J. Shanghai Jiaotong Univ. (Sci.), 2009, 14(6): 676-680
DOI: 10.1007/s12204-009-0676-0

Cyclostationary Property Based Spectrum Sensing Algorithms for Primary Detection in Cognitive Radio Systems

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Abstract: To implement the primary signal without interference in cognitive radio systems, cognitive radios can detect the presence of the primary user in low SNR. Currently, energy detector is the most common way of spectrum sensing because of its low computational complexity. However, performance of the method will be possibly degraded due to the uncertainty noise. This paper illustrates the benefits of one-order and two-order cyclostationary properties of primary user's signals in time domain. These feature detection techniques in time domain possess the advantages of simple structure and low computational complexity comparing with spectral feature detection methods. Furthermore, performance of the one-order and two-order feature detection is studied and the analytical results are given. Our analysis and numerical results show that the sensing performance of the one-order feature detection is improved significantly comparing with conventional energy detector since it is robust to noise. Meanwhile, numerical results show that the two-order feature detection technique is better than the one-order feature detection. However, this benefit comes at the cost of hardware burdens and power consumption due to the additional multiplying algorithm.
Key words: cognitive radio, spectrum sensing, cyclostationary feature detection
CLC number: TN 925 **Document code:** A

Introduction

Cognitive radio has been recently proposed as a solution of the conflicts between current spectrum congestion and spectrum under-utilization[1]. However, cognitive radios are considered lower priority for a primary user. The fundamental requirement is to avoid interference to potential primary users in their vicinity. To implement the primary signal without interference, spectrum sensing must be performed before the cognitive users access the channel. A great challenge of spectrum sensing to the cognitive radio is the ability to detect the presence of the primary transmitter with fast speed and precise accuracy. Currently, the spectrum sensing techniques mainly focus on primary transmitter detection and they usually can be classified by matched filter detection, energy detection and cyclostationary feature detection for single secondary user (SU) sensing.

The matched filter is optimal[2], but it needs the prior knowledge of the primary user signal such as the modulation type and order, pulse shaping and packet format. Energy detector based approach is the most common way for spectrum sensing because of the low computational complexity[3]. Moreover, it does not need any knowledge on the primary user signal. However, the sensing performance of the method will be possibly degraded due to the noise uncertainty. Cyclostationary feature detection can detect the signals in a very low SNR due to its robustness to the noise[4]. Recent researches on cyclostationary feature sensing mainly focus on two-order cyclostationary, i.e. auto-correction function, in frequency domain. These features are detected by analyzing a spectral correlation function in the frequency domain. The main advantage of the spectral correlation function is that it can differentiate the noise energy from modulated signal energy[5]. However, the computational complexity is the bottleneck for its implementation. As all the frequencies should be searched in order to generate the spectral correlation function, the calculation complexity is huge.

To meet the time and sensitivity requirements, in this paper, we propose a spectrum sensing scheme by using cyclostationary properties of the primary signals in time domain. Since they are performed in the time domain, low-power consumption and fast spectrum-detection can be achieved. Meanwhile, it drastically reduces

Received date: 2008-11-03
Foundation item: the National Natural Science Foundation of China (No. 60972039), the National High Technology Research and Development Program (863) of China (No. 2009AA01Z241), the Key Project of Nature Science Foundation of Jiangsu Province (No. BK2007729) and the National Postdoctoral Research Program (No. 20090451239)
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hardware burdens comparing with previous two-order feature detection in the frequency domain.

1 Cyclostationary Detection Technique

1.1 General Model for Spectrum Sensing

The detection problem of spectrum sensing at SU can be formulated as a binary hypothesis testing problem. H_0 corresponds to primary user (PU) signal absent, and the alternative hypothesis H_1 corresponds to PU signal. Therefore, the goal of spectrum sensing is to distinguish the following two hypotheses:

$$\left.\begin{aligned} H_0 &: x(t) = n(t), & 0 < t \leqslant T \\ H_1 &: x(t) = hs(t) + n(t), & 0 < t \leqslant T \end{aligned}\right\}, \tag{1}$$

where T denotes the observation time, $x(t)$ is the received signal by SU, h is the amplitude gain of the channel, $s(t)$ is the transmitted signal of the PU, and $n(t)$ is the additive white Gaussian noise (AWGN) with zero mean and variance δ^2.

1.2 Theoretical Expression of Cyclostationary

According to the cyclostationary signal processing theory, most modulated signals are characterized by cyclostationary since their means and autocorrelations exhibit periodicity. A detailed discussion on cyclostationary can be found in Ref. [6]. Common analysis of cyclostationary signal is based on autocorrelation function in frequency domain. However, we exploit the cyclostationary characteristics of the primary signal in time domain.

Consider a deterministic complex sine signal $s(t)$ which may be expressed as

$$s(t) = a\cos(2\pi f_0 t + \theta), \tag{2}$$

where a is the envelope, f_0 is the frequency, and θ is the initial phase.

In the transmission of $s(t)$ through an AWGN channel, $x(t) = s(t) + n(t)$. The mean function of $x(t)$ can be written as

$$M_x(t) = E[x(t)] = s(t), \tag{3}$$

where E denotes the expectation operator.

We observe that the mean is time-varying. Moreover, it is a periodic function of time with period $T_0(= 1/f_0)$. If we know the period T_0, there is a method called synchronized averaging to realize this periodicity. In such method, the process $x(t)$ is periodic sampled at the sampling instants $\cdots, t - kT_0, \cdots, t - T_0, t, t + T_0, \cdots, t + kT_0, \cdots$ for any time instant t and for any integer value of k[7]

$$M_x(t)_T \triangleq \frac{1}{2N+1}\sum_{k=-N}^{N} x(t + kT_0), \tag{4}$$

where the parameter N is the number of collected cyclic prefix used to do autocorrelation. $T \triangleq (2N+1)T_0$ still denotes the observation time. When T goes to infinite, this relation becomes

$$M_x(t) = \lim_{T\to\infty} M_x(t)_T = \lim_{N\to\infty} \frac{1}{2N+1}\sum_{k=-N}^{N} x(t + kT_0). \tag{5}$$

Obviously, $M_x(t)$ is also a periodic function with period T_0, i.e., $M_x(t) = M_x(t+kT_0)$ for $k = 0, \pm1, \pm2, \cdots$. Such a characteristic is called one-order cyclostationary.

The auto-correlation function is defined as[7]

$$R_{xx}(t; t-\tau) = \lim_{N\to\infty} \frac{1}{2N+1}\sum_{k=-N}^{N} x(t + kT_0)\times x^*(t + kT_0 - \tau), \tag{6}$$

where τ is the time difference, and $x^*(\cdot)$ denotes the complex conjugate of $x(\cdot)$.

Again, we observe that

$$R_{xx}(t; t-\tau) = R_{xx}(t + kT_0; t - \tau + kT_0) \tag{7}$$

for $k = \pm1, \pm2, \cdots$. Hence, the auto-correlation function of $x(t)$ is also periodic with period T_0. Such a characteristic is called two-order cyclostationary.

As an example, we consider noise $n(t)$. In this case, no periodicity can be found. $n(t)$ is the Gaussian random variable with zero mean and can be ignored after the periodic sampling. Therefore, cyclostationary feature detection is robust to the noise uncertainty. Moreover, there is a peak value within each period if PU is present. We search for the peak value in time domain and compare it with the predetermined threshold. If no periodicity is found, it means that there is no signal in the detected band. Otherwise, the band is used by the primary users.

The proposed one-order and two-order cyclostationary feature detection methods are described in Fig. 1. λ is the detection threshold.

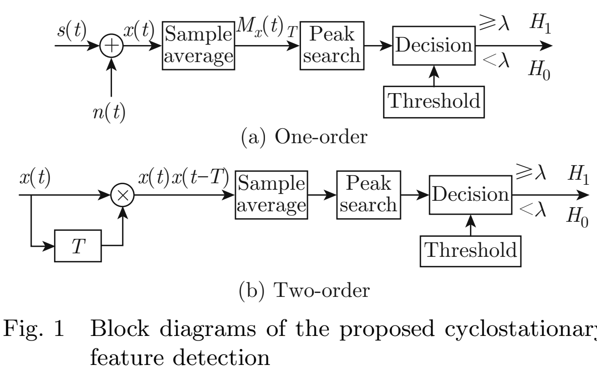

Fig. 1 Block diagrams of the proposed cyclostationary feature detection

2 Performance Analysis for Cyclostationary Feature Detection

In this section, we will deduce the relationship between the conditional detection probability P_{d} and the false alarm probability P_{f} of the cyclostationary feature, including one-order cyclostationary property, i.e. mean function, and two-order cyclostationary property, i.e. auto-correlation function, over AWGN and Rician fading channels, respectively.

2.1 One-order Cyclostationary Detection Technique

According to the Central Limit Theorem, when the number of terms added in Eq. (4) is sufficiently large, the probability distribution function (PDF) of $M_x(t)_T$ for both hypothesis H_0 and H_1 can be approximated by Gaussian distributions

$$P_{M_x(t)_T}(t : H_0) \sim C_N\left(0,\ \frac{\delta^2}{2N+1}\right), \tag{8}$$

$$P_{M_x(t)_T}(t : H_1) \sim C_N\left(\mu,\ \frac{\delta^2}{2N+1}\right), \tag{9}$$

where $C_N(\mu, \delta^2)$ represents the circularly symmetric complex Gaussian distribution with mean μ and variance δ^2.

Therefore, under hypotheses H_0 and H_1, the envelopes of $M_x(t)_T$ satisfy respectively Rayleigh and Rician distributions[8]

$$P(r : H_0) = \frac{r}{\delta_{\mathrm{A}}^2} \exp\left(-\frac{r^2}{2\delta_{\mathrm{A}}^2}\right), \tag{10}$$

$$P(r : H_1) = \frac{r}{\delta_{\mathrm{A}}^2} \exp\left(-\frac{r^2+A^2}{2\delta_{\mathrm{A}}^2}\right) I_0\left(\frac{Ar}{\delta_{\mathrm{A}}^2}\right), \quad A \geqslant 0, \quad r \geqslant 0, \tag{11}$$

where $\delta_{\mathrm{A}}^2 = \delta^2/(2N+1)$, $I_0(\cdot)$ is the zeroth-order modified Bessel function, and A stands for a non-centrality parameter.

Therefore, for a particular threshold λ, an approximate expression for the false alarm probability $P_{\mathrm{f,OFD}}$ and the detection probability $P_{\mathrm{d,OFD}}$ of one-order feature detector (OFD) over AWGN channel can be obtained as[8]

$$P_{\mathrm{f,OFD}} = \exp\left(-\frac{\lambda^2}{2\delta_{\mathrm{A}}^2}\right), \tag{12}$$

$$P_{\mathrm{d,OFD}} = Q_1\left(\frac{\sqrt{2\gamma}}{\delta},\ \frac{\lambda}{\delta_{\mathrm{A}}}\right), \tag{13}$$

where the parameter γ is the instantaneous signal-to-noise ratio (SNR), and $Q_1(\cdot,\cdot)$ is the generalized Marcum Q-function.

Since the signal strength follows a Rician distribution in such case, $f_{\mathrm{Ric}}(\gamma)$, the PDF of SNR, will be[9]

$$f_{\mathrm{Ric}}(\gamma) = \frac{K+1}{\bar{\gamma}} \exp\left[-K - \frac{(K+1)\gamma}{\bar{\gamma}}\right] \times I_0\left(2\sqrt{\frac{K(K+1)\gamma}{\bar{\gamma}}}\right), \tag{14}$$

where $\bar{\gamma}$ is the average SNR, and $K = A^2/(2\delta_{\mathrm{A}}^2)$ is the Rician factor. For $K = 0$, this reduces to the Rayleigh expression.

The average $P_{\mathrm{d,OFD}}$ in the case of a Rician channel, $\bar{P}_{\mathrm{d,OFD}}$, is then obtained by averaging Eq. (13) over Eq. (14). Then,

$$\bar{P}_{\mathrm{d,OFD}} = \int_0^\infty Q_1\left(\frac{\sqrt{2\gamma}}{\delta},\ \frac{\lambda}{\delta_{\mathrm{A}}}\right) f_{\mathrm{Ric}}(\gamma)\mathrm{d}\gamma = Q_1\left(\frac{\sqrt{2K\bar{\gamma}}}{\delta\sqrt{K+1+\bar{\gamma}}},\ \frac{\lambda\sqrt{K+1}}{\delta_{\mathrm{A}}\sqrt{K+1+\bar{\gamma}}}\right). \tag{15}$$

It is note that, $\bar{P}_{\mathrm{d,OFD}}$ is the special case for $u = 1$[10]. Meanwhile, $\sqrt{\lambda}$ is replaced by λ.

2.2 Two-order Cyclostationary Detection Technique

A correlation process may derive the amount of the similarity between two signals. Thus, the same waveform can reach the maximum correlation value. In practice, most of primary signals contain deterministic periodic features, i.e. cyclic prefix and preambles, etc. Meanwhile, waveforms modulated by random data have a small amount of correlation between those waveforms. Therefore, this correlation between the periodic signal waveform and the data modulated signal waveform can be ignored. As a result, we can utilize the correlation of two consecutive cyclic prefixes as the basic approach to perform spectrum sensing.

The correlation between the original input signal $x(t)$ and the corresponding delayed signal $x(t-T)$ is performed by multiplying these two signals. This correlation value is summed by an integrator. Then the resulting integrator output is compared with pre-threshold to decide whether or not the primary user is present. Using this benefit of the correlation characteristic, we define the decision statistic of the two-order feature detector (TFD) as

$$R_{\mathrm{TFD}} = |r_{\mathrm{TFD}}(t)|, \tag{16}$$

$$r_{\mathrm{TFD}}(t) = \frac{1}{2N+1} \sum_{k=-N}^{N} x(t+kT_0) \times x^*(t+(k-1)T_0). \tag{17}$$

The delay value T is predetermined and, for simplicity, we assume that $T = T_0$.

Then, from the Central Limit Theorem, for sufficiently large N, the PDF of $r_{\mathrm{TFD}}(t)$ for both hypotheses H_1 and H_0 will approach circularly symmetric complex Gaussian distributions, i.e.[11]

$$p_{r_{\mathrm{TFD}}(t)}(t:H_1) \sim C_N\left(\delta_{\mathrm{cp}}^2,\ \frac{2\delta_{\mathrm{cp}}^2\delta^2+\delta^4}{2N+1}\right), \qquad (18)$$

$$p_{r_{\mathrm{TFD}}(t)}(t:H_0) \sim C_N\left(0,\ \frac{\delta^4}{2N+1}\right), \qquad (19)$$

where the parameter δ_{cp}^2 is the average energy of the received signal cyclic prefix.

Therefore, the false alarm probability for variable R_{TFD} under H_0 can be evaluated as

$$P_{\mathrm{f,TFD}} = \exp\left[-\frac{(2N+1)\lambda^2}{2\delta^4}\right]. \qquad (20)$$

Similarly, for a given instantaneous SNR, γ_{cp}, the detection probability $P_{\mathrm{d,TFD}}$ over AWGN can be obtained as

$$P_{\mathrm{d,TFD}} = Q_1\left(\frac{\sqrt{2\gamma_{\mathrm{cp}}}}{\delta},\ \frac{\lambda}{\delta_{\mathrm{B}}}\right), \qquad (21)$$

where

$$\delta_{\mathrm{B}}^2 = \frac{2\delta_{\mathrm{cp}}^2\delta^2+\delta^4}{2N+1} = \frac{(2\gamma_{\mathrm{cp}}+1)\delta^4}{2N+1}.$$

Average $P_{\mathrm{d,TFD}}$ over Rician fading channel can be obtained by averaging $f_{\mathrm{Ric}}(\gamma)$ over Eq. (21),

$$\bar{P}_{\mathrm{d,TFD}} = Q_1\left(\frac{\sqrt{2K\bar{\gamma}}}{\delta\sqrt{K+1+\bar{\gamma}}},\ \frac{\lambda\sqrt{K+1}}{\delta_{\mathrm{B}}\sqrt{K+1+\bar{\gamma}}}\right). \qquad (22)$$

3 Numerical Results

Numerical results are presented in this section to demonstrate the spectrum sensing performance of one-order and two-order cyclostationary detection techniques. In the following simulations, we assume that average SNR, $\bar{\gamma}$, and Rician factor, K, are 10 dB through this paper.

In order to give a comparison between the one-order feature detection and energy detection (ED), simulations are carried out in AWGN environment for the case $\delta^2 = 1$, as shown in Fig. 2, where P_{m} is the probability of a Miss. The time-bandwidth product for energy detector is assumed to be 5. The numerical results indicate that better performance can be achieved using one-order feature detection compared to the energy detector due to its robustness to the noise uncertainty.

Figure 3 illustrates the complementary receiver operating characteristic (ROC) curve under Rician fading scenario for one-order feature detection. A plot for pure

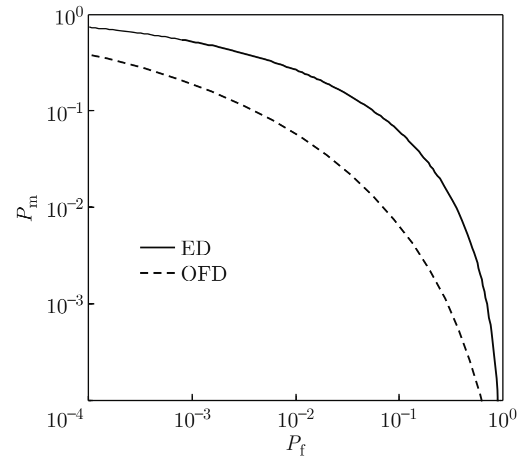

Fig. 2 Performance comparison between OFD and ED over AWGN channel

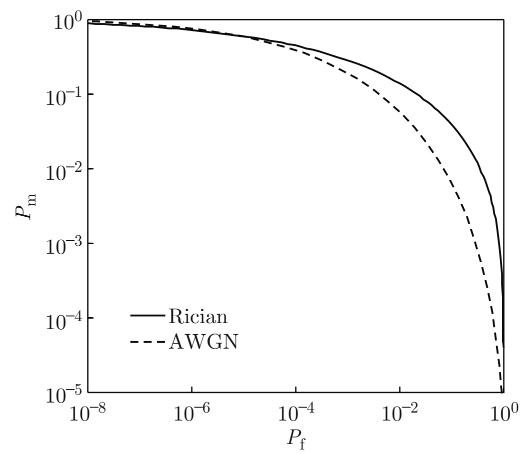

Fig. 3 Complementary ROC curves under Rician and AWGN channels for OFD

AWGN case is also provided for comparison. Simulation shows that the sensing performance is degraded due to Rician fading.

Figure 4(a) illustrates the complementary ROC performance of two-order feature detector (TFD) under Rician fading and AWGN channels for the case $\gamma = 10$ dB and $\delta^2 = 2$. We observe that Rician fading degrades performance of two-order detector significantly. In order to carry out a comparison with the one-order feature detection, simulation is completed in AWGN environment, as shown in Fig. 4(a). Figure 4(b) depicts the ROC (P_{d} versus P_{f}) for TFD and OFD under different channels. From Fig. 4, we find that the two-order feature detector can perform better than the one-order feature detector. Since the correlation between the original input signal $x(t)$ and the corresponding delayed signal $x(t-\tau)$ is performed by multiplying these two signals, this benefit may lead to the cost of hardware burdens and power consumption due to the additional multiplier, which increases the design complexity. Therefore, we can choose an appropriate spectrum sensing scheme according to the practical requirements to sensing accuracy and computational complexity for the secondary networks.

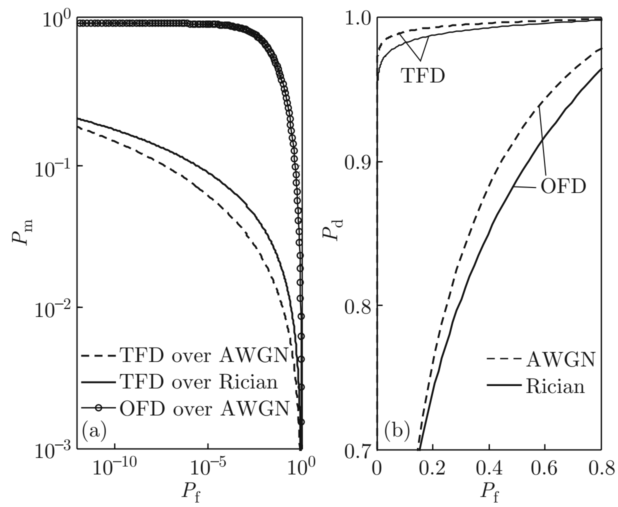

Fig. 4 Performance comparison between OFD and TFD over Rician and AWGN channels

4 Conclusion

A spectrum sensing scheme by using one-order and two-order cyclostationary properties of primary signals in time domain are proposed to meet with the requirements of accuracy in cognitive radio systems. Since the proposed feature detection is performed in time domain, the real-time operation and low-power consumption can be achieved. Numerical results show that the sensing performance of the cyclostationary feature detection is improved significantly comparing with conventional energy detector. The insights obtained in this paper are useful for the design of optimal spectrum in cognitive radio networks.

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