Cross $\Psi_B$-energy operator-based signal detection

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In this paper, two methods for signal detection and time-delay estimation based on the cross $\Psi_B$-energy operator are proposed. These methods are well suited for mono-component AM-FM signals. The $\Psi_B$ energy operator measures how much one signal is present in another one. The peak of the $\Psi_B$ operator corresponds to the maximum of interaction between the two signals. Compared to the cross-correlation function, the $\Psi_B$ operator includes temporal information and relative changes of the signal which are reflected in its first and second derivatives. The discrete version of the continuous-time form of the $\Psi_B$ operator, which is used in its implementation, is presented. The methods are illustrated on synthetic and real signals and the results compared to those of the matched filter and the cross correlation. The real signals correspond to impulse responses of buried objects obtained by active sonar in iso-speed single path environments.

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I. INTRODUCTION

Signal detection and time-delay (TD) estimation between signals are important problems in communications and signal processing. Typical applications involve radar, sonar, machine fault diagnostics, biomedicine, and geophysics. For example, in sonar signal processing the difference in arrival time of a signal at two or more spatially separated receivers is used to estimate range and bearing of the source. A common method of signal detection based on TD involves cross correlation of the receiver output, whereby an estimate of the TD is given by the argument that maximizes the cross-correlation (CC) function. Although the CC method is the simplest similarity measure used in signal processing, it is not sensitive to nonlinear dependence of the signals. In practice, the interaction between signals may be nonlinear. Furthermore, sensor characteristics may also cause nonlinear distortions. Consequently, the maximum CC may not correspond to the maximum signal interaction. In this case, the resulting TD may be erroneous. To tackle this problem a new similarity function that measures the nonlinear interaction between signals is proposed. The method is based on the cross $\Psi_B$-energy operator, recently introduced by the authors. The $\Psi_B$-energy operator is derived from a second energy-like function, called the Cross Teager-Kaiser Energy Operator (CTKEO) which measures the interaction between two real time functions. Compared to the CC, $\Psi_B$ includes temporal information and accounts for relative changes of the signal by using the first and the second derivatives of the signal. Since it is based on a nonlinear operator, $\Psi_B$, the proposed method can be viewed as a nonlinear matched filter and we note it MBF (for Matched $\Psi_B$ Filter). $\Psi_B$ can also be used to “detect” the presence of known signals as components of more complicated signals by measuring how much one signal is present in another one. Thus, the $\Psi_B$ operator can be used as a strategy for signal detection. As shown in Ref. 3, $\Psi_B$ or methods based on $\Psi_B$ are well suited for nonstationary signals such as mono-component AM-FM signals. Based on the CTKEO, both the $\Psi_B$ operator and the associated methods for TD estimation or signal detection are limited to narrowband signals.

The paper is organized as follows. In the following section the $\Psi_B$ operator is presented. Section III deals with the discrete form of $\Psi_B$ which is the basis of its numerical implementation. In Sec. IV a $\Psi_B$-based signal detection framework is introduced followed by the $\Psi_B$-based estimation approach in Sec. V. A pseudo code of the MBF is presented in Sec. VI. Simulation and experimental results are presented in Sec. VII and concluding remarks are stated in Sec. VIII. We illustrate the method with an underwater acoustics application, where each signal is the impulse response of a buried object obtained by active sonar in a single path environment.

II. THE CROSS $\Psi_B$-ENERGY OPERATOR

Recently the CTKEO has been extended to complex-valued signals and an operator called $\Psi_B$ introduced. Given two complex signals $x(t)$ and $y(t)$, $\Psi_B$ is dened as follows:
where

$$\Psi_c(x,y) = 0.5[\Psi_c(x,y) + \Psi_c(y,x)]$$  \hspace{1cm} (1)$$

and

$$= 0.5[x^*y + y^*x] - 0.25[x^*y + x^*y + y^*x + y^*x]$$  \hspace{1cm} (2)$$

where $$\Psi_c(x,y) = 0.5[x^*y + y^*x] - 0.25[x^*y^2 + x^*y]$$. It has been shown that the cross $$\Psi_B$$-energy operator of $$x(t)$$ and $$y(t)$$ is equal to the cross-Teager energies of their real and imaginary parts.\(^2\)

$$\Psi_B(x,y) = \Psi_B(x_r,y_r) + \Psi_B(x_i,y_i)$$  \hspace{1cm} (3)$$

Thus, from $$\Psi_B$$ we obtain

$$\Psi_B(x_r,y_r) = x_r y_r - 0.5[x_r^2 y_r^2 - x_r^2 y_r + y_r x_r y_r]$$,  \hspace{1cm} (4)$$

where $$x(t) = x_r(t) + jx_i(t)$$ and $$y(t) = y_r(t) + jy_i(t)$$. The complex form of the signals is obtained using the Hilbert transform. For $$\Psi_B$$ implementation, different derivative approximations can be used.

III. DISCRETIZING THE CONTINUOUS-TIME $$\Psi_B$$ OPERATOR

Discretized derivatives of the continuous $$\Psi_B$$ operator are combined to obtain an expression closely related to the discrete form of the operator $$\Psi_B$$, and operating on discrete-time signals $$x(n)$$ and $$y(n)$$. Different sample differences can be used, but only the two-sample backward difference is detailed herein. For simplicity, we replace $$t$$ by $$nT_s$$ where $$T_s$$ is the sampling period, and $$x(t)$$ with $$x(nT_s)$$ or simply $$x(n)$$.

$$\dot{x}(t) \rightarrow [x_k(n) - x_k(n-1)]/T_s$$

$$\ddot{x}(t) \rightarrow [x_k(n) - 2x_k(n-1) + x_k(n-2)]/T_s^2$$

$$\Psi_B(x_k(t), y_k(t)) \rightarrow x_k(n-1)y_k(n-1)/T_s^2$$

$$\Psi_B(x_k(t), y_k(t)) - 0.5 [x_k(n)y_k(n-2) + y_k(n)x_k(n-2)]/T_s^2$$

$$\Psi_B(x_k(t), y_k(t)) \rightarrow \Psi_B(x_k(n-1), y_k(n-1))/T_s^2$$

$$k \in \{i,r\}$$.  \hspace{1cm} (5)$$

The discrete form of $$\Psi_B(x(t), y(t))$$ is given by

$$\Psi_B(x(t), y(t)) \rightarrow [\Psi_B(x_k(n-1), y_k(n-1))$$

$$+ \Psi_B(x_k(n-1), y_k(n-1))]/T_s^2$$,  \hspace{1cm} (6)$$

where $$\rightarrow$$ denotes the mapping from continuous to discrete. Thus, from $$\Psi_B$$ we obtain $$\Psi_B$$ shifted by one sample to the left and scaled by $$T_s^2$$. It is easy to show that using the two-sample forward difference from $$\Psi_B$$, we obtain $$\Psi_B$$ shifted by one sample to the right and scaled by $$T_s^2$$. For both asymmetric two-sample differences, $$\Psi_B$$ is shifted by one sample and scaled by $$T_s^2$$. If we ignore the one-sample shift and the scaling parameter, we transform $$\Psi_B(x(t), y(t))$$ into $$\Psi_B(x(n), y(n))$$ as follows:

$$\Psi_B(x(t), y(t)) \rightarrow \Psi_B(x_k(n), y_k(n)) + \Psi_B(x_k(n), y_k(n))$$,  \hspace{1cm} (7)$$

The three-sample symmetric difference can also be used but it leads to a more complicated expression compared to asymmetric two-sample differences. Indeed, the asymmetric approximation is less complicated for implementation and faster than the symmetric one because it requires fewer operations.

IV. $$\Psi_B$$-BASED SIGNAL DETECTION

We motivate $$\Psi_B$$-based detection by considering the classical binary hypothesis testing problem encountered in radar or in sonar.\(^8\) Let $$s(t)$$ denote a baseband transmitted signal. The received signal $$R(t)$$ is processed over an interval $$[T_i, T_f]$$ to detect the presence of a target. The hypotheses on $$R(t)$$ are

$$H_0: R(t) = n(t), t \in [T_i, T_f]$$,

$$H_1: R(t) = \alpha s(t-t_0) + n(t), t \in [T_i, T_f]$$.  \hspace{1cm} (8)$$

Under the null hypothesis, $$H_0$$, the received signal contains only an additive noise, $$n(t)$$. Under the alternative hypothesis, $$H_1$$, a received time-shifted version of the transmitted signal $$\alpha s(t-t_0)$$ is received in the presence of noise where $$\alpha$$ denotes an unknown gain parameter. The unknown time, $$t_0$$, represents the delay of the received signal and corresponds to the unknown distance of the target. Let $$T = [T_{min}, T_{max}]$$ denote the possible range of values for $$t_0$$. The required observation interval is $$[T_i, T_f] = [T_{min}, T_{min} + T_{max}]$$ in this case. For any given value of $$t_0 \in T$$, the decision whether to reject $$H_0$$ is given by computing

$$T_B = \arg \max_{t \in T} \int_T \Psi_B(R(t), R(t))dt$$,  \hspace{1cm} (9)$$

where $$T_B$$ corresponds to the time of maximum of interaction between $$R(t_0)$$ and $$R(t)$$.\(^3\) $$T_B$$ is then compared to a threshold to determine the presence ($$H_1$$) or absence ($$H_0$$) of a target. Thus, the best detector calculates the interaction between the received signal and all possible time-shifted versions of the transmitted signal and picks the largest energy interaction as the basis for the detection decision. The location of the peak is the estimate of the unknown parameter $$t_0 = T_B$$.  

V. $$\Psi_B$$-BASED TIME-DELAY ESTIMATION

We have shown that the TD estimation problem measures the interaction between two FM signals by using the $$\Psi_B$$ operator.\(^3\) Consider a signal from a remote source being received in the presence of noise at two spatially separated receivers. The time histories of the receiver outputs, denoted by $$r_m(t)$$ and $$r_r(t)$$, are given by

$$r_m(t) = s(t) + n_m(t)$$,

$$r_r(t) = \beta s(t - (k-m)r) + n_k(t)$$,

where $$s(t)$$ is the signal waveform, $$n_m(t)$$ and $$n_k(t)$$ are the noise waveforms at the respective receivers, $$\beta$$ is an attenuation factor, and $$r$$ is the difference in wavefront arrival times at two consecutive receivers ($$k = 2, m = 1$$). We assume that
$n_m(t)$, $n_k(t)$ and $s(t)$ are mutually uncorrelated.\(^3\) Propagation TD between receivers $k$ and $m$ is given by $\Delta d/c = (k-m)\tau$, where $\Delta d = d_k - d_m$ is the path length difference, and $c$ is the sound speed in the medium. When the target is sufficiently distant from the receivers the wavefronts can be approximated by plane waves and the theoretical TD, $T_{\text{Theor}}$, is given by
\[
T_{\text{Theor}} = (k-m)d \sin \theta/c,
\]
where $d$ is the distance between two consecutive receivers and $\theta$ is bearing angle (See Fig. 1).

VI. PSEUDO CODE OF MBF

The complex form of the signal is obtained using the Hilbert transform. The MBF implementation involves the following steps:

**Inputs:**

*Emitted signal:* $s(p) = s_x(p) + js_y(p)$, $p \in \{1,2,\ldots,w\}$

*Received signal:* $r(n) = r_x(n) + jr_y(n)$, $n \in \{1,2,\ldots,N\}$

where $a$ and $b$ indicate the real and imaginary parts of the complex signal respectively. $w$ and $N$ are time durations of $s(t)$ and $r(t)$, respectively, and $z=1$ to $M$ denotes the index of the sliding window.

**Outputs:** $T_B$.

![FIG. 1. (Color online) Geometry used to estimate the time delay associated with plane waves.](Image)

![FIG. 2. (Color online) Linear chirp test signals (left) and their respective IFs (right).](Image)

![FIG. 3. (Color online) Interaction measure between $s(t)$ and $y(t)$.](Image)

![FIG. 4. (Color online) Interaction measure between $x(t)$ and $y(t)$ using CC.](Image)
the emitted signal and the echo of the second signal. The IF of the first signal increases linearly with time while that of the second signal decreases with time. The interaction between \( x(t) \) and \( y(t) \) is calculated using Eq. (2). Figure 3 shows the energy of each signal and the energy of their interaction. The maximum of interaction corresponds to the instant when the two IFs intersect (Figs. 2 and 3) and also where the energy of \( x(t)(\Psi_B(x,x)) \) and that of \( y(t)(\Psi_B(y,y)) \) are equal (Fig. 3). The point where the IFs intersect is located at \( t=125 \). Maximums of interaction between \( x(t) \) and \( y(t) \) occur at \( t=125 \) and \( t=240 \) for \( \Psi_B \) and CC, respectively, as shown in Figs. 3 and 4. The CC fails to point out, as expected, the point of maximum of interaction. This result shows that the CC measure is insensitive to nonlinear dependency between \( x(t) \) and \( y(t) \). Away from the point where the IFs cross, the amplitude of the interaction decreases because there is less similarity between the two signals. As the IFs converge from the time origin to their intersection, the interaction intensity of the signals increases and the maximum of interaction is achieved at the intersection. This example shows that \( \Psi_B \) is more effective with nonstationary signals than the CC. This is due to the fact that \( \Psi_B \) is a nonlinear operator while CC is linear.

In this simulation an example of delay estimation performed between two signals is presented. Two synthetic ref-

\[
\begin{array}{cccccccc}
\text{SNR}=-6 \text{ dB} & \text{SNR}=-2 \text{ dB} & \text{SNR}=1 \text{ dB} & \text{SNR}=3 \text{ dB} & \text{SNR}=5 \text{ dB} & \text{SNR}=9 \text{ dB} \\
\hline
\text{Signals} & \text{MBF} & \text{MBF} & \text{MBF} & \text{MBF} & \text{MBF} & \text{MBF} \\
\text{SNR} & \text{MF} & \text{MF} & \text{MF} & \text{MF} & \text{MF} & \text{MF} \\
\hline
s_1(t) & 300 \pm 1 & 300 \pm 1 & 300 \pm 1 & 300 \pm 1 & 300 \pm 1 & 300 \pm 1 \\
\hline
s_2(t) & 300 \pm 2 & 300 \pm 2 & 300 \pm 1 & 300 \pm 1 & 300 \pm 1 & 300 \pm 1 \\
\end{array}
\]
reference signals, $s_1(t)$ and $s_2(t)$, with size window (in samples) \(w\) set to 65 and 81, respectively, are shown in Figs. 5(a) and 5(d). The received signals, $r_1(t)$ and $r_2(t)$ shown in Figs. 5(b) and 5(e), are obtained by time-shifting 300 samples, adding noise, and attenuating the reference signals. In this example, \(\alpha\) is set to 0.95. Outputs of the MBF are shown in Figs. 5(c) and 5(f) indicating a net maximum at \(t=T_B\). As expected, the peak of the function \(f(t)\) is located at \(T_B=300\). Table I lists the \(T_B\) values calculated for $s_1(t)$ and $s_2(t)$ with different signal-to-noise-ratios (SNRs) ranging from −6 to 9 dB, with \(\alpha\) set to 0.7. Each value of Table I corresponds to the average of an ensemble of 25 trial TD estimates. These results show that the performance of the MBF is very close to that of the MF. Both methods point to the same theoretical value for \(T_B\).

In this third example we consider the TD estimation in the case of a linear array composed of 20 uniformly spaced sensors. Each received signal sensor corresponds to the back-scattered echo of a punctual target (acoustic source). Observed sensor output signals are shown in Fig. 6(a) as two-dimensional plots (time sensors). The delay estimation is performed between the first sensor of the array, taken as the reference sensor, and each sensor of the array. The MBF delay estimation, of the received signal on two sensors, versus the sensor indexes along the linear array is shown in Fig. 6(b). Estimated delays, for all array sensors, are plotted as a function of the position indexes of the sensors along the array. Figure 6(b) shows a perfect agreement between theoretical estimation using Eq. (10) and that of the MBF.

B. Real signals

We demonstrated the performance of the $\Psi_B$ operator on real data. These data are acoustic measurements conducted in a tank with a linear array of 20 sensors (\(n=20\)) where air-filled cylindrical objects are slightly buried under the sand bottom. Two cylinders were used for Data set 1 and one cylinder was used for Data set 2. The remaining parameter settings were \(c=1485\) ms\(^{-1}\), \(d=2\) mm, \(\theta_{\text{Data1}}=22^\circ\), \(\theta_{\text{Data2}}=64.15^\circ\), frequency band [150 kHz, 250 kHz] and sampling rate = 2 MHz. Sensor outputs of signals (backscattered echoes) arriving from one (Data2) and two (Data1) cylinders are shown in Figs. 7 and 8, respectively. These signals correspond to the output of the first sensor of the array and are selected as reference signals. Both the MBF and the MF

![FIG. 6. TD estimation results (solid line: MBF method, dotted line: Theoretical method). (a) Time-sensors representation in synthetic case. (b) Estimated TD versus the sensor position indexes along the array in synthetic case.](image)

![FIG. 7. Example of one buried target. The selected reference signal of Data2.](image)

![FIG. 8. Example of two buried targets. The selected reference signal of Data1.](image)
methods, applied to the filtered signals, are implemented in the time domain. Signals are smoothed using a third-order Savitzky–Golay filter over a moving window of width set to 51. TDs are estimated using the Theoretical [Eq. (10)], MBF, and the MF methods. In each case delay estimation is performed between the first sensor of the array and the remaining ones. Root mean square error (RMSE) between pair of sensors for Data1 and Data2 is reported in Table II:

\[
\text{RMSE}_{(A-\text{Theor})} = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (TDA(i) - T_{\text{Theor}}(i))^2}
\]  

C. Analysis

As shown in Fig. 9(b), for Data set 2 there is a perfect agreement (except for sensor 2 where the error is of one sample) between the MBF and the Theoretical method [Eq. (10)]. This is confirmed by the RMSE value (5.26%). The RMSE of the MF is 3.42 times higher than that of the MBF. Figure 9(a) shows that, for Data1, the TD estimated by the MBF and the MF deviate moderately from TD values given by Eq. (10). Note that the MBF performs slightly better than the MF with a RMSE of 21.7%. For both Data1 and Data2, globally, the MBF performs better than the MF. This may be due, even partially, to the nonlinear relationship between the signals that linear method such as the MF cannot account for. The mismatch between expected TD and MBF TD values may be due to the estimation error of the bearing angles \( \theta_{\text{Data1}} \) and \( \theta_{\text{Data2}} \) and to \( c \), the sound speed in the water, which depends on the temperature. It is important to keep in mind that there is no odd way for estimating the TD value. The method based on Eq. (10) can be used, in the far field case, as a reference method if we have good measures of both \( \theta \) and \( d \).

VIII. CONCLUSION

In this paper, two methods, based on the \( \Psi_B \) operator, for signal detection and time-delay estimation [called Matched \( \Psi_B \) Filter (MBF)] are introduced. MBF measures how much one signal is present in another one. The discrete version of the continuous-time operator \( \Psi_B \), which is used in its implementation, is presented. Preliminary results of signal detection and TD estimation and their comparison with matched filter and a reference method are presented. These signals show that \( \Psi_B \) is sensitive to nonlinear dependencies between signals compared to the classical CC method. For signal detection the MBF gives the same results as the MF for different SNRs. For TD estimation, the MBF results are very close to those of the reference method. The result for real acoustic signals shows that the MBF globally outperforms the MF. The processed signals are either noiseless or moderately noisy. For very noisy signals, the robustness of the MBF must be studied. As future work, we plan to use smooth splines to give more robustness to the MBF. To confirm the effectiveness of the MBF, the method must be evaluated with a large class of signals and in different experimental conditions including high noise levels and sampling rates and sample sizes. As future work, we plan to estimate TD values in the case of a nonuniform (distorted) array. We also plan to modify the proposed scheme to analyze situations in which signals and noises are mutually correlated.
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7However, we do not claim that MBF is an optimum detector of signal in white noise as the Matched Filter (MF). The main difference between the MBF and the MF is that the MF is linear measure while MBF is a nonlinear one.