Sedflux 2.0: An advanced process-response model that generates three-dimensional stratigraphy

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Abstract

Sedflux 2.0 is the newest version of the Sedflux basin-filling model. Sedflux 2.0 provides a framework within which individual process-response models of disparate time and space resolutions communicate with one another to deliver multigrain-sized sediment load across a continental margin. Version 2.0 introduces a series of new process models, and the ability to operate in one of two modes to track the evolution of stratigraphy in either two or three dimensions. Additions to the 2D mode include the addition of models that simulate (1) erosion and deposition of sediment along a riverbed, (2) cross-shore transport due to ocean waves, and (3) turbidity currents and hyperpycnal flows. New processes in the 3D mode include (1) river channel avulsion, (2) two-dimensional diffusion due to ocean storms, and (3) two-dimensional flexure due to sediment loading. The spatial resolution of the architecture is typically 1–25 cm in the vertical and 10–100 m in the horizontal when operating in 2D mode. In 3D mode, the horizontal resolution usually extends to kilometers. In addition to fixed time steps (from days to hundreds of years), Sedflux 2.0 offers event-based time stepping as a way to conduct long-term simulations while still modeling low-frequency but high-energy events.

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1. Introduction

The development of numerical models that explore the evolution of continental margins is comparatively recent. The modeling community has developed three main types of models to simulate the growth of continental margins (for a full overview, see Paola (2000), Overeem et al. (2005), and Tetzlaff and Priddy (2001)). The first set consists of individual process models intended to model the effects of a single process on (usually) a portion of a margin. For instance, Bonham-Carter and Sutherland (1967) and later Morehead et al. (2001) used river plume models to simulate the progradation of a river delta. Harris and Wiberg (2001) developed a detailed model to simulate the transport of shelf sediments by wave and current interactions (also, Wiberg and Smith, 1983; Li et al., 1997; Reed et al., 1999). The modeling of continental slope processes has received somewhat less attention. Imran et al. (2001) developed a numerical
model to simulate the movement (but not failure) of sediment down a continental slope as a debris flow. Again, while not modeling the inception of turbidity currents from sediment failure, Parker et al. (1986) modeled the transport of sediment through turbidity currents. This work then led to the modeling of submarine fan formation through turbidity currents (Imran et al., 1998).

Modeling of the entire continental margin has received less attention, and has taken two routes. The first route develops a small set of equations that govern the evolution of the entire continental margin (Ross et al., 1994; Steckler, 1999; Swenson et al., 2000; Granjeon and Joseph, 1999). These models do not attempt to simulate individual processes that act to form stratigraphy. Instead,
they model the long-term result of complex interactions between physical processes. The second route links individual process models into a component sequence-stratigraphic model (Martinez and Harbaugh, 1993; Syvitski and Alcott, 1995; Syvitski and Hutton, 2001; Ritchie et al., 1999). In this way, the major processes that form stratigraphy are able to interact with one another. Because of the complexity of the component models, this route is able to provide a detailed prediction of the evolving stratigraphy but at a cost of increased computation time and program complexity.

One such component sequence-stratigraphic model is Sedflux 1.0c (Syvitski and Hutton, 2001). It is a two-dimensional basin-filling model that generates stratigraphy that varies vertically, and in a single lateral dimension. The structure of Sedflux 1.0c provides an architecture within which a collection of individual process models are able to interact with one another, and with the sediment deposits they produce. Not only does this model produce stratigraphy, but it also tracks the geometrical properties of the deposits (grain size, bulk density, porosity, etc.; Hutton and Syvitski, 2003).

Some of the process modules that contribute to this stratigraphy are modeled using vertically averaged variables that only vary horizontally. For instance, the turbidity current model assumes vertically averaged flow characteristics but allows them to vary in the horizontal dimension. Other processes, such as sediment plumes, are averaged over a specified basin width. This width is user-defined and is constant in time but can vary in space. The suite of processes that Sedflux 1.0c modeled consisted of (1) river-mouth dynamics, (2) buoyant surface plumes, (3) hyperpycnal plumes, (4) ocean storms, (5) slope instability, (6) turbidity currents, (7) debris flows, (8) compaction, and (9) flexure of the lithosphere.

Although a two-dimensional basin-filling model may provide insights into how stratigraphy forms, it contains a basic limitation in that it is two-dimensional. In reality, sediment spreads laterally rather than simply within a plane. Certainly, there are situations where a two-dimensional model is adequate, but, in general, this is not the case and so a three-dimensional model is necessary.

Often one will use a two-dimensional model and assume that sediment moves along some profile. The profile need not be a straight line, only that sediment transport follows this line. Some two-dimensional models will introduce a width to approximate a third dimension (oftentimes, referred to as 1.5-dimension models). For example, Skene et al. (1997) adapt a one-dimensional model to simulate a hyperpycnal flow that follows a channel of non-constant width. In such cases though, the deposit parameters only vary in a single dimension, and the flow direction is also in a single dimension. In reality, the geometry of a location may be such that there is no single direction of flow and so 1.5-dimension models are not adequate. For instance, ocean currents may be such that a surface plume flows along bathymetric contours and so deposits sediment along these contours. Once on the ocean floor, gravity may become the major driver of sediment transport rather than ocean currents, in moving down-slope, perpendicular to bathymetric contours.

Two-dimensional basin-filling models are often based upon processes that are assumed to be one-dimensional (for example, Steckler, 1999; Swenson et al., 2000; Syvitski and Hutton, 2001). That is, the model assumes that the variables fluctuate only in one direction (the direction of transport). Again, this may be a reasonable assumption in some cases (the unidirectional flow of sediment down a narrow submarine canyon, for instance) but in general it is not. In fact, many transport processes are more complicated and a two-dimensional transport model will give a better approximation. A sediment gravity flow, for instance, may create a mound-like structure around which subsequent flows will have to travel (McAdoo et al., 2000). Thus, although the physics may be accurate, addition of more potential flow directions gives rise to different results. For instance, a two-dimensional basin-fill model may produce an aggradational basin-floor fan because successive flows attempt to run over previous flow deposits (Skene et al., 1997; Pratson et al., 2001). A three-dimensional basin-fill model can potentially produce a basin-floor fan that builds laterally as successive flows are allowed to move around previous flow deposits (Bouma, 2000, 2001; Richards et al., 1998). In this case, it is not the physics of the higher-dimensional process that gives rise to the different structure, rather it is the change in flow direction (and, in fact, simply geometry).

Since the publication of the Sedflux 1.0c model (Syvitski and Hutton, 2001), Sedflux has advanced to address this issue. The version of Sedflux described here (Sedflux 2.0) provides a modeling framework that is able to produce and track the development of three-dimensional structures. This three-dimensional
basin-fill model is able to generate simulations that can be compared with the three-dimensional data that are now common within oil exploration. In addition, it is able to produce output that can be compared to two-dimensional data that are still commonplace or even be run in a 2D mode. A seismic section may lie along the believed direction of flow, but in reality deposition may have occurred at angles that are oblique to this direction. A three-dimensional basin-fill model can provide insights into how a three-dimensional feature is represented in a two-dimensional section. Furthermore, a three-dimensional seismic model coupled with a three-dimensional basin-fill model will demonstrate how a seismic line will portray these features.

The three-dimensional basin-filling model Sedflux 2.0 is an extension of the two-dimensional basin-filling model Sedflux 1.0c (Syvitski and Hutton, 2001). As with the previous versions of Sedflux, the basis of Sedflux 2.0 is an architecture that allows various process modules to communicate with one another to create either two- or three-dimensional stratigraphy (depending upon which mode the user chooses to run the model in). This architecture allows for the straightforward addition of new modules. New modules can be added either by writing a “wrapper” that converts the new model’s framework to that of Sedflux or, more simply, the new model can be written using the application programming interface (API) of Sedflux.

The process models present in the previous Sedflux release are still present within the current version. Although the principal change between the two versions is the ability to track three-dimensional architecture, a number of new processes were added to the 2D mode. In addition, the 3D mode of Sedflux 2.0 required addition of new modules that were not necessary in two dimensions.

2. Development history

Since the publication of Syvitski and Hutton (2001), Sedflux has advanced along three major paths. The first is the expansion of process models that operate solely within the 2D mode of Sedflux. The second is the expansion of the Sedflux architecture to be able to track stratigraphy in three dimensions and the addition of processes necessary to do this. The third are additions that operate in both the 2D and 3D mode. We describe only new or extensively modified modules.

Sedflux 2.0 is a three-dimensional basin-filling model that is able to predict stratigraphy that varies laterally and vertically. The 3D mode of Sedflux records sediment deposition in two lateral dimensions while the 2D mode tracks sediment only in the cross-shore direction. The individual processes that operate within either mode of the Sedflux framework are either one- or two-dimensional. These processes produce sediment deposits that vary laterally (in either one or two dimensions) but use vertically averaged variables that only vary horizontally. The absence of three-dimensional models within Sedflux is not due to a limitation of its architecture but rather because these models are not yet incorporated.

Prior to a Sedflux simulation, the user specifies the spatial and temporal resolution of the architecture. The user defines the basin geometry at points on an initial surface (or line, in the case of the 2D mode) at a fixed resolution. Typically, this resolution is on the order of kilometers for 3D mode, and tens of meters for 2D mode. The vertical resolution defines the thickness over which sediment characteristics are averaged (this ranges from centimeters to meters). These resolutions only constrain the size of the cells that Sedflux uses to track sediment deposits. Each component module uses its own resolutions, which are independent of the architecture resolutions. For instance, Sedflux can run with a daily time step, but because of stability constraints, the debris flow module typically runs with time steps less than 1 s.

A series of rivers supply the basin with sediment (the 2D mode is limited to a single river). Input files specify the number of rivers, their location, and hydrologic characteristics. These rivers transport sediment composed of a user-defined number of distinct grain types. Each river introduces its suspended sediment through a surface plume or a hyperpycnal flow, depending on the sediment-laden density of the river water. The bedload component of the river’s load is spread over a user-specified distance.

3. Additions to Sedflux 3D

The transition of Sedflux to three dimensions requires the addition of a channel avulsion model. Because of the two-dimensional nature of Sedflux 1.0c, the river mouth was fixed to the land–ocean interface along a profile. In three dimensions, the river mouth can move laterally. The flexure and
diffusion modules were also improved to work within the three-dimensional framework. Both modules now solve their corresponding two-dimensional governing equations.

3.1. Channel avulsion

River deltas distribute sediment laterally by frequently changing the position of the river mouth and the main distributary. Existing delta models incorporate partially understood and complex physics in an attempt to simulate realistically the channel avulsions (Overeem et al., 2003; Sun et al., 2002). However, the angular location of the main distributary is more simply derived as a stochastic Brownian process and has a Gaussian distribution over short time scales, resulting in localized deposition and lobe-like formations. Over long time scales, the same process is uniform in distribution, resulting in sediment spread over the entire delta plain.

Using the distribution $X$ for the change in an angle after an avulsion ($\Delta \theta$), the angular position of the distributary channel after $n+1$ avulsions is given by

$$\Theta_{n+1} = \Theta_n + X_n$$

where $\Theta_n$ is the current angle and $X_n$ is the $n$th realization of $X$ (Fig. 1). In other words, the angular position is just the sum of angular jumps that are generated from the distribution $X$. Regardless of the underlying physics, some probability distribution must represent this change in angle. The precise distribution will not be known, but observations of large deltas suggest that the probability of avulsing somewhere nearby is high, while the probability of larger avulsions is low (Milliman et al., 1987).

A simple numerical model illustrates the realistic-looking deltas generated by such a stochastic process (Fig. 2). This model assumes that an avulsion happens every time step, the basin is flat-bottomed, and the grid scale is such that one cell is always filled by the river’s sediment with every time step. The model randomly generates angles from the distribution $X$, moves the mouth of the distributary by these angles around the coastline, and fills empty cells with sediment. The shoreline provides a reflection boundary condition at $\Theta = \pm 90^\circ$. Both diagrams show a delta building into the sea. The left figure picks $X_n$ from a uniform distribution, and the right figure uses a normal distribution (with a mean of 0 and standard deviation of 0.05°). Colors indicate the time of deposition (cool colors are oldest, hot colors are recent). A uniform distribution builds a symmetric and radial delta while the normal distribution creates a more lobe-like delta. These
river-dominated delta morphologies would change with the inclusion of waves, tides, and other processes (Orton and Reading, 1993).

3.2. River plumes

The momentum-driven hypopycnal plume described in Syvitski and Hutton (2001) is still the basis for the river plume used in the 3D mode of Sedflux 2.0. However, unlike implementations used in previous versions of Sedflux, the plume deposit is no longer averaged in the alongshore direction. Instead, Sedflux 2.0 tracks the deposit due to a two-dimensional surface jet that emanates from each river mouth. In addition, the plume module considers a set of open-coast plume scenarios able to handle upwelling and downwelling against strong and weak coastal currents.

When the plume enters the ocean and is met with an alongshore current, it deflects in the direction of the current. Chassaing et al. (1974) describes the deflection of a jet’s centerline upon encountering a crossflow. Modified for Coriolis force, the plume’s centerline is

$$\frac{x}{b_0} = 1.53 + 0.9 \left( \frac{u_0}{v_0} \right) \left( \frac{y}{b_0} \right)^{0.37}$$

where $v_0$ is the ambient coastal current velocity, and $b_0$ and $u_0$ are river-mouth width and velocity, respectively. The model solves the same governing equations described in Syvitski et al. (1998) but with the coordinate system warped to reflect the curved centerline. What was the cross-shore direction in the original coordinate system becomes distance along the plume’s (curved) centerline; alongshore distance becomes distance perpendicular to the plume’s centerline.

3.3. Bedload dumping

In the 3D mode of Sedflux 2.0, all of a river’s suspended sediment is delivered to the ocean as a hypopycnal plume. The river delivers the remaining sediment as bedload in much the same way as that of the 2D mode (Syvitski and Hutton, 2001). Instead of being distributed evenly over a specified distance, the river now spreads the sediment evenly over a cone with a user-specified angle and radius. This radius is meant to reflect the transport by both river momentum and tides.

3.4. Two-dimensional diffusion of seafloor sediments

Sedflux1.0c modeled the resuspension and transport of bottom sediments due to a host of effects as a single diffusive process. Sedflux 2.0 extends this model to include two-dimensional diffusion.

We assume that the amount of bottom sediments that can be reworked by these processes ($q_s$) is proportional to bathymetric slope and a diffusion coefficient, $k$:

$$q_s = k(t, z, D) \nabla z = k \left( \frac{\partial z}{\partial x} i + \frac{\partial z}{\partial y} j \right)$$

where the diffusion coefficient, $k$, is a function of time ($t$), water depth ($z$), and grain size ($D$). Although the model does not assume a specific depth (or $z$) dependence of $k$, typically we assume an exponential decline to reflect decreasing wave energy with depth. The user specifies the rate at which diffusion decreases with depth, and the current ocean storm conditions scale the value of $k$ at the water surface.

This flux is interpreted as the maximum volume of sediment that can be reworked under the local conditions; not all of this sediment will be moved down-slope however. For each grain size, the above flux is scaled by a user-defined index ($\beta_i$) between 0 and 1 that reflects the ability of this process to move the $i$th grain size. Thus, the amount and direction of transport of the $i$th grain size is

$$q_s = \beta_i q_s$$

This flux is drawn only from the resuspended sediment ($q_s$). Any sediment that may be left behind (due to $\beta_i < 1$) is mixed and redeposited.

Although geomorphological studies have measured diffusion coefficients, they do not directly relate to the above $k$. Instead, they often represent diffusion values integrated over entire margins, or over long time scales. For spatial scales of 10–100 km and time scales of $10^7$–$10^6$ years, diffusion coefficients range from $10^2$ to $10^3$ m$^2$ yr$^{-1}$.

3.5. Flexure of the lithosphere

Within the Sedflux 2.0 framework, a suite of processes deposit and erode sediment causing the load distribution over the region to evolve over the simulation. Thus, the lithospheric load changes as the model evolves. Depending upon how the load distribution develops, this flexure can result in the basin uplifting or subsiding (or both). The pattern
of subsidence in time and space largely determines the gross geometry of time-bounded units because it controls the rate at which space is created for sedimentation (Angevine et al., 1990).

It is typical to model the deflection of Earth’s crust due to loading by assuming that it behaves as a beam (or a plate, in the case of two dimensions) on an elastic foundation (for example, Lambeck, 1988; Anderson, 1994; Syvitski and Hutton, 2001). Lambeck (1988) solves this problem in two dimensions. The solution shows the deflection of Earth’s crust \( w(\lambda r) \) to be a function of the non-dimensional length \( \lambda r \) as

\[
w(\lambda r) = \frac{q \lambda}{2\pi \rho_d g} \text{Kei}(\lambda r)
\]

where \( \text{Kei} \) is the Kelvin function, \( q \) is the point load, \( \rho_d \) is the density of the asthenosphere, \( \lambda \) is the flexural parameter, and \( r \) is the distance from the point load. We define the flexural parameter as

\[
\lambda = \left( \frac{D}{\rho_d g} \right)^{1/4}
\]

where \( D \) is the flexural rigidity of Earth’s crust. The Kelvin function behaves as an exponentially damped sinusoid, and the flexural parameter acts to control the width of the deflection. To solve the Kei function numerically, we use the algorithm developed by Amos (1986).

Eq. (5) predicts the deflection due to a point load; it does not account for the additional weight of material that fills the deflection. For instance, a rise in sea level will cause a deflection that will be filled with additional water, which in turn will cause further deflection. If we know the density of material that fills the deflection, the total deflection is increased by a factor of \( (\rho_d/\rho_w)^{1.25} \) (Angevine et al., 1990). Using \( \rho_d = 3300 \text{ kg m}^{-3} \), and \( \rho_w = 1030 \text{ kg m}^{-3} \), this amount to an increase in deflection by a factor of 1.6. Unfortunately, in general, we do not know the density of the added (or removed) material as it could be either air or water. Thus, we solve for the total deflection by iteratively solving Eq. (5) and updating the load after successive iterations until the solution converges.

Because the viscous asthenosphere has to flow out of the way before the lithosphere can deflect, there will be a time delay between the addition of load and the lithosphere’s response. Eq. (7) expresses this time delay as an exponential (Peltier, 1998):

\[
w(t) = w_0 (1 - \exp(-t/t_0))
\]

where \( w_0 \) is the equilibrium deflection as determined by Eq. (5), \( t \) is the time since the load was applied, and \( t_0 \) is the response time associated with mantle viscosity. In reality, response time is a function of the viscosity of the underlying mantle (Peltier, 1998) Typical relaxation times vary from about 1500 to 5000 years (Peltier, 1998; Huybrechts, 2002; Paulson et al., 2005).

As an example of the flexure model, we estimate the deflection of the lithosphere due to the weight of the modern (last 5500 years) Mississippi delta (Fig. 3; New Orleans is indicated by the vertical red line). For this experiment, we estimated the sediment distribution and the timing of lobe switches from the work of Fisk and McFarlan (1955). The simulation began at the Last Glacial Maximum with rising sea level and no sedimentation. A plane dipping with a gradient of .0004 formed the initial bathymetry (the grid resolution was 1 km). We include this initial period of rising sea level and no sedimentation to include any residual loading effects from the increased water load. Any sediment deposited during this time would augment subsidence rates by an amount directly related to the density difference of sediment and water. However, we assume that this signal is drowned by that of the larger, more distributed water load. At 5500 years BP, sea-level rise was stopped and sediment added one lobe at a time. Fig. 3 shows the final deflections due to the additional sediment load.

Fig. 4 shows the subsidence rate at New Orleans throughout the model simulation. The dashed line shows the response of the lithosphere due to
increased water loading from rising sea level. Subsidence rates increase for 6 kyr until sea-level rise stops. Because of the relaxation time (Eq. (7), with $t_0 = 2500$ years), deflection continues even though no new weight is added.

The solid line again shows subsidence rates through the model run but now the effects from both sediment and water are included. The lines match one another for the first 6 kyr, but then diverge with the addition of sediment. The sharp jumps in subsidence rates are due to sudden shifts in the location of sediment loading (due to lobe switching). At the end of the simulation, we see that the total subsidence rate is about $0.002 \text{ m yr}^{-1}$ and that 25% of that ($0.0005 \text{ m yr}^{-1}$) is due to water loading that ended about 5400 years ago.

4. Additions to Sedflux 2D

When using sequence stratigraphy to analyze the formation of a continental margin, it is critically important to track the position of the shoreline. Thus, it is also critically important to model the processes that deposit and erode sediment near the shore. This includes the deposition and erosion of sediment within a river channel and the resuspension of shallow marine sediments.

The previous version of Sedflux models the deposition and erosion of sediment along a river using a geometric model based on an assumed equilibrium profile of a river’s thalweg. This module needed improvement because (1) it relies on knowing beforehand what the equilibrium profile for a region is and (2) it is a steady-state model. In general, the equilibrium shape of a river’s longitudinal profile depends on factors (for instance, discharge, sediment load, and grain size) that vary from region to region (Mackin, 1948) and so is difficult to determine in advance. Also, a river does not immediately attain its equilibrium shape but is constantly changing in response to external forces (Mackin, 1948; Leopold and Bull, 1979). A dynamic model developed by Paola et al. (1992) replaces the previous steady state.

Cross-shore processes driven by wave energy govern the evolution of wave-dominated coastal systems (Storms et al., 2002). Sedflux 1.0c modeled this process using a diffusion-based model. Although this is a common way of modeling coastal evolution (Kaufman et al., 1992; Niedoroda et al., 1995; Swenson et al., 2000; Schlager and Adams, 2001), it had two drawbacks that made it inadequate. Firstly, the evolving cross-shore profile depended on an imposed (site-specific) diffusion coefficient. Secondly, such models do not adequately resolve the inner shelf. The work of Dean (1991) and Bruun (1962) suggest that the cross-shore profile is concave upward and follows a power law. This is not the case for the diffusive model used within Sedflux 1.0c or with those listed above.

A third cause of sediment deposition and erosion in the near-shore environment is through surface and subsurface plumes. For low river suspended sediment concentrations Sedflux 1.0c implemented a surface plume module (Syvitski et al., 1998) to deliver suspended sediment to the shelf as “sediment rain”. For river concentrations greater than the density of seawater, Sedflux 1.0c used a hyperpycnal plume module (Mulder et al., 1998) to deliver the sediment to the basin. Both of these models show good agreement with field observations (Morehead et al., 2001; Mulder et al., 1998) and use real-world measures as input parameters. However, Kubo et al. (2005) added a new hyperpycnal plume model to Sedflux, giving the user a choice as to which model to use.

4.1. Subaerial erosion and deposition by rivers

Sedflux 2.0 uses a model based on previous work of Paola et al. (1992) to model the transport of sediment along a river. This new module allows both erosion and deposition along the stream profile without having to define an equilibrium profile (as was the case in previous Sedflux versions). Because
the diffusion equation forms the basis of the module (Eq. (8)), the river’s equilibrium profile is linear along its entire length (after setting time derivatives to zero, the only remaining term is that of topographic curvature). The diffusion coefficient controls the rate at which the streambed moves toward this equilibrium.

Accurate modeling of erosion and deposition on the delta plain is critical for a number of reasons. This process controls the position of the shoreline, particularly during periods of rising sea level. For instance, typical equilibrium river profiles over large distances are often represented as an exponential or a logarithmic function (Peckham, 1995). The coefficients that define these curves may change from river to river but their basic form will remain the same—they are concave upward with shallow and nearly linear slopes near the river mouth (Peckham, 1995; Muto and Steel, 2000). Because of this equilibrium geometry, a rapid sea-level fall would result in an extreme amount of incision if the stream profile were to instantaneously move to its new steady-state position. In reality, it takes a stream on the order of hundreds of years to reach grade (Thorne and Swift, 1991). Thus, by reaching grade immediately, a subsequent sea-level rise will see larger than expected accommodation. However, because the new model is based on the diffusion equation, it limits the amount of erosion based upon the profile’s current distance from equilibrium. Thus, Sedflux 2.0 is capable of providing insight into both the magnitude and rate of sea-level change on stratigraphy.

While controlling the amount of accommodation, the new model better controls the amount of sediment transport to the marine basin. An equilibrium model consistently provides an excess of sediment to the basin during times of lowering sea level. Because of the interplay between sediment supply and accommodation, it is critical to model them well.

To predict the erosion and deposition along a river channel, the derivation of Paola et al. (1992) does not assume beforehand that the diffusion equation is the correct model; rather, they begin from first principles with conservation of mass and momentum. The final result is

$$ \frac{\partial \eta}{\partial t} = v \frac{\partial^2 \eta}{\partial x^2} $$

(8)

where \( \eta \) is the height of the bed, \( t \) is time, \( x \) is position along the river channel, and \( v \) is the diffusion coefficient. The diffusion coefficient, \( v \), is expressed in terms of measurable quantities:

$$ v \equiv -\frac{8\langle q \rangle A \sqrt{c_r}}{C_0(s - 1)} $$

(9)

where \( \langle q \rangle \) is the long-term average water discharge, \( c_r \) is a drag coefficient, \( C_0 \) is the sediment concentration of the bed, \( s \) is sediment specific gravity, and \( A \) is a river-type dependent constant. The value of \( A \) is user-defined and takes on one of two values depending on the river type. For a meandering river, \( A = 1 \), while for a braided river, \( A \equiv (\epsilon/(1 + \epsilon))^{3/2} \). The value \( \epsilon \) relates the shear stress (\( \tau \)) in the center of a braided channel to the critical shear stress (\( \tau_c \)) needed for bank erosion (\( \tau = (1 + \epsilon)\tau_c \)). Measured values of \( \epsilon \) are typically about 0.4 (Parker, 1978) for gravel bed rivers.

Sedflux 3D expands this erosion and deposition routine to operate in a two-dimensional framework. In this framework, the \( x \) dimension in Eq. (8) becomes the along-channel distance. Elevation changes are calculated in the same way, but are assumed to be averaged over a user-defined channel-belt width.

Eq. (8) transports eroded river sediments into the basin. Based on grain size, the sediment will move as either bedload or suspended load. The process is able to cause a change in both the amount of sediment that the basin receives, as well as the way that it receives it. For instance, during a period of falling sea level (and channel incision), erosion could conceivably cause the suspended sediment concentration to increase enough to trigger a hypopycnal plume, where the normal mode of dispersal would be a surface plume.

Fig. 5 shows an example of a Sedflux simulation (operating in 3D mode) that demonstrates the use of the new avulsion and channel erosion modules. For this theoretical experiment, Sedflux ran for 1000 (model) years, using five distinct grain-size classes, a grid resolution of 1 km, and a 1-year time step. Throughout the model run, sediment supply was held constant and the sediment distributed to the basin through a surface plume. The snapshot of basin topography in the left image of Fig. 5 is at a low sea-level stand (~25 m) and shows two incised valleys in the delta topsets and several smaller channelized features in the delta slope. The fall in sea level caused marine sediments to be exposed, which were then eroded by the new stream erosion module. To simulate the incised valley, we specified a decrease in the frequency of channel avulsion for this period.
The right image of Fig. 5 shows the topography at the end of the simulation when sea level has risen 12 m. For rising sea level, the channel is once again allowed to avulse over the entire delta. One can still see the flooded delta plain but plume sedimentation has mostly filled in the incised valleys there. Base-level rise has also caused deposition on the new delta plain. However, unlike plume sedimentation, deposition on the delta plain was not as effective in obscuring the incised valleys.

Fig. 6 shows the internal stratigraphic architecture of the deposits this experiment generated. There are three horizontal slices at elevations of −15, −125, and −195 m and one dip profile at 0 km. The colors correspond to average grain size along the slices. One is able to distinguish the coarse near-shore sands (yellow) in the top slice and more distal clays (blue) in the bottom slices. In the top slice, two pockets of coarser sediment separated by an undisturbed ridge of finer sediment distinguish the two valley fills.

4.2. Cross-shore transport due to ocean storms

**Sedflux 1.0c** modeled the resuspension, transport, and mixing of bottom sediments (wave energy, bottom currents, and bioturbation, for example) as a single diffusive process. **Sedflux** now separately models the resuspension and distribution of bottom sediments due to ocean-wave energy. The new model is a composite of two models. The first is a modified version of the marine component of the model presented by Swenson et al. (2005) to predict the reworking of the seafloor by offshore waves. The second is a diffusion-based model used only within the near-shore.

For the purposes of this model, we define the near-shore as the region of depths less than the closure depth, \( h_c \), as defined in Nicholls et al. (1998):

\[
h_c = 2.28H_{ss} - 6.85 \left( \frac{H_{ss}^2}{gT} \right) \quad (10)
\]

where \( H_{ss} \) is the height of the storm wave that is exceeded only 12 h each year and \( T \) its associated period. This depth provides a boundary for the two wave resuspension models.

For the outer shelf (depths greater than \( h_c \)), we estimate the sediment flux, \( q_s \), at each position along the profile as

\[
q_s = \frac{16}{3\pi} \frac{\rho}{\rho_s - \rho} C_{fs} \varepsilon_{ss} I_s \frac{U_{om}^2}{w_s} \left( v_0 + \frac{U_{om}^2}{5w_s} \frac{\partial h}{\partial x} \right) \quad (11)
\]

where \( C_{fs} \) is a constant drag coefficient, \( \varepsilon_{ss} \) is the efficiency of suspended sediment transport, \( I_s \) is the
time fraction (intermittency) of ocean storms, \( U_{om} \) is the near-bed velocity due to waves, \( w_s \) is settling velocity, \( v_0 \) is the velocity of cross-shore currents, and \( h \) is the local water depth. The settling velocity describes the rate at which grains settle from the flow. For each grain size, Sedflux converts user-defined removal rates to settling velocities (Bursik, 1995). Eq. (11) is from Coco (1999) and is the basis for the marine component of the Swenson et al. (2005) model. To accommodate a seafloor containing a range of grain sizes, we assume that the total energy applied to the flow is distributed equally among the multiple grain types. That is, an equal fraction of energy is given to each sediment grain size, which consumes the energy at its own rate governed by its settling velocity.

In this equation, cross-shore currents and bathymetric slope (through the depth dependence of shoaling waves) drive the flux of suspended sediment along the seafloor. However, the sediment flux is also proportional to grain size (through settling velocity). For each grain type, the user specifies a settling velocity. The water depth along the profile specifies the wave orbital velocity through the equation for shoaling waves:

\[
U_{om}(h) = \frac{g\gamma_b}{2} \sqrt{gh_b} \left( \frac{h}{h_b} \right)^{-3/4}
\] (12)

where \( \gamma_b \) (assumed to be 0.6) is the ratio of wave height to water depth where the wave will break, \( h_b \) (Swenson et al., 2005). To limit the size of sediments that a wave is able to mobilize, we use Komar’s (1998) equation for the threshold of sediment motion:

\[
\frac{\rho u_t^2}{(\rho_s - \rho)g} = \begin{cases} 
0.21 \left( \frac{d_0}{\bar{d}} \right)^{1/2} & \text{for } D \leq 0.5\text{ mm} \\
0.46 \pi \left( \frac{d_0}{\bar{d}} \right)^{1/4} & \text{for } D > 0.5\text{ mm}
\end{cases}
\] (13)

where \( D \) is grain size, \( u_t \) is near-bottom threshold velocity, and \( d_0 \) is orbital diameter of the wave motion. Airy wave theory (Komar, 1998) gives a relationship of \( d_0 \) to wave height (\( H \)), period (\( T \)), length (\( L \)), and water depth (\( h \)):

\[
u_t = \frac{\pi d_0}{T} = \frac{\pi H}{T \sinh(2\pi h/L)}
\] (14)

In this equation we calculate the changing characteristics of a wave as it travels over shallow-bathing bathymetry. With these modifications, Sedflux predictions of seafloor morphology are similar to that of the Swenson model, but Sedflux is able to go a step further by predicting the grain-size structure of the seafloor sediments. For instance, lags of coarser grained sediments form at shallower depths where waves have moved the finer grained sediments to calmer waters.

Within the near-shore zone (depths less than \( h_c \)), sediment transport is more complex due to non-linear waves and alongshore current. Thus, we approximate the combined effects of these processes using a diffusion equation with a non-constant diffusion coefficient

\[
q_s = k_c x^{1-m} \frac{dh}{dx}
\] (15)

where \( x \) is offshore position normalized by the position of the near-shore boundary. We use a diffusion coefficient with this specific \( x \)-dependence to ensure the equilibrium profile will be a Bruun profile of the form \( h \propto x^m \) (\( m = 2/3 \)). The constant \( k_c \) is chosen so that the sediment flux at the near-shore boundary matches that of the Swenson equation at the same water depth.

Based on the work of Kubo et al. (2005), we used Sedflux to simulate the building of the Po River delta over the last 8 kyr. Fig. 7 shows the results from two of these experiments. The two simulations ran under the same conditions, varying only wave height. Both used a yearly time step, a horizontal resolution of 400 m, and a vertical resolution of 20 cm. Kubo et al. (2005) provide the sea-level history and Kettner and Syvitski (2005) the sediment supply over this period. Surface plumes distribute sediment into the ocean. Once deposited, waves and bottom currents resuspend and transport the sediment. Wave action is able to control the slope of the delta as it builds out into the ocean. In the first case (Fig. 7A), waves were limited to about 0.5 m in height and averaged about 0.4 m to reflect present-day conditions (Kubo et al., 2006; Cavaleri et al., 1997). For the second case (Fig. 7B), average storm conditions produced waves of about 3.4 m.

Fig. 8 compares the final bathymetry of these two runs with the modern-day bathymetry of the Po River delta. The dashed curves are three representative profiles from the present-day Po delta, and the solid curves are the final bathymetry from the simulations in Fig. 7. The bathymetry generated through high wave conditions matches well with a profile taken far from the major sediment supply of...
the current delta (dashed line (a)). The low wave condition simulation matches better with profiles that were taken nearer the present Po River outlet (dashed lines (b) and (c)).

4.3. Turbidity current model, sakura

Sedflux 1.0c used the one-dimensional steady-state turbidity current model INFLO (Skene et al., 1997). Kubo et al. (2005) added the turbidity current model, called sakura, to Sedflux (Kubo et al. (2005) give a complete description of the model). The user must specify which of the two models Sedflux will call when a turbidity current or hyperpycnal flow is triggered. This choice provides a means for comparison of basin evolution due to different implementations of the same process. In addition, the user may decide on one model based on computation time (INFLO is computationally faster) or the appropriateness of one theory over the other. Sedflux triggers a turbidity current if the clay fraction of a seafloor failure is greater than a user-specified amount (typically, 10%), and a hyperpycnal flow if the river density exceeds that of the ocean.

The layer-averaged, three-equation model of Parker et al. (1986) is the basis of the sakura model.
Accordingly, the three governing equations are
\[
\frac{\partial h}{\partial t} + \frac{\partial}{\partial x}(uh_t) = E_wu \tag{16}
\]
\[
\frac{\partial}{\partial t}(uh_t) + \frac{\partial}{\partial x}(u^2h_t) = -C_d(1 + z)u^3 - C_a(Ch_t^2) + \frac{(\rho_s - \rho_w)gh_tC}{\rho_w} \tag{17}
\]
\[
\frac{\partial}{\partial t}(Ch_t) + \frac{\partial}{\partial x}(uC_t) = -F_d + F \tag{18}
\]
where \(u\) is the flow velocity, \(h_t\) is the flow thickness, \(g\) is the acceleration due to gravity, \(E_w\) is the water entrainment coefficient, \(\rho_s\) and \(\rho_w\) are the densities of the sediment and ambient water respectively, \(C\) is the volume concentration of sediment, \(S\) is the bottom slope gradient, \(C_d\) is the drag coefficient \((C_d = 0.004)\), \(z\) is the ratio of the drag force at the upper flow surface to that at the bed \((z = 0.43)\), and \(F_d\) and \(F_e\) are the flux of sediment deposition and erosion, respectively. Fukishima et al. (1985) provide an empirical relationship between the entrainment coefficient and Richardson number:
\[
E_w = \frac{0.00153}{0.0204 + Ri} \tag{19}
\]
where the Richardson number is
\[
Ri = \frac{(\rho_s - \rho_w)gh_tC}{\rho_w u^2} \tag{20}
\]
This system of three equations contains five unknowns \((h_t, u, C, F_d, \text{ and } F_e)\) and thus two more equations are necessary to close the problem. These final closure equations describe the erosion and deposition rates using the flow's ability to carry sediment. The flow will begin depositing sediment once its concentration exceeds its capacity \((G)\). Likewise, the flow will erode sediment if its concentration is below capacity and its shear stress is greater than that of the bed. The equations used for the deposition and erosion rates are
\[
F_d = \begin{cases} 
   w_sC(2 - 1/p_z), & \text{for } p_z < 0.5 \\
   0, & \text{for } p_z \geq 0.5
\end{cases} \tag{21}
\]
\[
F_e = ((C_d\rho_t u^2 - b)/(a \cdot 86,400 s)) \tag{22}
\]
where \(w_s\) is particle settling velocity and \(\rho_t\) is the flow density. The constants \(a\) and \(b\) \((a = 3.5 \text{ N m}^{-2} \text{ and } b = 0.2 \text{ N m}^{-1})\) define the depth profile of the bed shear strength. The shear strength at the bed is \(b\) and increases linearly with burial depth at a rate, \(a\) (Mulder et al., 1998). Hiscott (1994) provides an empirical approximation for \(p_z\) with respect to the Rouse number \(Z_0\):
\[
|\log p_z|^{1/4} \approx 0.124\log_2 Z_0 + 1.2 \tag{23}
\]
\[
Z_0 \equiv w_s/(k u_*) \tag{24}
\]
where \(k\) is the von Karman constant \((k = 0.4)\) and \(u_*\) is the shear velocity of the flow.

As an example of the turbidity current model, we present results of Kubo et al. (2005). The aim of this study was to apply Sedflux 2.0 to a tank experiment performed in the experimental earthscape (XES) facility of the St. Anthony Falls Laboratory. The tank controlled basin subsidence, and sediment input, making it possible to simulate turbidity current sedimentation in a subsiding basin while varying the rate of sediment supply. The experiment was intended to emphasize the impacts of subidence on the evolution of a turbidite fan.

Although Sedflux usually operates with time steps of days to years, this study used a time step of 1 min to simulate the pulse-type flows that were as short as 2 min in the laboratory experiment. Sediment concentrations of each grain-size fraction, flow velocity, depth, and width of the injection pipe were set to match that of the experimental setting. Sedflux was then set to run in a basin of the size of the XES tank \((5 \text{ m} \times 2 \text{ m})\), using only the turbidity current model, sakura, as a means of distributing sediment. Fig. 9 plots grain size and sediment age of the final deposits of the Sedflux experiment. Overall, the deposit thins with distance from the source. However, within the mini-basin the deposit briefly thickens. Grain size shows a more monotonic decrease with distance due to disparate settling rates of the component grains.

5. Event-based time step

Primarily, high-energy events control the distribution of sediment due to wave resuspension and river transport (Storms, 2003; Paola et al., 1992). That is, short-lived but high-energy ocean storms are able to move more sediment than is moved during the long periods of low energy that separate these events. This is also the case for intermittent but large river floods that become ‘morphology-forming’ events (Paola et al., 1992). Thus, to correctly model these processes, one must operate at a time step that captures these high-energy events.
A daily time step may be able to adequately resolve individual storms and so correctly model the various storms that cause large movements of sediment. However, such a small time step is not practical when conducting simulations covering geologic time scales. The solution is to introduce a time step that varies in length depending upon the magnitude of energy applied to the system (Storms, 2003).

When using a variable time step, these processes will require a time scale over which the process is morphology forming. The ratio of the amount of time during stormy conditions to the total length of a time step defines the time scale \( \frac{n}{m} \). Here, a ‘stormy’ day is any day that is energetic enough to cause significant morphologic changes. For the case of the distribution of sediment within a river, this may be the number of days that the river is bank-full. For the resuspension of shallow marine sediments, this may be the number of days that ocean waves are large enough to result in significant bed shear stress.

This procedure assumes that the long-lasting calm periods move an insignificant amount of sediment compared to the short-lived storms. This may not always be the situation however. In such a case, the model will simulate the calm periods as well. Here, a second time scale is necessary, which is defined as the ratio of the length of a single stormy period to the length of the time step \( \frac{m}{n} \). The energy conditions are constant over each of these time steps. By also modeling the calm periods the number of time steps increases linearly with the number of stormy periods \( \frac{n}{m} \). Even by modeling both the calm and stormy events, the computational saving is significant. Say the average length of a large storm is a week, each year contains approximately one of these storms, and we wish to use a 100-year time step, then there will be 200 time steps within the larger 100-year time step (100 storms separated by 100 calm periods). On the other hand, if we reduced the time step to a week to match the length of the storms, over a 100-year period there would be over 5000 time steps. Thus, only modeling the storms and averaging over the calm periods represents a speed increase of 25 times.

**Fig. 10A** shows a year’s worth of simulated discharge data for the Eel River in Northern California. Because the Eel is a ‘flashy’ river, this technique works well since every year only a few large flood events carry most of the sediment. The dashed line plots the discharge for every day of the year, while the solid line plots only the events that Sedflux will model. Not every river exhibits the flashy behavior of the Eel. For instance, **Fig. 10B** shows a similar time series but this time for the Klinaklini River. In this case, to account for 90% of the total sediment, one must consider at least 150 discharge events (compared to 15 for the Eel).

For wave action, we see a similar result (**Fig. 11A** and B). We have conducted two simulations that together demonstrate the effect that time averaging has on wave resuspension. Both simulations ran under the same conditions except the left simulation (A) used an event-based time step that modeled the
largest ocean storms and averaged over calm periods. The right simulation (B) used yearly averaged wave conditions. Larger storms produce a larger subaqueous delta, and more variable internal architecture than average conditions. As with flood events, the variability of storm events within the year also has an affect on the structure of the delta. The simulation shown in Fig. 11C and D used ocean-wave distributions with large (C) and small (D) variability. Although the final bathymetry is similar in both cases, the internal structure generated by the highly variable storm environment produced bedding patterns with greater variability.

Fig. 10. Time series of simulated sediment load from Eel River (A) and Kliniklini River (B). Solid lines represent events that carry 90% of sediment for the year, while dashed lines plot daily values. For Eel River, the largest 15 events carry 90% of sediment as compared to 150 events for Kliniklini.

Fig. 11. Experiment to demonstrate affect of ocean-storm averaging and variability on shape and internal structure of a delta. Upper-left simulation (A) used an event-based time step, while upper-right (B) used yearly averaged ocean-storm conditions. Bottom two simulations use ocean-storm distributions with high variability (C) and low variability (D). Mean of distributions is the same in both cases. Both simulations result in similarly shaped deltas but with different internal architecture.
6. Program notes and structure

In this section we give a brief description of the main input and output files for Sedflux. A more detailed user manual is included with the Sedflux distribution (available via an anonymous ftp from iamg.org).

6.1. Input files

There are four principal input files required for a Sedflux simulation. These are ASCII files that describe the computational environment, boundary conditions, sediment properties, and process-specific variables. The user specifies the name of the file that describes the computational environment on the command line. This file then references all subsequent input and output files.

6.1.1. Computational environment

This file describes global parameters for a Sedflux simulation as well as the location of the other input files. The file consists of at least two groups. The first defines the spatial resolution, and the location of the sediment and bathymetry files. Subsequent groups define the time step, duration, and process definition files for periods within the simulation. A simulation consists of at least one such period.

6.1.2. Bathymetry

The bathymetry file is a comma-delimited text file that describes the starting elevations of the nodes used in the Sedflux grid. In 2D mode, this file contains two columns of data: horizontal position and elevation. If necessary, Sedflux will linearly interpolate between points to create an array with the specified horizontal resolution. In 3D mode, bathymetry is provided as a matrix with each element specifying the elevation of a particular node in the model grid. The horizontal resolutions define the positions of the nodes.

6.1.3. Sediment

The sediment tracked in Sedflux is composed of a number of distinct grain types. The user defines these grain types in a sediment file. This file consists of a series of groups that define various grain type specific parameters. The parameters that define a grain type are grain size, grain density, saturated density, void ratio in closest compacted state, diffusion index ($\beta$ in Eq. (4)), removal rate, and a compaction coefficient.

6.1.4. Process definitions

The process definition file specifies which processes are active and sets any necessary parameters for the processes. This file consists of a series of groups (one for each process) that contain parameter/value pairs that define the process-specific constants.

6.2. Output files

There are two types of output files for Sedflux, both of which consist of binary data. The first is a three-dimensional grid of sediment property data for each cell of sediment within the simulation. The second records seafloor properties over the entire simulation grid.

6.2.1. Property file

Sedflux is able to track a large number of sediment properties throughout a simulation (Hutton and Syvitski, 2003). A property file records a specified property for all of the sediment cells of a simulation. For large simulations, these files contain a large amount of data and so they are written in a compressed format. Appendix A (available via an anonymous ftp from iamg.org) provides a matlab m-file that is able to read these output files.

6.2.2. Measuring station file

Sedflux enables the user to set up a series of ‘tripods’ on the simulation grid. Each tripod measures a user-specified property at regular intervals. Bathymetric slope, water depth, mean grain size, and percent clay are examples of such properties. Again, a matlab m-file used for reading these files is provided in Appendix A.

7. Application programming interface

The Sedflux project is aimed toward helping two distinct sets of users. The first consists of those whose interest is in conducting numerical experiments. The second group (which oftentimes overlaps with the first) consists of those that write numerical models. Thus far, this paper has described the interface that a model user sees. However, Sedflux also provides a set of libraries that contain an API aimed toward the model developer.

The Sedflux API consists of functions that allow developers to interact with the Sedflux modeling environment without having to worry about specific
implementation details. The benefit of the interface is that it provides the developer with a wide range of functions to use and so reduce the amount of duplicate code written. The interface eliminates dependencies of a developer’s program with the underlying implementation of the Sedflux architecture. Thus, the programmer need not worry that the implementation will change since the interface will not. The programmer can program to an interface, rather than to an implementation.

The Sedflux architecture consists of many objects for developers to access. Sediment cells hold packages of sediment that can be compacted, added to (or subtracted from) one another, queried for geotechnical properties, or stacked on top of one another to form columns of sediment. These columns of sediment combine with one another to form cubes of sediment. The Sedflux framework contains many other objects that describe, for instance, types of grains, sediment distributions, waves, and rivers. Through the API, the user can access and manipulate these structures without having to worry about how the structures are actually built.

8. Summary

Sedflux 2.0 provides the earth science community with a new version of a stratigraphic simulation model. Based on the architecture of previous versions, Sedflux is now able to track the evolution of stratigraphy in three dimensions. However, it can still be run in a two-dimensional mode that functions much like previous versions.

Although it may seem as though the three-dimensional mode might render a two-dimensional mode obsolete, this is not the case. Since the publication of Sedflux 1.0c, Sedflux has seen advances in both versions. The 2D version is faster, requires less memory, is easier to set up, and simulates a wider range of processes. Apart from simulating regions that are nearly two-dimensional, the 2D modeling environment is ideal for obtaining a rough estimate of inputs that will later be used within a larger three-dimensional run. In addition, it is a testing environment for new models that will eventually progress to become modules that are run within the 3D environment.

Sedflux 1.0c contained many of the important processes that distribute sediment within the marine environment. However, it lacked a shelf sediment transport model, and a fluvial deposition and erosion subroutine. Both of these processes are now present within Sedflux 2.0. In addition, Sedflux now contains two hyperpycnal plume models for the user to choose from. The Sedflux environment provided an ideal place to test the new model (sakura) by providing a system to run many simulations with a large range of boundary conditions and to explore the feedbacks both between previous flows as well as other processes.

Many processes are presently not part of the new three-dimensional Sedflux architecture. For example, although Sedflux is able to simulate turbidity currents and debris flows along a profile, it lacks equivalent models that are able to travel over a two-dimensional grid. However, the structure of Sedflux allows these additions to be made and researchers are invited to contribute to this community modeling effort.

The Sedflux API provides a means for modelers to access the underlying architecture. Researchers are now able to communicate with Sedflux without having to worry about the details of the implementation of the architecture. This makes it easier to either write wrappers around existing models so that they can communicate with Sedflux or to write models from scratch within the Sedflux framework. In addition, the modeler need not duplicate the work of others by rewriting commonly used pieces of code such as grid interpolation, or matrix solvers. All of this code is open source and so we encourage other developers to contribute to this growing library. As the library grows, the amount of duplicate work done in the community will decline, and the quality of the library will increase, which will benefit everyone.

Appendix A. Supplementary materials

Supplementary data associated with this article can be found in the online version at doi:10.1016/j.cageo.2008.02.013.

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