Noise sensitivity of the exponential histogram ADC test

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Abstract- This paper deals with some error effects caused by additive noise at analogue-to-digital converters (ADCs) testing based on the histogram method and the exponential shape of input testing stimulus. The histogram method with the exponential stimulus has been an alternative test method for ADC that was developed by the authors of this paper. Here, the theoretical analysis of some errors in estimations of code bin widths and quantisation levels caused by additive input Gaussian noise is performed. The theoretical results are verified by simulations and experimental tests. Finally the noise sensitivity of the exponential histogram test method is compared with the known noise sensitivity of the standardised sinewave histogram test method.

Keywords: A/D converters, ADC histogram test method, noise distortion, exponential stimulus test

I. Introduction

Analogue-to digital converter (ADC) testing is an expensive and time consuming process which needs a high quality instrumentation according the standards ([1], [2], [3]). This fact leads to the suggestion of new, non standardised methods based on different types of calibrating signals such as Gaussian noise ([4], [5]) or exponential pulses [6]. The main advantage of the last mentioned method, based on exponential stimulus pulses and consecutive processing of acquired ADC code histogram, is a simple realisation of stimulus generating circuit [6].

The presence of additive input noise can deteriorate the results of ADC testing using any test method. Analyses of noise influence were performed for the sinewave histogram test in [7] and [8] and for the Gaussian noise stimulus test in [9]. This paper deals with errors in estimations of code bin width and transition levels caused by an input superimposed noise at the exponential stimulus histogram test method [6].

Fig. 1. The exponential stimulus signal.

The exponential stimulus signal (Fig. 1.) can be described by:

\[ x(t) = (FS + B) \exp \left( -\frac{t}{\tau} \right) - B, \]

where \( \tau \) is the time constant of the exponential pulse, \( FS \) and \(-FS\) determine the full-scale input range.
of ADC under test and \(-B\) is the final value of the exponential signal for \(t \to \infty\). The distribution function \(P(x)\) and the density function \(p(x)\) for such a signal according to [6] are:

\[
P(x) = \begin{cases} 
0 & \text{for } x < -FS \\
1 - C \ln \frac{B + FS}{B + x} & \text{for } -FS \leq x \leq FS \\
1 & \text{for } x > FS
\end{cases}
\]

\[
p(x) = \begin{cases} 
0 & \text{for } x < -FS \\
\frac{C}{B + x} & \text{for } -FS \leq x \leq FS \\
0 & \text{for } x > FS
\end{cases}
\]

where constant \(C\) is: \(C = 1 / \ln \frac{B + FS}{B - FS}\)

II. Influence of additive noise

Let the exponential signal be deteriorated by an additive Gaussian noise with the bias \(\mu = 0\), the variance \(\sigma^2\), and the probability density function \(g(x)\). Such noise can cause errors in estimation \(\hat{T}_k\) of ADC code transition levels \(T_k\) and estimation \(\hat{W}_k\) of code bin width \(W_k\) (\(k = 0, 1, 2^{N-1}\), where \(N\) is the nominal number of ADC bits). The errors are assessed thereinafter.

A. Error in estimation of code bin width

The probability density function (pdf) of the ideal exponential stimulus signal \(p(x)\) is distorted by an additive noise. The pdf of sum of the calibrating signal (1) and the noise is given by the convolution \(f(x) = p(x) * g(x)\) that can be approximated according to the lemma 1 in [7] and substituting \(p(x)\) from (2):

\[
f(x) = p(x) * g(x) \equiv p(x) + \frac{\sigma^2}{2} p^*(x) = C \left( \frac{(B + x)^2 + \sigma^2}{(B + x)^2} \right)
\]

The distortion of resulting pdf is the reason of relative errors of code bin width estimation and it can be assessed by formula:

\[
E_{\hat{W}_k} = \frac{\hat{W}_k - W_k}{W_k} = \frac{\hat{W}_k}{W_k} - 1 \equiv \frac{f(x)}{p(x)} - 1 \equiv \frac{\sigma^2}{(B + x)^2} \leq \frac{\sigma^2}{(B - FS)^2}
\]

The maximal error of estimation is being reached for the input signal level \(x\) closing to \(-FS\) and it can be minimised by increasing \(B\). This analytically predicted error was verified by simulations performed in LabVIEW and LabWindows/CVI. They proved a good match between the analytically predicted values and the simulation results as it can be seen in Fig. 2.
Let the maximal acceptable relative error of testing due to the additive noise for estimation of code bin width be $\beta$. Then the stimulus generating circuit must generate the signal with value $B$ as follows

$$B \geq \frac{\sigma^2}{(B - FS)} \Rightarrow B \geq \frac{\sigma}{\sqrt{\alpha}} + FS$$

It means that the small overdriving of the ADC input range can ensure a needed acceptable accuracy of test results for the exponential stimulus signal spoiled by the additive noise. The needed overdriving is relatively small and it is only a bit larger than the overdriving needed for sinewave test according to [7]. If the second derivation of the density function is not constant and it is rapidly changing, what occurs for relatively small $B$, the real error will be a bit larger than the predicted one by (4).

**B. Error in estimation of code transition levels**

The estimation $\hat{T}_k$ of ADC code transition levels $T_k$ can be achieved from the cumulative histogram built from $M$ samples in the record that is proportional to the distribution function $F(x)$ of the input stimulus signal.

$$\hat{T}_k \equiv (B + FS) \exp \left( \frac{H_c(k)}{MC} + \frac{1}{C} \right) - B = (B + FS) \exp \left( \frac{F(T_k)}{C} \right) - B$$

where $H_c(k)$ is the cumulative histogram for the code value $k$ and $F(T_k)$ is the corresponding distribution function for the discrete ideal code transition levels. In the case of the noise free exponential stimulus, the estimated code bin levels $\hat{T}_k$ signal from the expression (6) are equal to their ideal values $T_k$ ($\hat{T}_k = T_k$).

The resulting distribution function $F(x)$ of the exponential stimulus signal with added Gaussian noise is given by the convolution of the distribution function of signal $P(x)$ and the density function of noise $g(x)$, i.e. $F(x)=P(x)*g(x)$. According to [7], the convolution can be approximated as follows:

$$F(x) = P(x) + \frac{\sigma^2}{2} P^*(x)$$

The estimated code bin level $\hat{T}_k$ changes according to the formula (6) taking in account the distortion of the distribution function $F(x)$ of the exponential signal by the additional noise (7). Substituting for code bine levels $x=T_k$ we obtain:

$$\hat{T}_k \equiv (B + FS) \exp \left( \frac{P(T_k)}{C} + \frac{\sigma^2}{2} P^*(T_k) \right) - B$$
Taking a first-order Taylor expansion of \( \exp(.) \) function about \( \frac{P(T_k)}{C} \) and supposing that \( \frac{\sigma^2}{2} p^*(T_k) \) is relatively small lead to the estimation of the transient code levels:

\[
\hat{T}_k = (B + FS) \left( \exp \left( \frac{P(T_k)}{C} \right) + \frac{\sigma^2}{2C} p^*(T_k) \exp \left( \frac{P(T_k)}{C} \right) \right) - B = (B + FS) \exp \left( \frac{P(T_k)}{C} \right) \left( 1 + \frac{\sigma^2}{2C} p^*(T_k) \right) - B
\]

Substituting (2) for the code levels \( x = T_k \) gives deviation between the estimated and the ideal code bin levels caused by the additive noise as follows:

\[
\hat{T}_k = \frac{\sigma^2}{2} \frac{1}{B + T_k} + T_k \Rightarrow \hat{T}_k - T_k = \frac{\sigma^2}{2} \frac{1}{B + T_k}
\]

The maximal error of the estimation is reached for the values of input signal \( x \) closing to \( -FS \) and, analogously to the error of code bin width estimation, the error can be minimised by increasing \( B \). A good match between the predicted and the simulation results was found as it can be seen in Fig. 3.

Fig. 3. Comparison of errors in \( T_k \) estimation acquired from the analytical prediction (10) (solid line) and from the simulation (dots), the number of samples 100 000 and the number of test rerunning 500. The figure a) shows the result for \( \sigma = 0.1 \) and the figure b) for \( \sigma = 0.5 \).

Let the maximal acceptable error \( |\hat{T}_k - T_k| \) of testing due to the additive noise for the estimation of code transition level \( T_k \) be \( \beta \), \( \beta \geq |\hat{T}_k - T_k| \) for any \( k \). Taking in account (10), for the worst case \( T_k = -FS \), the stimulus signal must be generated with the value \( B \):

\[
B \geq \frac{\sigma^2}{2\beta} + FS \tag{11}
\]

The result shows that the relatively small overdriving of the ADC input range can ensure a needed
acceptable accuracy of ADC testing at the presence of additive noise.

C. Variance of transition code levels estimation

Let suppose the uniform sampling with the sampling period $\Delta t$ and with the time interval $T_{MEAS}$ while the noiseless exponential testing signal is within the ADC input range. This time interval is divided into $M$ equal time steps with the width $\Delta t$. The time, precisely corresponding to crossing transition code level $T_k$, is $t(T_k) = (M - (n + \alpha))\Delta t$, where $n$ is an integer, $\alpha$ is a number between 0 and 1 and $M$ is the number of samples in the record. Then the probability of case $H_c(k) = n + 1$ is $\alpha$ and complementary $H_c(k) = n$ is $1 - \alpha$. The mean value of such binomical distribution is $n + \alpha$ and variance $\alpha(1 - \alpha)$. The density of samples around the transition level $T_k$ is given by (2). Using these facts we obtain the variance of the transition level estimation as follows

$$\sigma^2_{\hat{T}_k} = \alpha (1 - \alpha) \frac{1}{M^2 \rho(T_k)} \left( \frac{1}{M^2 \rho(T_k)} \right)^2 = \alpha (1 - \alpha) \frac{1}{M^2 \rho(T_k)} \left( \frac{T_k + B}{T_k + B} \right)^2$$

where $\alpha = \left\{ t(T_k) \over \Delta t \right\}$ and where $\{ \cdot \}$ is the fractional part operator.

Now let a Gaussian noise be added to the input exponential testing signal. Let consider the probability that a sample is inside the time interval for a cumulative histogram bin at a distance $x$ in Volts from an edge of the interval. If it is from the right edge then $G(x)$ is the probability that the noise voltage is smaller or equal to $x$. If the distance is considered from the left edge, then $G(x)$ is the probability that the noise voltage is bigger or equal to $x$. In either case:

$$G(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{z^2}{2\sigma^2}} dz$$

Let $c_j$ be a random variable that takes the value one if $j$-th sample point is recorded in the cumulative histogram bin in question and takes the value zero if it is recorded outside. Then the variance $\nu(x)$ of random variable $c_j$ is $G(x) (1 - G(x))$. Let $\lambda$ be a probability density of samples around the border of the sampling interval for code bin $k$. Using (2) we obtain:

$$\lambda(T_k) = \frac{1}{M \rho(T_k)} = \frac{(T_k + B)}{MC}$$

Because of two contributing regions around the border of code bin, the variance of transition level estimation must be multiplied by the factor 2. Finally, combining the previous facts, we obtain:

$$\sigma^2_{\hat{T}_k} = 2 \lambda(T_k) \int_{0}^{\infty} v(x) dx = \frac{2}{M \rho(T_k)} \int_{0}^{\infty} v(x) dx = \frac{1.13}{2} \frac{\sigma}{MC} (T_k + B)$$

where, according to [7], $\int_{0}^{\infty} v(x) dx = \int_{0}^{\infty} G(x)(1 - G(x)) dx = \frac{1.13}{4} \sigma$

The analytical prediction of variance (15) proved the good match with the simulation results as it can be seen in Fig 4.
Fig. 4. Comparison of variances of $T_k$ estimation acquired from the analytical prediction (15) (solid line) and from the simulations (dots), the number of samples 100 000 and the number of test rerunning 500. The figure a) shows the result for $\sigma=0.1$ and the figure b) for $\sigma=0.5$.

The total variance of estimation of transition levels is given by addition of equation (12) and (15) because of independence of these two effects. The dominant component of variance at the presence of a noise is given by (15) because of square of $M$ in the denominator in (12).

D. Variance of code bin width estimation

A code bin width is the difference between two adjacent code transition levels. If the code bin width is larger than twice the noise level then the errors in the code transition level can be supposed to be nearly independent and that is why the variance of code bin width can taken twice variance of code transition levels estimation:

$$\sigma_{\hat{B} T M_k}^2 = 1.13 \frac{\sigma}{M C} (T_k + B)$$

The analytical prediction of variance (16) proved the good match with the simulation results as it can be seen in Fig 5.
Fig. 5. Comparison of variances of code bin width $W_k$ estimation acquired from the analytical prediction (16) (solid line) and from the simulation (dots) for the number of samples 100 000 and the number of test rerunning 500. The figure a) shows the result for $\sigma=0.1$ and the figure b) for $\sigma=0.5$.

If the noise level is higher we would have to take into account the covariance of adjacent levels but because of the natural low pass filtering in generation circuits of slow exponential stimulus the case of the large noise seems to be very rare in praxis and it is out of our scope in this paper.

As shown in Fig. 5 the variance of $W(k)$ estimation increases for the higher code bins. It is caused by the higher slope of the testing signal. Moreover the value of the variance $W(k)$ calculated from the simulated data is lower than the analytically derived one. This deviation increases for the higher level of the added noise. The increased deviation for the increased level of the additive noise is caused by the correlation effect (covariance) in the estimation of the variance in the processed record of samples.

II. Experimental results

The theoretical and simulation results were verified by experiments, too. The test stand consisted of the multifunction PC plug-in board PCI-6036 with 16 bits resolution ADC under test, arbitrary DDS generator Agilent 33220A and the control and data-processing software. The control and data processing software was developed in LabWindows/CVI.

The test signal – exponential stimulus with an additive Gaussian noise was generated by the arbitrary DDS generator with 14 bit DAC resolution controlled by the software in PC. To avoid the imperfection of DAC in the generator and ADC under test in PCI-6036 plug-in-board (nonlinearity and the additive noise of analogue output and input circuits), the ADC resolution was adjusted on 8 bits by rounding the acquired 16 bits samples in the control and data processing software.

The achieved results from some tests in comparison with the predicted values are shown in Fig. 6 and Fig. 7. The figures confirm the good match between the predictions and the results of the experimental tests.
Fig. 6. Comparison of errors in $T_k$ estimation acquired from the analytical prediction (4) and from the experimental test for the number of samples 10,000 and the number of test rerunning 500. The figure a) shows the results for $\sigma=0.1$ and the figure b) for $\sigma=0.5$.

Fig. 7. Comparison of variances of code bin width estimation acquired from the analytical prediction.
(16) (solid line) and from the experimental tests (dots) for the number of samples 10 000 and the number of test rerunning 500. The figure a) shows the results for $\sigma=0.1$ and the figure b) for $\sigma=0.5$.

III. Comparison of noise influence on histogram test for sinewave and exponential signal

The variance of transition level estimation for the sinewave histogram test according to [7] and [8] is:

$$\sigma^2_{SHT} = \alpha (1 - \alpha ) \frac{\pi^2}{M^2} (A^2 + \hat{T}_i^2) + \frac{1.13 \pi}{2M} \frac{\sigma}{\sqrt{A^2 + \hat{T}_i^2}}$$

(17)

$$\alpha_i = \left\{ \frac{2\psi_i}{\Delta \varphi} \right\}, \quad \Delta \varphi = \frac{2\pi}{M}, \quad \psi_i = \arccos \left\{ -\frac{T_k}{A} \right\}, \quad \hat{T}_k = T_k - d$$

The dominant component at the presence of a noise in (17) is its second part [9]. The ratio of variances for the sinewave histogram test $\sigma^2_{SHT}$ and for the exponential stimulus histogram test $\sigma^2_{EHT}$ for DC component of sinewave $d=0$ is:

$$\frac{\sigma^2_{SHT}}{\sigma^2_{SHT}} = \frac{1.13 M}{2} \frac{\sigma}{\pi} (T_k + B) \frac{\pi C}{2\sqrt{A^2 - T_k^2}}$$

(18)

The equation (18) indicates that this ratio does not depend on the noise level and the number of samples but it depends only on shape parameters of the test stimulus signals. The minimum can reached for $T_k = -FS$ or anywhere within the ADC input range as it can be seen from some examples in Fig. 8a. It means that neither the sinewave nor the exponential stimulus have principally better resistance to the additive noise disturbance.

![Graph](image)

Fig. 8. Comparison of variance ratio for the exponential and the sinewave histogram test according to (18). The figure a) shows the comparison for $A=1.05FS$ and $B$ equal to 1.05 (solid line), 1.55 (dashed), 2.05 (dotted) and 2.55 (dot-dashed). The figure b) shows the comparison at the condition $B = A = 1.05$ (solid line), 1.55 (dashed), 2.05 (dot-dashed).
To simplify partially the comparison let $B=A$. Analysing (18) one can simply find that the minimum is always reached for $T_k=-FS$ and the maximum for $T_k=FS$. If $B=A>1.581$ the ratio (18) is always smaller than 1 for any $T_k$, i.e the variance of code transition level estimation is always smaller for the exponential stimulus test than for the sinewave test in such conditions. If $B=A<1.581$, the smaller variance is achieved by the exponential stimulus test for lower $T_k$ and by the sinewave test for higher $T_k$ (Fig. 8b).

IV. Conclusions

The influence of additive noise on the accuracy of estimation of code bin width and transition code levels for the exponential stimulus histogram test method of ADC has been analysed. The analytical expressions for estimation errors have been derived and verified by simulation with a good match. The results show that the errors are relatively small and closed to those for the sinewave test. Neither the sinewave nor the exponential stimulus have principally and generally better resistance to the additive noise. The test errors due to the additive noise can be minimised by a convenient construction of stimulus signal generating circuit with a large value of value $B$ of the exponential test signal.

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References