermosyphon effect, which happens often during drastic temperature drops. The reverse-thermosyphon effect can cause flooding.

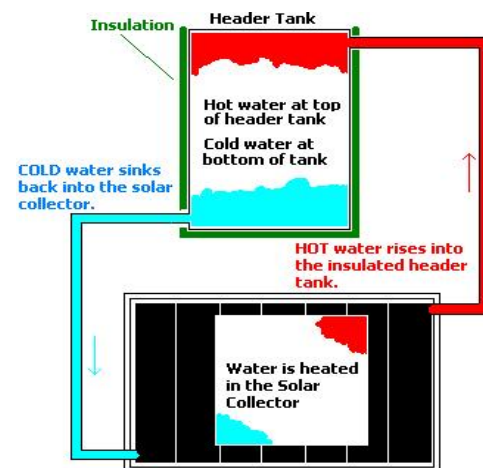


Fig. 1: Thermosyphon Solar Hot Water Heating System

A bubble pump, also known as a geyser pump, STWH system, as shown in Figure 2, is an improved version of a closed loop thermosyphon system. It is not an active system, but works as one. The unique design of the system runs like a pump but does not require mechanical work as input to run the pump. When heat is added to the system the liquid working fluid begins to vaporize, creating a two-phase flow condition. The buoyance of gas creates a pump-like effect that pushes the boiling working fluid to a higher elevation. Unlike the thermosyphon system, the position of the tank does not depend on the solar panel. With an initial hand pump to get the

pressure in the bubble tank to be under vacuum, cold working fluid enters to the bottom of the panel. Similar to a thermosyphon system, the working fluid heats up through natural convection and hot fluid rises to the top of the panel due to buoyancy. This hot fluid then leaves the panel through lifting tubes and enters a second reservoir, which then drains down to the top of the tank and creates a pressure difference in the system. Due to the pressure difference, the cold working fluid in the tank heat exchanger will rise up into the panel. When the working fluid in the panel is overheated, the pressure in the system is unstable and the valve of the header at the second reservoir opens. This then is connected to a third reservoir and allows vapor gas to escape the closed system. The third reservoir is also connected to the entrance of cold fluid in the panel. Once the vapor gas condenses, it flows together into the panel again with the cold fluid. By doing so, it also preheated the cold fluid entering the panel. The working fluid in the Sunnovations system is a standard mix of Propylene glycol and water. Although the geyser pump is slightly more costly than a thermosyphon system, it does prevent freezing, overheating, as well as reverse-thermosyphon effects.

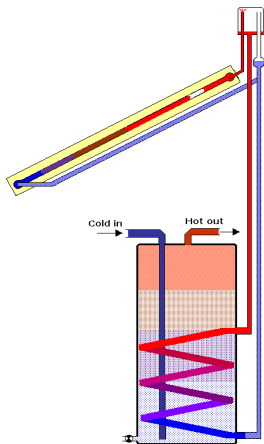


Fig. 2: Geyser Pump Flow Schematic

There are several parameters of the geyser pump which can be adjusted to optimize the STWH system. These parameters include: the diameter of the lift tube, the number of lift tubes, and the material of the lift tube. Slug flow is the optimal operating two phase flow regime for fluid pumping (White, 2001), but because this system will change dynamically with solar input fluctuations, it will be difficult to design the geyser pump to stay in this regime. For a given system operating temperature, previous studies have shown that a smaller than optimal diameter lift tube will experience churn flow and a much higher than optimal diameter tube will produce bubbly flow (White, 2001). As the temperature of the solar flat plate collector changes throughout the day, the two phase flow regime in the lift tube changes between bubbly flow, slug flow, and churn flow. Because the optimum diameter is limited in size for maximum efficiency, multiple lift tubes

may be needed to achieve the desired flow rate of heated working fluid through the system. Multiple lift tubes can help boost up the speed of removing heated fluid from the flat panel, which then increase the circulation speed of the system and can help the system stay in the slug two-phase flow regime for a longer period of time. Material properties may also influence the surface tension related forces in the two phase flow structure which thus influences the performance of the system.

There are very few studies in the literature regarding the operation of a geyser pump driven solar thermal system, however two companies are offering such systems in the U.S. market: Sunnovations and SOL Perpetua. There are geyser pump SHWS patents by Haines (1984) and van Houten (2010) related to these two companies. Both involve copper lifting tubes. Haines (1984) uses a mixture of water and methanol as the working fluid of the system, van Houten (2010) uses a mixture of water and propylene glycol, whereas SOL Perpetua (2011) uses water propylene as the working fluid. Li et al (2008) believed the diameter and friction factor of the lifting tube have an inverse relationship to each other, and as the diameter of the lifting tube increases, the efficiency of the geyser pump would also increase.

### 3. METHODOLOGY

Two-phase flow is the main mechanism for running the geyser pump in a STWH system. As the water boils, the water vapor coalesces and pushes slugs of liquid water through the pipe into another water storage reservoir. Although the geyser pump has been used greatly in other products, such as the percolating coffeemaker, rarely any research was done on the optimization of the system (White, 2001). Some key terminology associated with two-phase flow is listed in Table 1. There are four types of basic flow patterns: bubbly, slug, churn, and annular.

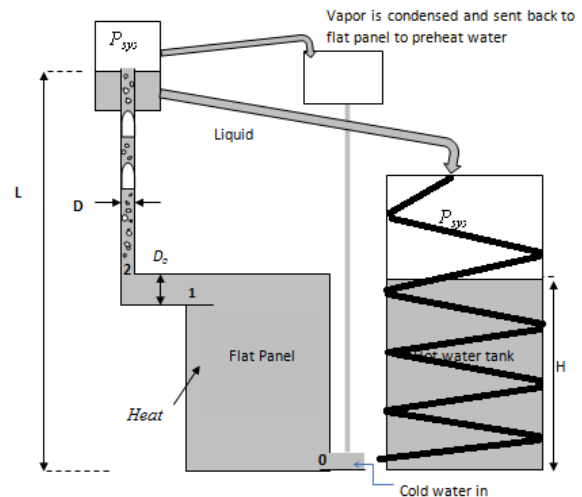


Fig. 3: Geyser Pump System Layout

TABLE 1: Two-Phase Flow Parameters

Parameter	Units (SI)	Definition
$\rho_G$	kg/m <sup>3</sup>	Density of gas phase
$\rho_L$	kg/m <sup>3</sup>	Density of liquid phase
$D$	m	Diameter of lift tube
$A=\pi D^2/4$	m <sup>2</sup>	Total cross sectional area of pipe
$A_G$	m <sup>2</sup>	Cross sectional area gas occupies
$A_L=A-A_G$	m <sup>2</sup>	Cross sectional area liquid occupies
$\varepsilon = A_G/A$	-	Gas void fraction of the flow
$Q_G$	m <sup>3</sup> /s	Gas volumetric flow rate
$Q_L$	m <sup>3</sup> /s	Liquid volumetric flow rate
$Q=Q_L+Q_G$	m <sup>3</sup> /s	Total volumetric flow rate
$j_G=Q_G/A$	m/s	Gas superficial velocity
$j_L=Q_L/A$	m/s	Liquid superficial velocity
$j = j_L + j_G$	m/s	Total average velocity of flow
$V_G = j_G / \varepsilon$	m/s	Velocity of the gas
$V_L = j_L / (1-\varepsilon)$	m/s	Velocity of the liquid
$\dot{m}_G$	kg/s	Mass flow rate of gas
$\dot{m}$	kg/s	Total mass flow rate
$x = \dot{m}_G / \dot{m}$	-	Quality
$S = V_G / V_L$	-	Slip between phases

A series of momentum and mass flow balances can be performed to model the operation of the geyser pump system based on the definitions of the state locations in Figure 3. These equations are provided in detail as follows: Momentum equation from Psys to 0 gives:

$$P_0 = P_{sys} + \rho_L g H - \rho_L \frac{V_0^2}{2} \quad [1]$$

Where:

$V_0$  is the velocity (m/s) at point 0 (liquid solution)

H is the height of the working fluid level in the tubes as it passes through the water tank

Momentum equation from 0 to 1 yields (including pressure drop from friction):

$$P_1 = P_0 - \rho_L V_0 (V_1 - V_0) - \rho_{TP} g H_{FP} \quad [2]$$

Where,  $V_1$  is the velocity (m/s) at state 1,  $D_0$  is the diameter (m) of the water entrance line,  $\rho_L$  is the density (kg/m<sup>3</sup>) of the water entrance line,  $\rho_{TP}$  is the density (kg/m<sup>3</sup>) of the two phase mixture in flat panel, and HFP is the height (m) of the water level in flat panel.

Conservation of mass from state 0 to 1 for the system yields:

$$\rho_L A_0 V_0 = \rho_{TP} A_0 V_1 \quad [3]$$

The homogeneous density follows from this equation after substituting for the areas at state 0 and state 1

$$\rho_{TP} = \frac{\rho_L A_0^2 V_0}{A_1^2 V_1} \quad [4]$$

At this point, the two-phase flow terminology from Table 1 is needed to proceed because the flow in the lift tube is most clearly defined in these terms. The definitions of superficial velocities and void fraction can be related to the terminology used thus far.

Since states 0 and 1 are under liquid conditions:

$$V_0 = \frac{Q_L}{A_0} \quad [5]$$

While the definition of the superficial liquid velocity,  $j_L$  is:

$$j_L = \frac{Q_L}{A} \quad [6]$$

Therefore:

$$V_1 = \frac{Q}{A_1} \quad [7]$$

Additionally, state 2 has two phases, but  $V_2$  still describes the total average velocity of the mixture:

$$V_2 = \frac{Q_L + Q_G}{A} = \frac{Q}{A_2} \quad [8]$$

This is precisely the definition of  $j$ . Therefore:

$$V_2 = j \quad [9]$$

It follows that:

$$V_2 - V_1 = j - j_L \left( \frac{A}{A_0} \right) \quad [10]$$

Conservation of momentum from state 1 to 2, neglecting friction in this transition is then:

$$P_2 = P_1 - \rho_{TP} V_1 (V_2 - V_1) \quad [11]$$

The void fraction is defined as the average cross sectional area occupied by the gas divided by the total cross sectional area of the pipe.

$$\text{Therefore: } \frac{A_L}{A_2} = 1 - \varepsilon \quad [12]$$

Now the momentum equation in the lift tube (from state 2 to  $P_{sys}$ ) can be stated as:

$$P_2 = P_{sys} + f_{TP} \frac{(\rho_L j_L + \rho_G j_G)^2}{2\rho_{TPH}} \left( \frac{L}{D} \right) + \rho_L L g (1 - \varepsilon) \quad [13]$$

where  $f_{TP}$  is the two-phase friction factor, based on average properties of liquid and gas and  $\rho_{TP}$  is the two-phase density of the fluid mixture in the lift tube. Here, for the frictional pressure drop term, a new two-phase density is now required for the conditions in the lift tube. This two-phase density can be found from the density definition applied to the lift tube volume:

$$\rho_{TPH} = \rho_G \varepsilon + \rho_L (1 - \varepsilon) \quad [14]$$

Therefore, combining Equations [1], [2], [11] and [13], a general equation for the submergence ratio ( $H/L$ ), which describes the average pressure gradient along the lift tube, can be solved as:

$$\frac{H}{L} = \frac{f_{TP} (\rho_L j_L + \rho_G j_G)^2}{2gD\rho_L\rho_{TP}} + \frac{j_L^2 \left( \frac{D}{D_0} \right)^4}{2gL} + \frac{j_L \rho_{TP} \left( \frac{D}{D_0} \right)^2 \left( j - j_L \left( \frac{D}{D_0} \right)^2 \right)}{\rho_L gL} + (1 - \varepsilon) \quad [15]$$

#### The Drift Flux Model

The drift flux model is a widely accepted method for analyzing void fractions in two-phase flow. This method, formalized by Zuber and Findlay in 1965, provides a means to account for the effects of the local relative velocity between the phases as well as the effects of non-uniform phase velocity and concentration distributions.

While many others contributed to the beginnings of two-phase flow theory, Zuber and Findlay's (1965) analysis establishes the basis of the drift flux formulation used today (Chexal 1997). It relates the average gas void fraction of the two-phase flow to: 1) the superficial velocities (the velocity each phase would have if they occupied the entire area of the pipe alone) of the gas and liquid phases; 2)  $C_o$ , the distribution parameter; and 3)  $V_{gj}$  ( $= V_G - j$ ), the drift velocity. The resulting drift flux model can be summarized by the following equation:

$$\varepsilon = \frac{j_G}{C_o(j_L + j_G) + V_{gj}} \quad [16]$$

Many authors have formulated empirical correlations for  $C_o$  and  $V_{gj}$  depending on the two-phase vertical flow regimes and other parametric effects. The only all-in-one model which accounts for transitions between these regimes is the correlation of Chexal and Lellouche (1996), which was used in this study.

The working fluid used in the current study is water, but may be changed with another working fluid in the future. Based on pressure data provided by Sunnovations, the SHWS modeled runs under a vacuum pressure of 20-25 inches of mercury. While the model of the solar thermal water heating systems require a solar flat plate panel collector to collect solar energy and heat up water in the panel, it was initially assumed that the panel is about 50% efficient and can collect a theoretical heat input of 700 W/m<sup>2</sup> from a standard 3 m<sup>2</sup> panel, and the temperature of the fluid entering the lift tube is at the saturated temperature under these pressure and heat flux conditions. The model was then also tied to a flat plate solar panel modeled in TRNSYS to provide more accurate conditions to simulate the input to the geyser pump under a variety of weather scenarios. The efficiency of the solar thermal system is measured by the mass flow rate of the hot water output of the geyser pump as the input to the system is free solar thermal energy. The minimum hot water mass flow rate output requirement from the geyser pump, based on data from Sunnovations, is 1 liter per minute for two lift tubes for a typical residential system operation. Only a single lift tube is being considered in the current model.

## 4. RESULTS & DISCUSSION

To solve all of the non-linear equations, Engineering Equation Solver (EES) software is used. EES is an equation solver software that can iteratively solve thousands of linear or non-linear equations simultaneously. It has built in libraries for thermodynamic and fluid transport properties, thus it is widely used in the Mechanical Engineering field. It can also solve differential and integral equations, which is helpful for the optimization of parameters for the solar thermal geyser pump water heating systems.

As a result, the mass flow rate output when the system uses pure water is slightly lower than of water and propylene glycol mixture, since only pure steam gets boiled out of the working fluid.

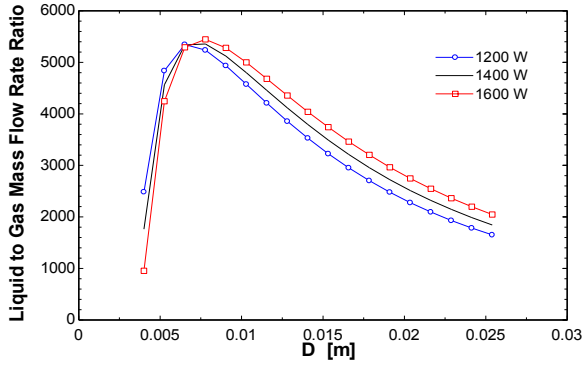


Fig. 4: Ratio of Liquid and Gas Mass Flow Rate VS Diameter

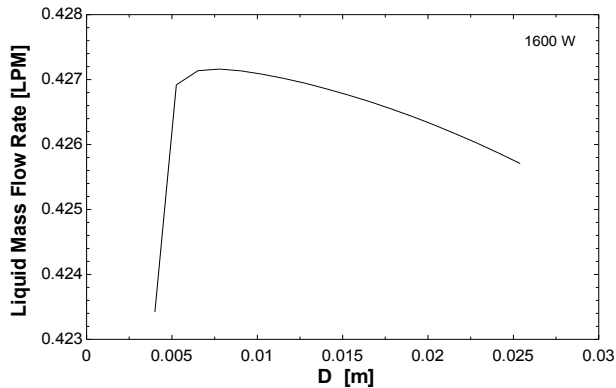


Fig. 5: Liquid Mass Flow Rate VS Diameter

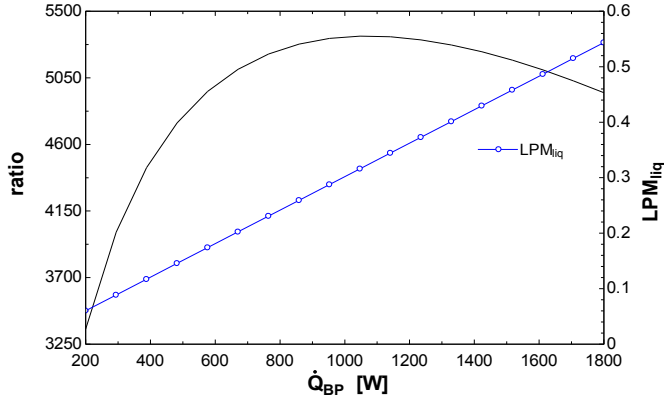


Fig. 6: Ratio of Liquid and Gas Mass Flow Rate, and Liquid Mass Flow Rate VS Useful Energy Input from Solar Panel

Figure 4 shows the results of an initial series of parametric studies for the geyser pump design for integration with a solar thermal hot water heating system. As shown in Figure 4, as the diameter for a geyser pump design increases, the mass flow rate of the liquid being pumped through the system increases to a peak “ratio” value, which is the ratio of liquid mass flow rate to gas mass flow rate, and then decreases again; as this occurs, the two phase flow regime changes from churn to slug and lastly to bubbly flow. It can also be seen in this figure that the optimum diameter of the lift tube changes with varying solar insolation conditions. The lower the radiative heat transfer conditions to the flat plate, the smaller

the optimum lift tube diameter. However, it is unrealistic to have a constant solar energy input throughout the day every day, which is why more meaningful results can be obtained from simulating the geyser pump’s operation over a year of transient conditions.

Figure 5 solidified the result such that when solar energy input is at 1600W, the optimal liquid mass flow rate occurs when diameter is about ¼ inch. The ratio of liquid to gas mass flow rate and liquid mass flow rate is plotted against useful energy input from solar panel while diameter being held constant at ¼ inch in figure 6. The result shows, the ratio of liquid to gas mass flow rate maximize when useful energy input from solar panel is around 1000 to 1200 W. Whereas, the liquid flow rate increases linearly as energy input from solar panel increase. Although the liquid flow rate increases when useful energy input from solar panel increase, it is unrealistic to have 1800 W of useful solar energy.

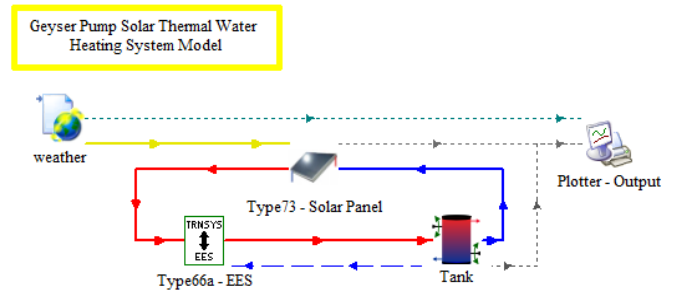


Fig. 7: TRNSYS-EES Simulation Setup

The EES model was then integrated into a STWH modeled in Transient System Simulation (TRNSYS) software, allowing for transient simulation of the geyser pump performance under varying meteorological conditions. The setup of this simulation is shown in Figure 7. The results from this integrated simulation are preliminary and shown in Figures 8-10.

In Figure 8, the starting temperature in the tank was 35.88 C and rose to 36.49 C. The constant irradiation situation shown here is very similar to that of the cases conducted within the isolated EES geyser pump model; however there is now a hot water demand load included into the analysis via TRNSYS. This demand prevents steady state behavior, and thus more erratic results from the simulation model are found. The erratic behavior is not entirely understood, but is the focus of our immediate future steps in this study.

Figure 9 shows a time period in which there is at first high irradiance and then a period of lower irradiance, which shows that the model is reacting appropriately to this sudden change in solar input. Meanwhile, Figure 10 shows the opposite situation as Figure 9, showing the temperature of the collector rising as the solar irradiance suddenly increases.

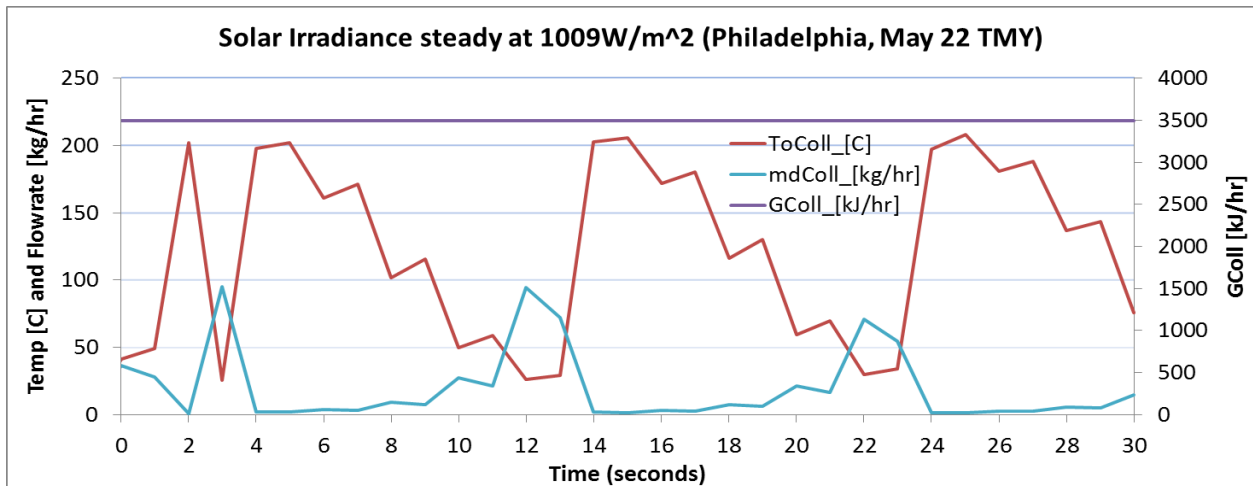


Fig. 8: TRNSYS-EES Simulation Results for a High Irradiance Condition

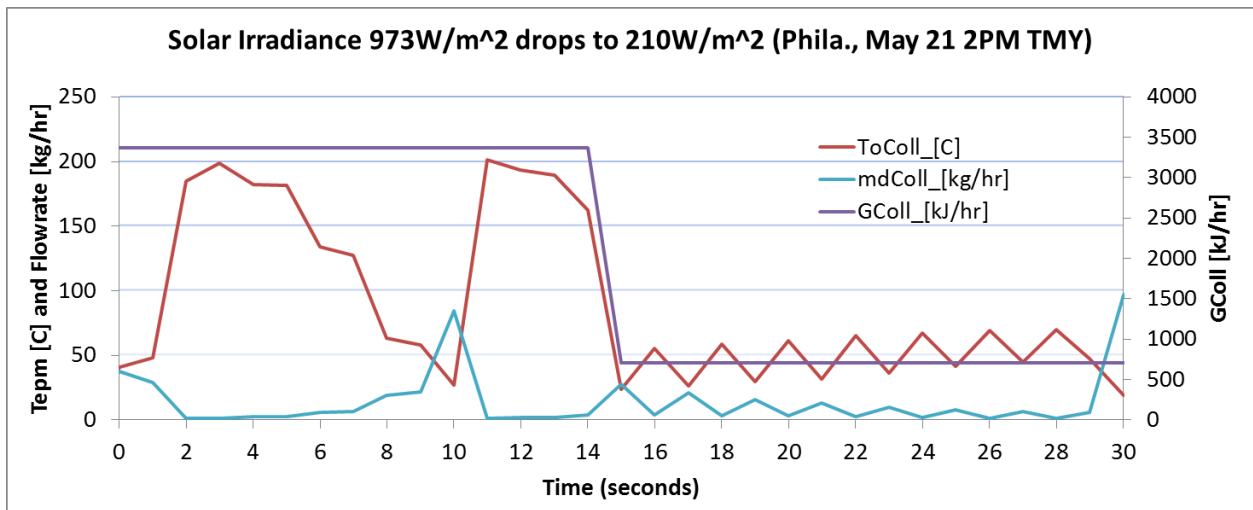


Fig. 9: TRNSYS-EES Simulation Results for a Period in which Passing Clouds Decrease Irradiance

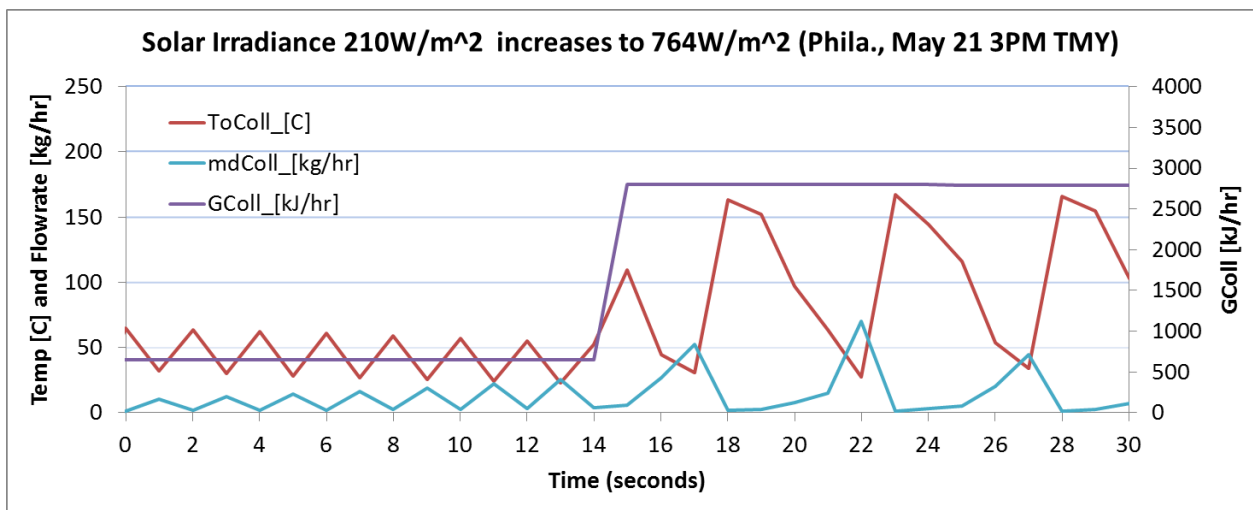


Fig. 10: TRNSYS-EES Simulation Results for a Period in which Clouds Clear from the Sky Increasing Irradiance

## 5. CONCLUSIONS AND RECOMMENDATIONS

The model results showed that having a ¼ inch lift tube operating at a pressure of 0.336 bars can boost up the mass flow rate output to .3 liters per minute. Initial results are provided from the EES simulation integrated with TRNSYS software, allowing for more detailed modeling of the time varying impacts of the solar irradiance fluctuation. Future work will entail improving the operation of this integrated simulation, simulating the results over the course of a year to study design impacts including the addition of multiple lift tubes. Experimental performance will also be compared with the resulting model before further analysis and implementation of system changes.

## 6. ACKNOWLEDGEMENTS

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