Dynamic Optimization of Human Stair-Climbing Motion

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ABSTRACT

The objective of this paper is to present our method of predicting and simulating visually realistic and dynamically consistent human stair-climbing motion. The digital human is modeled as a 55-degrees of freedom branched mechanical system with associated human anthropometry-based link lengths, mass moments of inertia, and centers of gravity. The joint angle profiles are determined using a B-spline-based parametric optimization technique subject to different physics-based, task-based, and environment-based constraints. The formulation offers the ability to study effects of the magnitude and location of external forces on the resulting joint angle profiles and joint torque profiles. Several virtual experiments are conducted using this optimization-based approach and results are presented.

INTRODUCTION

The field of human modeling and simulation is quickly growing and gaining momentum. Tools are being developed for modeling digital humans within virtual environments to facilitate designing products, evaluating capabilities, predicting injuries, and simulating real-world scenarios. Stair climbing is one of the important activities along with walking, running, pushing, and pulling, which are frequently encountered in daily life and must be modeled and simulated. This paper discusses our efforts to simulate stair climbing and determine the amount of effort being exerted by the virtual human to perform a simulated task. It is envisioned that such a dynamic simulation capability would allow one to perform many virtual experiments to study “what if?” scenarios.

A biomechanically accurate model of a human being must have large degrees of freedom (DOFs). Dynamic simulation of 3D human motions using such highly redundant models is a challenging problem from an analytical and computational point of view. Several different approaches have been applied towards solving this complex problem. Optimization-based methods were developed to use kinematic data available from the motion capture experiments to generate simulations without incorporating any dynamics (Zhang and Chaffin, 2000). A controls-based approach was also used with motion capture data to generate physics-based simulations (McGuan, 2001). Many commercial packages such as Visual 3-D and Anybody also allow dynamic simulations of digital humans. Recently, an open-source platform has been proposed to analyze dynamic simulations (Delp et al., 2007) driven by muscle excitations. While it is difficult to mention all efforts in this area, Piazza (2006) presents a nice review of work that uses muscle-actuated forward dynamic simulations. One important limitation of these approaches is their use of experimental motion capture data, which restricts their use to simulating only those situations for which the data exists. In contrast, Anderson and Pandy (2001) use a muscle-based approach to predict and simulate walking motion. While such muscle-driven simulations have advantages of their own, the computation involved in solving such detailed 3-D models is extremely time intensive.

A lot of research effort has been directed toward studying walking. A relatively small number of studies have focused on the stair-climbing task. Most of these studies are either from the robotics aspect or the clinical aspect (Kennedy et al., 2007). While robotics researchers have been looking at using wheeled (Morales et al., 2006), tracked (Mourikis et al., 2007), or hexapod (Moore et al., 2002) mechanisms to negotiate stairs, the most relevant work to simulation of the human stair-climbing motion is the use of biped mechanisms (Figliolini and Ceccarelli, 2001; Shih, 1999). Most biped research has been toward achieving a robot capable of dynamic walking on an even floor. Achieving stable gait has always been an issue with biped mechanisms. Figliolini and Ceccarelli (2001) use external control elements like suction cups to ensure equilibrium of the biped while walking or climbing stairs. Several implementations of biped robots satisfy Zero-Moment-
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Point (ZMP) (Vukobratovic, 1973; Sardain and Bessonnet, 2004) criterion to achieve dynamic equilibrium. In this work, we ensure that the ZMP lies within the Foot Support Region (FSR) to ensure the stability of the digital human. On the clinical side, several studies on stair climbing focus on determination of normal gait parameters to assist in the recovery decisions of patients (Costigan et al., 2002; Hamel et al., 2005; Nadeau et al., 2003). Most of these studies are performed using one particular design of stairs. Some studies have also been performed to determine the effect of different staircase inclinations on the gait parameters (Riener et al., 2002). However, not much has been reported on prediction and dynamic simulation of the stair-climbing motion for a full-body digital human model.

We use predictive dynamics (Kim et al., 2005; Kim et al., 2006), a constrained optimization-based approach, with dynamic effort as performance measure subject to different task-based, physics-based and environment-based constraints, to predict and simulate digital human stair-climbing motion. This approach to predicting and simulating physics-based motions has been validated (Xiang et al., 2007). It allows us to apply different loads (wearing a backpack, holding a box in the hands, etc.) and analyze their effects, not only visually in simulation, but also analytically in terms of changed joint torque profiles. Predictive dynamics does not require the motion capture experimental data corresponding to the full simulation cycle to generate the resulting motions. One more advantage of this approach is that it avoids solving the typical differential algebraic equations, thus allowing usage of highly redundant and anatomically correct joint-based full-body human models. In addition to predicting natural human stair-climbing motion, predictive dynamics also outputs the effort (joint torque profiles) required to complete a task. A detailed muscle model of different joints of interest can also be created. These joint torque profiles can be used to determine muscle forces and fatigue effects. Some task-specific and environment-based constraints have been added to the existing formulation. Mass and inertia properties of individual body segments have also been modified based on values obtained from the Generator of Body Data (GEBOD) program (Cheng et al., 1996). Physical limits have been placed on the maximum value of joint angles and torques to avoid predicting unrealistic motions.

DIGITAL HUMAN MODEL

The kinematic representation of the digital human, modeled as a three-dimensional 55-DOFs branched rigid link mechanical structure (including 6 global DOFs), is based on the Denavit and Hartenberg (DH) parameterization (Denavit and Hartenberg, 1955). The DH parameterization is a matrix transformation method to systematically describe the translational and rotational relationship between adjacent reference frames in an articulated chain. Multiple DOFs are collocated to adhere to the DH notation, which requires a single DOF between two consecutive links. Hence, for any joint in the human body that has more than one DOF, one or more virtual links with zero length are inserted between two consecutive joints. For instance, one virtual link is inserted between the two joints in the ankle, and two virtual links are inserted between the three collocated joints in the spine, as shown in Figure 1. The mass and inertia properties of each body segment are based on a 50th percentile male obtained from the GEBOD program with a body weight of about 770 N and a height of about 1.7 meters.

An accurate and efficient calculation of equations of motion is extremely important to generate physically consistent simulations of a digital human. We use computationally efficient recursive Lagrangian dynamics method to develop the equations of motion for the digital human.

The kinematics analysis in the recursive form leads to a simpler form for the transformation matrix \( A_i \):

\[
A_i = T_i^r T_i^t \ldots T_{i-1}^r T_i = A_{i-1} T_i
\]
where $T_i$ is the $i^{th}$ link transformation that relates the $i^{th}$ and $i-1^{th}$ local reference frames and is expressed as

$$
T_i = \begin{bmatrix}
\cos \theta_i & -\cos \alpha_i \sin \theta_i & \sin \alpha_i \sin \theta_i & a_i \cos \theta_i \\
\sin \theta_i & \cos \alpha_i \cos \theta_i & -\sin \alpha_i \cos \theta_i & \sin \alpha_i \\
0 & \sin \alpha_i & \cos \alpha_i & d_i \\
0 & 0 & 0 & 1
\end{bmatrix}
$$

Any point of interest in the $i^{th}$ frame, $x_i$, can be expressed in the global reference frame, $x_0$, as:

$$x_0 = A_i x_i$$

Different DH parameters like $\theta_i$, $d_i$, $\alpha_i$ and $a_i$ used in the transformation matrix $T_i$ are explained in Figure 2. Time derivatives of the transformation matrix $A_i$ are necessary to calculate the dynamics and can also be obtained in the recursive form as:

$$B_i = \dot{A}_i = B_{i+1} T_i + A_{i+1} \frac{\partial T_i}{\partial q_i} \dot{q}_i$$

$$C_i = \ddot{B}_i = C_{i+1} T_i + 2B_{i+1} \frac{\partial T_i}{\partial q_i} \dot{q}_i + A_{i+1} \frac{\partial^2 T_i}{\partial q_i^2} \ddot{q}_i + \frac{\partial T_i}{\partial q_i} \dddot{q}_i$$

where $q_i$ is the generalized coordinate for transformation $T_i$, $A_0 = I$ and $B_0 = C_0 = 0$.

The Lagrange’s equation is given as

$$\tau_i = \frac{d}{dt} \left( \sum L \right) - \frac{\partial L}{\partial q_i}$$

where $L = K - V$ (kinetic energy − potential energy), $q$ is the generalized coordinate vector (joint angles), and $\tau$ is the joint torque vector. Given the mass and inertia properties of each body segment, and the external force $f_i = [f_x, f_y, f_z, 0]$ and moment $h_i = [h_x, h_y, h_z, 0]$ for the link $k$ ($1 \leq k \leq n$) defined in the global coordinate system, the joint actuation torques $\tau_i$ for each of the joints can be computed using recursive backward dynamics as follows:

$$\tau_i = tr \left[ \frac{\partial A_i}{\partial q_i} D_i + \frac{\partial A_i}{\partial q_i} E_i - T_i \frac{\partial A_i}{\partial q_i} F_i - G_i A_i z_i \right]$$

where

$$D_i = J_i C_i^T + T_{i+1} D_{i+1}$$

$$E_i = m_i r_i T_{i+1} E_{i+1}$$

$$F_i = r_i \delta_i + T_{i+1} F_{i+1}$$

$$G_i = h_i \delta_i + G_{i+1}$$

$$D_{n+1} = E_{n+1} = F_{n+1} = 0$$

The following assumptions were made for solving the stair-climbing simulation problem:

**STAIRS CLIMBING**

The problem statement for the stair-climbing task can be described as: “Given step length, step height, human anthropometry, segment inertial properties, physical joint motion and actuation limits, and desired time for single step motion, generate visually appealing and dynamically consistent stair-climbing motion that minimizes dynamic effort for all joints subject to different task-based, physics-based, and environment-based constraints.”

A single step in stair climbing can be divided into two phases just as in case of walking: the single support phase and the double support phase. However, one of the important differences between normal walking and stair climbing is the point of contact after the swing (or single support) phase. When the double support phase starts, the fore foot contacts the step in the case of stair climbing as opposed to the heel in the case of walking (Riener et al., 2002).
- Symmetric and cyclic motion.
- Uniform staircase configuration, i.e., the step length and step height are constants.
- No skipping/jumping of the steps.

The first step in the stair-climbing motion requires acceleration to achieve normal joint velocities from zero initial velocity while the final step requires deceleration to zero final velocities. For the results presented in this paper, we assume that the digital human is already in the stair-climbing motion, as shown in Figure 3. The digital human, thus, has some initial joint velocity and accelerations that are predicted using predictive dynamics, as are the joint angle values at the initial and final simulation time. A single step is predicted from the left foot strike to the right foot strike. The resulting joint angle profiles are then mirrored to generate the full stride motion.

PREDICTIVE DYNAMICS

Predictive dynamics is a novel approach to predicting and simulating human motions considering joint-based activation. This optimization-based approach avoids solving typical differential algebraic equations (or ordinary differential equations) in order to create the resulting simulations for highly redundant systems.

Detailed and anatomically correct full-body human models with large DOFs can thus be used to create more realistic simulation of tasks with relatively less computation. The problem statement as described above lends itself to an optimization formulation, various components (design variables, performance measure, and constraints) of which are discussed below.

**DESIGN VARIABLES:**

Joint angle profiles, $q_i(t)$, are approximated as linear combinations of a cubic B-spline basis functions. The control points representing these B-splines are treated as design variables for optimization. Corresponding joint angle, velocity, and acceleration values are calculated at each iteration from these control point values.

**PERFORMANCE MEASURE:**

The goal of the optimization process is to reduce the dynamic effort at each joint. The performance measure (or objective function) is, therefore, to minimize the sum of the torques squared for all joints over the simulation time.

$$\min f(q) = \int_0^T \sum_{i=1}^n \tau_i^2(q) dt$$

where $n$ is the total number of joints of the human model, $\tau_i$ is the actuator torque of $i$th joint, and $T$ is the total simulation time. Joint actuation torques are calculated using the recursive Euler-Lagrangian formulation as a function of joint angles, velocities, and accelerations.

**CONSTRAINTS**

Several physics-based, task-based and environment-based constraints have been employed to predict the motions for stair-climbing. The physics-based constraints, common with simulating other tasks and discussed in detail in previous work (Xiang et al., 2007) are listed below:

- **Joint angle limits** to restrict the physical range of motion for each joint
- **ZMP stability** to ensure that the ZMP (defined as a point on the ground where the tipping moment acting on the digital human, due to gravity and inertia forces, equals zero) lies in the Foot Supporting Region (FSR) of the digital human.
- **Soft impact** to minimize the loss of energy
- **Arm-leg coupling** to produce visually appealing movement of arms.

Some redundant constraints employed in previous walking implementation, like pelvic velocity and knee flexion at mid-swing, have been removed from the formulation of stair-climbing. An average pelvic velocity is indirectly achieved since the time required to finish a step is an input to simulation. Below is the detailed description of other physics-based constraints added to the formulation to predict more realistic and physically consistent motions.

**Torque limits** – The torque profiles for each joint from the predicted motion can be compared against the limiting values of the corresponding joint as a post-processing step. However, by enforcing the torque limit constraint, new methods of performing a task can be “designed” without violating physical joint torque limits. Torque limits, in general, are a function of the joint angle position and joint angle velocity. With increasing velocity, the torque-producing capability of a joint reduces. However, in the current implementation, torque limits are assumed to be constant.

\[
\tau_i^\text{min} \leq \tau_i \leq \tau_i^\text{max} \quad \forall 0 \leq i \leq n
\]

where \(\tau_i^\text{min}\) and \(\tau_i^\text{max}\) are the lower and upper limits on the torque that can be produced by the \(i^{th}\) joint.

**Continuity/symmetry condition** – A cyclic motion assumption allows repeating the one stride (left and right step) motion multiple times, in order to generate a continuous stair-climbing motion. Current simulation also assumes that the motions of the left and right step are symmetric. Hence, to avoid any discontinuities of the joint angle profile in continuous motion, the initial and final postures of a step being simulated should satisfy the symmetry condition.

\[
q_{\text{left}}(0) = q_{\text{right}}(T) \\
q_{\text{left}}(0) = q_{\text{right}}(T) \\
q_{\text{left}}(0) = -q_{\text{right}}(T) \\
q_{\text{left}}(0) = -q_{\text{right}}(T)
\]

where \(q_{\text{left}}\) and \(q_{\text{right}}\) are the corresponding DOFs on the left branch and right branch of the legs and arms, subscripts \(x, y, \) and \(z\) correspond to the global axis, and \(q_j\) represents the DOFs for the global, spine, and head branches except the global DOFs corresponding to forward and vertical linear motion. A symmetry constraint was also implemented at velocity level to avoid any jerky motion.

\[
\dot{q}_{\text{left}}(0) = \dot{q}_{\text{right}}(T) \\
\dot{q}_{\text{left}}(0) = \dot{q}_{\text{right}}(T) \\
\dot{q}_{\text{left}}(0) = -\dot{q}_{\text{right}}(T) \\
\dot{q}_{\text{left}}(0) = -\dot{q}_{\text{right}}(T)
\]

**Self avoidance** – Since the different segments of the digital human are free to move with respect to each other, some segments like the hand and foot tend to penetrate other segments of the body. Imaginary spheres (proportional to the size of the segment) are placed on each individual segment like the hands, feet, hips, etc., as shown in Figure 5. The self-avoidance constraint is then implemented as:

\[
(s_{ix} - s_{ix})^2 + (s_{iy} - s_{iy})^2 + (s_{iz} - s_{iz})^2 \geq (r_i + r_j)^2 \\
\forall i, j (i \neq j)
\]

where \((s_{ix}, s_{iy}, s_{iz})\) are the global coordinates of the \(i^{th}\) sphere with radius \(r_i\).
staircase at different levels. Hence, the ground penetration constraint is modified accordingly.

**Obstacle (stairs) avoidance** – This constraint has been implemented to avoid the foot from penetrating the stairs during the swing phase. The most general method used for obstacle avoidance is filling the objects with spheres and then implementing a constraint similar to a self-avoidance constraint. However, the number of spheres required to fill the staircase will be large, which would increase the number of constraints, thus slowing down the optimization process. Since the stairs are uniform, two cylinders as shown in Figure 6 were used to avoid the stairs.

![Figure 6](image)

**Figure 6** Sequential snapshots of the digital human climbing stairs

During the swing phase of a foot as it goes from step 1 to step 3, while the other foot is supported on step 2, cylinder a and cylinder c can be used for obstacle avoidance. The size of cylinder c can be used to guarantee toe-clearance which varies with stair inclination (Riener et al., 2002).

\[
(c_{ci} - p_{pi})^2 + (c_{cj} - p_{pj})^2 \geq r_i^2 \quad \forall i, j
\]  

(12)

where \((c_{ci}, c_{cj}, c_{cj})\) are the global coordinates of the center of the cylinder, \(i \in a, c\) in Figure 6, and \((p_{pi}, p_{pj}, p_{pi})\) are the global coordinates of the reference point \(j\) on the foot in swing phase (refer to Figure 4).

**Foot strike position** – These are the terminal constraints on the position of the feet. Since the initial position of the feet and step length are known, the foot strike position is known for each foot during the contact phase. The distance between the foot strike position on the staircase and the contacting points on the foot should be zero at contact.

\[
\left(p_{pi} (0) - p_{pi} (T)\right)^2 + \left(p_{pj} (0) + l_i - p_{pj} (T)\right)^2 = 0
\]  

(13)

where \(T\) is the total simulation time for one step.

**NUMERICAL IMPLEMENTATION**

The optimization problem is solved using a large-scale sequential quadratic programming (SQP) solver within SNOPT (Gill et al., 2002) software. The parameters like various link lengths, mass and inertia properties, joint angle limits, joint torque limits, step length, step height, and desired time to finish a step-climbing motion are read from a text file, thus allowing user the flexibility to modify them easily.

Accurate sensitivity is a key factor for efficiently achieving an optimal solution. Although the finite difference approach can be used to approximate gradients, the computational expense becomes more serious as the number of variables increases. In addition, accuracy of the derivatives can affect convergence of the optimization process, thus leading to further computational expense. Hence, for all the constraints and objective function used in stair-climbing simulation, the gradients with respect to control points (design variables) are calculated analytically.

The mass and inertia properties of each body segment are based on a 50\(^{th}\) percentile male obtained from the GEBOD program with a body weight of about 770 N and a height of about 1.7 meters. The step length and the step height of the simulated staircase are 0.254 m (10 in) and 0.165 m (6.5 in). The desired time for climbing one step was 0.5 sec.

**SIMULATION RESULTS**

![Figure 7](image)

Figure 7 Sequential snapshots of the digital human climbing stairs

![Figure 8](image)

Figure 8 Snapshot of digital human climbing stairs (a) without 45 lb backpack and (b) with backpack

Many case scenarios were simulated using the formulation discussed above for different values of step length and step height. However, the staircase to be used for validating the results had steps 10 in long and 6.5 in high. Figure 7 shows sequential snapshots of the digital human climbing stairs. The motion appears visually realistic.

Several simulation case studies were also performed with different backpack weights carried by the digital human while climbing the staircase. Predictive dynamics not only feeds back the effect of increased weight...
analytically by outputting increased actuation torque requirements, but also visually as shown in Figure 8.

Initial simulations did not use stairs avoidance as a constraint. However, while visualizing results, it was evident that the foot in the swing phase would penetrate the step on which the other foot is supported. In order to avoid this penetration, a stairs-avoidance constraint was implemented.

![Figure 9](image)

**Figure 9** Comparison of the joint angle profiles for (a) hip and (b) ankle DOFs with and without implementation of the stairs-avoidance constraint

Figure 9 shows the plots of the simulation results for hip and knee angle profiles with and without the implementation of the stairs-avoidance constraint. The plots correspond to one stride data starting with the foot strike and ending at the successive foot strike. All of the data was normalized to percent of stride. Since we use a symmetry constraint, the corresponding plots from foot strike to successive foot strike for the left as well as the right foot will be similar. While the difference between the two profiles of the hip angle is small, a major difference is seen in the profiles for the knee angle. The knee angle profile for the case when the stairs-avoidance constraint is implemented matches the earlier studies.

**CONCLUSION**

Predictive dynamics, an optimization-based approach to solving large nonlinear dynamics problems, was used to predict visually appealing and physically consistent stair-climbing motion. Minimization of the dynamic effort was considered as the objective function subject to various physics-based, task-based, and environment-based constraints. The resulting simulations are presented and look visually appealing. An analytical comparison of the results with and without the inclusion of stairs-avoidance constraints is also shown. The analytical results seem to match the results published in the literature. An in-house validation effort to validate the stair-climbing motion is ongoing. Efforts to incorporate joint torque limit surface (which depends on the joint angle position and velocity) are also in progress (Laake and Frey Law, 2007). Simulation of stair-climbing is thus a work in progress, and further improvements will be made to current stair-climbing simulations based on feedback from motion capture experiments.

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**REFERENCES**


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