Outlining A New Method To Quantify Uncertainty In Nitrogen Critical Loads

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Abstract

We highlight deficiencies and improvements of a nitrogen critical load model. An original model using logistic regression augmented observations with fictitious data. We replace that with actual data, and show how to incorporate uncertainty in nitrogen measurement into the modeling process. In the end, however, we show a basic logistic regression model has irremovable deficiencies, giving positive probability of harmful effects of nitrogen even when no nitrogen is present.

Keywords: critical loads, logistic regression, nitrogen, uncertainty

1 Introduction

Nitrogen loads play a decisive role in environmental policies. Critical loads (CL) for nitrogen are usually defined as follows: "A quantitative estimate of an exposure to one or more pollutants below which significant harmful effects on specified sensitive elements of the environment do not occur according to present knowledge" [3].

As a means to fathom the actual modelling of CLs for nitrogen, we will explore the methods of [1]. We will subsequently present an improvement of their methods, and suggest new directions modeling should take.

Their method was as follows:

Collect data from a series of planned nitrogen experiments, in which known amounts of nitrogen were added to background atmospheric amounts placed on small plots. Various measures of growth of plant matter on these plots were then measured. If the growth was higher on experimental than control (no added nitrogen) plots, in a statistical sense, an adverse or harmful effect was noted. The amounts added were then scaled from the small plots up to the hectare.

They produced the following table:

Table 2. Summary of effects observed at given N deposition rate

Reference	Location	Effect (0/1)	First level of N deposition where effect observed (kg ha^{-1} yr^{-1})	Lowest estimated background deposition (kg ha^{-1} yr^{-1})
Aerts et al. (1992)	N Sweden	1	20.6	2
Aerts et al. (1992)*	S Sweden	0	49	9
Berendse et al. (2001)	Finland	1	34	4
Berendse et al. (2001)	Sweden	1	38	8
Berendse et al. (2001)	Switzerland	1	48	18
Berendse et al. (2001)	Netherlands	1	69	39
Gunnarsson & Rydin (2000)	S Sweden (Akhultmyren)	1	10	7.2
Gunnarsson & Rydin (2000)	S Sweden (Kopparasmyren)	1	10	7.2
Gunnarsson & Rydin (2000)	N Sweden (Luttumyren)	1	10	4.2
Norbakken et al (2003)	Norway	1	12.9	7.9
Bragazza et al. (2004)	Pan-Europe	1	10	1
Phuyal et al. (2008)	Scotland	1	64	8
Tomassen et al. (2003)	Netherlands	1	2.5	0
Wiedermann et al. (2009)	N Sweden (LD)	1	15	2
Wiedermann et al. (2009)	S-N Sweden	1	8	2
Breeuwer et al. (2009)	Sweden	1	40	0
Limpens et al. (2003)	Netherlands	1	40	0
Heijmans et al. (2001)	Netherlands	1	50	15
Redbo-Torstensson (1994)	Sweden	1	11	0.6

*Observational study. Height growth and productivity of *Sphagnum* increased with applications of P but not N; due to previous high background deposition of N it was considered that the system had become P limited and was therefore not responding to subsequent additions of N.

Figure 1: Table 2 from [1]. The explanation of the columns is in the text.

The "Reference" points to the papers from which the experimental data was extracted, and where the experiments were performed at the "Location." The "Effect" was whether the nitrogen-added plots had greater statistical plant growth (1) than the control plots, or not (0), as noted in the references.

The "First level of N deposition where effect was observed (kg ha^{-1} yr^{-1})" is the amount of nitrogen scaled up from the small-plot experiments, and "Lowest estimated background deposition (kg ha^{-1} yr^{-1})" was also gleaned from the references.

After reviewing the papers referenced, there is room for different interpretations of the statistical results, leading to values different to those presented in the Table. For example, Banin [1] used the lowest background nitrogen levels, but a good case can be made to pick the average value observed. However, for the purposes of our simple demonstration, we take all values here as they are presented in the Table.

The next step was to estimate a function at which a known amount of nitrogen, in (kg ha^{-1} yr^{-1}), corresponded to a probability of an harmful effect. The level of 20% was picked as a threshold requiring action. A standard logistic regression was picked for this function.

Banin used only the nitrogen-added data in the model data and not the background levels per se. This turns out to be a crucial point. Since there was only one instance of an effect = 0 in the added nitrogen column, the logistic regression's parameters could not be estimated (this is a standard statistical limitation). To overcome this difficulty, the authors *added* 90 zeros to both the effect and levels of added nitrogen. This represents a sort of pseudo data. In other words, the authors padded the 19 data points with 90 fictitious observations of nitrogen = 0, and 90 fictitious observations of effect = 0.

The authors gave no justification for the number of fictitious data points used (why not 80? why not 100?). As for adding the fictitious data itself, they surmised that no nitrogen would incur no defined harmful effect *of* nitrogen, which is surely true.

2 Suggested Modeling Approach

The data need not be padded with zeros. In place of the pseudo data, the observed background levels could be instead, which are actual measures and associated with effect = 0 (no harmful effects due to nitrogen).

The substitution of the fictitious zeros with the actual background rates represents the first point of departure from our new proposed method with theirs.

The second is to account for the uncertainty in the measures themselves. This arises in two ways.

First, in the references themselves, the nitrogen values are given not as certain values, but values with a plus-and-minus attached, or with standard deviations or other statistical measures of variability (usually because of variability in the background measurements). We intend to use this variability, though since we do not yet have a complete survey of all references (those from the Table plus quite a few others), we do not know what the variability is for all entries in the Table.

Merely for demonstration purposes, we take the square root of the Table nitrogen entries as representing the standad deviation (square root of the variance). This is because in the data we have collected so far, this is a reasonable if imperfect approximation. For example, a mean of 20.6 kg ha^{-1} yr^{-1} (the first entry) is assigned a standard deviation of 4.54 kg ha^{-1} yr^{-1}. Again, we stress this approximation is only for the purposes of illustration.

Second, to derive the correct variances from the reported data in the references, we need to account for the scaling of the plot-sized nitrogen values to the hectare. This scaling induces variability that must be accounted for. This is simply because we can't be certain the amounts added to a square-meter plot exactly scales up a hectare, which are 10,000 times larger. Here we use the approximation that the amounts of nitrogen can be represented with normal distributions.

The amounts of nitrogen added were in g m^{-2}, but with times of experiments not yet noted. Converting to kg ha^{-1} yr^{-1} amounts to a factor of 10, as long as we assume the time of the experiments is the same, which it likely was not for each. We have yet to explore this, but will in future efforts. In any case, given the plot-sized variance is v_p, it is easy to figure the variance of the hectare-scaled data, which is $v_h = 10^2 v_p$, a standard calculation. Using that on a few entries of the table gives rise to the square-root approximation mentioned earlier.

Here in Fig. 1 are the methods of [1] using added zeros (0-padding) compared to a second logistic regression using the background low rates instead (all with effect = 0). The variability in measurement is not yet accounted for here. The 0-padded data is in black, and the background-added-data is in green. This plot represents the central estimate of the logistic regression in a thick line, and the uncertainty due to the parameter estimation in thin lines; i.e. the 95% confidence intervals. A horizontal line at 20% is overlaid.

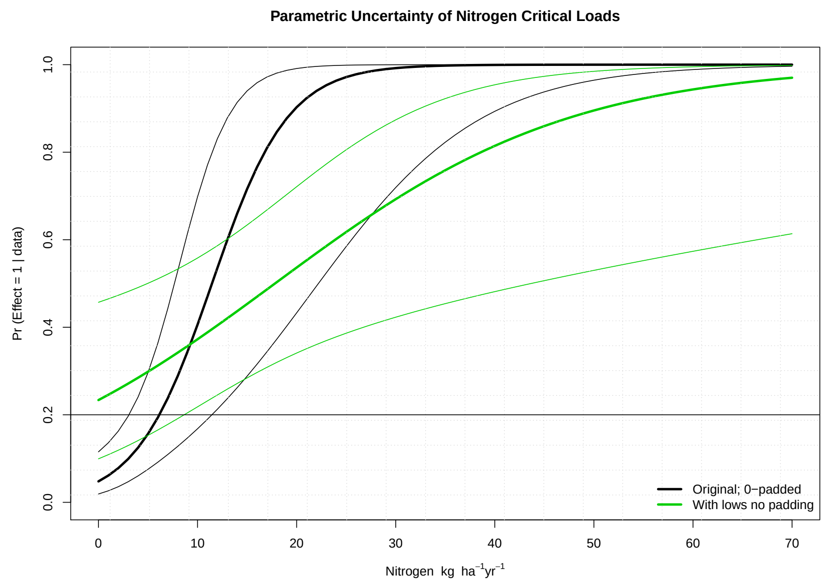

Figure 2: Parametric uncertainty of nitrogen critical load uncertainty, using 0-padded (black) and background low levels (green), with central estimates (thick lines) and 95% confidence intervals (thin lines).

The 0-padded estimate crosses the 20% threshold at about 7 kg ha^{-1} yr^{-1} of N, with a range of about 2 to 11 kg ha^{-1} yr^{-1} N. The background-level data central estimate *begins above* 20%, with a range of 0 to about 9 kg ha^{-1} yr^{-1} N. This means that, even with background levels, and with *no* nitrogen whatsoever, there is an estimated greater than 20% chance of an harmful effect due to nitrogen.

This is, of course, not possible. Obviously, the answer is not nearly enough data is available, or that different interpretations can be given to the presently measured data, or in inadequacies of the model form itself. We think all explanations are partly true. However, none of these ideas are explored in this paper.

In any case, it is clear something has gone wrong with a model that gives positive probability of nitrogen having an ill effect at 0 levels of nitrogen. This result is also found (but not noted) in [1], as there was a definite positive probability of an ill effect with 0 nitrogen in the 0-padded data. I.e., the black line at 0 kg ha^{-1} yr^{-1} N is about 5% in their Fig. 2 (not shown here).

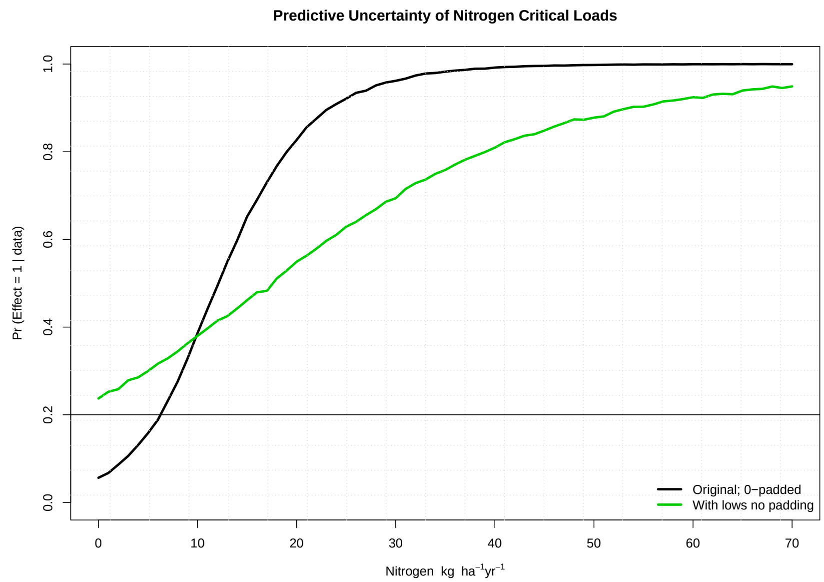

Figure 3: Predictive uncertainty of nitrogen critical load uncertainty, using 0-padded (black) and background low levels (green). This shows the probability of an effect with a given level of N.

Passing over these impossibilities, the next step is to account for the uncertainty in the parameter estimates, presenting the curves in a predictive way instead. This is pictured in Fig. 3.

This plot gives direct statements of Pr(Effect|N level, model, data) (the condition is shown as just "data" in the plots). This represents a Bayesian approach to the model, showing the predictive posterior distributions of the model, see [2]. This is a more actionable form than the standarad parametric uncertainty displays, because it's never clear what to with the confidence interval. Here, once a level of nitrogen is specified, direct probabilities are given, with no ambiguity in interpretation.

In any case, the story is the same. The level at which the threshold is crossed for the 0-padded data is about 5 kg ha^{-1} yr^{-1} N. And again, even with 0 nitrogen in the atmosphere, there is a greater than 20% of nitrogen causing an effect with the background-data model.

Even though it is by now clear the data, or the model or both, have difficulties, we present a picture of how to incorporate measurement uncertainty to the model, using the variance approximation mentioned above. In other words, the regression model now makes use of the plus-or-minus attached to each observation. This is pictured in Fig. 4

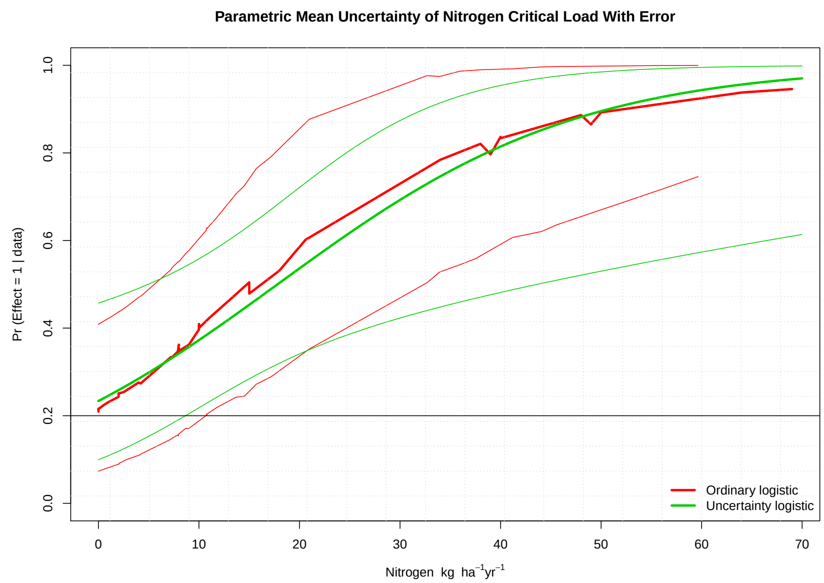

Figure 4: Measurement uncertainty added model.

Here we ignore the 0-padded data (which has no uncertainties). Because of the difficulties mentioned, and because our variance estimates are only approximations, it is the shape that is important here, and not the exact values, which are only an approximation. The green lines are the ordinary logistic regression with confidence interval, using the background low data. The red line is the model expanded to allow uncertainty in the measurements.

This model is choppier than the green because there is a great increase in the number of parameters due to the measurement uncertainties, which makes estimates a bit more difficult to make. In any case, it is clear there are many changes in the final model, compared against the ordinary, uncertainty-free model. These changes will very likely be present in the actual data, once it is compiled.

References

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