Adapting the mathematical task framework to design online didactic objects
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Designing didactic objects involves imagining how students can conceive of specific mathematical topics and then imagining what types of classroom discussions could support these mental constructions. This study investigated whether it was possible to design Java applets that might serve as didactic objects to support online learning where ‘discussions’ are broadly defined as the conversations students have with themselves as they interact with the dynamic mathematical representations on the screen. Eighty-four pre-service elementary teachers enrolled in hybrid mathematics courses were asked to interact with a series of applets designed to support their understanding of qualitative graphing. The results of the surveys indicate that various design features of the applets did in fact cause perturbations and opportunities for resolutions that enabled the users to ‘discuss’ their learning by reflecting on their in-class discussions and online activities. The discussion includes four design features for guiding future applet creation.

Keywords: didactic objects; online learning; applet creation

1. Introduction

Although web-based computer applets have been used by businesses to increase online sales and website usability for the last 15 years, their use in education is relatively new. Our experience in classrooms indicates that the proliferation of advanced technologies such as digital white boards and netbooks has created a new demand for well-designed applets that can enhance whole-class and small group discussions [1]. For example, teachers may want to demonstrate a topic such as graphing constant motion using a digital whiteboard, or they may want groups of students to use netbooks to engage in interactive explorations. The goal of this article is to discuss how we adapted a classroom-based design framework to create a series of mathematical applets that could be used for in-class demonstrations or with small group or individual explorations that would complement their in-class discussions.

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2. Design perspective

2.1. Creating didactic objects to support classroom-based learning

The design perspective we have adopted is based on constructivist views of learning. As von Glasersfeld notes, this perspective suggests that learners construct knowledge through successive cycles of action, perturbation and reflection [2]. In classrooms, the sources of these perturbations may come from any number of social catalysts including the teacher, activity, textbook, other students, etc. In all of these cases, students are challenged to resolve the conflict between their own expectations and the particular outcome or discussion experienced through whole-class and small-group discussions. Thompson [3] suggests that classroom teachers can support this equilibration process by creating thought-provoking questions that will form the basis for reflection and discussion – what he calls didactic objects. His view is that when a teacher skilfully poses a well-planned series of questions based on these didactic objects, he or she can support shifts in students’ constructions of the underlying mathematical ideas. This study examines the degree to which a set of well-designed didactic objects designed for online use might be able to support an analogous shift in students’ mathematical ideas.

According to Thompson, didactic objects can range from simple drawings to more complicated computer simulations. The critical issue is not the object itself – in fact Thompson argues that an object is not didactic in and of itself – it is the way that it is used to support students’ constructions of increasingly complex cognitive structures. Thus, the key to designing such objects lies in creating a conceptual analysis to imagine ‘What mental operations must be carried out to see the presented situation in the particular way one is seeing it?’ [2, pp. 78, 3, 196].

Given that Thompson’s theories are deeply rooted in the social setting of the classroom, it may seem overreaching to adapt the theory of didactic objects to the design of online learning activities – especially if learners are working at home or at individual computers. In order to resolve this dilemma, we found Sfard’s [4,5] commognitive perspective to be helpful. Sfard coined this term to suggest that educational researchers may do well to dispel the distinction between interpersonal communication and intrapersonal cognition. In her view, ‘...thinking is defined as the individualized version of interpersonal communication – as a communicative interaction in which one person plays the roles of all interlocutors’ [4, p. iv (italics in original)]. The implication for online learning is that if students are thinking about what they are viewing on the screen and making action choices based on their thinking, then they are engaging in a type of interpersonal communication. Our hypothesis is that during this communication, a self-negotiation process occurs in which students revise their current actions and perceptions.

2.2. Adapting didactic objects to the online environment

Although not a linear process, we can illustrate the process of designing didactic objects – and our adaptation of them to online environments as shown in Figure 1. In what follows, we describe Steps 1–3 of this framework. We then describe the results of our study to analyse Step 4: implementation. We focus on the example of qualitative graphing to illustrate one of the ways this process may play out for a particular topic.
Step 1. Creating a conceptual analysis. The first step in the design process is to consider how learners form ideas regarding the target mathematical topic. In our case, the goal of instruction was to examine how students’ natural conceptions of graphing evolve over time. We found the Covariational Framework created by Carlson et al. [6] to be helpful because it outlines the mental actions involved in developing a conceptual understanding of functions generally. Carlson et al. hypothesize that it is helpful to begin by supporting students’ efforts to identify the two quantities that are changing, and then to coordinate the direction of change of one variable in relation to the other. The third mental action involves coordinating the amount of change of one variable with a constant increment of the other. Mental actions at the highest levels involve coordinating the instantaneous rate of change of the function with continuous changes in the independent variable for the entire domain of the function. The ways in which these mental actions served as a conceptual analysis to guide interface design are discussed in the following section.

Step 2. Designing tasks that might elicit perturbations. Once the conceptual analysis was framed, our goal was to create a provocative didactic model that would support at least the first three mental actions of the conceptual analysis outlined above. Given that the goal of didactic objects is to support in-class synchronous discussions, the challenge was to imagine conversations that might take place within a student’s own interactions with a dynamic object. This was conceptualized as a design challenge that involved providing activities that might catalyse a perturbation between what was expected and what emerged and to offer feedback that might help the individual learner to resolve this perturbation. In order to think more deeply about how to design an activity that would engage students in these modes of action and reflection, we consulted Stein et al.’s [7] Mathematical Task Framework for assessing cognitive demand.

Stein et al. [7] developed their Mathematical Task Framework to help teachers identify tasks that require high-level thinking, which is useful in the selection as well as creation of tasks. Their Task Analysis Guide divides activities into two main categories: low- and high-level tasks. Low-level tasks are characterized by memorization and the application of procedures without connections to the underlying mathematical ideas. In terms of online applets, examples of low-level tasks might include ‘drill and kill’ or flash-card type exercises that do not involve
making novel conceptual connections or deep mathematical thinking. It is important to note that these programmes have been shown to improve skills such as fact recall [8], but are not designed to challenge users to think about the underlying mathematics or how the mathematical arguments might play out in a classroom discussion. Therefore, they do not help develop mathematical reasoning and higher order thinking.

Higher demand cognitive tasks involve the development of novel procedures that have connections to the underlying mathematics. These tasks are designed to focus students’ attention on the use of procedures for developing deeper levels of understanding and require some degree of cognitive effort. Tasks at the highest level require a student to apply non-algorithmic thinking and demand self-monitoring or self-regulation of one’s own cognitive processes.

It is important to note that perceived task difficulty is not necessarily correlated with cognitive demand. In fact, a task can be quite easy in terms of execution, but still cognitively demanding. For example, one could argue that the quadratic formula may be difficult to memorize, but restating it is a memorization task, and using it without connections keeps it at a low level of cognitive demand. On the other hand, finding the area of an irregularly shaped object may not be arithmetically difficult, but it can be cognitively demanding to mentally manipulate and break down the shape into more familiar simpler shapes for which the area is easier to find. In both these cases, the focus is placed on the intended nature of the solution strategy, not the procedural component of executing it.

One example of a high-cognitive demand task for graphing is to ask groups of students to walk in front of a motion detector to create various target position-time graphs [9]. This activity is very powerful for helping students begin to focus on how the dependent variable – distance from the motion detector – can be identified as a measured quantity that is tracked over time. The target graphs serve as didactic objects if they are sequenced in a way that supports group conversations involving planning, executing and reflecting on the resultant graphs for resolution.

Although this task has high-cognitive demand, it has some drawbacks that an online activity could supplant. First, it cannot be easily revisited in later classes once the motion detectors are put away (not to mention the fact that these devices are not always readily available in schools). In contrast, applets are more easily accessible at any time and can be used at the students’ leisure and pace. Second, it can be difficult for students to produce ‘perfect’ graphs using the motion detector because graphs generated from empirical data have bumps or distracting marks that students may conflate with the actual concepts the instructor hopes they will construct. In contrast, computer applets can be designed to provide ‘cartoon worlds’ in which objects can move at mathematically accurate rates, thus producing more ‘perfect’ position-time graphs [10,11].

Step 3. Generating specific mathematical goals. The genesis for the series of applets we designed emerged over the course of 3 years of professional development experience. The original task, which was adapted from Bassarear [12] had been presented to teachers in many in-service workshops via pencil and paper.

Based on our classroom experience using this task during face-to-face workshops with in-service teachers, we developed the following four goals and four successive online applets.
Goal 1: Create applets that will allow users to focus on particular points on the graph to support the development of an appreciation of how the two quantities co-vary. The desired mental action that we were aiming for was to engage students in the activity of identifying quantities involved in the context of the flag-raising scenario (time and flag height), and the direction of how one changes as the other changes (e.g. constant change versus jagged change as might occur if raising the flag by motor versus by hand). Our prior classroom experiences indicated that one way to help teachers realize this goal was to have the instructor mark specific points on the graph and ask the in-service teachers to identify the time at which the flag was at the indicated location or the level of the flag at the specific time. The first applet in our series (shown in Figure 2) was designed to enable users to drag time to see corresponding flag height. We did not allow users to drag the flag at this point to reinforce the idea that time is independent, while flag height is dependent. The use of the multiple-choice question was to allow for individual student investment (and potentially initiate a perturbation).

Other features that we predicted might hold potential for perturbation and resolution via high cognitive demand were the requirement that the user monitor both quantities (although not simultaneously) in order to identify various points, and the hide/show feature to heighten anticipation rather than revealing the answer right away.

Goal 2: Create engaging task that will involve having students predict the graph based on viewing/imaging activity. The second applet (shown in Figure 3) invites users to predict the shape of two function plots using coloured ‘pencils’ (traced points) to once again engage predictions before revealing. This type of work is cognitively demanding because it requires non-algorithmic thinking and self-regulation. The possible perturbations we anticipated were how students would equate speed with
slope, and whether they would include a horizontal segment at the top of graph of the faster flag to reflect staying at one height.

**Goal 3:** Enable users to interpret constant speed and acceleration by comparing actual motions with their graphical representations in a conceptual way. Predicting graphs of nonlinear movement involves coordinating the instantaneous rate of change of the function with continuous changes in the independent variable for the entire domain of the function (Figure 4).

Applet 3 was designed to engage students in a task that involves matching two nonlinear graphs with their respective flag motions. Our experiences from our prior work using this applet in person with in-service elementary teachers led us to anticipate several possible perturbations. In order to capitalize on these, we created a video to support students’ online explorations. First, the video narrator (the second author of this article) asks students to watch the motion of the two flags (by clicking ‘raise flags’ or watching as the instructor does so in the video). Next, the students are asked to make a sketch of the two graphs using pencil and paper. The video pauses at this point mimicking the pedagogical move a teacher would make to encourage students to take the time needed to think about the problem. After the student has made a sketch, he or she is instructed to click the screen to continue the movie. At this point, the actual graphs are revealed and the narrator encourages asks the student to compare his or her sketched graph to the actual graph. The narrator then points out two features of the applet designed to support resolution of the potential perturbations between the sketched graphs and the shape of the actual graphs. First, she mentions the hide/show buttons designed to hide one graph so that the user can focus on just one motion at a time. Second, she points out the time ‘slider’ that allows the user to compare the positions of the two flags at any given time in relation to each other. Most importantly, she models how to pick out characteristics of the graph to tell the story of each flag motion. For example, she states, ‘When time is
about halfway, we see that the orange flag is further along than the green one'. She also discusses how to further differentiate the two graph motions by calling attention to the differences in height changes over a given increment in time by stating ‘As I move the time about halfway across the total time, I can see that the orange flag has traveled more than halfway up the flagpole. If I look at the second flag during that same time interval, we see that it has not traveled halfway’. The goal of using this type of modelling is to help students focus on developing vocabulary for talking about the causality of what they see rather than just focusing on getting the correct answer [9,13]. In other words, students believe that the graphs are correctly linked, and therefore they accept that the animation is linked to the graph. However, we wanted to support the graph to animation link by focusing on the co-variation of the time and position of the two flags over equal increments of time. In essence, we attempted to model how qualitative observations could be used to support a reverse link in which students pick out characteristics of the graph to tell the story. Such conceptual thinking is rarely encouraged or modelled in online applets. Instead, students are often encouraged to rush through to get an immediate answer.

**Goal 4: Revisiting the idea of co-variation by involving students in identification and manipulation of variables.** The idea for the fourth and final applet (shown in Figure 5) came from an in-service teacher who had used an earlier version of the applet series. Her suggestion was that we create the opportunity for the user to manipulate the virtual flag to see what graph is created from the movement. Unlike the other applets where the users were required to move time, this activity involves manipulating flag height as time moves automatically in order to trace out each of the three pictured
target graphs. This supports several conceptual goals including the idea that a graph is a series of points in time rather than immutable line [3].

Possible sources of perturbation from this applet include the requirement of constant self-monitoring, non-judgemental feedback (i.e. the computer does not state that a user’s graph is ‘accurate enough’, and the question of why some target graphs are more difficult than others. A third anticipated perturbation/resolution potential embedded in Applet 4 that users may experience is that each of the target graphs represents mathematical motion that is basically impossible to attain in real life. For example, in the first stair-step graph, it is impossible to move a flag instantaneously from one height to another to attain a perfectly vertical jump. In the second graph, it is very difficult to maintain a perfectly steady rate of increase or decrease. When emulating the third target graph, users realize that it is even more difficult to maintain a rate of speed that increases at a constant rate.

3. This study
This study is part of an ongoing developmental research programme. Over the past several years, we have attempted to build applets that include pedagogical moves that have been effective in face-to-face classrooms. Therefore, the goal of this particular research is to explore the degree to which the flag applets had become didactic objects that supported learners’ efforts to resolve perturbations and engage in reflective abstraction. More specifically, we wanted to determine if the design features in the flag applet series had provided sufficient interest, tools and feedback to support learners’ construction of deeper conceptual understandings based on mathematical goals 1–4.
3.1. Setting and methods

The setting for this particular data collection effort was a math class for pre-service elementary teachers at a large, urban university. The course contained 84 students (broken into two sections) with a total of 78 females and 6 males. The course was designed to examine ideas of early algebra from a conceptual point of view. The students had taken and passed enough high school algebra classes to pass a calculation-based college entrance exam. However, a large percentage of the comments on the open-ended sections of the responses and our prior experience with this population of students who have only taken minimally demanding math courses led us to believe that their conceptual understandings of the relation between functions, algebraic notation and graphing were very weak. The students had engaged in a motion detector activity during their first day of class and were asked to engage in the flag-raising activities (and related survey) for their first homework assignment.

The survey, which is included in the appendix, was completed online after the students had interacted with each of the applets. The students knew that they were not graded on their work with the applets and that their answers to the survey were anonymous, although they were given one point of homework credit for completing both tasks. The results were automatically tabulated using the survey feature of the Blackboard Learning Management System.

4. Results

Goal 1: Did the applet series enable students to focus on specific points on the graph and the co-variation of the two quantities? Results from the survey indicated that this question was viewed as ‘very easy’ by over 65% of the respondents. However, some respondents did indicate that they were able to utilize the ‘move time’ feature to illustrate the way in which the flag motion was represented on the graph. One student wrote:

I raised the flag about three times before I saw that the height and time were moving in constant and equal increments. Graph A shows this relationship. It shows that as time moves forward so does height. I moved the time slider on the graph and that confirmed that the time going by was making the height increase. At first I had confusion with what the flag had to do with the graph. I explored and thought about what the graph was representing in relation to the flag to clear up my confusion.

Goal 2: Did the hypothesize-before-reveal feature serve as a catalyst for eliciting perturbation? Open-ended comments from Applet 2 indicated that the use of the ‘cyber pencils’ seemed to engage users at a deeper level than if they were simply asked to ‘imagine’ what the graphs might look like, yet did not overburden them by asking them to find paper and pencil and make a completely accurate graph of each trip. This ‘sketch’ approach also enabled them to focus on the important characteristics of the graphs such as relative positions and starting and stopping placements as opposed to being worried about getting the graphs ‘exactly right’. In addition, the ability to quickly erase the traces also invited hypothesis revisions. In all, 72% of the respondents stated that they were surprised by the extra ‘horizontal’ part of the graph that emerged when the first flag hit the top of the pole, but all reported that they were able to make sense of this perturbation once they reflected on it.
Two examples of comments reflecting how the tools supported their efforts to resolve their perturbations follow:

The drawing pencils didn’t work as accurate as I wanted them to, but they still helped to more or less draw an idea of what I had in mind. I didn’t quite expect for the red flag to be graphed at such a constant rate but the line drawn had a constant slope. I think seeing both flags raised at the same time made me compare the two instead of looking at them individually and drawing them that way. It makes sense after looking at the flags individually.

I was able to use the drawing pencils to sketch a graph. This was helpful because instead of just picturing the graph in my head, I was able to look at the flag motion and right away put results down. I didn’t really understand why the yellow graph had not slope towards the end [sic]. After resetting the flag a couple more times and using the move time, I was able to understand the flat line at the top of the yellow graph. The flat line represents the time which the yellow flag had already reach the top of the flag pole while the red flag still needed more time to reach the top of the flag pole. I really liked this activity. Once getting the hang of all the apps [sic], I was able to manipulate the graphs and take a deeper look and making connections.

Goal 3: Did the interactive simulations enable users to interpret constant speed and acceleration by comparing actual motions with their graphical representations in a conceptual way? Of the 83 respondents, over 40% rated Applet 3 (nonlinear plots) as somewhat or very difficult. Student comments revealed that this was the first applet that provided new, and sometimes surprising, mathematical ideas. As noted earlier, we anticipated that this might be the case and therefore decided to make a video based on one instructor’s prior experience with this graph in her face-to-face sessions with in-service teachers. We then asked the pre-service teachers for their feedback on this applet and whether they found the video useful. Here are a few sample responses:

This activity was by far the most difficult for me because I never thought the graphs could turn out the way they did. It’s not linear because the lines are not straight. After watching the video, these graphs made a lot more sense. The video helps explain the different outcomes of these graphs, which without having watched the video; I couldn’t have answered this correctly.

I liked this graph because the flags did the opposite thing. I was able to match which graph went with what because I watched the time on both of them and the slower flag had more time in the beginning and sped up at the end [sic]. I definitely enjoyed watching the demonstration video because I felt like not only did it help with this graph, but with the other graphs because you have to think about what is going on before graphing.

The question of whether this student now ‘understands’ the full concept of nonlinear qualitative graphs remains to be seen. However, this answer, and several others like it, demonstrates one of our stated goals: having students think about the mathematical causality behind how the graph appears on the screen from a conceptual point of view. The student’s words, ‘I watched the time on both of them and the slower flag had more time in the beginning and sped up at the end’ indicate the type of thinking we hoped to elicit from the video.

Goal 4: Did the ‘drag-the-flag’ activity serve to highlight the flag height as a quantity that is co-varying with time? Comments on Applet 4 indicate that the visual feedback (trace of height of flag plotted as time moves forward) did scaffold users to first identify the two variables that were co-varying (time and distance) and to ‘feel’ how
their co-variation represented speed [14] and how this speed was reflected in the three different target graphs. Participants indicated that this activity was particularly insightful because it caused them to shift perspective. Instead of thinking about what the graph would look like, they had to get inside the motion to appreciate how the graph correlated position at any given time. For example, one participant stated, ‘It took me a couple of tries to get the speed close on the second graph. That was the most difficult for me. I had to find the right speed and stop to continue that speed back’. Another student commented on her self-monitoring process:

This was an interesting activity because I was moving the flag and making the graph according to the movements. Mimicking the graph with the stairs was a little challenging, but I figured out that if you paused the flag for a moment that it would make a horizontal line because you are not traveling at any speed over a certain amount of time. After I worked through the graph I had no problems. The pyramid-looking graph meant that the flag’s increase in speed at the same rate of decreasing speed. I understood what the graph was representing so it wasn’t too hard to copy although my manipulation of the flag was a little more rounder at the top then the pyramid’s peak.

Several others noted that the motion-detector activity that preceded this activity was very helpful. For example,

I believe the motion detector on the first day of class was very helpful because it allowed me to really understand the graph. Usually, I just know what the graph is saying, but there is no real meaning behind it. With this activity, I saw WHY the graph made sense. The homework activity was very helpful as well. It was good practice and allowed me to see how different graphs were created.

5. Conclusions

The answer to the overall question of whether it is possible to adapt Thompson’s [3] framework for creating didactic objects to the online environment appears to be a qualified ‘yes’. Although the quantitative data indicate that the activities may not have been rated as particularly difficult, they were still cognitively demanding in the sense that they required self-monitoring and the resolution of various perturbations. Given that the students were working alone, it seems as though the activity maintained their interest and served as a way to inspire what Sfard [4] calls ‘intrapersonal communication’. In other words, this study supports Sfard’s contention that interaction (considered interpersonal communication followed by action) can be social and is, in fact, educative. The open-ended data provide a glimpse into the ‘conversations’ that students were having with themselves as they made hypotheses and interacted with the features of the applets to resolve their perturbations.

As shown in Figure 1, the process of adapting Thompson’s [3] didactic object construct to the design of online applets is not straightforward. When teachers are using didactic objects in a live classroom, they can modify the question and the objects on-the-fly. Adding information – or hiding it at a critical juncture in the ongoing discussion – is a critical part of that process. However, this dynamic gets changed in an online version where designers have to anticipate the problems before they occur. This situation forces designers, at times, to remove some of the task’s cognitive demand. On the other side of the argument, designers can exploit the
interactive nature of the feedback to engage the user in some of the assessment tasks
the teacher would normally use to gauge next steps and feedback.

We conclude by listing some of our critical findings regarding specific design
considerations.

5.1. **Create tasks that are cognitively demanding**
As indicated in step 2 of our design scheme (pictured in Figure 1), we found the
Mathematical Task Framework [7] to be very helpful in determining the types of
activities that are sufficiently rich to engage students in mathematical excursions
without being so difficult that the task degenerates into a ‘guess and click’ formality.

5.2. **Include instructive videos that model conceptual thinking and provide language
for conceptualizing causality**
As many of the comments indicated, students found the video very useful either as an
introduction to the software, to the concept, or to the idea of picking out specific
characteristics of a graph to tell a story about motion a conceptual way. The intent of
the video – as well as the contents – can be derived by examining results from steps 1
and 3 in the design scheme.

5.3. **Provide non-judgemental feedback requiring the user to assess the accuracy of
an answer**
This finding, which aligns with step 4 of the design scheme, supports other research
we have conducted regarding the degree to which pre-service teachers prefer to make
their own decisions regarding the accuracy of an answer versus those who prefer
explicit feedback [12]. As with the other study, the results were mixed. While the
majority did appreciate the open-ended nature of the task and feedback, several did
suggest that a ‘correct’ or ‘incorrect’ feedback mode might have been more helpful in
terms of highlighting the conflict explicitly. One of our goals for further research is to
delve more thoroughly into this aspect of educational design. One potentially
promising idea is to use a feedback meter that indicates how close an answer is to the
‘actual’ answer. For example when students are asked to use the ‘cyber pencil’ to
sketch a graph of each flag (Figure 4), some students indicated that they would have
preferred to know if their sketch was ‘close enough’. Our view is that the two goals of
the assignment were (1) to highlight different slopes and (2) to indicate that while the
first flag had reached the top, its graph continues as time continues. A thermometer
might be placed in the frame that would indicate how ‘close’ the student’s response
is to these two aspects of the problem.

5.4. **Build investment by asking students to hypothesize before revealing the correct
answer**
This feature, which also aligns with step 4 of the design scheme, has proven to be a
critical aspect for engaging students online. When students are asked to make visual
predictions such as drawing a function plot or placing a point on number line, they
become more engaged in finding out the causality behind the correct answer. Some
of the applets that we have designed require students to input an initial answer before one is revealed while others (such as this flag series) do not. We chose to allow for a reveal then hypothesize action to support students who have no conception of what is expected but want to work backwards from the revealed answer. This process also supports the idea of having a conversation and resolution with oneself to construct new understandings.

We conclude with a caveat: we firmly maintain that online didactic objects will never be able to replace the spontaneity and generativity of a well-conceived, whole-class discussion. For example, many of the more subtle ideas, such as the fact that a graph is a locus of points rather than a set line [14] – which could have been brought out in a synchronous classroom discussion – did appear to elude students’ attention. However, we are inspired to continue with future efforts to design online didactic objects that will involve creating cognitively demanding tasks that have features such as hypothesize before reveal buttons, non-judgemental feedback and questions that require monitoring of one’s own physical actions (as opposed to just mental actions), such as the processes involved in Applet 4. This type of bodily engagement reflects Nemirovsky’s [15] claim that engaging students in gesturing and body movement may further support cognitive development. Regardless of the technology on which it is delivered, the critical question for us is to continue to research how various design features that engage today’s ‘digital natives’ in the process of driving their own learning.

References

Appendix

Survey questions for flag applet

Question 1: Essay
The goal of the first question (which is pictured below) was to introduce the flag scenario with just one flag moving. Describe how you solved this first task. You may want to consider some of the following ideas in your answer: 1) Did you raise and lower the flag several times before making a choice, or did you raise the flag right away? 2) Did you move the slider on the graph? 3) Describe any perturbations (confusions) that you experienced. How did you resolve them?

Which of the following graphs best represents the realized motion that you just saw? Once you have made a choice, click "new graph" to see the graph be drawn as the animation continues.

Time

Height

Other Graph

New Graph

Question 2: Open-Ended
Rate the difficulty of activity #1 (1 flag as pictured above).

Question 3: Essay
In the second question (pictured below) you were asked to create a graph that reflected each of the two flag motions. Please answer the following questions: 1) How long did you feel comfortable using the "drawing pencil" to sketch a graph, or did you just imagine what the two graphs would look like? 2) Describe any differences between what you expected and what you saw. If the answer surprised you, what was surprising and how did you reconcile your expectations with what you saw? 3) Does it make sense now? 4) Do you have any suggestions for changing/adjusting this activity?

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Question 1: Opinion:
Rate the difficulty of activity 1 (1-5 flags, as pictured above).

Question 2: Essay
In the second activity, the flags moved at non-constant rates. Please describe your reflections on the set of scenarios. Consider the following: a) Did you see the graphs that were created in response? If so, how? b) Were you able to match the correct graph to the appropriate flag? If not, did this help you? c) Did you view the video modeling how to interpret these graphs? What reactions do you have to the video? d) Would you suggest that we place an introductory video before the first activity?

Question 3: Multiple Choice
Rate the difficulty of activity 2 (2 flags, as pictured above).

Question 4: Essay
In the matching activity, you were asked to match graphs to stories. Describe how you solved these tasks. Consider: a) Did you attend to specific wording in the descriptions? b) Did you visualize a graph that went with a description, or did you read the description and look for an appropriate graph? c) Which (if any) of the scenarios were problematic or surprising to you? Please explain.

Question 5: Multiple Choice
Rate the difficulty of activity 3 (2 flags, as pictured above).

Question 6: Essay
The "Challenge" activity was added later on the suggestion of one teacher who said that it would be neat to control the flag movement and see the resulting graph. Describe your reactions to this activity. a) What did you find most challenging about this activity? b) Which of any graphs were particularly difficult to model? Please explain any insights you got from your work. c) Consider the second graph. How did you go about modeling this one? d) Do you have any other suggestions for modifications of this activity?

Question 7: Multiple Choice
Rate the difficulty of activity 4 (matching, as pictured above).

Question 8: Essay
The difficulty of the challenge activity (you move flag to make specific target graphs, as pictured above).