Energy-Efficient Multi-Hop Transmission for Machine-to-Machine Communications

Christian Dombrowski*, Neda Petreska§, Simon Görtzen‡, Anke Schmeink*, James Gross‡

*UMIC Research Centre, RWTH Aachen University, Germany
§Fraunhofer Institute for Communication Systems ESK, Munich, Germany
‡School of Electrical Engineering, Royal Institute of Technology, Sweden
dombrowski@umic.rwth-aachen.de  neda.petreska@esk.fraunhofer.de

Abstract—Emerging machine-to-machine communication scenarios are envisioned to deal with more stringent quality-of-service requirements. This relates mainly to outage and latency requirements, which are for example for safety-critical messages quite different than for traditional applications. On the other hand, it is widely accepted that machine-to-machine communication systems need to be energy-efficient due to battery-power devices but also due to their huge deployment numbers. In this paper we address these issues with respect to multi-hop transmissions. Specifically, we deal with minimizing the consumed energy while transmitting a packet with end-to-end outage and latency requirements. We account for the case where the system can utilize instantaneous channel state information as well as only average channel state information and develop an optimization based on convex programming. We can show numerically that despite accounting for the energy consumption of acquiring instantaneous channel state information, especially as the outage and latency requirements become tough it is by a factor of up to 100 more energy efficient to convey a packet with instantaneous channel state information than with average one.

Minimizing energy consumption for a wireless multi-hop transmission that has to comply to Quality-of-Service constraints (reliability and latency) is not an easy task. This is of eminent relevance if challenging application scenarios are considered, e.g., machine-to-machine communications. This paper focuses on the impact of time domain aspects on energy efficiency. The second topic addressed is the question whether it is beneficial to collect instantaneous channel information or to adapt to an average channel knowledge, depending on several transmission parameters. A closed-form, analytical energy expression is derived for the average case, which is based on the solution of an optimization problem. The optimal allocation has similarities with waterfilling for multi-carrier systems. The optimal time spent on acquiring channel state information is numerically determined by means of simulations. Furthermore, it is also shown that the number of relays bridging a certain, fixed distance has to be carefully selected to avoid undermining energy efficiency.

I. INTRODUCTION

The human demand for communication has been the major source driving the evolution of wireless networks over the last decades. As communication over these networks became more and more popular, mass-market systems like cellular or local area networks evolved to meet this increasing demand. As a consequence, this has lead to a steady increase in rate over the last 25 years. Associated to this steady increase in rate has been the ability of networks to support best-effort or delay-sensitive data flows as the major Quality-of-Service (QoS) classes.

In contrast, in the newly emerging area of machine-to-machine communication, we find scenarios with more challenging QoS requirements. In this case, the required level of reliability can become quite high, i.e., the acceptable probability that a message is corrupted is in the range of $10^{-5}$ and below while tight deadlines (in the range of milliseconds) have to be met. Applications with such QoS requirements are typically encountered in industrial automation or in different kinds of distributed control systems. To date, such communication is typically carried over wired networks due to reliability and security issues. However, recently there is more and more interest in feasible designs of wireless networks to substitute wired links in such application scenarios [1], [2]. Apart from the reliability requirements, it is also widely accepted that wireless networking solutions for machine-to-machine type of applications have to be energy-efficient. This is mainly due to the fact that networking devices might be battery-driven. Hence, energy-efficient operation ensures a long lifetime.

Surprisingly, energy-minimization under outage and latency requirements have not found much attention so far in related work. Especially regarding multi-hop networks, the typical assumption is that the sum transmit power is minimized while some end-to-end outage constraint needs to be fulfilled [3]–[5]. Such works typically show that transmit power can be saved if more hops are employed between a source and a destination. This implies, however, that the latency increases. As energy is the product between power and time, it is apparent that the implications for energy consumption under both outage and latency constraints are not clear from related work.

Hence, in this work, we study a relatively basic question: What is the minimum energy that it takes to transmit a packet of a certain size from a source to a destination such that on the one hand a certain end-to-end outage probability is not violated, while on the other hand the transmission also respects a given deadline. As mentioned, these parameters can become quite demanding in machine-to-machine type of applications for which high reliability levels have to be reached while meeting short deadlines. We study this question mostly with respect to multi-hop forwarding from a source to a destination. In this context, we study two different approaches: Forwarding the packet either based on average Channel State Information...
(CSI) or, instead, forwarding it based on instantaneous CSI. While in the first case the entire time span is available for payload transmission from source to destination, in the case of using instantaneous CSI we first have to obtain the channel states. This consumes time and transmit power (i.e., energy), but gains the nodes afterwards a big advantage for packet forwarding: The nodes are able to invest as much transmit power as necessary to achieve the QoS requirements, and therefore, avoid doing power over-provisioning and wasting energy. Based on convex optimization, we develop for both cases the optimal allocation of transmit power along a multi-hop route, and afterwards we numerically study the comparison between both schemes. We can show that the duration of the channel acquisition phase has a big impact on the energy consumption in case of exploiting instantaneous CSI. Moreover, for more demanding transmission scenarios (large packets, short deadlines, high reliability requirements) the energy savings from working with instantaneous CSI are quite large (up to a factor of 100). Finally, we numerically show that for a given distance between source and destination there is an optimal number of hops to use with respect to minimizing transmit energy. Using other than the optimal number of hops leads to a significant increase in the consumed energy.

Our work is structured as follows: In Sec. II we present the system model and problem statement, and summarize related work. In Sec. III we introduce the foundation of the power and energy minimization framework. Then we present evaluations of the derived analytical and numerical expressions in Sec. IV. Sec. V concludes the paper.

II. PRELIMINARIES

In this section, we first present the system model. Then, we give a more formal description of the problem we are interested in and at the end discuss related work.

A. System Model

We consider the transmission of packets of size $D$ from a source to a destination over a set of $n$ links $(n-1$ intermediate nodes). All nodes in the system are static. The packets might belong to a flow, however, we only focus on a single packet transmission. This transmission is constrained by QoS parameters. Namely, we define by $\mathcal{P}$ the required success probability of the transmission, and by $\mathcal{T}$ an associated deadline. A successfully transmitted packet implies that it reaches the destination within the time span $\mathcal{T}$ correctly. In all other cases (too late arrival, bit errors) an outage occurs.

In order to forward the packet, all transceivers use certain resources. First of all, as there are $n$ links in the multi-hop route, each node can utilize a specific, bounded time for forwarding the packet to the next node. We refer to this time unit as slot in the following. During its slot, node $i$ utilizes a transmit power of $P_i$ to forward the packet. Finally, all nodes utilize the same bandwidth of $B$ Hz. This spectrum is not subject to any external interference.

The major source of unreliability in the network stems from the random behavior of the wireless links along the path. Forwarding link $i \in \{1, \ldots, n\}$ (from node $i$ to $i+1$) is characterized by an instantaneous channel gain $h_i^2$. This instantaneous channel gain is composed of an average channel gain $\bar{h}_i^2$ as well as a random fading component. The average channel gain $h_i^2$ consists of a path loss factor and a random (but constant) shadowing factor. For the path loss we assume a straightforward model in which the gain is given by $d_i^{-\alpha}$ with $d_i$ being the distance between the transmitter and the receiver of link $i$. Link lengths $d_i$ are arbitrary and bridge a total distance $d$. For the shadowing component we assume a lognormal distribution which is parameterized by its standard deviation $\sigma_{SH}$. Finally, any instantaneous channel gain sample $h_i^2$ deviates from the average gain due to random small-scale fading. This fading is modeled by a stationary Rayleigh process such that the instantaneous Signal-to-Noise Ratio (SNR) $\gamma_i$ is an exponentially distributed random variable with mean

$$\mathbb{E} [\gamma_i] = \bar{\gamma}_i = P_i \cdot \bar{h}_i^2 / \sigma^2,$$

where $\sigma^2$ denotes the noise power. We assume a slowly varying block-fading process such that over the time span $\mathcal{T}$ the instantaneous channel gains remain constant.

Due to the fading, packet transmissions are potentially subject to errors. If the transmitter $i$ does not have information about the instantaneous channel state $\gamma_i$, we account for the transmission errors by a threshold error model [6]. Given the random SNR $\gamma_i$ the corresponding error-free transport capacity $c_i$ of the link is a random variable as well. Taking the slot duration $T_i$ into account, the instantaneous transport capacity is given by

$$c_i = T_i \cdot B \cdot \log_2 [1 + \gamma_i \beta].$$

Transport capacity represents the amount of data (in bits) that can be sent error-free for an SNR of $\gamma_i$ over the corresponding link and depends directly on the applied transmit power $P_i$. For the transmission of a packet of size $D$ the packet is lost on link $i$ whenever $c_i < D$. Based on the definition of transport capacity and the stochastic SNR model, we can derive the success probability $p_i$, which is the probability that the random transport capacity is bigger than the packet size $D$. To determine this, we first need to derive the Probability Density Function (PDF) of $c_i$ based on the exponential distribution of the link SNR. This can be obtained by straightforward PDF transformation as

$$f_{c_i}(x) = \frac{2x/(T_i \cdot B)}{T_i \cdot B \cdot \bar{\gamma}_i} \cdot \exp \left[ -\frac{2x/(T_i \cdot B)}{\bar{\gamma}_i} - 1 \right].$$

Given this characterization of random transport capacity, the

\footnote{The $\beta$ factor accounts for different modulation / coding types [3]–[8], that practical systems are able to use. This gap factor can be used to match Shannon rates to practical systems. For the sake of simplicity, throughout the paper we assume $\beta = 1$.}
success probability \( p_i \) is obtained as

\[
p_i = \Pr \{ \epsilon_i \geq D \} = \int_{-\infty}^{\infty} f_{\epsilon_i}(x) \, dx
\]

\[
= \exp \left[ -\frac{1}{\tau_i} \left( 2^{\frac{\epsilon_i}{\gamma}} - 1 \right) \right] = \exp \left[ -\frac{K[D_s T_s]}{P_i h_i^2} \right],
\] (3)

in which \( K[\cdot, \cdot] \) is a strictly positive scaling factor given as

\[
K[\delta, \tau] = \sigma^2 \left( 2^{\delta/(\tau B)} - 1 \right).
\]

B. Problem Statement

In this paper we are interested in fundamental insights how to transmit a packet over multiple hops in an optimal way. Optimality refers in the following to the total consumed power whereas the transmission is constrained by the QoS pair \( \{P, T\} \) of target success probability and deadline. As we consider the same slot duration \( T_s \) for every link along the path, the total energy \( E \) is directly given by

\[
E = P \cdot T_s,
\] (4)

where \( P = \sum_i P_i \) denotes the sum of the individual transmit powers. Energy necessary for packet reception (as well as idling) at each node will not be considered in our work as these values are strongly dependent on the implementation of the transceiver. However, note that such terms could be simply added to Eq. (4).

A key aspect in order to reduce energy consumption is the availability of channel state information at the nodes. By knowing the channel conditions, nodes are going to be able to adjust the exactly necessary transmit power. Hence, we consider two different approaches for which we are interested in minimizing the energy spent on packet transmission from a source to a destination. We assume that the source node possesses knowledge about all average channel states of the links along the route to the destination. Important in our consideration is that this information does not need to be acquired separately for each packet sent along the route, but is updated infrequently. Both considered approaches differ in the type of channel state information available to the nodes:

- **Average CSI Approach**: In this case, the source can determine transmit powers for all links only based on average channel states. The transmit powers have to be selected such that the required energy is minimized while satisfying the demanded end-to-end QoS constraints. Note that in this case all forwarding nodes have to perform significant power over-provisioning to cope with the random channel behavior. On the other hand, the entire time span up to the deadline can be dedicated exclusively to packet forwarding. We deal with energy minimization for this approach in Subsec. III-A.

- **Instantaneous CSI Approach**: In this case, all links first acquire the instantaneous channel states by exchanging control packets. Note that this consumes energy, as well as a certain time amount from the overall available time.

Once this information has been successfully acquired, the payload packet is sent from the source to the destination, adapting to the current channel states. We consider the energy minimization problem of this case in Subsec. III-B.

A part of the novelty of our work comes from the consideration of both end-to-end success probability \( P \) and deadline \( T \) as QoS parameters. Applications for which such a QoS model applies are typically multimedia streams. Moreover, in the upcoming domain of machine-to-machine communications, we also witness such applications. In comparison to multimedia flows, here the required reliability is usually much higher while the deadlines are quite short. This applies, for instance, to safety critical applications. A major motivation for our investigations is given by such scenarios.

C. Related Work

Optimizing the energy consumption of wireless networks has received significant attention over the last several years. In [9]–[12], the scenario setups differ from the one elaborated in this work.

In the context of multi-hop networks, only few works have been considering related issues. Efficient resource allocation schemes in wireless multi-hop networks are discussed in [3]–[5]. [3] considers energy-constrained multi-hop links subject to an end-to-end outage constraint. A closed-form expression for the minimum total transmit power is derived. The authors show an \( n \)-fold reduction of the total power for an \( n \)-hop route compared to a power distribution according to individual outage requirements. However, they do not consider an end-to-end deadline as QoS parameter nor do they include channel state information in the system model. In [4], the authors define an optimal power allocation scheme in a multi-hop amplify-and-forward system considering an end-to-end instantaneous SNR as target QoS requirement. The paper analyzes a QoS-aware multi-branch relaying power allocation problem which aims at minimizing the total power consumption of all transmission nodes. It demonstrates a gain of up to 5 dB when applying the optimal power allocation compared to an equal power allocation over all links. However, no end-to-end deadline is considered. Power allocation for regenerative and non-regenerative relayed systems is investigated in [5].

The authors derive a closed-form expression by means of convex optimization for the power per hop while minimizing the end-to-end outage probability subject to various power constraints. The results show that optimizing the power allocation is required for systems with highly unbalanced links or with a large number of hops. Thereby, a gain of up to 2 dB over an equal power allocation has been achieved, but again, not considering end-to-end deadlines nor instantaneous CSI. Finally, [13] investigates the minimization of end-to-end outage probability under different assumptions regarding the knowledge of the instantaneous channel state. It shows that optimal power allocation derived by means of convex optimization leads to significant enhancements in outage probability if instantaneous channel state information can be used.

This information might be available through a previous routing decision.
However, the authors do not consider the cost of obtaining those instantaneous channel states.

III. ENERGY MINIMIZATION FRAMEWORK

In this section we present details on how to minimize energy consumption for both transmission cases (average CSI and instantaneous CSI). Recall from Subsec. [II-A] that we deal with a QoS-constrained packet transmission over $n$ hops where the objective is to minimize the required transmission energy. The QoS constraints are composed of a minimum success probability $\mathcal{P}$ and an associated deadline $\mathcal{T}$. Furthermore, the nodes use knowledge about the average channel gain of each link to optimize the multi-hop transmission. This information is assumed to be available at the nodes. If instead the nodes utilize instantaneous CSI, they have to acquire it first. We start with the minimization of energy consumption for the average CSI case in Subsec. [III-A] before we present the energy minimization in case of utilizing instantaneous CSI in Subsec. [III-B].

A. Optimal Energy Consumption with Average CSI

If the nodes only utilize average channel state information, the entire available time can be spent on the transmission of the payload. However, each forwarding link will need to perform a significant over-provisioning of the transmit power (and hence the consumed energy) to account for potentially very bad link states. We obtain for the minimization of the energy the following theorem.

**Theorem 1:** In case of using only average channel state information, the minimum total energy $E$ required for a transmission of a packet of size $\mathcal{D}$ with probability of successful transmission $\mathcal{P}$ and time slots of length $T_i$ over $n$ hops is given as

$$E = -\frac{\sigma^2}{\ln \mathcal{P}} \left(2^{\mathcal{D}/(T_i \cdot \mathcal{B})} - 1\right) \left(\sum_{i=1}^{n} \frac{1}{h_i}\right)^2 \cdot T_c. \quad (5)$$

**Proof:** Since the average CSI knowledge is given a-priori, the entire deadline $\mathcal{T}$ is used for data transmission, resulting into equally long time slots of length $T_i = \frac{\mathcal{T}}{n}$ to be used at each link. Hence, Theorem 1 can be proven by considering the generalized power allocation problem in the Appendix and letting $k = 0$ in Eq. (17). Finally, the result is multiplied by $T_i$ to obtain the minimal transmit energy.

B. Optimal Energy Consumption with Instantaneous CSI

In contrast to using only average CSI for transmission, nodes might also send data along the route based on instantaneous CSI. If a node knows the actual channel state of its link, it can forward the data packet without transmit error due to the threshold error model introduced in Subsec. [II-A]. Therefore, each node along the route has to first acquire the instantaneous channel state. In the following, we account for this acquisition by a dedicated two-phase model. Prior to the actual data transmission, nodes exchange small control packets in order to estimate the current channel state in a two-way handshake fashion. The transmission of these control packets becomes now subject to the end-to-end success probability constraint. Due to the control packet exchange, the available time $\mathcal{T}$ to pass the payload data packet from source to destination is shortened. The two phases are given as follows:

- **Channel Acquisition Phase:** A control packet of size $\mathcal{D}_c$ is successively exchanged between every node $i$ and its successor based on average CSI. If channel reciprocity is given, the response control packet might already be sent with instantaneous CSI on its way back (mode $m = 1$); otherwise with average CSI as well (mode $m = 2$). In the first mode, sending the control packet with instantaneous CSI is assumed to cause no errors\(^3\) whereas in the latter mode, setting per-link success probabilities needs to respect the fact that the control packet traverses each link twice. This phase ends at time instance $T_c$, resulting into slot lengths of $T_i = \frac{T_c}{2n}$.

- **Payload Transmission Phase:** In the remaining time, $\mathcal{T} - T_c$, the data packet of size $\mathcal{D}$ is forwarded such that it always reaches the next hop reliably. This is possible as each node now holds the exact channel state information and can set the transmit power accordingly. The payload transmission phase is divided into equally sized time slots, as well.

We now proceed to determine the associated energy consumption of this scheme. Instantaneous channel gains $h_i^n$ allow a node to perfectly adapt to the current channel state by inverting the gains and transmitting with a modified power level. The required power level of node $i$ can be derived from Eq. (2). Hence, the total energy consumed during the payload transmission phase is given by

$$E_{\text{data,inst}} = K \left[ \mathcal{D}_c, \frac{\mathcal{T} - T_c}{n} \right] \sum_{i=1}^{n} \frac{1}{h_i^2} \cdot \frac{T_c - T - \mathcal{T}}{n}. \quad (6)$$

The minimum energy for transmitting a control packet with average CSI only one-way ($m = 1$) or both ways ($m = 2$) is calculated according to Eq. (5). The resulting energy is

$$E_{\text{c,avg}}(m) = -K \left[ \mathcal{D}_c, \frac{T_c}{2n} \right] \frac{m}{\ln \mathcal{P}} \left(\sum_{i=1}^{n} \frac{1}{h_i}\right)^2 \cdot \frac{T_c}{2n}. \quad (7)$$

In case control packets are sent with instantaneous CSI on their way back, the energy for this part is given as

$$E_{\text{c,inst}} = K \left[ \mathcal{D}_c, \frac{T_c}{2n} \right] \sum_{i=1}^{n} \frac{1}{h_i^2} \cdot \frac{T_c}{2n}. \quad (8)$$

For both above described operation modes, a further issue is how to divide the totally available time $\mathcal{T}$ into channel acquisition and payload transmission phase. The corresponding

\(^3\)Due to fading effects, knowledge about actual channel states might already be partially outdated once a node starts transmitting with an adjusted power level. This can be accounted for by another imperfection factor. Based on the channel coherence time and the respective $T_c$, a safety margin can be put on top of the SNR, e. g., 3 dB. As we will show in the evaluation section, utilizing instantaneous CSI is still advantageous for a wide range of parameters.
minimization problems can be stated as follows

\[
\begin{align*}
\arg\min_{0 < T_c < T} & \left( E_{c,\text{avg}}(m = 1) + E_{c,\text{inst}} + E_{\text{data,inst}} \right), \\
\arg\min_{0 < T_c < T} & \left( E_{c,\text{avg}}(m = 2) + E_{\text{data,inst}} \right),
\end{align*}
\]

Unfortunately, both problems can only be solved numerically, because they are inherently non-linear in \( T_c \). Furthermore, even if a closed-form expression that approximately resembles the analytical solution can be found, an evaluation is causally infeasible since determining the optimal length \( T_c \) of the channel acquisition phase would require a-priori knowledge of instantaneous CSIs, and thus, render the channel acquisition phase meaningless.

IV. NUMERICAL EVALUATION

In this section we evaluate the two different approaches against each other. We first give a brief overview of the applied methodology, and then present results regarding the energy consumption for several different parameter variations.

A. Methodology

Both approaches (instantaneous CSI and average CSI) are evaluated and compared against each other. In case of instantaneous CSI, a further important question to investigate on relates to the optimal length of the channel acquisition phase. Our primary metric of interest is the total energy consumption required to transmit a packet with respect to the QoS constraints. We consider the transmission of a packet of 2500 bit within a deadline of \( T = 10 \) ms requiring a high success probability of \( P = 1 - 10^{-5} \). Source and destination are separated by 20 m and the path consists of \( n = 3 \) links.

This is our reference scenario. More parameters are presented in Table I. Starting from the reference scenario, we vary several parameters sequentially to investigate on the total energy consumption. We thereby consider different packet sizes, deadlines and number of links in the path, as well as different bandwidths.

In case of employing average CSI we simply rely on numerical evaluations of the equations from Subsec. III-A. In case of applying instantaneous CSI, the evaluation is more involved as we have to account for the instantaneous channel states. Hence, we generate a set of 10,000 instances for the channel states along the route, calculate the energy consumption for the instantaneous values, add the share of energy of the channel acquisition phase (based on the derivations in Subsec. III-B), and finally aggregate everything by averaging over the instances. All our evaluations are done with MATLAB. Intervals for the 0.95 confidence level are plotted as green curves if their size exceeds 0.1% of the obtained average value.

B. Results

We start in Fig. 1 with a basic comparison of the approaches of average CSI and instantaneous CSI for the reference scenario. The figure shows the total required energy for packet transmission versus the choice of the channel acquisition phase duration \( T_c \). As can be seen, the average CSI approach (red dashed line) is insensitive to \( T_c \) selection since it does not involve obtaining instantaneous channel states. It ends up at around \( 7 \cdot 10^{-9}\) Joule. However, the duration of the channel acquisition phase has a significant impact on the energy consumption if instantaneous CSI (red solid and dash-dotted curves) is going to be used. Then, the energy consumption varies extensively, and reaches its minimum (marked with a cross in the graphs) at approximately \( 10^{-9}\) Joule, with the minimum achieved at an acquisition phase duration of about 7.5 ms (while the deadline \( T = 10 \) ms). Energy consumption can be further reduced if the channel states are reciprocal (red solid curve). In the remainder of the paper, this mode of the instantaneous CSI approach will be used as reference. Two remarks should be noted. First of all, acquiring CSI performs better than using average CSI by a factor of about 18. However, if the optimal duration of the channel acquisition phase is not respected, the relationship might easily turn around. Second, for the minimum energy consumption with instantaneous CSI the optimal duration of the channel acquisition is comparably long. This shows that most time is spent on reliably acquiring CSI and obtaining channel states dominates the total energy consumption for this approach. This can also be seen by referring to the black curves in Fig. 1. They depict the fractions of the reference scenario’s total energy that are spent on transmitting the control packet based on average CSI (black dash-dotted curve) in the forward direction, backwards based on instantaneous CSI (black dotted curve), and the payload transmitted with instantaneous CSI (dashed black curve).

Next, in Fig. 2 we study various modifications regarding the parameters of the reference scenario. The plot again shows

---

**Table I**

<table>
<thead>
<tr>
<th>Abbrev.</th>
<th>Explanation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( D )</td>
<td>Message size</td>
<td>2500 bit</td>
</tr>
<tr>
<td>( T_c )</td>
<td>Control packet size</td>
<td>250 + 8 \cdot \text{n bit}</td>
</tr>
<tr>
<td>( B )</td>
<td>Bandwidth</td>
<td>300 kHz</td>
</tr>
<tr>
<td>( d )</td>
<td>Total distance</td>
<td>20 m</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>Path loss coefficient</td>
<td>3</td>
</tr>
<tr>
<td>( \sigma_{\text{SH}} )</td>
<td>Shadowing variance</td>
<td>0 dB</td>
</tr>
</tbody>
</table>
total energy consumption versus the length of the channel acquisition phase. Hence, all horizontally dashed lines relate to the average CSI approach, whereas all curves with a minimum relate to the instantaneous CSI approach. We first study a payload size reduction to 300 bit (black curve). In this case, total energy consumption is in general lower as compared to the reference scenario (for the average CSI approach significantly). However, if the payload size is that small, it does not pay off to work with instantaneous CSI any more. Instead, at the optimal duration of the channel acquisition phase (which is close to 10 ms), the energy consumption is slightly worse than for the average CSI scheme. On the other hand, if we vary other parameters, e.g., decreasing the bandwidth (green curve), the achievable gain increases (factor of about 100).

A comprehensive list of gain factors (ratios of the average CSI approach to the minimum of the instantaneous CSI approach) for all plotted curves can be retrieved from Table II. Further observation of Fig. 2 reveals that the sensitivity of determining $T_c$ depends on system parameters. For some, the corresponding curve is V-shaped, resulting into a remarkable degradation in energy efficiency if the optimal duration of the channel acquisition phase is not determined; while for other parameters, the results are U-shaped, and hence, almost insensitive to the selection of the channel acquisition time over a wide range of values.

In Fig. 3 we consider different deadlines for the packet transmission. Again, the plot compares the total energy consumption against the length of the channel acquisition phase.

However, note that the scaling of the x-axis is different, as it shows the duration in percentage of the total packet deadline. The plot shows that as the deadline becomes more restrictive (green curve) the energy consumption increases, while the gain between the instantaneous CSI and the average CSI approach increases also (factor of 95). If we increase instead the deadline (black and cyan curves), for both schemes the total energy consumption decreases while the gains remain around a factor of 10. Interestingly, the consumption converges against a threshold for both approaches if the available deadlines $T$ are extended: The achieved power level reduction is exactly compensated by the longer transmission time. Considering the optimal channel acquisition phase duration, the percentages decrease and increase, respectively, from about 72% (for the reference scenario) to about 37% (if $T = 2$ ms) and 95% (if $T = 500$ ms). The reason for this is twofold. First, enlarging the time available for packet transmission with average CSI linearly, exponentially reduces the required energy (see Eq. 5). Since the energy budget of sending the control packets based on average CSI is the dominant part (compare black curves in Fig. 1), extending the channel acquisition phase tends to improve energy efficiency. Second, given a certain set of instantaneous channel states, the system’s only choice is to deal with the actual states by inverting gains. This may lead to high power levels. Reducing the associated energy can only be accomplished by transmitting for a diminishing time span. Consequently, sending the payload at an acceptable energy level requires a short, almost constant amount of time, regardless of the totally available time.

Furthermore, we want to look into the proper selection of number of intermediate (relaying) nodes to bridge a certain distance. In Fig. 4 the total energy is plotted against a varying number of links for both approaches (red curves) and for a different channel acquisition phase of $T_c = 0.75$ ms. As can be seen, mistakenly selecting too few or too many relay nodes may lead to increased energy consumption. The curves obey to a convex shape (proof omitted due to space constraints). The optimal number of intermediate nodes depends on the approach and on the scenario parameters.

As part of a larger numerical analysis, we show in Table III exemplary results regarding optimal $T_c$ values, as well as gain factors (ratios). As can be seen, different paths (characterized
by total path length, single path lengths, pathloss, shadowing) have no impact on the optimal duration of the channel acquisition phase $T_c$ and on the ratio that can be achieved by exploiting instantaneous CSI if $T_c$ is chosen optimally. However, note that absolute energy values may differ.

V. CONCLUSIONS

We have presented an approach which minimizes the transmit energy required to carry out a QoS constrained, multi-hop transmission. The time domain plays an important role in the process of energy minimization. The second investigated aspect covers the comparison of energy consumption in case of two approaches for determining the transmit power: Relying only on average CSI or first acquiring instantaneous CSI before the actual payload transmission takes place. As a first step, we derive a closed-form solution for a generalized power allocation problem that requires only knowledge of average channel states, and respects QoS requirements and a node power limit. Based on that, we formulate and numerically evaluate energy minimization problems providing insights into how much time should be spent on obtaining instantaneous CSI. We show that, depending on system parameters, gains of multiple magnitudes can be achieved in comparison to using average CSI. The duration of the channel acquisition phase has to be selected carefully to actually achieve a gain from utilizing instantaneous CSI. Although the solution for such optimal duration can only be obtained numerically, the scheme can be applied to real systems efficiently by storing proper durations in look-up tables since the optimum only depends on system parameters, but not on channel states. Furthermore, we show that for some scenarios conducting a transmission based solely on average CSI is outperforming a transmission scenario that requires instantaneous CSI, regardless of the length of the channel acquisition phase.

Results provided by the framework are a first step in the design for transmission devices fulfilling QoS demands. These systems may cover a wide spectrum from hard real-time message exchange to less constrained VoIP traffic. The applicability of the results to practical systems can be achieved by regarding several imperfection factors.

APPENDIX

GENERALIZED POWER OPTIMIZATION PROBLEM

In the following we first formulate the power allocation problem as a convex optimization problem and present afterwards a solution based on Lagrangian duality theory. We generalize the problem with respect to practical systems: The applicable transmit power per link is always upper bounded by a maximum $P_{\text{max}}$, due to technical limitations.

Without loss of generality, we assume that all $n$ links of the path are sorted in ascending order according to their channel gain, i.e., $h_1^2 \leq \ldots \leq h_i^2 \leq \ldots \leq h_n^2$. For the sake of clear notation, we simplify the $K[\cdot, \cdot]$ expression by dropping its arguments. We start with deriving the required transmit power $P_i$ for link $i$ to achieve a packet success probability of $p_i$ according to Eq. (3). Hence, the total transmit power along the path can be formulated depending on the per-link success probabilities as

$$P = \sum_{i=1}^{n} P_i = \sum_{i=1}^{n} \frac{-K}{h_i^2 \ln [p_i]}.$$  

Taking into account the maximum transmit power restriction per node, $P_{\text{max}}$, we obtain the following optimization problem

$$\text{minimize} \quad P_1, \ldots, P_n \quad \sum_{i=1}^{n} P_i$$  

subject to \quad $\prod_{i=1}^{n} p_i \geq \mathcal{P}$  

$$P_i \leq P_{\text{max}} \quad \forall i \in \{1, \ldots, n\}.$$  

Constraint (12a) preserves the end-to-end reliability $\mathcal{P}$. Any optimal solution to problem (12) will fulfill this constraint with equality.

**Theorem 2:** Problem (12) is feasible if and only if

$$\sum_{i=1}^{n} \frac{1}{h_i^2} \leq -\ln[\mathcal{P}] \frac{P_{\text{max}}}{K}.$$  

An optimal solution to the minimization problem is specified by the number of nodes $k$ operating at $P_{\text{max}}$, i.e.,

$$P_i^* = P_{\text{max}} \quad \forall i \in \{1, \ldots, k\}.$$  

For a given $k \in \{0, \ldots, n\}$, the optimal power assignment $P_i^*$ at the remaining nodes, which minimizes problem (12), is given by

$$P_i^* = \frac{c}{h_i} \sum_{j=k+1}^{n} \frac{1}{h_j} \quad \forall i \in \{k+1, \ldots, n\},$$  

### TABLE II

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Fluctuating links</th>
<th>Equal links</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$d_i \sim \text{exp}$</td>
<td>$d_i = d/n$</td>
</tr>
<tr>
<td></td>
<td>$\sigma_{\text{SH}} = 3, \text{dB}$</td>
<td>$\sigma_{\text{SH}} = 0, \text{dB}$</td>
</tr>
<tr>
<td>$T_c$ [ms]</td>
<td>Factor</td>
<td>$T_c$ [ms]</td>
</tr>
<tr>
<td>---------------------------</td>
<td>-------------------</td>
<td>-----------------------</td>
</tr>
<tr>
<td>Reference ($m = 1$)</td>
<td>7.26</td>
<td>17.7</td>
</tr>
<tr>
<td>Reference ($m = 2$)</td>
<td>7.70</td>
<td>4.5</td>
</tr>
<tr>
<td>$D = 300, \text{bit}$</td>
<td>9.65</td>
<td>0.96</td>
</tr>
<tr>
<td>$B = 50, \text{kHz}$</td>
<td>3.32</td>
<td>96.8</td>
</tr>
<tr>
<td>$n = 1, \text{link}$</td>
<td>8.09</td>
<td>11.9</td>
</tr>
<tr>
<td>$n = 15, \text{links}$</td>
<td>4.50</td>
<td>38.6</td>
</tr>
<tr>
<td>$T = 2, \text{ms}$</td>
<td>0.73</td>
<td>95.8</td>
</tr>
<tr>
<td>$T = 50, \text{ms}$</td>
<td>44.85</td>
<td>10.2</td>
</tr>
<tr>
<td>$T = 500, \text{ms}$</td>
<td>474.5</td>
<td>9.0</td>
</tr>
<tr>
<td>$d = 20,000, \text{m}$</td>
<td>7.26</td>
<td>17.6</td>
</tr>
<tr>
<td>$\alpha = 2$</td>
<td>7.25</td>
<td>17.7</td>
</tr>
<tr>
<td>$P = 1 - 1 \times 10^{-10}$</td>
<td>8.37</td>
<td>20.1</td>
</tr>
</tbody>
</table>
in which the (positive) constant $c$ is defined as
\[
c = -\left(\frac{\ln[P]}{K} + \frac{1}{P_{\text{max}}} \sum_{j=1}^{K} \frac{1}{h_j^2}\right)^{-1}.
\] (16)

A closed-form expression for the minimum total transmit power along the path, $P^*$, is given by
\[
P^* = kP_{\text{max}} + c \left(\sum_{i=k+1}^{n} \frac{1}{h_i}\right)^2
\]
\[
= kP_{\text{max}} + \frac{\left(\sum_{i=k+1}^{n} \frac{1}{h_i}\right)^2}{\sigma^2(2^{\beta/(\ln \beta)} - 1) - \frac{1}{P_{\text{max}}} \sum_{j=1}^{K} \frac{1}{h_j^2}}.
\] (17)

Proof: Problem (12) can be stated as a convex optimization problem. By taking the logarithm of Eq. (12) we obtain
\[
\text{minimize } \Pi \sum_{i=1}^{n} P_i
\]
subject to
\[
\Pi \sum_{i=1}^{n} \frac{-K}{h_i^2 P_i} \geq \ln[P]
\]
\[
P_i \leq P_{\text{max}} \quad \forall i \in \{1, \ldots, n\}. \tag{18b}
\]

The feasibility of the problem is easily checked by setting $P_i = P_{\text{max}}$ for all $i \in \{1, \ldots, n\}$ and evaluating Eq. (18a). This immediately yields Eq. (13), which ensures that the solution space is nonempty, guaranteeing the existence of an optimal solution. Otherwise, success probability $P$ cannot be achieved without violating the transmit power constraint $P_{\text{max}}$ in one or more links. For the remainder of this proof, we assume the problem to be feasible.

The convex formulation of the problem allows us to solve it with Lagrangian duality theory [14]. We introduce a Lagrangian multiplier $\mu \geq 0$ corresponding to constraint (18a) and multipliers $\lambda_1, \ldots, \lambda_n \geq 0$ corresponding to the set of constraints given by (18b). With $\lambda = (\lambda_1, \ldots, \lambda_n)$, the Lagrangian of problem (18) is defined as
\[
\mathcal{L} \{P_i\}, \lambda, \mu) = \sum_{i=1}^{n} P_i + \mu \left(\sum_{i=1}^{n} \frac{K}{h_i^2 P_i} + \ln[P]\right)
\]
\[
+ \sum_{i=1}^{n} \lambda_i (P_i - P_{\text{max}}).
\] (19)

Differentiating Eq. (19) with respect to $P_j$, $j = 1, \ldots, n$, yields the stationarity conditions
\[
\frac{\partial}{\partial P_j} \mathcal{L} \{P_i\}, \lambda, \mu) = (1 + \lambda_j) - \mu \frac{K}{h_j^2 P_j^2} \equiv 0.
\] (20)

For the variables $P_i$, $\lambda$, and $\mu$ to be primal and dual optimal, Eqs. (18a), (18b), and (20) have to hold. In addition, the complimentary slackness conditions
\[
\lambda_i (P_i - P_{\text{max}}) = 0 \quad \forall i \in \{1, \ldots, n\}
\] (21)

have to be fulfilled. In conclusion, the optimal solution to the optimization problem (18) is given by
\[
P_i^* = \min \left\{ \mu \frac{K}{h_i^2 P_{\text{max}}} \right\}
\] (22)

for all $i \in \{1, \ldots, n\}$, with the Lagrangian multiplier $\mu$ chosen such that Eq. (18a) holds with equality
\[
\sum_{i=1}^{n} \frac{-K}{h_i^2 P_i} \min \left\{ \mu \frac{K}{h_i^2 P_{\text{max}}} \right\}^2 = \ln[P].
\] (23)

Having found a value of $\mu$ that satisfies Eq. (23), we can directly derive the number of nodes $k$ transmitting with $P_{\text{max}}$
\[
k = \arg \max \left\{ \frac{h_i^2}{h_i^2} \geq P_{\text{max}} \right\}.
\] (24)

Once $k$ is computed, Eq. (14) immediately follows. In the next step, the success probabilities achieved on these $k$ links have to be deducted from $P$. Then for the $n-k$ remaining links, Eq. (23) can be simplified to
\[
\sum_{i=k+1}^{n} \frac{-K}{h_i \sqrt{\mu}} = \ln[P] - \sum_{j=1}^{k} \frac{-K}{h_j^2 P_{\text{max}}}.
\] (25)

Solving Eq. (25) with respect to $\mu$ and applying it to the unconstrained part of Eq. (22), yields Eq. (15). Finally, Eq. (17) is obtained by computing the sum of the transmit powers $P_i^*$ in Eqs. (14) and (15). This concludes the proof.

The problem of minimizing the summed transmit power can also be thought of as finding the optimal distribution of success probabilities along the path. This allows for an interpretation of the per-link success probabilities as assignable resources.

Lemma 1: Given the optimal distribution of transmit power according to Eq. (15), the corresponding success probabilities compute to
\[
p_i = \exp\left[-\frac{K}{h_i^2 P_i^*}\right].
\] (26)

REFERENCES


