

# SHEAR MODULUS REDUCTION CURVES OF GUAYURIBA SANDS BY CYCLIC TRIAXIAL AND BENDER ELEMENT TESTS

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## ABSTRACT

A laboratory experimental programme based on cyclic triaxial and bender element tests was performed on three particle sizes of sands from Guayuriba River (Colombia). The material from the Guayuriba River consists mostly of quartzite of the group Quetame (Paleozoic shale formation). Tests performed under different confinements (50, 100, 200 and 400kPa) allow obtaining a general view of the cyclic behavior of this material. The influence of the grain size on the shear modulus reduction curve was identified. There were good continuity between the bender element and the cyclic triaxial results. Moreover, it was verified the applicability of a unified model to predict the shear modulus reduction curves for this sand for normalized shear modulus greater than 0.4. For the material tested, this model could be used to obtain a reasonable approximation of the shear modulus reduction curve. Two parameters are required: the initial shear modulus, for example by bender element testing, and the shear strain for a normalized shear modulus equal to 0.7, which is easily recognizable by a cyclic triaxial test without internal instrumentation. These tests are less expensive than the resonant-column method and other specialized techniques.

*Keywords: Bender Element Test, Cyclic Triaxial Test, Shear Modulus, Soil Stiffness*

## INTRODUCTION

Around the world, many authors have investigated the dynamic behavior of granular soils by laboratory testing. Kokusho [1], Santos and Gomes Correia [2], Zhou y Chen [3], Jafarzadeh and Sadeghi [4] are examples of that kind of investigations. In Colombia, there are some works with local materials, providing advances in this subject, for example Yamin et al [5], Garcia [6], Chaves [7], Rivas [8], who have made similar investigations about the behavior towards the phenomenon of liquefaction of sands of Tumaco (Chocó, Colombia) or the dynamic behavior in schists of Sabaneta (Antioquia, Colombia).

The shear modulus is one of the parameters required to estimate the dynamic response of soil layers. For example, the shear modulus is used in the assessment of seismic behavior of a sand deposit, to assess the risk from cases like the one in the middle of Atrato River (Colombia), where it presented the liquefaction of a dam formed by sandy material product of an earthquake presented on 17 and 18 October 1992 [9].

This paper aims to study the shear modulus in fluvial sands of the Guayuriba River (Colombia), by obtaining the shear modulus reduction curves. These curves were obtained by cyclic triaxial and bender element tests with different confining pressures. The results were analyzed, and a unified shear modulus reduction curve based on the model of Santos and Gomes Correia [2] has been obtained.

## THEORETICAL BACKGROUND

In this section, the most relevant concepts related to the subject of this research are presented, starting by the definition of the initial shear modulus and the reduction of the modulus along different strain levels. Special attention is devoted to the unified model used in this work.

### Initial Shear Modulus

The initial shear modulus ( $G_0$ ), is the parameter that defines the stiffness of a soil subjected to cyclic stresses in the very small range of deformations.  $G_0$  has been studied by different authors like Kokusho [1], Ishihara [10], Santos and Gomes Correia [2] and Richart et al. [11]. These researchers have found that a number of factors influence  $G_0$  in sandy soils. The most important factors are the effective confining pressure ( $\sigma'_0$ ) and the void ratio ( $e$ ), as shown in Eq. (1).

$$G_0 = A \left[ \frac{(B-e)^2}{(1+e)} \right] (\sigma'_0)^n \quad (1)$$

Where A, B and n are experimental constants that depend of the material. A compilation of different materials presented by some authors, shows that the most common values of B and n are 2.17 and 0.5, respectively. A fluctuates between 7000 and 14100 for  $G_0$  and  $\sigma'_0$  expressed in kPa [12].

**Modulus Reduction Curves**

The modulus reduction curve is a representation of the secant shear modulus divided by the initial shear modulus (i.e. the shear modulus at very small strain) as a function of the strain level [12]. The normalized curve is an advantageous representation because it can be simply multiplied by an appropriate experimentally determined  $G_0$  to obtain the shear modulus for any strain level. This curve synthesizes the nonlinear behavior of soils. Figure 1 shows the modulus reduction curve along the shear strain [13].

The modulus reduction curve is not unique. In the case of sands, its shape and location depend mostly of the confining pressure, number of cycles and drainage conditions [1, 14].

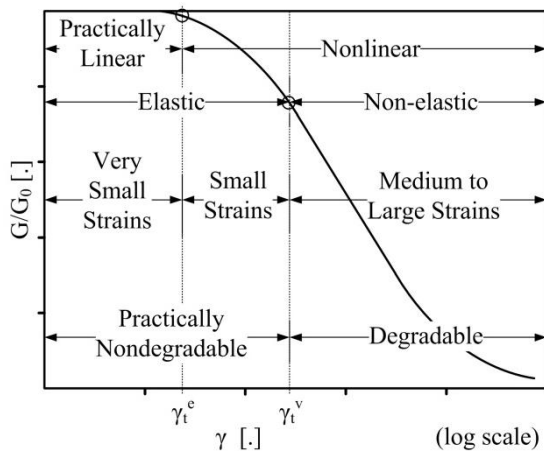


Fig. 1 Modulus reduction curve and zones of cyclic shear strain. Adapted from [13].

**Elastic And Volumetric Threshold Shear Strains**

The shear strain level can be classified in very small strains, small strains and medium to large strains [10]. The limit between very small and small strains was defined arbitrarily as the shear strain corresponding to  $G/G_0 \approx 0.99$  [13]. This strain receives the name of linear elastic cyclic threshold strain ( $\gamma_t^c$ ). The limit between small and medium strains is defined as the shear strain above which the microstructure is irreversibly altered by cyclic shearing [13]. This limit is known as the volumetric threshold shear strain ( $\gamma_t^v$ ) because from this value appear irreversible changes in the volume of the sample. Kokusho [15] concludes that  $\gamma_t^v$  is the shear strain for which  $G/G_0$  is between 0.6 and 0.85. Ishihara [10] proposes a mean value of  $G/G_0 = 0.7$ . Given the difficult to measure  $\gamma_t^v$  in practice, Santos [16] proposed to use the reference threshold strain ( $\gamma_t^r$ ), defined as the shear strain causing  $G/G_0 = 0.7$ . Relative locations of  $\gamma_t^c$  and  $\gamma_t^v$  are shown in Fig. 1.

**Unified Shear Modulus Reduction Curve**

A first proposal of a unified model was presented by Ishibashi and Zhang [17]. Based in many collected data, they proposed a set of formulas for all soil types, taking into account the effect of plasticity and confinement.

Santos and Gomes Correia [2] proposed a unified model by the normalization of the strain axis of the modulus reduction curve. The normalized shear strain ( $\gamma^*$ ) is computed by:

$$\gamma^* = \frac{\gamma}{\gamma_t^r} \tag{2}$$

Experimental data from Vucetic and Dobry [18] and Darendeli [19] was normalized using Eq. (2) and the results are presented in Fig. 2, showing that the model works for different soil plasticity and effective confining pressures.

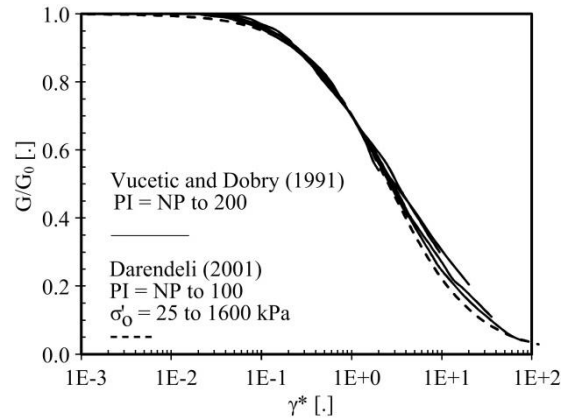


Fig. 2 Unified modulus reduction curves computed using data from Vucetic and Dobry [18] and Darendeli [19].

Based on a series of tests [2], it was demonstrated that data matches in a narrow zone delimited by the curves given by Eq. (3) and Eq. (4).

$$\frac{G}{G_0} = 1 \quad \text{for } \gamma^* \leq 10^{-2}$$

$$\frac{G}{G_0} = \frac{1 - \tanh\left[0.48 \ln\left(\frac{\gamma^*}{1.9}\right)\right]}{2} \quad \text{for } \gamma^* > 10^{-2} \tag{3}$$

$$\frac{G}{G_0} = 1 \quad \text{for } \gamma^* \leq 10^{-1}$$

$$\frac{G}{G_0} = \frac{1 - \tanh\left[0.46 \ln\left(\frac{\gamma^* - 0.1}{3.4}\right)\right]}{2} \quad \text{for } \gamma^* > 10^{-1} \tag{4}$$

Moreover, experimental data for plastic and non-plastic soils fit with good agreement with Eq. (5) [20].

$$\frac{G}{G_0} = \frac{1}{1 + (0.385\gamma^*)} \tag{5}$$

**MATERIALS, EQUIPMENT AND TESTING METHODS**

**Materials**

The material from the Guayuriba River consists mostly of quartzite of the group Quetame [21]. Three particle sizes of the same sand were used in the experiments, in order to analyze the influence of grain size on the results. The physical characterization included the determination of specific gravity of soil solids, particle-size analysis, maximum index density and minimum index density according to the ASTM standards [22]. Results of these laboratory tests are presented in Table I.

The cylindrical specimens (14 cm in height and 7 cm in diameter) were prepared by the dry pluviation method. Size and proportions of the specimens are in agreement with suggestions of previously-reported works [23-25].

Table I Physical properties of the materials

Property	Material		
	1	2	3
Minimum diameter [mm]	0.18	0.43	0.85
Maximum diameter [mm]	0.43	0.85	2.00
Uniformity coefficient [.]	1.58	1.42	1.39
Curvature coefficient [.]	0.93	0.94	1.00
Specific gravity of solids [.]	2.68	2.65	2.68
Void ratio [.]	0.81	0.91	0.99
Dry unit weight [kN/m <sup>3</sup> ]	14.8	14.1	13.5

**Equipment**

The dynamic test program was performed in a cyclic triaxial apparatus (Dynatriax, WF-Controls) with automatic control of cell and back pressure, equipped with submersible load cell and external displacement, volumetric change and pore pressure sensors, data acquisition unit and driver software with autonomous control of the tests stages.

The triaxial cell was equipped with a set of BE manufactured by the University of Western Australia [26]. The transmitter and receiver are located at the base cap and top cap of the triaxial device, respectively. The input signal is generated by a function generator (RIGOL, DG1022). A current amplifier stabilizes the signal and send it to the BE transmitter. The output signal of the BE receiver is amplified and both input and output signals are collected by a digital oscilloscope (Tektronics, 3S2012B). The BE system was calibrated using aluminum rods [27]. A schematic description of the BE system is presented in Fig. 4.

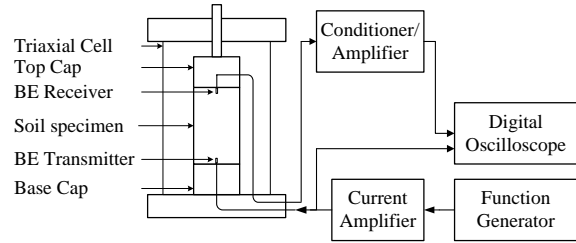


Fig. 4 Schematic of the BE system [27].

**Testing Methods**

Each soil specimen was installed in the cyclic triaxial cell, saturated, and then consolidated under a confining pressure of 50, 100, 200 or 400 kPa.

*Bender element tests (BE)*

BE tests were conducted by the following methodology:

- The function generator produces a linear sine sweep signal with frequency band between 1–20 kHz with duration of 40 ms and amplitude of 20 Vpp. This signal is sent to the BE transmitter.
- The digital oscilloscope acquires both input and output signals. The Fast Fourier Transform (FFT) computed in real time by the oscilloscope allows to identify the specific frequency in which the response of the BE receiver is higher.
- The function generator is turned to single-sine pulse mode with the frequency previously determined. The period of the signal is selected in order to allow enough time for the attenuation of the BE response before the next pulse. The output signals of a number of pulses are stacked in order to cancel out random noise, obtaining a clean response from the BE receiver.
- The arrival time is identified by the first inversion method [28]. The travel time (tt) is the difference between the arrival time and the time at the start of the sine pulse.
- The travel distance ( $L_{TT}$ ) is the height of the specimen minus the length of each BE. The initial shear modulus ( $G_0$ ) is computed by Eq. (6), where  $\rho$  is the mass density.

$$G_0 = \rho \left( \frac{L_{TT}}{tt} \right)^2 \tag{6}$$

Figure 5 shows a typical screen of the oscilloscope in the BE test. Channel 1 exhibits the single-sine pulse use to excite the BE transmitter and Channel 2 shows the output signal acquired by the BE receiver. Vertical cursors indicate the initial and arrival times used to compute the travel time.

According to different studies [12, 25, 29, 30], the shear strain level associated to the BE shear modulus is assumed to be  $5 \times 10^{-6}$ .

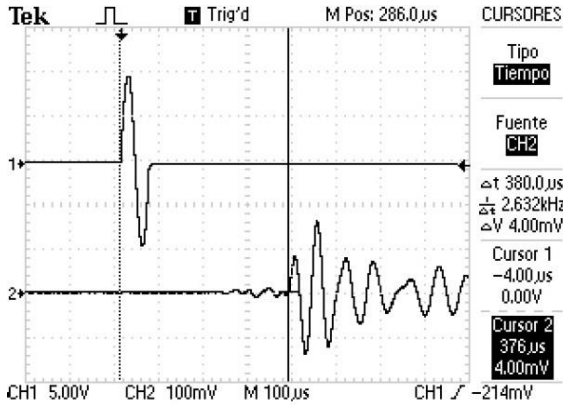


Fig. 5 Typical BE test showing input and output waveforms.

*Cyclic triaxial tests (CTX)*

After the bender element tests, cyclic triaxial tests were carried out according to the Method B (strain controlled) of ASTM D 3999 standard [31], with nine cyclic shear stages, each one with 40 sinusoidal cycles at frequency of 0.5 Hz. The strains were selected in a range of 0.028 mm ( $\gamma \approx 1.33 \times 10^{-4}$ ) to 1.4 mm ( $\gamma \approx 6.67 \times 10^{-3}$ ). The shear stress and shear strain are computed by Eq. (7) and Eq. (8), respectively.

$$\tau = \frac{q}{2} \quad (7)$$

$$\gamma = \frac{\epsilon}{1+\nu} \quad (8)$$

Where  $q$  is the deviator stress,  $\epsilon$  is the axial strain and  $\nu$  is the Poisson's ratio, assumed to be 0.5 for saturated sands. Fig. 6 shows an example of the hysteresis loop obtained by the cyclic triaxial test, where the shear modulus is obtained as the slope of the line connecting the two extreme points of the loop.

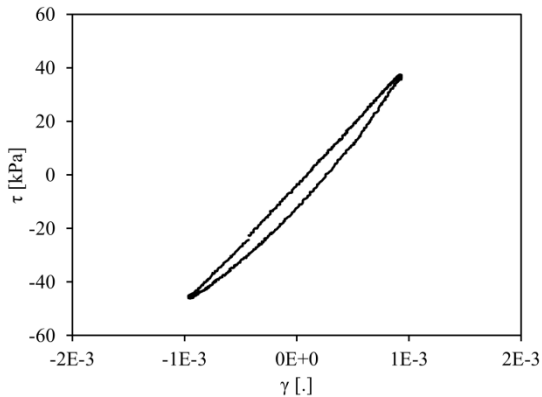


Fig. 6 Hysteresis loop of the cyclic triaxial test. Material 3,  $\sigma'_0 = 100$  kPa.

**RESULTS AND DISCUSSION**

Figure 7 shows the curves obtained for the three sands and the different effective confining pressures. The results obtained by bender element and cyclic triaxial tests show the influence of the particle size and the confining pressure on the shear modulus reduction curves. The gap between BE and CTX tests is remarkable, not only regarding to the shear strain level, but also concerning the magnitude of the shear modulus. The use of internal instrumentation on the CTX test could reduce this difference.

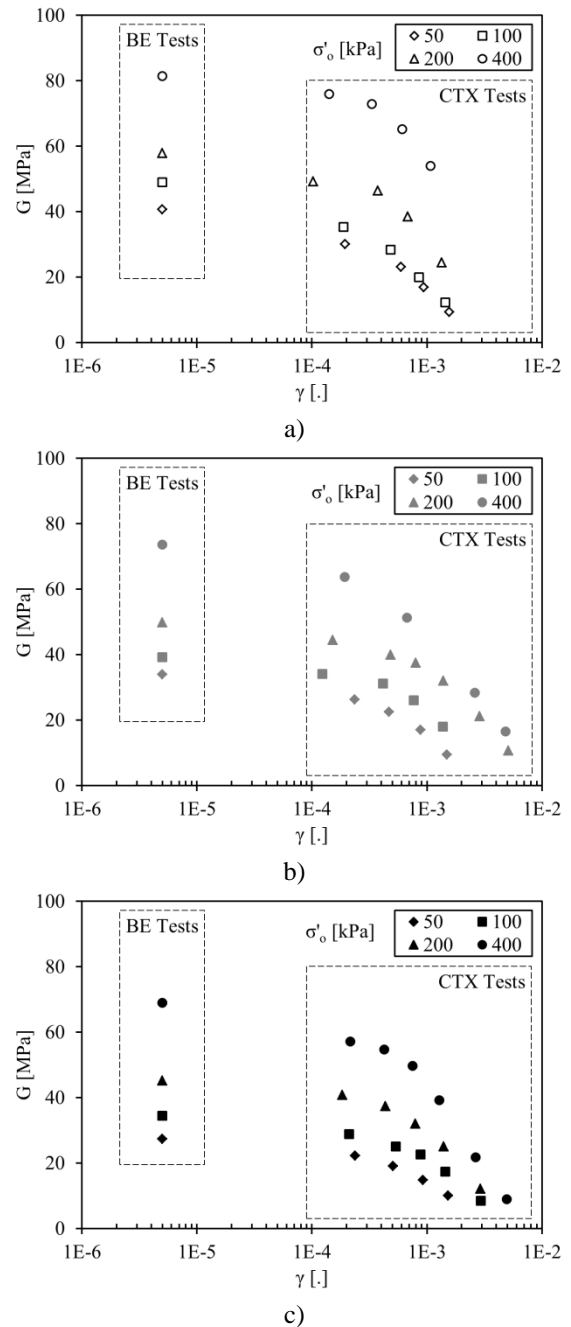


Fig. 7 Experimental modulus reduction curves: a) Material 1; b) Material 2; c) Material 3.

Shear moduli were normalized by their respective maximum shear modulus. These results are presented in Fig. 8. After a careful analysis of the figure, it is possible to observe that in general, the higher the confinement, the larger the normalized modulus for a given shear strain. This observation is in agreement with previous reported works [1]. However, the dispersion of result does not allow proposing a clear trend on this issue.

The reference shear strain ( $\gamma_t^r$ ) was estimated by interpolation of the data and the normalized shear strain ( $\gamma^*$ ) was computed by Eq. (2). These double-normalized curves are presented in Fig. 9, as well as the limits expressed by Eq. (3) and Eq. (4).

The influence of the assumed shear strain level in BE tests in results presented in Fig. 8 is insignificant because the normalized shear strain is computed using only CTX results and the magnitude of the shear modulus obtained by BE tests.

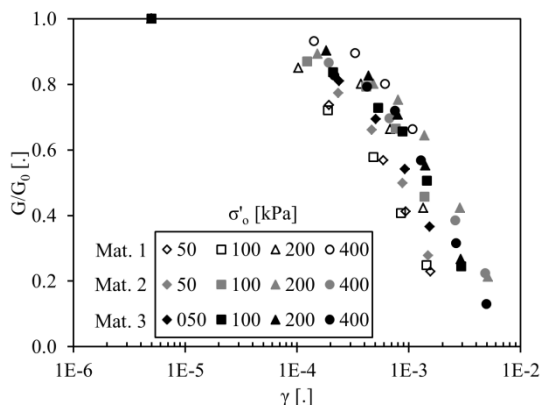


Fig. 8 Shear modulus reduction curves

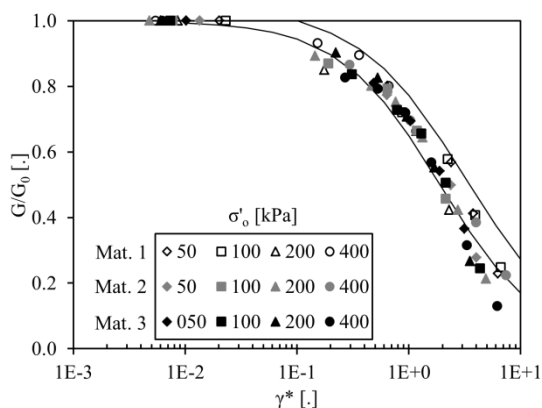


Fig. 9 Unified model for materials tested and comparison with limits proposed by Santos and Gomes Correia [2].

Deviation of the limits increases for normalized shear strains greater than 3, which is in average equivalent to a shear strain level of  $3 \times 10^{-3}$ . This

could be due that at this strain level, and for these loose specimens, the soil starts to exhibit cyclic liquefaction.

Results presented here suggest that a practical approximation of a shear modulus reduction curve could be obtained by a combination of BE tests and CTX tests.

## CONCLUSION

The procedures for the cyclic triaxial and the bender element tests were adequately implemented. Results on Guayuriba sands were suitable when compared with the literature and the models proposed by other authors.

The model presented by Santos and Gomes Correia, is applicable to the Guayuriba sands. This model has the advantage that only needs the maximum shear modulus and the shear strain corresponding to a modulus equal to 70% of the maximum shear modulus. This simple approximation could be useful to obtain initial estimates of the soil behavior or to check values obtained by more advanced tests.

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