THEORY AND MEASUREMENTS OF ANGLE-OF-ARRIVAL OF DIFFRACTION-LIMITED ELECTROMAGNETIC WAVE BEAMS IN THE TURBULENT ATMOSPHERE

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ABSTRACT

Atmospheric turbulence has been shown to have measurable effects on the angle-of-arrival (AOA) of electromagnetic wave beams, but these effects are on the order of a few microradians, which is a negligible level for most applications. In this paper, we present a theory describing this phenomenon and compare this theory to one-way measurements made over a 3.5 km path and two-way measurements made over a 25 km path using an X-band interferometric radar.

1. INTRODUCTION

The effects of atmospheric turbulence on radar angle errors are generally small, but in those cases where a radar is required to guide a missile without a seeker (command guide), a few microradians of error may be enough to cause the missile to miss its target. In this paper we compare two methods of calculating these angle-of-arrival (AOA) errors and show that the geometrical optics approach derived by Churnside and Lataitis [1], the first method, gives larger angular errors than those determined by experiment. This method is characterized by perfectly collimated and focused beams and ideal plane waves. The second method involves the adaptation of the approach described in [1] to a physical optics model in which the beams have a Gaussian profile. This method of calculation gives results reasonably close to experimental values. We determine the AOA using both approaches and compare the results obtained to an experiment conducted using an interferometric radar capable of measuring the small angles expected for this scenario.

2. THEORY

In Reference [1], the authors have derived the AOA of a focused beam in the geometrical optics limit for a one-way path and have shown that their result reduces to those obtained for plane and spherical waves in the limits of infinite and zero focal lengths, respectively. They also show that the AOA approaches infinity in a predictable way as range approaches the focal length of the focusing optic. They used the simple model shown in Figure 1 as the basis of their calculations. The tilt angle $d\alpha$ is given by

$$d\alpha = \Delta n(z)dz / w(L), \tag{1}$$

so that the total tilt angle over path L is

$$\alpha = \frac{1}{w(L)} \int_0^L \Delta n(z)dz. \tag{2}$$

Using this result, and assuming that there is no average gradient of refractive index, Churnside and Lataitis derive an expression for the one-way AOA based on geometrical optics.

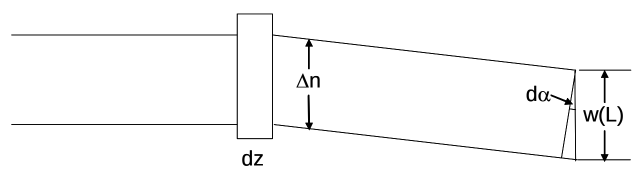

Figure 1. A thin atmospheric layer with varying index causes beam steering

Extending the arguments leading to Equation (1) to the two-way case, we find that the tilt back at the transmitter is given by

$$d\alpha = \frac{\Delta n_t(z) + \Delta n_r(z)}{w_r(0)}, \tag{3}$$

where Δn_t and Δn_r are the refractive index differences across the transmitted and reflected beams, respectively, and $w_r(0)$ is the diameter of the reflected beam at the transmitter. Summing contributions along the entire path as for Equation (2) gives a total tilt angle of

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$$\alpha = \frac{1}{w_r(0)} \int_0^L \left[\Delta n_t(z) + \Delta n_r(z)\right] dz \qquad (4)$$

Just as Equation (2) is the basis for derivation of the expressions for AOA for one-way propagation, Equation (4) is used to derive expressions for two-way propagation. The details of these derivations are given in [1] and will not be repeated here.

In deriving the AOA for the physical optics case, we proceed exactly as for the geometrical optics case using the procedure given in [1]. For one-way transmission, we have shown [2] that the variance of the AOA measured at the target is

$$\sigma_t^2 = 2.92 \frac{C_n^2}{w^2(L)} \int_0^L [w(z\ \ )]^{5/3} dz\ , \qquad (5)$$

where w(L) and w(z_1) are the beamwidths measured at ranges L and z, respectively and C_n^2 is the index of refraction structure parameter. In the physical optics (PO) case, the beam has a Gaussian profile and its width varies as [3,4]:

$$w(z) = 2w_0 \left[1 + \left(\lambda z / \pi w_0^2\right)^2\right]^{1/2}, \qquad (6)$$

where λ is wavelength and w_0 is the 1/e beamwidth at the transmitter.

For a reflected beam, this approach is extended to 2-way transmission using the method described in [1]. The result for the AOA variance back at the radar receiver is:

$$\sigma_r^2 = 2.92 \frac{C_n^2}{w_r^2(0)} \int_0^L \left\{ \begin{array}{l} [w_t(z)]^{5/3} + [w_r(z)]^{5/3} \\ + \dfrac{1}{2^{2/3}} \left(\begin{array}{l} [w_t(z) + w_r(z)]^{5/3} \\ - \left|[w_t(z) - w_r(z)]\right|^{5/3} \end{array} \right) \end{array} \right\} dz \qquad (7)$$

where $w_r(0)$ is ½(1/e) times the diameter of the radar antenna, $w_t(z)$ is the diameter of the transmitter beam, and $w_r(z)$ is the diameter of the reflected beam. These latter parameters are determined by substituting ½(1/e) times the diameter of the transmitter and the reflector, respectively. The reflector is assumed to be a circular mirror normal to the direction of propagation of the transmitter beam for these calculations.

To determine the one-way AOA using the physical optics approach, we must integrate Equation (5) numerically using the beam diameter given by Equation (6). Substituting Equation (6) into Equation (5), we get for the AOA variance

$$\sigma_t^2 = \frac{2.92 C_n^2}{(2w_0)^{1/3} \left[1 + \left(\dfrac{\lambda L}{\pi w_0^2}\right)^2\right]} \int_0^L \left[1 + \left(\frac{\lambda z}{\pi w_0^2}\right)^2\right]^{5/6} dz, \qquad (8)$$

where all parameters have been defined previously. In the derivation of Equation (8) we assume that all of the radiation from the transmitter is collected.

For the two-way case, we simply substitute Equation (6) for the Gaussian beam profile into the two-way AOA expression (7). The resulting equation must be solved numerically as must Equation (8).

To obtain the geometrical optics result we use an equation derived in [1] that is based on the geometrical optics scenario of a collimated transmitter beam illuminating a reflector with a negative focal length. We choose the focal length to be negative because most targets of interest will be convex. This result is Equation (31) of Reference [1] and gives the variance of the AOA, σ_r^2 as:

$$\sigma_t^2 = 7.01 C_n^2 L D_t^{-1/3}, \qquad (9)$$

where L is slant range, D_t is antenna diameter, and the parameter C_n^2 is the refractive index structure parameter. In deriving this result, we have used the approximation $L >> f_r$, where f_r is the (negative) focal length of the reflector. We do not have to know the value of the focal length because if we use this approximation, the entire bracketed term in Equation (31) of Reference [1] reduces to 2.40.

3. EXPERIMENT

Measurements of AOA were made on two separate occasions at a location in Brea, CA. The first series of measurements was made over a path length of 25 km using a passive reflector located on a hillside such that the elevation angle was about four degrees. Results from this series of measurements are presented in this paper. The second series of measurements was made in the same locality over a range of 3.5 km but with active repeaters instead of reflectors. Since these latter measurements were made with repeaters, they are considered to represent a one-way path, so that Equation (8) is applicable. These latter measurements also gave very useful values of C_n^2, which were used in the calculations presented herein, both for the one-way and the two-way paths.

Figure 2 shows the experimental arrangement used for the long-range measurements and Figure 3 shows the setup for the short-range measurements and those used to determine C_n^2. Both of these experiments use interferometric radars for measurements of the very small AOAs characteristic of microwave frequencies. For determination of C_n^2, the fluctuations from a single channel were used and C_n^2 was calculated from the log amplitude variance σ_χ^2 using the relation

$$\sigma_\chi^2 = 0.31 C_n^2 k^{7/6} L^{11/6}, \tag{10}$$

where k is wavenumber $2\pi/\lambda$.

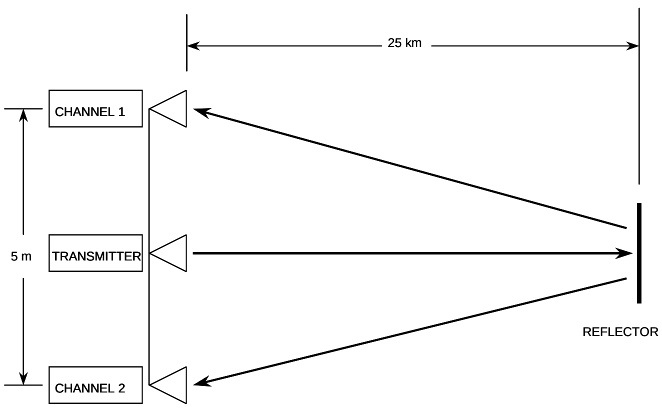

Figure 2. Diagram of the interferometric radar used for long-range AOA measurements.

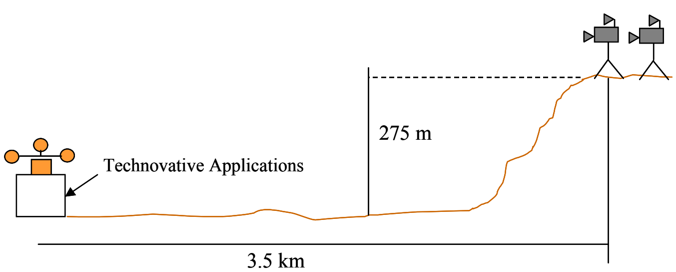

Figure 3. Diagram of the interferometric radar used for short-range AOA measurements and determination of C_n^2. These measurements were made at the Technovative Applications facility in Brea, CA.

Figure 4 is a photograph of the repeaters used for short-range one-way measurements. The antenna apertures are 12.3 X 9.1 cm. Figure 5 is a photograph of the interferometric radar. The transmitter is in the center and the receivers are at the ends of the boom with a spacing of 5 m. The transmitter frequency was 9523 MHz and the ntenna diameter was 56 cm.

Figure 4. Photograph of the repeaters used for short-range AOA and C_n^2 measurements.

Figure 5. Photograph of the interferometric radar used for long-range AOA measurements.

4. RESULTS

It was first necessary to determine the values of C_n^2 for the short-range experiment. Calculations of AOA were then made using Equation (8) with these values and compared to measured results. Other measurements of C_n^2 made at millimeter waves [5] have shown that this parameter can be as high as 5.5 X 10^{-12} $m^{-2/3}$ under hot, humid conditions. These parameters are much smaller in the visible and infrared range and partially explain the result that turbulent fluctuations are still observable in the microwave bands even though theory predicts that they should decrease as $f^{7/6}$, where f is frequency.

We have solved Equation (8) numerically for ranges between 500 and 5000 meters, a transmitter aperture of 0.1 m, a frequency of 9.5 GHZ, and values of C_n^2 varying from 1.4 X 10^{-13} to 6.8 X 10^{-13} $m^{-2/3}$, and the results are shown in Figure 6.

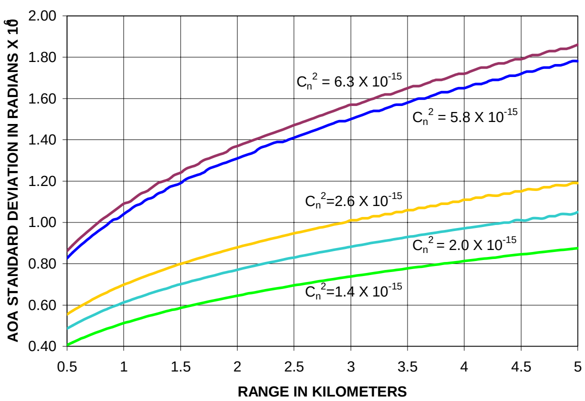

Figure 6. Calculated AOA variance as a function of range for the one-way path for the conditions described in the text.

Table I summarizes these results for the 3.5 km range used in the measurements. These are the physical optics (PO) results shown in the table. Calculation of the geometrical optics (GO) numbers were made using the equations of Reference [1] and the values of C_n^2 were determined using Equation (1). This level of agreement is considered good for this type of experiment for both sets of calculations, but the results show that the PO formalism gives better agreement with experiment.

We have used Equation (7) to calculate the AOA expected for the 25 km two-way path. Although simultaneous measurements of C_n^2 were not made, the value of this parameter corresponding to Figure 5 above was considered typical and was used in these calculations. Figure 7 shows the results obtained for values of the diameter of the remote reflector of 0.2, 0.4, 0.6, and 0.8 meters. For 0.6 meters at 25 km range, these curves show that the AOA variance is 7.3 X 10^{-10} rad^2 corresponding to a standard deviation of 27 microradians. The value of C_n^2 used in these calculations was 3.3 X 10^{-13} $m^{-2/3}$ and the transmitter antenna diameter used was 0.6 m.

Figure 9 is a scatter plot of cross range errors measured with the interferometric radar. This figure shows that the standard deviation of these errors is about 10 microradians. This agreement between measured and calculated values of AOA is considered to be fair. In the next section, we will discuss the possible reasons for this level of disagreement. Note that the parameter C_n^2 was not measured for the long-range results. We used the value 3.3 X 10^{-13} $m^{-2/3}$ for these calculations. If we had used 1.0 X 10^{-13} $m^{-2/3}$ for this parameter, which is more consistent with reality based on Figure 6, we would have obtained a standard deviation of 9 microradians, in good agreement with the results shown in Figure 9.

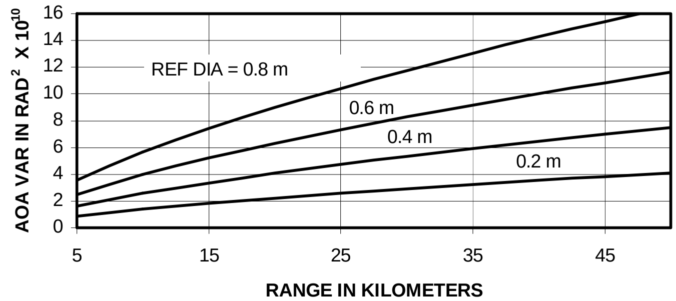

Figure 7. Results of calculating AO for two-way paths varying from 5 to 50 km for various receiver reflector diameters. The parameter of interest for this experiment are 25 km range and 0.6 m reflector diameter.

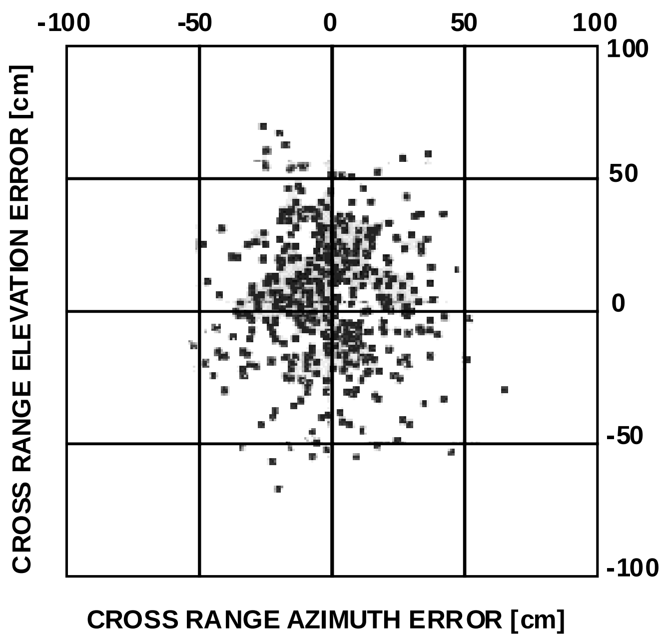

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Figure 8. Scatter plot of AOA measurements made with a 0.6m transmitter and reflector over a 25 km path. The measured standard deviation is 10 microradians

It is instructive to compare the results depicted above to those obtained by using the one-way equation $\sigma_t^2 = 2.92 L C_n^2 D_r^{-1/3}$, where D_r is receiver diameter, and the two-way equation as derived in [1]. Using the same parameters, we get an AOA standard deviation of 63 microradians for the one-way case and 262 microradians for the two-way case. Even though the analysis given above gives values of AOA that are consistently higher than measured values, they represent a considerable improvement over the results obtained by using the earlier accepted techniques for calculating AOA.

Table I. Results Of AOA Measurements Over A One-Way, 3.5 Km Path.

AOA Standard Deviation in Microradians					
C_n^2 in $m^{-2/3}$	$5.8X10^{-13}$	$2.0X10^{-13}$	$1.4X10^{-13}$	$2.6X10^{-13}$	$6.3X10^{-13}$
GO Calculated	5.9	3.5	2.9	3.9	6.1
PO Calculated	1.6	0.9	0.8	1.1	1.7
Measured	0.9	0.5	0.3	1.2	2.8

5. CONCLUSIONS

We have developed a method of calculating radar AOA based on the physical optics principle of Gaussian beam profiles. In this approach, the radar beams are allowed to diverge normally instead of being perfectly collimated as in the geometrical optics case.

We find that our theory predicts AOAs that are consistently slightly higher than measured values. Part of this error can be ascribed to the slight elevation of the propagation path and the resulting slight reduction in C_n^2 as a function of altitude for which our theory does not account. Other possible sources of error are noise generated in the repeater receivers and transmitters and noise in the radar receiver. Other problems include repeater or radar motion caused by wind or by ground vibrations, although all of these potential errors would tend to increase AOA standard deviation instead of reducing it.

Despite this lack of agreement with measured AOA, our results are still better than those obtained by using a geometrical optics approach. The poor agreement using geometrical optics results from the lack of diffractive beam spreading inherent in the assumptions used in deriving this theory. We expect to continue measurements of AOA using the apparatus described herein and will refine our theory to attempt to get better agreement between theory and measurement.

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Outline

- Purpose
- Procedure
- Theory
- Experiment
- Results
- Conclusions

Purpose

- Precise radar applications require high angular accuracy
- It is necessary to determine the atmospheric contribution to angular error
- Knowledge of the index of refraction structure parameter C_n^2 is necessary to quantify angular errors
- Measurements of angle-of-arrival are necessary to support our theoretical basis

Procedure

- Measure fluctuations in X-band signal over a path of suitable length L
- Calculate C_n^2 using the equation

$$\sigma_\chi^2 = 0.31 C_n^2 k^{7/6} L^{11/6}$$

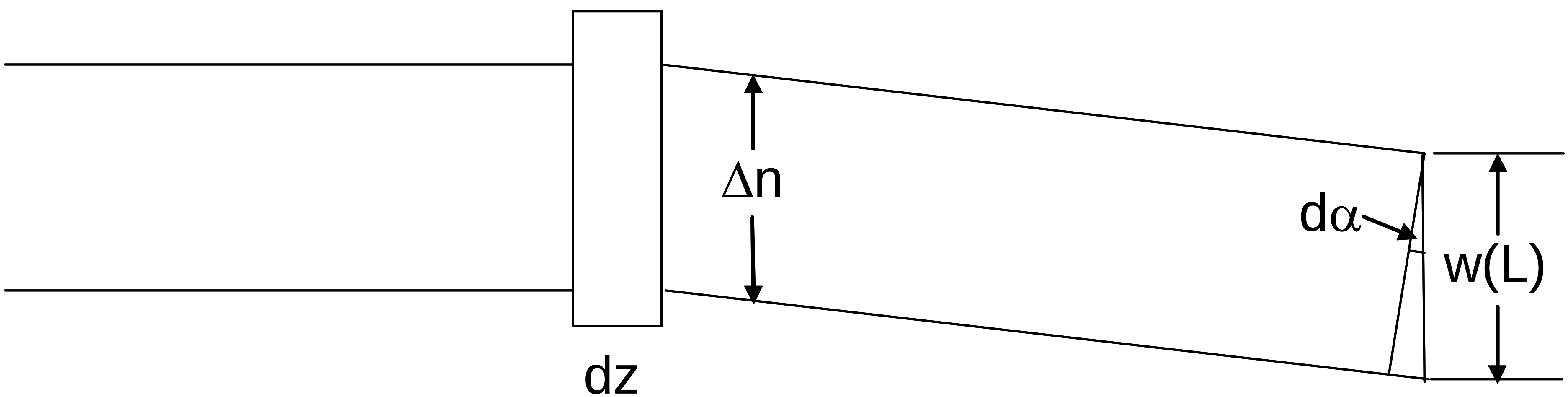
  - Where σ_χ^2 is log amplitude variance and k is wavenumber
- Calculate AOA using our theory
- Compare theory and experiment

A Thin Atmospheric Layer With Varying Index Causes Beam Steering

$$d\alpha = \Delta n(z)dz / w(L)$$

$$\alpha = \frac{1}{w(L)} \int_0^L \Delta n(z)dz$$

In deriving these equations, we assume that there is no average gradient of refractive index



One-Way Path

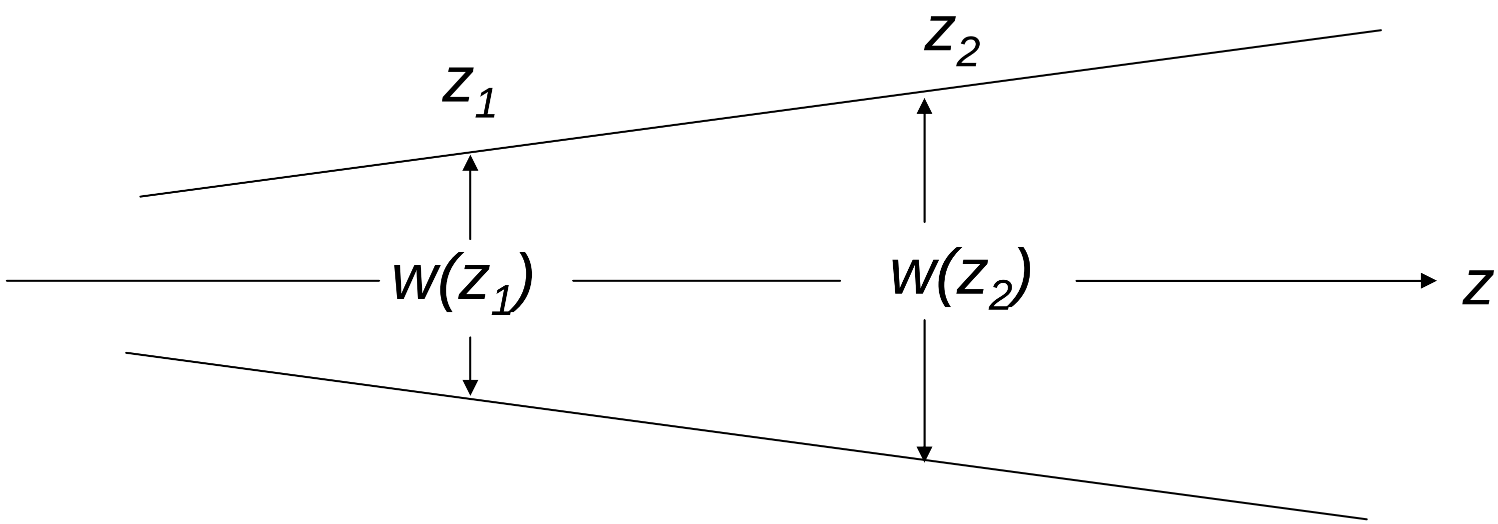
The variance of AOA fluctuations for a one-way path is then given by:

$$\sigma_t^2 = \frac{1}{w^2(0)} \int_0^L \int_0^L \langle \Delta n(z_1) \Delta n(z_2) \rangle dz_1 dz_2 ,$$

The index gradient can be written in terms of two spatial coordinates as

$$\Delta n(z) = n\left[z, \frac{w(z)}{2}\right] - n\left[z, -\frac{w(z)}{2}\right],$$

Beam Geometry



One-Way Path (Continued)

The quantity in the angle brackets can be expanded and rearranged to get

$$\left\langle \Delta n(z_1)\Delta n(z_2)\right\rangle = 1/2\left\langle \begin{array}{l} \{n[z_1, 1/2w(z_1)] - n[z_2, 1/2w(z_2)]\}^2 \\ + \{n[z_1, -1/2w(z_1)] - n[z_2, 1/2w(z_2)]\}^2 \\ - \{n[z_1, 1/2w(z_1)] - n[z_2, -1/2w(z_2)]\}^2 \\ - \{n[z_1, -1/2w(z_1)] - n[z_2, -1/2w(z_2)]\}^2 \end{array}\right\rangle .$$

Using the equation for the calculation of the distance between two points and and the definition of the index of refraction structure parameter $C_n^2 = \left\langle n(r_1) - n(r_2)\right\rangle^2 / (r_1 - r_2)^{2/3}$ we find that for homogeneous, isotropic turbulence, the above quantity becomes

One-Way Path (Continued)

$$\langle \Delta n(z_1)\Delta n(z_2)\rangle = C_n^2\left(\{(z_2 - z_1)^2 + 1/4[w(z_2)+w(z_1)]^2\}^{1/3} - \left[(z_2 - z_1)^2 + 1/4[w(z_2)-w(z_1)]^2\right]^{1/3}\right).$$

Now assume that the magnitude of the focal length of the optical system that determines these beamwidths is much larger than the beam diameter so that the quantity is a slowly varying function of position along the path. For a radar beam, this condition is easily met because the focus is at infinity. We may therefore consider the beam diameter to be constant unless the separation becomes much greater than the diameter. In this case, the above equation becomes very small because the turbulent fluctuations become decorrelated. Under these assumptions, $w(z_1) \approx w(z_2)$ and the above integral may be written

One-Way Path (Continued)

$$\frac{C_n^2}{w^2(0)}\int_0^L\int_0^L w^{2/3}(z_1)\left(\left[1+\frac{(z_2-z_1)^2}{w^2(z_1)}\right]^{1/3}-\left[\frac{(z_2-z_1)}{w\ (z_1)}\right]^{2/3}\right)dz_1dz_2$$

Most of the contribution to the integral comes from values of z_2 that are very nearly equal to z_1, so there is little error involved in extending this integral from negative infinity to infinity. Note that the above integral is then of the form

$$\int_0^L\int_{-\infty}^{\infty} w^{5/3}(z_1)[(1+y^2)^{1/3}-(y^2)^{1/3}]dydz_1$$

Integrating over z_2 numerically gives simply

$$\int_0^L\int_0^L\left[\langle\Delta n_t(z_1)\Delta n_t(z_2)\rangle\right]dz_1dz_2=\frac{2.92C_n^2}{w_0^2}\int_0^L w_t^{5/3}(z_1)dz_1.$$

For One-Way Path

$$\sigma_t^2 = 2.92 \frac{C_n^2}{w^2(L)} \int_0^L [w(z)]^{5/3} dz$$

From the theory of Gaussian beam propagation,

$$w(z) = 2w_0 \left[1 + \left(\lambda z / \pi w_0^2 \right)^2 \right]^{1/2}$$

Substituting

$$\sigma_t^2 = \frac{2.92 C_n^2}{\left(2w_0\right)^{1/3} \left[1 + \left(\lambda L / \pi w_0^2 \right)^2 \right]} \int_0^L \left[1 + \left(\lambda z / \pi w_0^2 \right)^2 \right]^{5/6} dz,$$

J. H. Churnside and R. J. Lataitis, “Angle of Arrival Fluctuations of a Reflected Beam in Atmospheric Turbulence”, *J. Opt. Soc. Am*. **4**, July 1987, 1264.

For Two-Way Path

$$d\alpha = \frac{\Delta n_t(z) + \Delta n_r(z)}{w_r(0)},$$

$$\alpha = \frac{1}{w_r(0)} \int_0^L \left[\Delta n_t(z) + \Delta n_r(z)\right] dz$$

For Two-Way Path

Subject to several assumptions, the expression for a reflected beam (two-way path) becomes:

$$\sigma_r^2 = 2.92 \frac{C_n^2}{w_r^2(0)} \int_0^L \left\{ \begin{aligned} & \left[w_t(z)\right]^{5/3} + \left[w_r(z)\right]^{5/3} \\ & + \frac{1}{2^{2/3}} \left(\begin{aligned} & \left[w_t(z) + w_r(z)\right]^{5/3} \\ & - \left|\left[w_t(z) - w_r(z)\right]\right|^{5/3} \end{aligned} \right) \end{aligned} \right\} dz$$

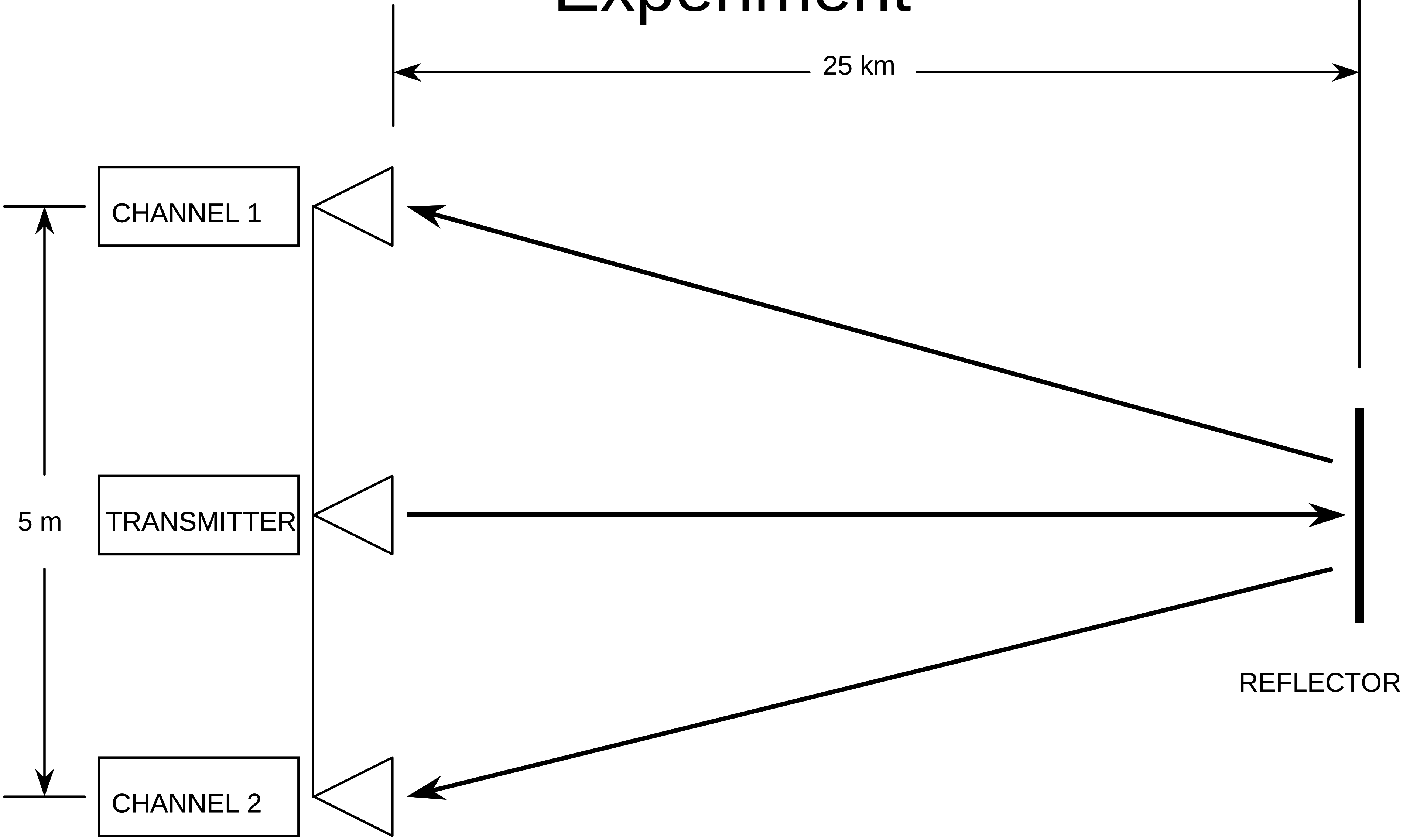
If we now substitute the expression for Gaussian beamwidth given above for $w_t(z)$ and $w_r(z)$, we obtain an equation that Can be solved numerically.

Geometrical Optics AOA Theory

$$\sigma_t^2 = 2.92 C_n^2 L w^{-1/3}$$

For a one-way path and aperture w

Experiment

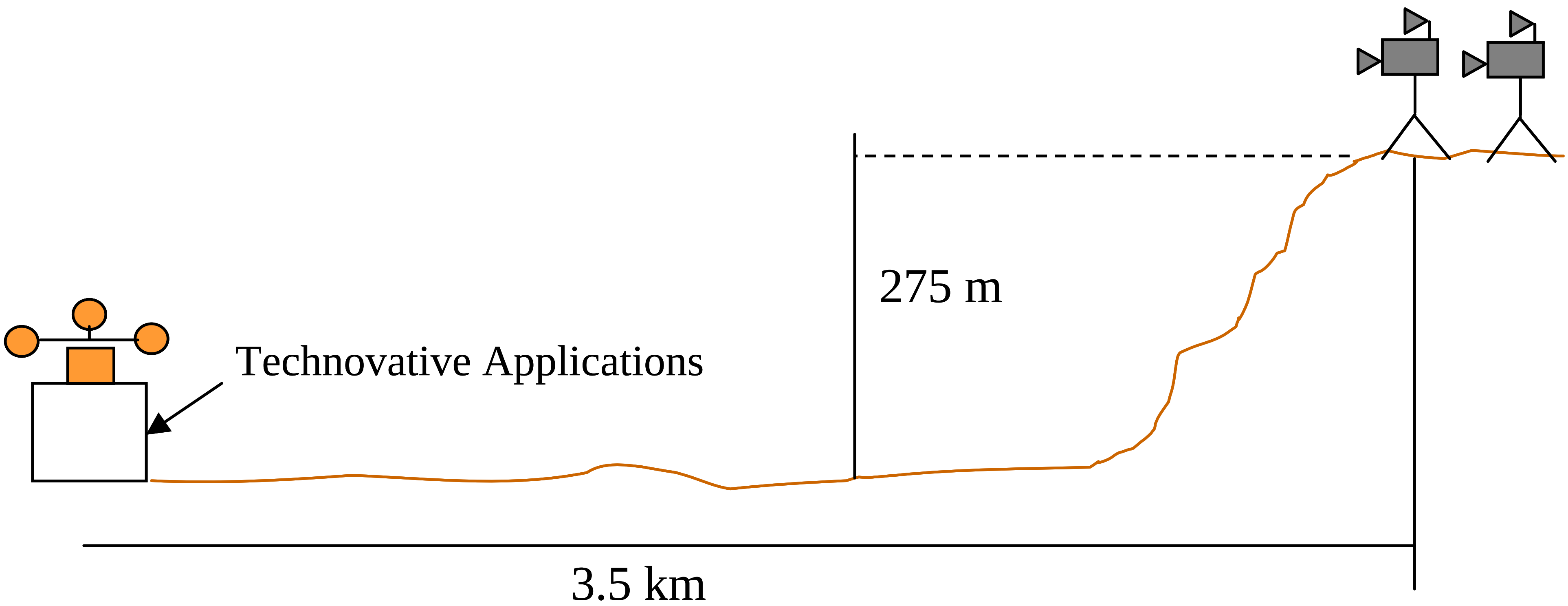


Transmitter (center) and Interferometric Receiver

One-Way Measurements

We used repeaters (one at a time)

Repeater Diagram



Determination of C_n^2

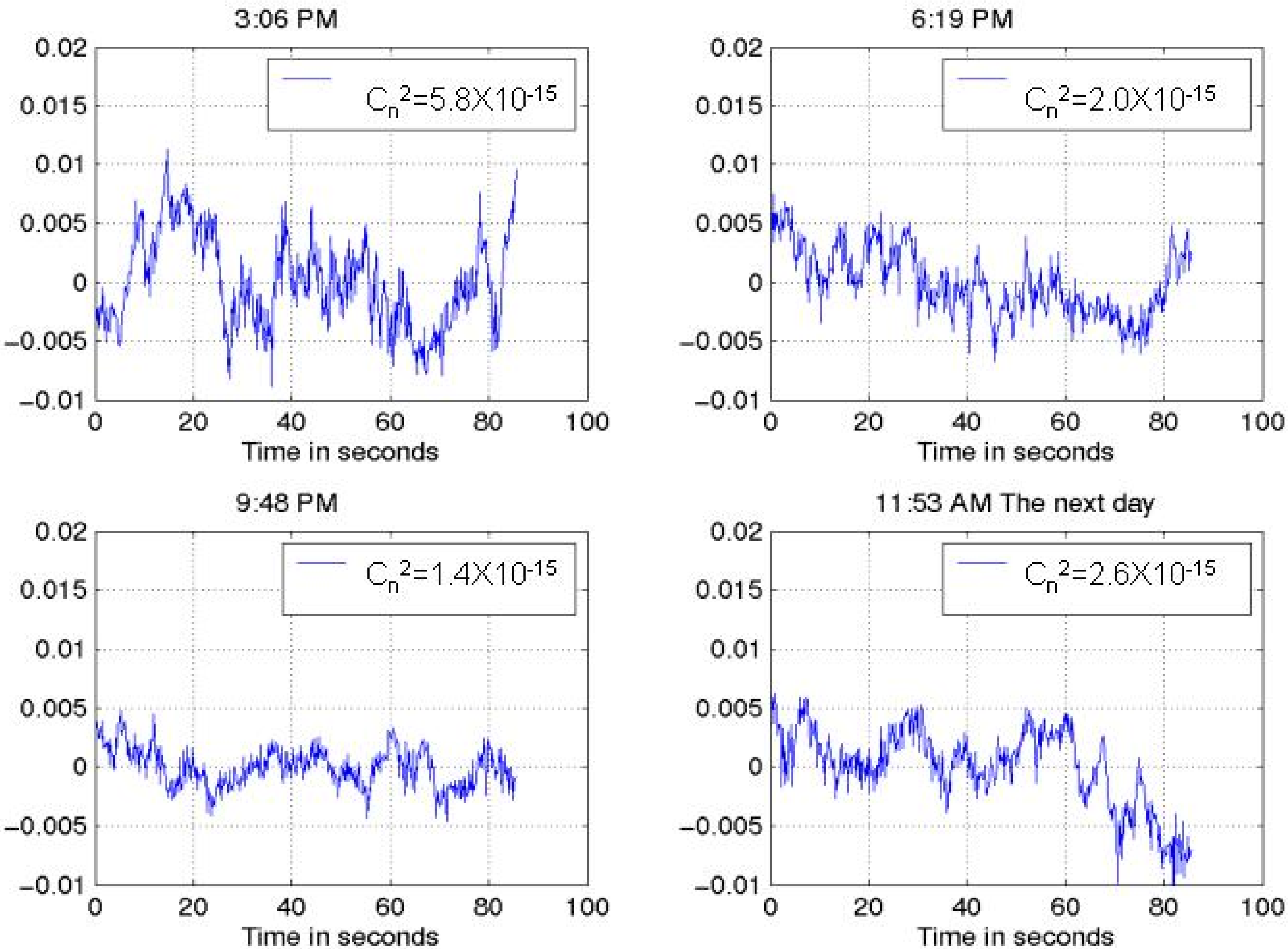
- No instrumentation to measure C_n^2 directly
- Used signal from one of the receivers and the equation

$$\sigma_\chi^2 = 0.31 C_n^2 k^{7/6} L^{11/6},$$

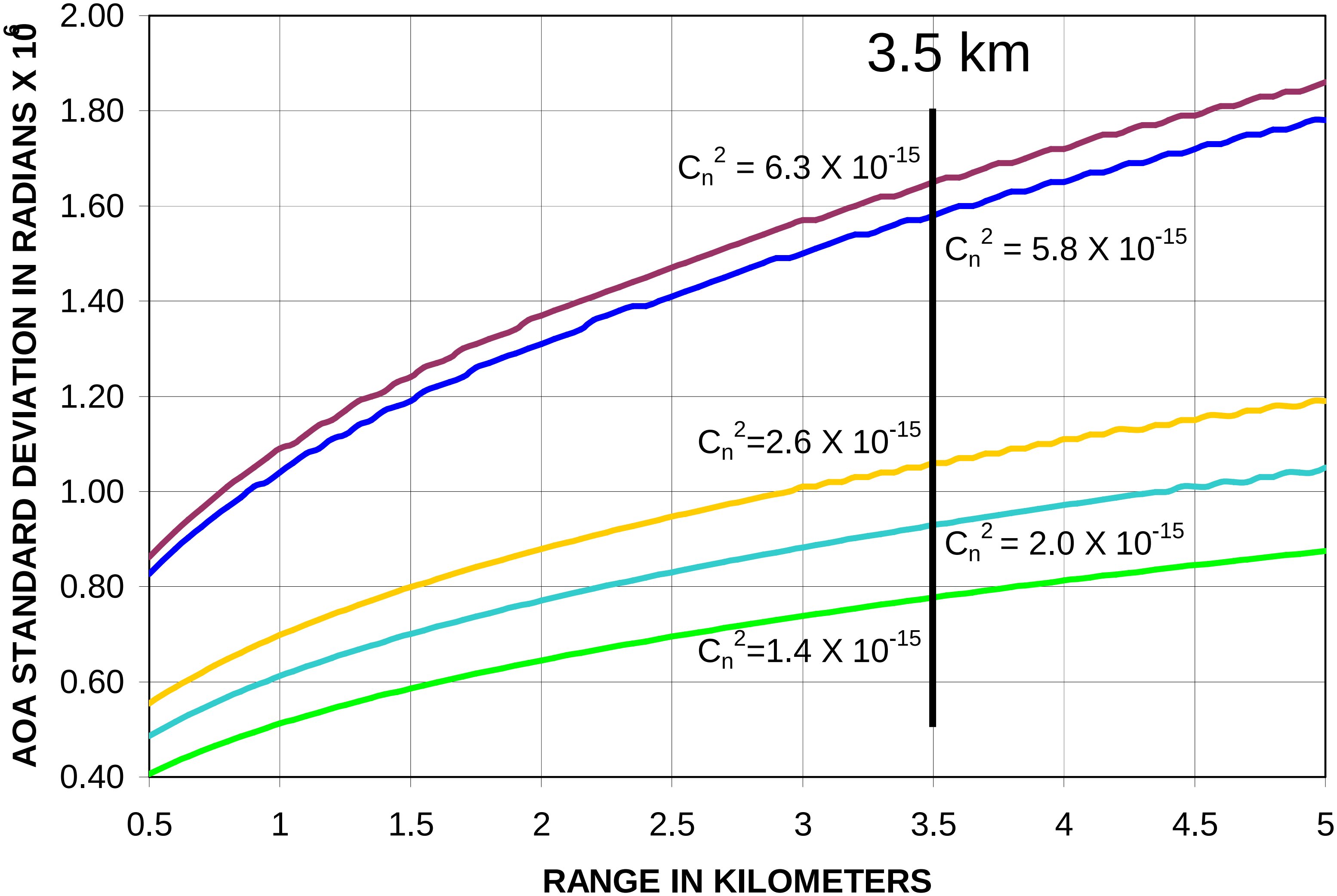
•Values measured were somewhat lower than those generally observed for microwave systems, which is inconsistent with earlier work*

*R. J. Hill, R. A. Bohlander, S. F. Clifford, R. W. McMillan, J. T. Priestley, W. P. Schoenfeld, "Turbulence-Induced Millimeter-Wave Scintillation Compared with Micrometeorological Measurements", *IEEE Trans. Geosciences and Remote Sensing,* 26, May 1988, 330.

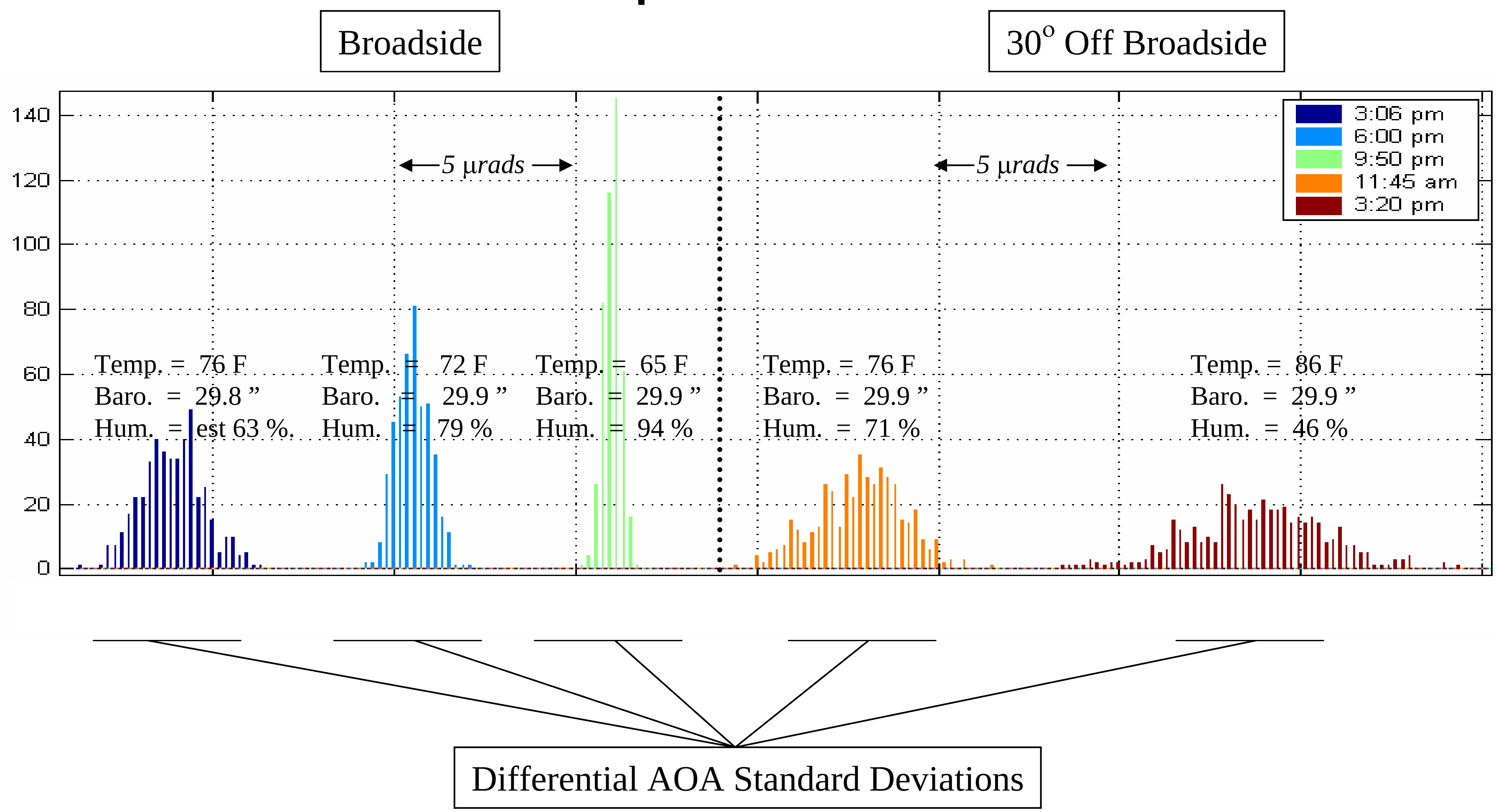
Results - C_n^2


3:06 PM
C_n^2=5.8X10^{-15}
6:19 PM
C_n^2=2.0X10^{-15}
9:48 PM
C_n^2=1.4X10^{-15}
11:53 AM The next day
C_n^2=2.6X10^{-15}
0.02
0.015
0.01
0.005
0
−0.005
−0.01
0
20
40
60
80
100
Time in seconds

Calculated One-Way AOA Standard Deviation



Histograms of AOA
Colors Correspond to Previous Slide



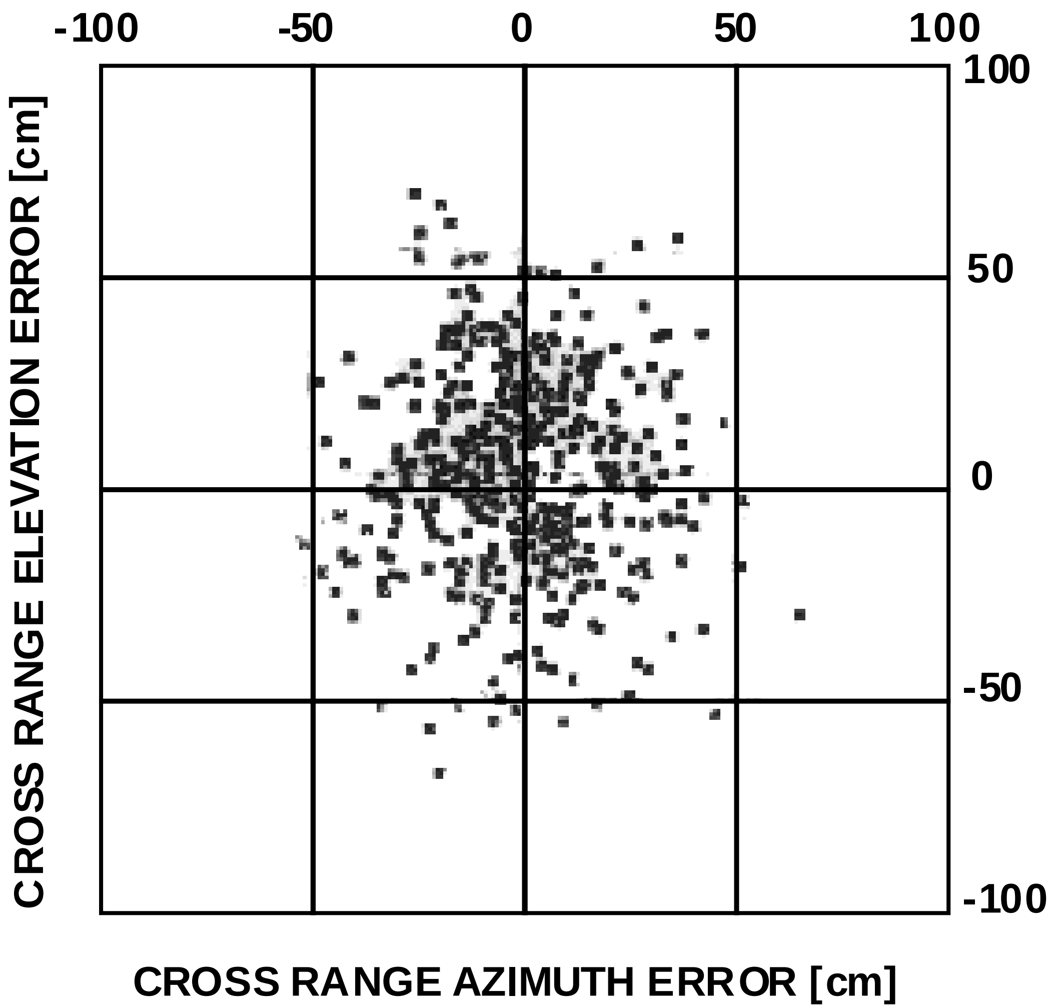
Tabular Results

Geometrical Optics (GO), Physical Optics (PO)

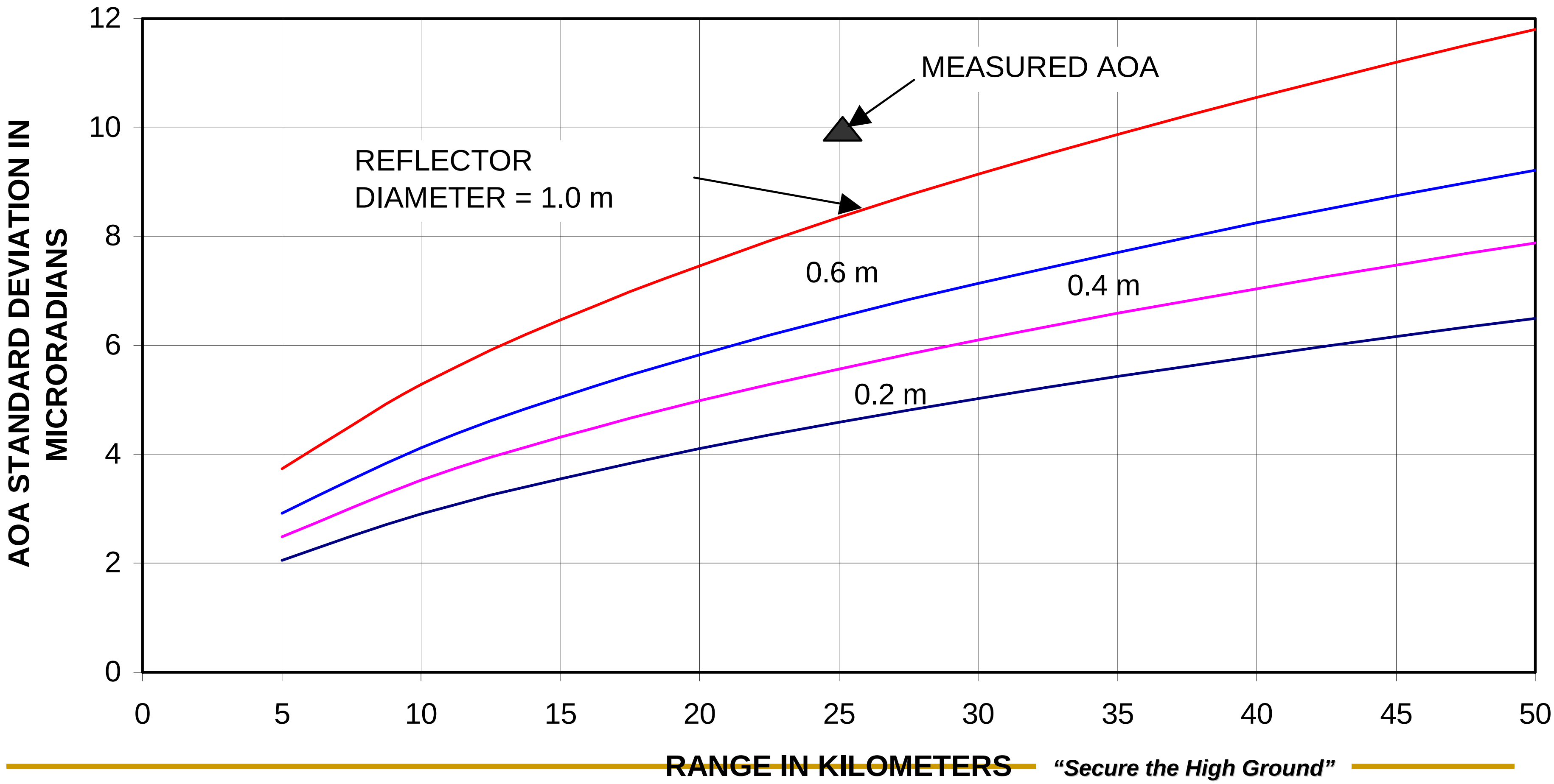
AOA Standard Deviation in Microradians					
C_n^2 in $m^{-2/3}$	$5.8X10^{-13}$	$2.0X10^{-13}$	$1.4X10^{-13}$	$2.6X10^{-13}$	$6.3X10^{-13}$
GO Calculated	5.9	3.5	2.9	3.9	6.1
PO Calculated	1.6	0.9	0.8	1.1	1.7
Measured	0.9	0.5	0.3	1.2	2.8

Scatter Plot of AOA Measurements

25 km Range



Calculated 2-Way AOA Versus Range



Conclusions

- We have measured C_n^2 using x-band signals over 3.5 km path
- Correlation with AOA gives good results
- We used C_n^2 results to calculate 2-way AOA over 25 km path
  - Good agreement with measured results
  - Geometrical optics calculations also give good results