The retailer multi-item inventory problem with demand cannibalization and substitution

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Available online 5 June 2006

Abstract

Retailers of products with limited shelf life are faced with the dilemma of stocking the right mix of standard product and its customized stock keeping units, in each product category. In this paper we model the retailer multi-item inventory problem with demand cannibalization and substitution. The model focuses on the twin problems of optimal portfolio selection as well as optimal stocking under retailing context. Owing to analytical complexity in determining optimal solution, we develop heuristics for solving the problem. Using set of numerical examples we compare the heuristic solutions against the optimal solutions. In addition, we also attempt to understand the impact of important parameters on retailer profits through a series of sensitivity analysis.

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Keywords: Inventory planning; Multi-item with cannibalization and substitution; Retailing; Retail portfolio planning

1. Introduction

Increase in competition is accompanied by the twin trends of product price erosion and product proliferation. Retailers cater to the price sensitive customer by providing low-priced standard products. They also cater to the customer segments that are willing to pay a premium for customization by offering customized extensions of the standard product. While profit margin for a standard product is generally low and profit depends on volumes, its customized extensions carry higher profit margins. However, the standard product typically has a longer shelf life while its customized extensions are often promoted as flavours of the season and generally have short shelf lives. Introduction of customized extensions in many situations helps firms to add new customers and increase the total sale volume. Examples of this are (i) plain breakfast cereal (standard product) with flavours for different climatic seasons, (ii) chocolate (standard product) with flavours for different occasions like vacations and festivals, and (iii) sportswear (standard product) with special designs for different game seasons/occasions like basketball season, cricket World-Cup and Olympic Games. Demand uncertainty increases with increase in customized extensions. Demand forecasting accuracy of individual stock keeping units (SKUs) reduces with increase in demand uncertainty. This in turn implies higher possibility of stock outs or oversupply.
Retailers of products with limited shelf life are faced with the dilemma of stocking the right mix of standard product and its customized extensions or SKUs, in each product category. On one hand, stocking only a standard product ensures logistics that can be managed easily and efficiently, lower manufacturing cost, and stable demand with minimal stock-out or oversupply. The profit is driven by sales volume. It is also owing to lesser stock-out or oversupply losses. On the other hand, stocking only customized products implies higher revenues and margins, higher logistics complexity owing to higher variety, higher manufacturing costs and higher stock-out and oversupply losses. The standard product retailer can aim for further profits through additional retailing of the customized extensions of the standard product. The possible increase in profit is owing to: (i) retailing products of higher margins (though, retailing them could also result in higher losses owing to clearance sales or stock-outs) and (ii) increase in total volume (if introduction of a customized extension does not cannibalize the demand for the standard product). Further, if a particular customized extension is out of stock, customers for whom this product is first choice might purchase the standard product. This phenomenon is widely known in literature as one-way substitution wherein a customized extension demand gets substituted as standard product demand in case of stock-out of that customized extension. In this paper, we attempt to derive the optimal allocation of a standard product and its customized SKUs that would maximize the retailer’s profit.

The above problem belongs to a class of problems known as the single period multi-product inventory models with stochastic demand. Over the last decade, several researchers have worked on the single period multiple-product inventory problem. In case we assume that all items have independent demand, and products cannot act as a substitute in stock out situation, such a problem is known as multi item newsboy problem. Refer Khouja (1999) for excellent survey on capacitated multiple-product single period inventory problem.

Analysis of a two-product substitution, where one product can substitute for the other, appears in a number of papers starting with McGillivary and Silver (1978). One set of papers deals with manufacture controlled substitution wherein higher quality product can substitute low quality level product, but not vice versa (Pasternack and Drezner, 1991; Bassok et al., 1999). Another set of papers deals with customer led demand substitution where customers purchase the substitute product when their preferred product is out of stock (Smith and Agrawal, 2000; Mahajan and van Ryzin, 2001). While one set of papers allows one way substitution (Bassok et al., 1999), another set of papers allows two way substitution (Pralar and Goyal, 1984; Pralar, 1988; Khouja et al., 1996). Eynan and Fouque (2003) deals with a special case of customer led substitution where the retailer puts in a special effort in inducing customer substitution of high variance item by low variance item even when high variance item is in stock.

Most of the earlier works deal with two products case and do not consider impact of cannibalization owing to introduction of an additional product. Unlike Annupindi et al. (1997), and Mahajan and van Ryzin (2001) who focus on optimal stocking policies for a given portfolio, Smith and Agrawal (2000) deals with optimal portfolio selection problem as well as optimal stocking problem. Our research differs in many ways from earlier research. Like Smith and Agrawal (2000) we focus on the twin problems of optimal portfolio selection as well as optimal stocking. However, unlike Smith and Agrawal (2000) our paper considers demand cannibalization and downward substitution. We explicitly bring in the demand cannibalization effect by defining a demand cannibalization index. By defining a substitution index, we also capture the downward substitution of customized products by standard products. Unlike Smith and Agrawal (2000) who model demand as a negative binomial process, we model demand as a Poisson process on the lines of Annupindi et al. (1997). Further, our focus is on deriving a heuristic solution that can provide a reasonably good solution in a multi-product portfolio case.

The problem is modelled in Section 2. The solution methodology by heuristics approach with numeric examples is described in Section 3. Numeric examples and evaluation of the heuristics are covered in Section 4. Discussions and conclusions are in Section 5.

2. The problem

In the newsboy problem, each period starts with a certain quantity of the product (opening stock) from which the product demand for that period is met. The newsboy cannot replenish the stock during the period in case the demand is more than the opening
stock. Hence, the demand that is in excess of the opening stock is lost sales for the newsboy. Also, when demand is less than the opening stock, the newsboy has to clear the unsold stock at a salvage or clearance price at the end of the period. The objective of the newsboy problem is to determine the optimal opening stock that minimizes the above losses.

In this section, we model the retailer’s SKU allocation problem as a multi-item single period inventory planning problem. The retailer may stock perishable or non-perishable items. As already described, the single-period inventory problem is valid for inventory planning of perishable goods. However, even inventory planning of non-perishable items—items that can be carried over to future periods in case of left over—can be modelled as a single period problem. The cost of overstocking of the single period problem in such a case would at the minimum be the inventory carrying cost per period. The specific period under study is described as model period henceforth.

A single product category comprising of a standard product and its customized extensions are the multiple items that are modelled in this problem. A specific application illustration of our problem is that of the English soccer team and players’ jerseys (product category). The standard product is the team jersey that comes with neither the player’s name nor his jersey number. The customized extensions are the players’ jerseys that come with the tournament logo, player name and player number (and even player picture). These additional features may mean higher value for the fans, which implies a higher unit revenue opportunity for the retailer. However, there is a high perishability associated with these features. Other examples have already been pointed out in the previous section.

The solution to the retailer’s SKU allocation problem comprises of arriving at the following decisions; (i) whether the retailer should retail standard product only or also retail customized extensions of the standard product (optimal retail portfolio); and (ii) optimal stocks of standard product and the customized extensions of the standard product that are included in the retail portfolio, in order to maximize his expected profit.

As already described in the previous section, the retailer may just stock only standard products and aim to make profits on volumes. By following this strategy, owing to retailing only one product, there would be higher demand forecasting accuracy, which would imply lesser stock outs or oversupply losses. However, the retailer may aim further profits through additional retailing of the customized extensions of the standard product. The possible increase in profit is owing to (i) retailing products of higher margins (though, retailing them could also result in higher losses owing to clearance sales and lost sales) and (ii) increase in total volume (if introduction of a customized extension does not cannibalize the demand for standard product). We capture the cannibalization of standard product demand by defining a demand cannibalization index.

We assume that the unit cost of procuring and retailing as well as the unit profit margin is higher for the customized extensions with respect to that of the standard product. We also assume that the base demands of the standard product and its customized extensions (base demand for a product is its demand from customers if that is the only product offered by the retailer) are independent of each other. If a particular customized extension is out of stock, the customers whose first choice is this product may purchase the standard product. We assume a potential one-way substitution wherein a customized extension demand may get substituted as standard product demand in case of stock-out of that customized extension. There is no substitution among the customized extensions. The one-way substitution also means that stock-out of standard product would not result in demand for any of its customized extensions; instead it would result in a lost sale. Going back to our example above, these assumptions imply that fans willing to buy the English team jersey will not buy a player jersey in case of a stock out of the team jersey (standard product). However, a fan of David Beckham may buy the English team jersey when the David Beckham jersey is out of stock. He or she will, however, not purchase any other player jersey if the David Beckham jersey is out of stock.

2.1. Notations

\[ I = \{i | i = 1, ..., p\} \]

= set of customized extensions (custom SKUs) of standard product offered by the retailer in model period

\[ r \]

unit retail price of standard product in model period
Maximize \( \Pi^J \)

\[
\Pi^J = \sum_{(x^b, x_1^b, ..., x^b_n)} \left[ \sum_{i \in J} \left\{ \frac{(r_i - c_i)(\min(x_i^b, s_i))}{(c_i - m_i)(s_i - \min(x_i^b, s_i))} \right\} \right] P(x^b, x_1^b, ..., x^b_n),
\]

subject to

\[ s, s_i \geq 0 \quad \forall i \in J. \]

For portfolio \( J \), let \( \Pi^{J^*} \) indicate the maximum retailer profit obtained by solving P1. The next stage
of the solution involves determining the optimal portfolio (portfolio that maximizes retailer profit). This is described by problem P2, which is formulated as follows:

Problem P2.

Maximize \[ \prod_{j=1}^{p} f_{i}(y_{i}) - \sum_{i=1}^{p} H_{i} y_{i}, \] (3)

Subject to

\[ J = \{ i \mid y_{i} = 1, \forall i = 1, \ldots, p \}, \] (4)

\[ y_{i} \in (0, 1) \quad \forall i = 1, \ldots, p. \] (5)

Solving problem P2 involves exhaustive enumeration of \( 2^{p} \) portfolios. For each enumeration, the optimal stock levels that results in maximum retailer profit are determined by solving P1.

3. Solution methodology

It can be seen from above formulations of P1 and P2 that solving the retailer’s SKU allocation problem is analytically complex. In this section, we propose a heuristics approach for solving the retailer’s SKU allocation problem.

The formulations in previous section are an adaptation of the multi-product newsboy problem (Hadley and Whitin, 1963). For the customized product \( i \), when demand \( d_{i}^{J} \) is less than the stock level, \( s_{i} \), a unit profit of \( (r_{i} - c_{i}) \) is realized on quantity \( d_{i}^{J} \) and an unit loss of \( (c_{i} - m_{i}) \) is incurred on quantity \( (s_{i} - d_{i}^{J}) \). When demand \( d_{i}^{J} \) is greater than or equal to the stock level, a unit profit of \( (r_{i} - c_{i}) \) is realized on quantity \( s_{i} \). The unsatisfied demand, \( (s_{i} - s_{i}) \), is met by sale of standard product, subject to availability.

Let,

\[ CF = \text{critical fractile for standard product} \]
\[ CF_{i} = \text{critical fractile for custom SKU } i \]
\[ CF_{i}^{J} = \text{Max } (CF_{1}, CF_{1}, CF_{2}, \ldots, CF_{n}) \]
\[ CF_{i}^{J} = \text{Min } (CF_{1}, CF_{1}, CF_{2}, \ldots, CF_{n}) \]
\[ d^{J} = \text{aggregate demand at retailer for portfolio } J \text{ in model period} \]
\[ S^{J} = \text{total stock at retailer for portfolio } J \text{ in model period} \]

The notations UB and LB are respectively used for indicating upper bound and lower bound of different variables. Let \( f(x) \) denote the probability distribution function of random variable \( x \) and let \( F(x, k) \) denote the cumulative probability function such that \( x \leq k \). We also use \( \mu(x) \) and \( \sigma(x) \) for describing mean and standard deviation, respectively, of \( x \). We define a function \( P(x, k, z) \) which returns the value of the smallest integer \( k \) that satisfies \( F(x, k) \geq z \).

Proposition 1. Total stock, \( S^{J} \), has an upper bound \( S_{UB}^{J} \) such that,

\[ P(D^{J}, k, CF_{\text{max}}^{J}) = S_{UB}^{J}. \] (6)

As the highest value among the critical fractiles of all the products stocked by the retailer is taken in (6), the optimal stock cannot exceed \( S_{UB}^{J} \). This upper bound is valid even in a situation where substitution is both ways, standard product to custom SKU and custom SKU to standard product. It may be noted, however, that in our model we assume only one-way substitution, from custom SKU to standard product, and that there can be partial substitution (\( 0 \leq \beta \leq 1 \)).

Proposition 2. Under total customized product demand substitution (\( \beta = 1 \)) and complete standard product demand cannibalization (\( \alpha = 1 \)), lower bound on optimal stock level of standard product is such that

\[ P(d^{J}, k, CF) = s_{LB}^{J} + \sum_{i \in J} s_{i}^{J}. \] (7)

In a complete standard product demand cannibalization situation, aggregate demand distribution is same as base demand distribution for standard product. It can be easily shown that \( d^{J} \) will follow distribution with \( \mu(d^{J}) = \mu(d^{J}) - \sum_{i \in J} \mu(d_{i}^{J}) \) and \( \sigma(d^{J}) \geq \sigma(d^{J}) \). Hence, it is obvious that

\[ s^{J} \geq s^{J} - \sum_{i \in J} s_{i}^{J}. \] (8)

In case retailer stocks only standard product, its optimum stock, \( s^{J} \), would be such that \( P(d^{J}, k, CF) = s^{J} \). Hence, lower bound on standard product stock for \( \alpha = 1 \) and \( \beta = 1 \) follows (7).

Proposition 3. Under the situation of no standard product demand cannibalization (\( \alpha = 0 \)), lower bound
on optimal level of custom SKU stock is given by
\[ P(d^*_i, k, MF_i) = s_{i, LB}^i. \]

where
\[ MF_i = [(r_i - c_i) - b(r - c)]/[(r_i - m_i) - b(r - c)]. \]  

(9)

Optimal stock level for custom SKU is obtained by critical fractile \((r_i - c_i)/(r_i - m_i)\) (Pasternack and Drezner, 1991). The cost of under-stocking custom SKU is different in this model context from the one in Pasternack and Drezner (1991), which results in a tighter lower bound for the custom SKU stock level as shown below:

Cost of under-stocking custom SKU \(i\)\)
\[ = (r_i - c_i) - b(r - c) \quad \forall i = 1, \ldots, n, \]

Cost of over-stocking custom SKU \(i\)
\[ = c_i - m_i \quad \forall i = 1, \ldots, n. \]

The above definition of cost of under-stocking custom SKU \(i\) assumes that standard product is always available in case the custom SKU is out of stock. Since the stock of standard product is finite, there is a finite probability that standard product stock is not available for substitution under a stock out situation. In such a situation, cost of under-stocking is higher than shown above. Hence, Proposition 3.

In case of standard product demand cannibalization \((\alpha > 0)\), the above need not be valid as demand of standard product (before substitution) is negatively correlated with custom SKUs demands.

The heuristic-based solution methodology comprises of two parts: (i) heuristic for determining optimal stock levels of standard product and custom SKUs in retail portfolio \(J\); and (ii) heuristic for determining optimal portfolio. To simplify our exposition we take \(\beta = 1\), that is the complete substitution case. However, it is not difficult to incorporate other values of \(\beta\) in our methodology.

3.1. H1. Heuristic for determining optimal stock levels for retail portfolio \(J\)

Step 1: Set \(\prod^J_0\), initial portfolio profit, equal to zero.

Step 2: Generate demand distribution for aggregate demand \(D^J\).

Since demand for each custom SKU is independent, generating their demand distributions is easy for extreme values of \(\alpha (\alpha = 0\) and 1). For \(\alpha = 0\), since all the base demands are independent, \(f(D^J)\) is such that:
\[ \mu(D^J) = \mu(d^b) + \sum_{i \in J} \mu(d^h_i), \]

and
\[ \sigma^2(D^J) = \sigma^2(d^b) + \sum_{i \in J} \sigma^2(d^h_i). \]

For \(\alpha = 1\), demand distribution for total demand is same as demand distribution for standard product in a base situation. Or,
\[ f(D^J) = f(d^b). \]

For intermediate values of \(\alpha (0 < \alpha < 1)\), which implies negative correlation between standard product and custom SKU demands, the correlation coefficient should also be considered while determining aggregate demand distribution.

Step 3: Assign values of \(s^J_i\).

Proposition 3 is used to assign values of \(s^J_i\).

\[ s^J_i = s_{i, LB}^J. \]

Though the above lower bound has been derived for \(\alpha = 0\), it is felt that it would give reasonably good solution for other values of \(\alpha\) including \(\alpha = 1\).

Step 4: Obtain initial value of total stock.

Proposition 1 is used to obtain initial value of total stock.

\[ S^J = S_{UB}^J. \]

Step 5: Obtain value of \(s^J\).

Proposition 2 is used to obtain value of \(s^J\).

\[ s^J = S^J - \sum_{i \in J} s^J_i. \]

Step 6: Obtain \(\prod^J\) for above values of standard and custom SKU stocks
If \(\prod^J - \prod^J_0 < \delta\), where \(\delta\) is a very small number, go to Step 7. Otherwise, set \(\prod^J_0\) equal to \(\prod^J\) and go to Step 7.

Step 7: Set \(S^J = S^J - 1\) and go back to Step 5.

Step 8: Stop H1.

The above methodology is valid for discrete demand distributions. In case of continuous demand distributions, a lower bound on total stock is set and a grid search is done for obtaining optimum value of \(S^J\). Since objective function is concave with respect to values of stock, above methodology is
valid. Lower bound on \( S' \) could be found using methodology similar to Proposition 1.

\[
F(d^i, S_{LB}^j) = CF_{\min}^j.
\]

(12)

3.2. H2. Heuristic for determining optimal portfolio

Though retailer faces \( 2^p \) choices, we restrict our focus on \( p \) portfolios to start with. The \( p \) portfolios are constructed by choosing only one custom SKU in each of the \( p \) portfolios.

Step 1: Set \( i = 1 \).

Step 2: Let \( J = 1 \).

Step 3: Determine \( \prod^J \) using \( H1 \) as described above.

Step 4: If \( \prod^J - H_i > \) Profit for portfolio with only standard product (no custom SKUs), then set \( y_i = 1 \), otherwise set \( y_i = 0 \).

Step 5: Set \( i = i + 1 \).

If \( i = p + 1 \), then go to Step 6. Otherwise, go to Step 2.

Step 6: Let \( J = i \) if \( y_i = 1 \) \( \forall i = 1, \ldots, p \). Stop H2.

4. Numerical example

In this section, we compare the heuristics solutions with the optimal solutions using a numerical example. We attempt to gauge the heuristic accuracy through these comparisons. For the sample problem, optimal solution is obtained by complete enumeration.

We consider a retailer who sells a standard product and has a set of two custom SKUs as potentials to be added to his retail portfolio. The standard product base demand in the model period is 40 units. For simplicity we assume that this base demand is constant for all time periods. Also, \( r, c \) and \( m \) take values 57.5, 50 and 45, respectively. The discrete demands for custom SKUs 1 and 2 follow a Poisson distribution with mean described by \( \lambda_1 \) and \( \lambda_2 \).

We carry out 72 simulations by considering three different demand situations (two only one custom SKU situations and one two custom SKUs situation), two levels of fixed cost per period for retailing (\( H_1 = 0 \) and \( H_2 = 20 \) \( \forall i = 1, 2 \)), two levels of standard product demand cannibalization index (\( z = 0 \) and 1), two levels of customized product demand substitution index (\( \beta = 0.5 \) and 1) and three levels of custom SKU critical fractile values (\( 3 \times 2 \times 2 \times 2 \times 3 = 72 \)). Three sets of \( r_i, c_i \) and \( m_i \) values are defined that correspond to three custom SKU critical fractile values, as shown in Table 1 below. As mentioned in Section 1, custom SKUs are likely to have higher price realization but lower clearance price. The same has been kept in mind while generating the three possible values of \( CF_i \). The three \( CF_i \) values are such that they are lower, similar and higher compared to the standard product critical fractile value (\( CF = 0.6 \)).

The software @Risk was used for running the simulations and arriving at expected profit of retailer and the optimal decision variables (\( x, s_i \) and \( y_i \)). @Risk is a powerful versatile simulation model which allows building of spreadsheet based simulation models (Winston, 2001). For given stock values, multiple simulations were carried out and each simulation run was carried for large number of iterations to ensure that steady state results are obtained. Each simulation on @Risk took just a few seconds of computer time on a personal computer.

The heuristic and optimal solution results for the two Only One Custom SKU demand situations are as shown in Table 2 (48 simulations). The optimal solution is determined by exhaustive enumeration for all the possible stock values of custom SKUs and standard product.

For the same aggregate demand, consider a two customized products scenario with \( \lambda_1 \) and \( \lambda_2 \) equal to 8 and 4, respectively. The heuristic and optimal solution results are as shown in Table 3.

Of the 72 simulations in Tables 2 and 3, the highest percentage between maximum profit difference between optimal solution and heuristic solution is 0.59%. The average error for the 72 simulations works out to 0.073%. In 47 out of 72 situations, the heuristic solution is same as the optimal solution. The errors were similar in the one custom SKU and two custom SKU situations (refer Table 4), which indicates that the heuristic performance does not deteriorate with increase in custom SKUs. Hence, it is reasonable to conclude that the proposed heuristic generates solutions that are close to optimal solution.
5. Discussion and conclusions

5.1. Impact of retail parameters on retailer profitability

From the numeric examples shown above in Tables 2 and 3, one can draw insights on impact of different parameters on profitability of retailer. We focus on impact of three parameters: (i) critical fractile, which captures relative profitability, (ii) standard product demand cannibalization index, and (iii) custom SKU demand substitution index. Fig. 1 captures impact of these parameters on expected profit for fixed cost of retailing additional unit \( (H_i) \) equal to zero. As can be expected, with increase in value of critical fractile, reflecting relatively higher marginal profitability of custom SKU, expected profit increases. Similarly, with increase in value of standard product cannibalization index, \( x \), expected profit declines. As discussed earlier, increase in profit due to addition of customized item to portfolio is owing to (i) retailing products of relatively higher marginal profits and (ii) increase total volume of demand. In a situation where \( x \) is high, bulk of demand of customized products comes at the expense of standard product demand. As a result, one would see only marginal increase in total volume of demand, finally resulting in a relatively lower increase in value of total profit compared to scenario of low \( x \). As one can expect intuitively, expected profit increases with increase in value of custom SKU demand substitution index.

It is interesting to see the interaction effect of critical fractile and custom SKU demand substitution index on expected profits. As can be seen in Fig. 1, the impact of value of \( \beta \) on profitability starts declining with increase in value of critical fractile. At low values of critical fractile, value of \( \beta \) does have significant impact on profits. For example for \( CF_i = 0.25 \) and \( \beta = 1 \), profit declines by 18.87 units when value of \( \beta \) goes down from 1 to 0.5. However, at \( CF_i = 0.75 \) and \( \beta = 1 \), decline in profit is just 6.41 units because of reduction in value of \( \beta \) from 1 to 0.5. This can be explained as follows. At low values of \( CF_i \), the retailer would stock smaller quantities of customized products and the same would result in

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### Table 2
Results for One Standard Product and One Custom SKU

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</tr>
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</table>

(continued...
higher values of expected stock out of custom SKUs. In such a scenario, high value of $\beta$ would result in higher level of demand substitution by standard products while in case of lower values of $\beta$ substantial portion of these demands would get lost. As a result, at a low value of $CF_i$, value of $\beta$ parameter plays important role. But at high values of $CF_i$, one would stock higher quantity of custom SKU items, which would result in decline of value of expected custom SKU shortage. Hence, impact of demand substitution index would be lower in magnitude.

As one would intuitively expect, presence of fixed cost $H_i$ (fixed cost of introducing custom SKU in portfolio) would have significant impact on portfolio composition. As can be seen in Table 2, when $H_i$ is not zero, there are a number of instances where custom SKUs do not get included in the portfolio because marginal increase in profit is less than the fixed cost of adding that particular custom SKU in the retail portfolio. However, our numerical analyses show that even when fixed costs are zero, cannibalization and substitution effects can reduce the size of the portfolio. As can be seen in Table 1, at high values of $\alpha$ and low values of $CF_i$ and $\beta$ it does not make sense to include custom SKU in the portfolio. This is because at high values of $\alpha$, demand of custom SKU comes at the cost of standard product demand. Given low values of $CF_i$, the retailer would stock a custom SKU at a level lesser than expected demand and bulk of this supply shortfall would get lost because of low value of demand substitution index. As a result, expected increase in profit on account of sale of custom SKU is lower than expected value of reduction in profit from sale of standard products.

### Table 3
Results for one standard product and two custom SKUs

<table>
<thead>
<tr>
<th>$H_i$</th>
<th>$\beta$</th>
<th>$\alpha$</th>
<th>$CF_i$</th>
<th>Heuristic</th>
<th>Optimal</th>
<th>Diff (%)</th>
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<td></td>
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<td></td>
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<td>$s_1$</td>
<td>$s_2$</td>
<td>$s$</td>
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<td>43</td>
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### Table 4
Comparison of heuristics for one custom SKU versus two custom SKUs cases

<table>
<thead>
<tr>
<th>Highest error (%)</th>
<th>Average error (%)</th>
<th>Percentage of times when $\Pi_0 = \Pi_h$</th>
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<tbody>
<tr>
<td>Single product</td>
<td>0.57</td>
<td>0.080</td>
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<tr>
<td>Two products</td>
<td>0.29</td>
<td>0.055</td>
</tr>
<tr>
<td>Overall</td>
<td>0.57</td>
<td>0.073</td>
</tr>
</tbody>
</table>

As one would intuitively expect, presence of fixed cost $H_i$ (fixed cost of introducing custom SKU in portfolio) would have significant impact on portfolio composition. As can be seen in Table 2, when $H_i$ is not zero, there are a number of instances where custom SKUs do not get included in the portfolio because marginal increase in profit is less than the fixed cost of adding that particular custom SKU in the retail portfolio. However, our numerical analyses show that even when fixed costs are zero, cannibalization and substitution effects can reduce the size of the portfolio. As can be seen in Table 1, at high values of $\alpha$ and low values of $CF_i$ and $\beta$ it does not make sense to include custom SKU in the portfolio. This is because at high values of $\alpha$, demand of custom SKU comes at the cost of standard product demand. Given low values of $CF_i$, the retailer would stock a custom SKU at a level lesser than expected demand and bulk of this supply shortfall would get lost because of low value of demand substitution index. As a result, expected increase in profit on account of sale of custom SKU is lower than expected value of reduction in profit from sale of standard products.
5.2. Independent versus joint inventory decision making

Most of the retailers take decisions about stock levels independently and do not take into account cannibalization and substitution in case of stock out situations (Smith and Agrawal, 2000). We compare performance of optimal solution with a naïve model where each item in the portfolio is stocked by solving newsboy problem independently. As shown in Table 5, for a case where \(H_i\) is 0, we find that the optimal solution is significantly superior to that of the naïve model.

It can be seen that in 10 out of 12 cases, the heuristic gives substantially better solutions (average improvement of 8.7% with maximum improvement of 21.7%). Only in two cases profit obtained in the naïve solution is same as the one obtained in optimal solution. In general, difference between optimal solution and naïve model solution increases with increase in value of standard product cannibalization index. Similarly, with higher level of

Table 5
Comparison of Optimal and Naïve Solutions

<table>
<thead>
<tr>
<th>(H_i)</th>
<th>(\beta)</th>
<th>(\alpha)</th>
<th>(\text{CF}_i)</th>
<th>(s_1)</th>
<th>(s_2)</th>
<th>(s)</th>
<th>(\Pi_h)</th>
<th>(s_1)</th>
<th>(s_2)</th>
<th>(s)</th>
<th>(\Pi_n)</th>
<th>((\Pi_h - \Pi_n)/\Pi_n)</th>
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Fig. 1. Impact of product parameters on profit.
custom SKU substitution index, optimal solution starts showing better performance compared to naïve model solution. Also, at a lower level of CF, difference between optimal solution and naïve solution widens further.

5.3. Other managerial implications

Given that retailers are forced to carry broader portfolios, they need to understand the impact of cannibalization and demand substitution. With increased competition in retailing, profit margins are under pressure. Higher profit margins to the tune of 8% as shown in the numerical example cannot be neglected during times of tough competition.

From the numeric examples shown in the previous section, one can draw insights about portfolio selection and inventory management under the situation of cannibalization and demand substitution. As expected, the fixed cost of retailing custom SKU is a crucial factor in deciding whether the SKU should be retailed or not. However, our numerical analysis shows that even when fixed costs are zero, cannibalization and substitution effect can reduce the size of portfolio.

The formulation and the results clearly indicate that the stock-level of customized SKU will depend on the cost of under-stocking and cost of over-stocking the SKU. As we saw in the formulation, the cost of under-stocking a custom SKU depends not only on the profit margin of the custom SKU but also on the profit margin of the standard product. Hence, portfolio decisions should factor the fixed cost of retailing of custom SKUs, the increase in profit margin owing to creation of the custom SKU and the clearance cost of custom SKU in case of lower than expected demand situation. It is important that this cost is also considered by the retailer. When fixed cost of retailing a custom SKU is high and the incremental profit margin of the same is negligible and clearance costs are high, it does not make sense to retail customized SKUs.

In many retail situations, it is not uncommon to see only custom SKUs being retailed. In such situations, if a customer does not get the custom SKU he seeks, he has no option but to forsake purchase. However, when there is one standard product available, customers who do not get the SKU of their choice can opt for that. For the retailer, a lost sale gets converted into a low-margin sale. Of course this strategy is valid only if custom SKU demand substitution index is high. It also implies that where a standard product is not available currently, introduction of one could be an ideal strategy for the retailer that would enable him to weed out slow moving customized SKUs and provide a hedge against high demand uncertainty of individual SKUs.

Even though it is possible to understand how each of these individual variables affects optimal portfolio and inventory decisions individually, it is difficult for retailer to work out intuitive solutions for various combinations of the relevant parameters. The model in this paper provides a comprehensive framework for multi-product inventory management in retail environment that helps retailers in selecting optimal portfolio and making optimal stocking decisions. Heuristic described in the article provides reasonably good solution with minimal computational effort.

References


