Granular Stochastic Modeling of Robot Micrometric Precision

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Abstract— The paper aims at modeling and quantifying robot precision when it is possible to obtain information from external sensors in the operating area. The author proves that in this situation, neither the pose repeatability, nor the pose accuracy are adequate to calculate the maximum position error. A new paradigm is then proposed: granulous space modeling which combines spatial resolution and actuators’ repeatability. This stochastic modeling is first detailed in unidimensional space then in the case of bidimensional space. The methodology to compute the maximal position error is given and compared with other approaches.

I. INTRODUCTION

Robot precision is usually characterized by two indices: repeatability and accuracy. When the move is repeated several times in the same conditions, the final attained positions are all slightly different and instead of being concentrated on one unique point, they constitute a cloud. Repeatability is used to characterize the dispersion of the points around the barycenter of the cloud. Accuracy is the distance between the barycenter of the cloud and the desired goal in the absolute coordinate system. Most of the performance indices related to precision are described in detail in the norms ANSI [1] and ISO [2]. This paradigm is used when the robot is controlled in an absolute coordinate system.

In this paper, we investigate robot precision when it is possible to obtain position information in the operating area. External sensors, for instance micrometers, a vision system, etc. are used to acquire relative position information with a resolution that is much better than the robot repeatability. Today many industrial robots claim to have a $10 \mu m$ repeatability performance whereas micrometers can reach a $1 \mu m$ resolution.

In the proposed strategy, after a first rough positioning, the corrected target is calculated. Then the robot comes back to what we named “the harmonization point” (HP). The distance between HP and the target is usually at least ten times larger than the lost motion zone width. From HP the trajectory is replayed with a slight change in the target to get closer to the desired position, as illustrated in fig. 1. In this context, neither repeatability nor pose accuracy are sufficient to compute the maximum position error as will be proved on a key example in section II. Then a new paradigm is developed that enables the determination of the maximum position error. This granulous space modeling is not only a theoretical tool but gives extremely accurate results when applied to robot control as experimented on an EPSON RS450.

In this section, let us consider a two link planar robot. In the micrometric scale, four different closest targets that it is possible to enter on the control panel are associated with four different repeatability disks $(D_1)$ to $(D_4)$ with centers $O_1$ to $O_4$ as illustrated in fig. 2. All the final robot endpoints will lie inside the repeatability disk corresponding to the chosen targets. Using position information from external sensors, it is possible to change the final target slightly so as to be as close as possible to the desired point. For instance, if the desired point is $P_0$ and the first attained point is $P_1$ in the repeatability disk $(D_1)$, then we can change the target in the controller to go to a closer repeatability disk $(D_2)$ and the position error is then lower.

In this scheme, for a commanded point $P_0$, the maximum position error can be expressed as $\varepsilon_{\text{max}}(P_0) = \inf \{d(P_0; O_i)/i \in \mathbb{N}\} + \text{Rad}(D_i)$ where $d(P_0; O_i)$ is the distance between the points $P_0$ and $O_i$ and $\text{Rad}(D_i)$ is the radius of the repeatability disk $(D_i)$ if the optimal target is
The maximum position error $\varepsilon_{\text{max}}(P)$ would occur when $P$ lies in the center $\Omega$ of the circle passing through the points $O_1, O_3, O_4$ and corresponds to $\varepsilon_{\text{max}}(P) = \varepsilon_{\text{max}}(\Omega) = \Omega O_1 + \text{Rad}(D_i)$.

The analysis of this example which is based on the real case of the Samsung robot FARA2, proves clearly that in the micrometric scale, position error cannot be linked directly to repeatability and accuracy. The usual understanding of these concepts would have misled us to conclude that the maximum error was the sum of accuracy and repeatability.

To explain why accuracy is not relevant in computing the maximum error, let us base our analysis on fig.3 describing the process that transforms a target in the control panel to a real position in Cartesian space.

In the first step, the inverse kinematics function is used to compute the target $T_i$ in the actuator space corresponding to the commanded position $P_C$. In the second step, this target is the input in the servo-control of the process. The servo-control will use the transfer function of the effector to transform the power signal into a position and some errors will appear in this phase: they are essentially due to servocontrol, internal sensors offset, and non-geometrical effects (thermal effects, static loads provoking flexion or elasto-deformation of the links). Because of these errors, a distortion appears between the expected position which corresponds to a perfect transfer function, and the reality. The actual positions of the interval centers cannot be predicted without experimentation. They will be estimated statistically after a sample is measured and here uncertainty linked to the statistical estimation must be taken into account.

At this stage, it is possible to quantify the maximum position error $\varepsilon = \|P - P_C\|$ between the commanded position $P_C$ and a final possible position $P$ as the sum of the accuracy and the repeatability $\varepsilon_{\text{max}} = \text{Acc} + \text{Rep}$, the definitions given above remaining the same.

But, if we consider now that external sensors provide information to improve the final positioning, it is clear that the maximum error can be reduced significantly by choosing the better target $T_{i-1}$ that will bring the final position distribution closer to the commanded position. Consequently the maximum error will not be the sum of accuracy and repeatability anymore.

If the supervisor has enough information concerning spatial resolution (SR) and a clear understanding of the underlying phenomena, it will be easy for him to reduce the error by choosing a better target. Then the maximum error will be $\varepsilon_{\text{max}}(P) = \text{SR} / 2 + \text{Rep}$.

So we have proved that in our specific context, accuracy and repeatability are not sufficient to describe the position error. Therefore a new modeling of precision is necessary. In this paper we propose a granulous stochastic modeling.

### III. Granulous space modeling

#### A. Definitions of the random variables

Here we give indepth mathematical definitions of the random variables (rv) involved and outline our previous research results on the modeling of robot micrometric precision. More details can be found in [3] and [4]. Let us consider a one-dimensional actuator. Let $X_n$ be the random variable (rv) corresponding to the final position of the end effector and let $f_n$ be the associated probability density function (pdf).

For a given target $T_n$, the end effector position varies on a subset $\Omega_n$. Let $X_n$ be the random variable (rv) corresponding to the final position of the end effector and let $f_n$ be the associated probability density function (pdf).

For the same target $T_n$, the rv $X_n$ varies also with time. Let $X_n(t_i)$ be the rv corresponding to the final position of the end effector when the target $T_n$ is set at time $t_i$.

We performed an exhaustive statistical study of the random process $X_n(t_i)$ and found the following characteristics:

$X_n(t_i)$ and $X_n(t_k)$ are independent rv when $i \neq k$. This result was obtained by calculating the autocorrelation function of the process.

$X_n(t_i)$ is a Gaussian rv with an expectation $E(T_n, t_i)$ and a variance $\text{Var}(T_n, t_i)$. The Gaussian characteristic was...
proved with $\chi^2$ adequation tests performed on three different robots and thousands of measurements.

When the trajectory is stabilized, variance $Var(T_n, t_i)$ do not depend on $t_i$, as was proved in an F-test procedure.

On the contrary, the expectation $E(T_n, t_i)$ cannot be considered constant. Its fluctuations according to time cannot be ignored and are in the same range as the robot repeatability index. This was proved in a z-test on two different portions of the trajectory and corresponds to physical phenomena such as thermal dilatations, hysteresis, wear...

All the above statistical work on real experimental data lead to the following conclusions:

The rv $X_n^{(t_i)}$ associated to the target $T_n$ set at time $t_i$ follows a Gaussian law with an expectation $E(T_n, t_i)$ affected by slight but not negligible fluctuations and a constant standard deviation $\sigma$.

B. The granulous space paradigm

The granulous space paradigm is a modeling of robot precision that takes into account simultaneously spatial resolution via the $E(T_n, t_i)$ expectation mesh and joint repeatability via the covariance matrix $C$.

The ratio $\tau = \frac{\sigma}{\Delta}$ is defined as the granularity ratio and is useful to characterize the typology of stochastic behavior of the angular space. If $\tau < \frac{1}{6}$, most of the space cannot be attained apart from 'hot spots'. If $\tau > \frac{1}{6}$, the same area can be attained with a different target but the probability is lower [5]. There is a kind of 'stochastic redundancy'.

C. Spatial resolution estimation

Spatial resolution is the pattern mesh of the different expectations $E(T_n, t_i)$ associated at time $t_i$ with the different possible targets $T_n$. It is sometimes assimilated with input error pattern but in reality, it is quite different. We proposed a special procedure called the sliding cycle test to evaluate it and proved that statistically speaking, the differences between the experimental means are directly linked with the Jacobian matrix $E(T_{n+1}, t_i) - E(T_n, t_i) = J(T_n) \times \Delta$. This procedure was applied to three different robots illustrating the three different behaviors of granular space. Fig.5 is an example of three trajectories corresponding to targets $T_1, T_2, T_3$ in the case of the fourth axis of a Samsung robot FAR2. This robot has an angular control panel resolution of $0.01^\circ$. The data is then processed statistically to draw a histogram displayed in fig.6.

Fig.7 and fig.8 correspond to the same idea but with the first axis of a Kuka robot IR384 with an angular control panel resolution of $0.001^\circ$.

Fig.9 and fig.10 are the results for the first axis of a EPSON RS450 with an angular control panel resolution of one pulse corresponding to $0.0011^\circ$ and trajectories of 50 samples.

D. Standard deviation estimation

A closer look inside the $E(T_n, t_i)$ expectation fluctuations over a long period of time show that the expectation drift can be set aside for a time window of a dozen shots but must be considered hereafter. This observation gives a good hint to build a robust estimator of the standard deviation.

Let $S_n = X_n^{(t_{i+1})} - X_n^{(t_i)}$ be the jump process. If $t_{i+1} - t_i$ is small, then the expectation $E(T_n, t_{i+1})$ and $E(T_n, t_i)$ stay very close. Consequently their difference can be ignored compared to the standard deviation value $\sigma$. In this case, the
rv process $S_n$ is time-independent and can be considered as one Gaussian rv with a nil expectation and a $\sqrt{2}\sigma$ std. This results also from the fact that the two rv are independent.

An important advantage of this estimation is the possibility of taking into account larger portions of the trajectories and not just 30 samples as in the ISO norm, increasing the precision of the estimator [6].

At this stage, let us point out that the standard deviation is not necessarily proportional to the joint resolution and it is worth noticing this before comparing granular space modeling with other approaches.

IV. BI-DIMENSIONAL SPATIAL RESOLUTION

Let us study the concept of spatial resolution in the case of a two link robot using rotational actuators controlled by the angles $(\theta_1, \theta_2)$ displayed in fig.11. We are interested in a small subset around a given location $P_0$ in the workspace and want to describe the micrometric stochastic structure of the space in this area. The width of this subset is a few micrometers and is very small compared to the surface of the space in this area. The width of this subset is a few

The forward kinematic function links Cartesian coordinates with angular coordinates $(x, y) = f(\theta_1, \theta_2)$. The Jacobian matrix $J$ resulting from the differentiation of the function $f$ links the small angular variations to the small Cartesian variations: $(dx, dy) = J(\theta_1, \theta_2) \times [d\theta_1, d\theta_2]^T$.

The first and second actuator are controlled using the same kind of process we studied previously. So it is possible to define an angular resolution for the first actuator that will then correspond to a spatial resolution $SR_1$ in the Cartesian space for the point $P_0$. Similarly $SR_2$ can easily be defined corresponding to the second actuator influence. But $SR_1$ and $SR_2$ must be modeled as vectors $SR_1$ and $SR_2$ and can easily be expressed using the Jacobian function: $SR_1 = J(P_0) \times [\Delta_1, 0]^T$ and $SR_2 = J(P_0) \times [0, \Delta_2]^T$.

$SR_1$ and $SR_2$ define a parallelogram displayed in fig.12 that constitutes the spatial resolution pattern for this bi-dimensional situation.

Now let us suppose that in order to hit point $P_0$, a target $T_1 = (\theta_1, \theta_2)$ is chosen. All the final positions corresponding to this target will be enclosed in a confidence ellipse $(E_1)$. Let $O_1$ be the center of $(E_1)$. If the target is slightly changed by increasing the 1st actuator target by a panel control resolution increment $\Delta_1$, then all the final positions corresponding to the new target $T_2 = (\theta_1 + \Delta_1, \theta_2)$ are enclosed in the confidence ellipse $(E_2)$ whose center $O_2$ is built using the relation $O_1O_2 = SR_1$. Similarly ellipses $(E_3)$ and $(E_3)$ with centres $O_4$ and $O_3$ respectively correspond to the targets $T_3 = (\theta_1, \theta_2 + \Delta_2)$ and $T_3 = (\theta_1 + \Delta_1, \theta_2 + \Delta_2)$.

For a better understanding of the situation, the size of the ellipses chosen are small but of course they can be larger depending on the granularity ratios of the first and second actuators.

It is easy to understand that the situation is close to the one analysed above, with a slight difference however, due to the fact that the repeatability confidence area is no longer a disk. Unfortunately this slight difference will not allow us to give a simple analytic expression based on the repeatability ellipse characteristics (semi-axes for instance) or spatial resolution pattern.

The maximum error position for a commanded position $P_0$ can be found in two steps. The first step is to determine the best target $i_0 = \arg \min_x \{\sup \{\text{dist}(P_0; U)/U \in (E_i)\}\}$ and in the second step the result will be $\varepsilon_{\text{max}}(P_0) = \sup \{\text{dist}(P_0; U)/U \in (E_{i_0})\}$.

Now finding the absolute maximum error position is a more difficult problem, but it can be solved by searching for the solution of an optimisation problem: to find the position $P$ in the parallelogram $O_1O_2O_3O_4$ in order to maximise $\varepsilon_{\text{max}}(P) = \sup \{\text{dist}(P; U)/U \in (E_{i_0})\}$ where
\[ i_0 = \arg\min_{i \in \mathbb{N}} \{ \text{Sup}\{ \text{dist}(P; U)/U \in (E_i)\} \}. \]

It is of course possible to generalize the computation of the maximum position error in the case of 6 dof robots aimed to positioning and orientating a tool in the extended Cartesian space \( \mathbb{R}^6 \). The methodology to compute the maximum position error will be quite similar to the case detailed above.

But, the case where the robot is redundant and has more than 6 dof is more complex. The difficulty is that the size and orientation of the ellipsoid associated with the targets can no longer be considered constant and moreover the spatial resolution vectors themselves cannot be considered constant; at this stage, the Jacobian matrix must be abandoned and the forward kinematic function should be used to find all the ellipsoid centers close to the commanded position. Numerical errors and uncertainty in the forward kinematic function can worsen the problem. Generally, here we can consider that the micrometric topology will evolve to a highly redundant structure and the computation of the maximum position error will in fact correspond to the repeatability error.

V. COMPARISON WITH OTHER MICROMETRIC POSITIONING MODELINGS

In this section, we will compare granulous space modeling to the other usual modelings of micrometric positioning and analyse the main differences between them.

A. Input and Output error modeling

In recent papers [7], [8], some authors have tried to characterize accuracy and employ the expressions “input and output error”, but the definitions of these concepts remains vague. It seems that they consider a target in the control or sensor space to be associated with an area in the Cartesian space. In this case, the output error would be closer to repeatability than accuracy. Spatial resolution is not taken into account in this approach so it is impossible to link the results to the maximum error except in one special case where the width of the repeatability confidence set and the spatial resolution are equal.

Fig.13 illustrates this modeling in the case of a two-dimensional process. It corresponds to the preceding fig.12 where the ellipses are replaced with a pavement of parallelograms \((P_1)\) to \((P_4)\). Hence the maximum position error is the diameter of the parallelogram and in the general case, it is the diameter of the convex envelop of the \(2^n\) points \( f(\Theta \pm \Delta) \) as explained in more detail in [9].

This modeling can be used in a first approach but is not accurate in general, because most of the time, the width of the repeatability confidence set and the spatial resolution are different. It cannot be used for all the topologies of granulous space but can give an upper limit of the maximum position error if the granularity ratio is high: \( \tau \gg 1/6 \).

B. Micrometric positioning modeling based on interval analysis

Some authors [10], [11] have used interval analysis to design robots with higher accuracy. More precisely, joint clearance and manufacturing tolerances are studied to compute the worst error in the workspace. It does not apply to the context of the present paper where extra information from external sensors is available.

However, this modeling could be adapted to give a good approximation of the maximum error. It can accept and take into account a difference between the width of the repeatability confidence set and the spatial resolution. For example, in the case of a small granularity ratio, the repeatability parallelograms \((P_1)\) to \((P_4)\) are smaller than the parallelogram \(O_1O_2O_3O_4\) modeling the spatial resolution, as displayed in fig.14. Analysing this scheme it is possible to compute the maximum position error using geometry and convex optimisation. Let us point out here that the output error parallelogram is not necessarily homothetic to the spatial resolution parallelogram.

So this modeling can be useful if the repeatability phenomenon and the spatial resolution are clearly identified. The Jacobian matrix could be used to compute the spatial resolution pattern and the interval analysis to draw the repeatability parallelograms.

In this case, it is important to notice that the concept of granulous space is also present. Indeed when the granularity ratio is low, it can be easily understood that some locations
are impossible to attain in the Cartesian space, for instance the point $O$ would never be reached.

VI. GRANULOUS AND HYPERREDUNDANT SPACES

Let us study now the two specific cases corresponding to a highly granulous space or a hyperredundant space and try to simplify the computation of the maximum position error.

If the space is highly granulous, $\tau \ll 1/6$, the final positions are concentrated in one small area and the maximum position error is directly linked with spatial resolution. For this, we can build for instance the Voronoi structure attached to the spatial resolution pattern. It consists of polyhedra whose centers are the spatial resolution pattern corners and divides the space into areas where the points are closer to one corner than the other. The maximum position error is then the largest dimension of the Voronoi polyhedra (maximal distance from one point of the polyhedra to the center).

If the space is highly redundant, $\tau \gg 1/6$, the ellipsoid centers are so close that their relative distance can be ignored considering their largest semi-axis. Then the maximum position error will be inferior to the ellipsoid's largest dimension termed maximax [12] and is directly linked with repeatability. But the difficulty here remains choosing the best target in order to minimize the maximum position error.

So these two extreme cases show that we need to consider both spatial resolution and repeatability to be able to compute the maximum position error. Once these concepts have been clearly defined, the difficulty of designing precise robots becomes apparant. Doing so implies taking into account other parameters than the usual kinematic parameters (topology and geometry), for instance the sensors’ location and resolution, the quality of the servocontrol, the control panel resolution. But this data depends on the manufacturer’s choices and is difficult to take into account in the a priori design stage. One compromise solution was proposed in [13] consisting of considering simultaneously geometry and control.

VII. CONCLUSIONS

In this paper, we considered that position information was available via external sensors and that it was possible to use it to chose a better target. We tried to quantify the maximum position error. First we proved that the maximum position error could not be computed using the usual accuracy and repeatability indices. It was necessary to introduce a definition of spatial resolution. A proper modeling of micrometric positioning must take into account both spatial distribution and repeatability phenomenon.

We chose to term this innovative paradigm of micrometric positioning 'granulous space modeling'. Granulous space is a major change in the way we apprehend the workspace micrometric structure. It was previously thought that the workspace was uniform in terms of being hit by the robot endpoint, but in reality, as proved in this paper, the workspace has holes, aggregates and hyperredundant areas even when the robot is supposed to be non-redundant! The granularity ratio was introduced to draw a typology and is useful to apprehend how large the holes and aggregates are in the micrometric structure. The granulous space paradigm can be used to compute the maximum position error in the micrometric scale and will complement efficiently the usual approaches (input/output errors and interval analysis) which cannot describe the variety of the stochastic topology.

This granulous space modeling also paves the way to a better control of robots to achieve minute positioning and we already have interesting results in this field and are able to quantify very precisely the final confidence area.

REFERENCES