Optimal Subsidies in the Competition between Private and State-Owned Enterprises*

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Abstract

Recent policy discussions have debated whether governments should treat state-owned and private enterprises equally or adopt different policies towards each type of enterprise. Such questions are pertinent for difficult economic climates in which government subsidy towards struggling state-owned enterprises seems natural, given their fundamental state-supported structure. However, should the government in turn also offer subsidies to the private sector, and how large should the subsidy be? We analyze this question in a mixed oligopoly setting, in which the government can award subsidies of different amounts to state-owned and competitive private enterprises, respectively. In a setting in which the state-owned enterprise seeks to maximize a weighted sum of social welfare and their own profits while private enterprises maximize their own profits, we find that the optimal subsidy policy is equal treatment of the different types of firms, regardless how much weight the state-owned enterprise puts on social welfare. The result suggests that equal subsidizing treatment of state-owned and private firms may be the socially efficient policy, regardless of the differences in objectives between the state-owned and private enterprises, as long as all firms share the same production technology. We show that our result is also robust to the functional form of production technology, the functional form of market demand, the composition of different types of firms in the market, and the heterogeneity of the objectives of firms. Finally, we show that heterogeneous cost structures among firms yields

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non-uniform optimal subsidy among the firms, and solve for the subsidy as a function of each firm's socially optimal production level.

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1 Introduction

State-owned enterprises have played a crucial role in China's economy and development, and they continue to hold substantial influence in a broad range of industries including financial, energy, metals, and transportation, among other sectors. While on an international level, foreign companies seek regulation of the government's support of state-owned firms out of concern for internationally competitive practices, another policy debate arises out of a domestic industrial concern.[1] Given that the government naturally tends to heavily subsidize state-owned enterprises during economic downturns, should the same economic aid be provided to private enterprises?[2]

This question is currently an intensely debated one, while China's government seeks to reassure private firms that they also have official support during slowing economic times. However, subsidizing of private firms not only expends government resources, but could distort the efficiency typically obtained in a competitive marketplace. Therefore, it is unclear whether the best policy of the government is one of fairness across firms, or of favorability to certain types of firms such as the state-owned enterprise.

Our main result demonstrates that even when the government has the choice to differentiate subsidies across firms in an industry, the optimal policy subsidizes the state-owned firm and all the private firms equally. The reason is that the efficiency gains from equalizing the playing field across firm types exceed the potential distortionary effect of the subsidy. This result suggests that even though a universal subsidy is costly to the government, it is theoretically preferable to a policy which targets only the state-owned firm.

Our paper contributes to the theoretical literature on mixed oligopoly and optimal policies (see DeFraja and Delbono, 1990 for a survey). One of commonly addressed topics is about the timing of privatization of mixed oligopolies and subsidy policies. Fjell and Heywood (2004) examine the optimal subsidy and welfare results in a case where privatization leads to either sequential or simultaneous move oligopoly. White (1996) examines the timing of subsidies in the privatization process. Poyago-Theotoky (2001) and Myles (2002) extend analysis of this question, finding an identical optimal subsidy under simultaneous or sequential moves, and extended to the case of different objectives of private firms by Kato and Tomaru (2007).

[1] For example, the G20 international business lobby has pressured the Chinese government to moderate favorable policies such as subsidy, debt relief and advantageous loans to state-owned enterprises; https://www.scmp.com/economy/china-economy/article/2167475/china-clashes-g20-business-lobby-group-over-support-state

[2] A recent example is the favorable loan given to state-owned automobile manufacturer FAW; https://www.scmp.com/economy/china-economy/article/2170253/chinas-state-owned-carmaker-gets-huge-lifeline-what-about

Other notable studies examine the mixed oligopoly question from different angles. Pal and White (1998) examine the effects of privatization of mixed oligopoly in an international trade setting. Heywood and Ye (2009) examine mixed oligopoly in a spatial price discrimination setting. Kato (2008) examines the government's privatization decision based on preferences over social welfare and tax revenue.

Finally, a subset of literature on mixed oligopoly examines the subsidy policy in a research and development setting. Lee and Tomaru (2017) find that the degree of privatization influences the optimal R&D tax, but not the optimal output subsidy. Haruna and Goel (2017) examine the optimal output subsidy in the case of research spillovers, finding that such subsidies may not attain efficiency.

Compared to the above studies, our model focuses on the static setting, and differs crucially in the ability of the government to assign heterogeneous subsidies across different firms. Prior literature has assumed a uniform subsidy across the entire industry, regardless of the ownership structure of the firm. Kato and Tomaru (2007) show that the optimal uniform subsidy is robust to heterogeneity in the weight of profit maximization in firms' objective functions. However, their analysis, like much of the prior literature, assumes a uniform subsidy exogenously. Our model by contrast, endogenizes the choice of subsidy level across firms, and shows that the uniform one is in fact optimal. In this sense, the theoretical contribution of our work can be seen as a generalization of the settings presented by Kato and Tomaru (2007), as well as the preceding studies Myles (2002), Povago-Theotoky (2001), and White (1996). However, in contrast to these studies, our paper does not focus on a privatization process.

We begin by analyzing a baseline model with linear demand, quadratic cost functions of firms, and n +1 firms, n of which are fully private and profit maximizing, and one of which is at least partially state-owned and therefore partially welfare-maximizing. Our main result in the baseline case shows that the optimal subsidy policy by the government is a uniform per-unit production cost subsidy across the two types of firms.

We then demonstrate the robustness of this result to generalizations in the model features, firstly any downward sloping demand function and secondly, any increasing and convex cost function. Additionally, in terms of market structure, we show that the uniform subsidy result is robust to both the number of partially state-owned firms, as well as heterogeneity in the objectives of those state-owned firms. Thus, the optimal uniform subsidy result is quite general and robust.

The optimality of the uniform subsidy however, is not robust to heterogeneity in firms' cost structures. The reason is that in the case of heterogeneous costs, the socially efficient production level differs for those firms with different costs, and therefore the optimal per-unit production subsidy is not the same across firms.

The remainder of the paper is organized as follows: Section 2 describes the benchmark model set-up; Section 3 describes the benchmark analysis and main results; Section 4 provides robustness results; Section 5 considers the case of heterogeneous production technology or costs; Section 6 concludes and discusses.

2 Baseline Model

There are $n(\geq 1)$ identical private firms ($i = 1, \cdots, n$) and 1 firm that is jointly owned by the public and private sectors (indexed by $i = 0$). Private firm i ($i = 1, \cdots, n$) maximizes its own profit π_i while firm 0 maximizes a weighted average of social welfare W and its own profit π_0, denoted as $u_0 = \alpha W + (1-\alpha)\pi_0$, where $\alpha \in [0, 1]$. It is common for state-owned enterprises to care to some extent about their profitability, which is represented by this weighted objective function. The public sector component is owned by the government, and maximizes social welfare W.[3] Note that in the extreme cases when $\alpha = 0$ firm 0 is simply a private firm while when $\alpha = 1$ firm 0 is a public firm fully owned by the government.

Firms compete in a market for a homogeneous good. The demand is linear, denoted by $Q = a - p$, where Q is total output and p is the market price. Note that $Q = \sum_{i=0}^{n} q_i$, where q_i is the output of firm i, $i = 0, \cdots, n$.

Following the literature (Poyago-Theotoky, 2001; Fjell and Heywood, 2004), we assume that all firms share the same producton technology with increasing marginal cost, denoted by $C(q_i) = c + \frac{1}{2}kq_i^2$, where $c \geq 0$, $k > 0$ and $i = 0, \cdots, n$. Since we do not focus on the firm entry issue, we let $c = 0$ without loss of generality.

We consider a two-stage game as follows. In stage 1, the government chooses the optimal output subsidy levels s_0 and s_1, where s_0 is the subsidy to firm 0 and s_1 is the subsidy to firm i, $i = 1, \cdots, n$. In stage 2, given the government's subsidy, firms compete simultaneously by chosing their own output level q_i, $i = 1, \cdots, n$.

Private firm i's profit is given by

$$\pi_i = q_i \left[a - \textstyle\sum_{i=0}^{n} q_i\right] - \frac{1}{2}kq_i^2 + s_1 q_i, \;\; i = 1, \cdots, n. \tag{1}$$

Firm 0's profit is given by

$$\pi_0 = q_0 \left[a - \textstyle\sum_{i=0}^{n} q_i\right] - \frac{1}{2}kq_0^2 + s_0 q_0. \tag{2}$$

The social welfare, defined as the sum of firms' profits and consumer surplus, is given by

[3] This weighting function also reflects the status of many state-owned enterprises in China, being partially state-owned and partially publicly traded.

$$W = \frac{1}{2}\left[\sum_{i=0}^{n} q_i\right]^2 + \sum_{i=0}^{n} \pi_i - s_0 q_0 - s_1 \sum_{i=1}^{n} q_i \quad (3)$$

$$= a\left[\sum_{i=0}^{n} q_i\right] - \frac{1}{2}\left[\sum_{i=0}^{n} q_i\right]^2 - \frac{1}{2}k\sum_{i=0}^{n} q_i^2. \quad (4)$$

3 Analyses and Results

The two-stage game is solved by backward induction. In stage 2, given the optimal subsidy levels s_0 and s_1, all firms ($i = 0, \cdots, n$) simultaneously choose their output levels q_i to maximize their respective objectives. The first order conditions are given by

$$\frac{d\pi_i}{dq_i} = a - \sum_{j=0}^{n} q_j - (k+1)q_i + s_1 = 0, \; i = 1, \cdots, n. \quad (5)$$

$$\frac{du_0}{dq_0} = \alpha\left[a - \sum_{j=0}^{n} q_j - kq_0\right] a + (1-\alpha)\left[a - \sum_{j=0}^{n} q_j - (k+1)q_0 + s_0\right] = 0 \quad (6)$$

Solving the above $n+1$ equations for q_i's, we obtain firms' outputs for the given subsidy levels (s_0, s_1):

$$q_0(s_0, s_1) = \frac{(k+1)a + (1-\alpha)(n+k+1)s_0 - ns_1}{(k+1)^2 + nk + (1-\alpha)(n+k+1)} \quad (7)$$

$$q_i(s_0, s_1) = \frac{(k+1-\alpha)a - (1-\alpha)s_0 + (k+2-\alpha)s_1}{(k+1)^2 + nk + (1-\alpha)(n+k+1)}, \; i = 1, \cdots, n. \quad (8)$$

Proposition 1 *Firm 0's output $q_0(s_0, s_1)$ is increasing in its subsidy s_0 and decreasing in private firms' subsidy s_1. Private firm i's output $q_i(s_0, s_1)$ is increasing in its subsidy s_1 and decreasing in firm 0' subsidy s_0.*

Note that setting $s_0 = s_1 = 0$ in $q_i(s_0, s_1)$ $(i = 0, \cdots, n)$ we obtain the standard result that firm 0's output exceeds the private firm's output, as $q_0(0,0) = \frac{(k+1)a}{(k+1)^2+nk+(1-\alpha)(n+k+1)} \geq \frac{(k+1-\alpha)a}{(k+1)^2+nk+(1-\alpha)(n+k+1)} = q_i(0,0)$.

In stage 1, the government, taking into account the firms' optimal output as a function of the subsidy, that is $q_i(s_0, s_1)$ $(i = 0, \cdots, n)$, maximizes the social welfare by choosing the optimal subsidy (s_0, s_1). The first order conditions imply the following two equations:

$$kq_0(s_0, s_1) + (1-\alpha)q_1(s_0, s_1) = (k+1-\alpha)s_1 \quad (9)$$

$$-k(n+k+1)q_0(s_0, s_1) + \left[(k+1)^2 + nk\right] q_1(s_0, s_1) = (k+1)s_1 \quad (10)$$

Note that both $q_0(s_0, s_1)$ and $q_1(s_0, s_1)$ are linear functions of s_0 and s_1, so we can solve for s_0 and s_1 by using the above two equations. The results are characterized by the following proposition.

Proposition 2 *When firm 0 is at least partially privatized ($\alpha \in [0,1)$), the optimal subsidy levels for all firms are the same: $s_0^* = s_1^* = s^* \equiv \frac{a}{n+k+1}$; When firm 0 is fully owned by the government ($\alpha = 1$), the optimal subsidy level for private firms is $s_1^* = \frac{a}{n+k+1}$ and the optimal subsidy level for firm 0 can be any number, that is $s_0^* \in R$.*

Note that although our equilibrium results are identical to the results in Poyago-Theotoky (2001), the key difference is that we endogenize the uniformity of the subsidy strategy, allowing for the government to potentially set different subsidy levels between firm 0 and other firms. Surprisingly, the optimal subsidy turns out to be the same for all firms in the case that the state-owned firm has even a minimal weight on its profitability in its objective function. In the special case that the state-owned firm is fully owned by the government, the subsidy for private firms is the same as in the general case, while any subsidy holds for the state-owned firm. The equilibrium result for output, price, profit and social welfare is given in the following corollary.

Corollary 1 *In equilibrium, all firms have the same output level $q^* = \frac{a}{n+k+1}$, the equilibrium price is $p^* = \frac{ka}{n+k+1}$, each firm's profit level is $\pi^* = \frac{(k+2)a^2}{2(n+k+1)^2}$, and the social welfare is $W^* = \frac{(n+1)a^2}{2(n+k+1)}$.*

An example of the equilibrium results for output, price, profit and social welfare is given below.

Example 1 *Suppose $k = 1, n = 1, a = 1$. The demand function becomes $Q = 1 - p$, and the cost function becomes $C(q_i) = \frac{1}{2}q_i^2$, $i = 0, \cdots, n$. The government's optimal subsidy will be $s^* = \frac{1}{3}$, the firm's output will be $q^* = \frac{1}{3}$, the market price will be $p^* = \frac{1}{3}$, the firm's profit will be $\pi^* = \frac{1}{6}$, and the social welfare will be $W^* = \frac{1}{3}$.*

The comparative statics results of the main equilibrium variables of interest with respect to market characteristics are provided as follows.

Corollary 2 *The equilibrium subsidy s_0^*, output q^*, and profit π^* are increasing in market size a, and decreasing in n and k; The equilibrium price p^* is increasing in a and k, and decreasing in n; The equilibrium social welfare W^* is increasing in a and n, and decreasing in k.*

The corollary states that market size has an increasing effect on the subsidy, while the number of firms and marginal cost parameters affect the subsidy negatively. Output and profits bear the same direction of comparative statics to the market characteristic variables as the optimal subsidy. Equilibrium price follows the intuitive comparative statics, increasing in market size and marginal cost parameter, while decreasing in the number of private firms. The social welfare result also follows the intuitive comparative statics, increasing with respect to market size and number of private firms, while decreasing in the marginal cost parameter.

4 Robust Extensions

4.1 General Production Technology

One may wonder whether the result of equal treatment in terms of optimal subsidy is due to the specific assumption of quadratic functional form for production technology. We show in this subsection that this is not the case. Assuming a general convex production function for all firms, denoted by $C(q_i)$, where $C'(q_i) > 0$, $C''(q_i) \geq 0$, and $C'(0) < a$, $i = 0, \cdots, n$, we can rewrite profit and social welfare as follows.

$$\pi_i = q_i \left[a - \sum_{j=0}^{n} q_j \right] - C(q_i) + s_1 q_i, \;\; i = 1, \cdots, n. \tag{11}$$

$$\pi_0 = q_0 \left[a - \sum_{j=0}^{n} q_j \right] - C(q_0) + s_0 q_0. \tag{12}$$

$$\begin{aligned} W &= \frac{1}{2} \left[\sum_{i=0}^{n} q_i \right]^2 + \sum_{i=0}^{n} \pi_i - s_0 q_0 - s_i \sum_{i=1}^{n} q_i & (13) \\ &= a \left[\sum_{i=0}^{n} q_i \right] - \frac{1}{2} \left[\sum_{i=0}^{n} q_i \right]^2 - \sum_{i=0}^{n} C(q_i). & (14) \end{aligned}$$

Similarly, the two-stage game is solved by backward induction. In stage 2, given the optimal subsidy levels s_0 and s_1, all firms simultaneously choose their output levels q_i to maximize their respective objectives. The first order conditions are given by

$$\begin{aligned} \frac{d\pi_i}{dq_i} &= a - \sum_{j=0}^{n} q_j - q_i - C'(q_i) + s_1 = 0, \;\; i = 1, \cdots, n. & (15) \\ \frac{du_0}{dq_0} &= \alpha \left[a - \sum_{j=0}^{n} q_j - C'(q_0) \right] + (1-\alpha) \left[a - \sum_{j=0}^{n} q_j - q_0 - C'(q_0) + s_0 \right] = 0 & (16) \end{aligned}$$

By rearranging the above $n+1$ equations, we have the following conditions:

$$\left[a - \sum_{j=0}^{n} q_j - C'(q_i) \right] + (s_1 - q_i) = 0, \;\; i = 1, \cdots, n. \tag{17}$$

$$\left[a - \sum_{j=0}^{n} q_j - C'(q_0) \right] + (1-\alpha)(s_0 - q_0) = 0 \tag{18}$$

Instead of following the standard backward induction method to solve for government's optimal subsidies and firms' equilibrium outputs, we now construct the equilibrium strategy directly.

First, note that $\frac{\partial W(q_0, \cdots, q_n)}{\partial q_i} = a - \sum_{j=0}^{n} q_j - C'(q_i)$ and $\frac{\partial^2 W(q_0, \cdots, q_n)}{\partial q_i^2} = -1 - C''(q_i) < 0$, for $i = 0, \cdots, n$. Therefore, the necessary and sufficient conditions for socially optimal output profile $(q_0, \cdots, q_n)$ are such that $a - \sum_{j=0}^{n} q_j - C'(q_i) = 0,\ i = 0, \cdots, n.$, implying $q_i = q^{O_1}, \forall i$, where q^{O_1} is uniquely determined by $a - (n+1)q^{O_1} - C'(q^{O_1}) = 0$.[4]

Second, note that if we set $s_0 = s_1 = q^{O_1}$, then $q_i = q^{O_1}, \forall i$ is a solution to the system of equations (17)-(18). This means when the government chooses the uniform subsidy policy $s_0 = s_1 = q^{O_1}$, the firms' equilibrium outputs are socially optimal. Since the government's objective is to maximize the social welfare, the best the government can achieve are the socially optimal outputs, therefore $s_0 = s_1 = q^{O_1}$ is an optimal subsidy policy for the government.

4.2 General Demand

In the baseline model, we assumed that the linear demand function is such that $Q = a - p$. We now further allow for the demand to take a more general form, and we denote the inverse demand function by $p(Q)$, where $p'(Q) < 0$, $p(0) > C'(0)$, and $\lim_{Q\to+\infty} p(Q) = 0$, $i = 0, \cdots, n$. Assuming convext production function $C(\cdot)$, we can write profit, consumer surplus, and social welfare as follows.

$$\pi_i = q_i p(\textstyle\sum_{j=0}^{n} q_j) - C(q_i) + s_1 q_i,\ i = 1, \cdots, n. \tag{19}$$

$$\pi_0 = q_0 p(\textstyle\sum_{j=0}^{n} q_j) - C(q_0) + s_0 q_0. \tag{20}$$

$$CS = \int_{t=0}^{\sum_{i=0}^{n} q_i} p\,(t)\, dt - (\textstyle\sum_{i=0}^{n} q_i) p(\sum_{i=0}^{n} q_i). \tag{21}$$

$$\begin{aligned} W &= CS + \textstyle\sum_{i=0}^{n} \pi_i - s_0 q_0 - s_1 \sum_{i=1}^{n} q_i & (22) \\ &= \int_{t=0}^{\sum_{i=0}^{n} q_i} p\,(t)\, dt - \textstyle\sum_{i=0}^{n} C(q_i). & (23) \end{aligned}$$

Similarly, the two-stage game is solved by backward induction. In stage 2, given the optimal subsidy levels s_0 and s_1, all firms simultaneously choose their output levels q_i to maximize their respective objectives. The first order conditions are given by

$$\begin{aligned} \frac{d\pi_i}{dq_i} &= p(\textstyle\sum_{j=0}^{n} q_j) + q_i p'(\sum_{j=0}^{n} q_j) - C'(q_i) + s_1 = 0,\ i = 1, \cdots, n. & (24) \\ \frac{du_0}{dq_0} &= \alpha \left[p(\textstyle\sum_{j=0}^{n} q_j) - C'(q_0)\right] + (1-\alpha)\left[p(\sum_{j=0}^{n} q_j) + q_0 p'(\sum_{j=0}^{n} q_j) - C'(q_0) + s_0\right] = 0 & (25) \end{aligned}$$

By rearranging the above $n + 1$ equations, we have the following conditions:

[4] Note that $a - (n+1)q$ is strictly decreasing in q and $C'(q)$ is weakly increasing in q, thus $f(q) \equiv a - (n+1)q - C'(q)$ is strictly decreasing in q. Since by assumption $f(0) = a - C'(0) > 0$ and $f(\frac{a}{n+1}) < 0$, we know by continuity of $f(\cdot)$ that $f(q) = 0$ has a unique solution $q^{O_1} \in (0, \frac{a}{n+1})$.

$$[p(\sum_{j=0}^{n} q_j) - C'(q_i)] + (s_1 + q_i p'(\sum_{j=0}^{n} q_j)) = 0, \;\; i = 1, \cdots, n. \tag{26}$$

$$[p(\sum_{j=0}^{n} q_j) - C'(q_0)] + (1-\alpha)(s_0 + q_0 p'(\sum_{j=0}^{n} q_j)) = 0 \tag{27}$$

Similar to the analysis in the previous subsection, note that the necessary and sufficient conditions for socially optimal output profile $(q_0, \cdots, q_n)$ are such that $p(\sum_{j=0}^{n} q_j) - C'(q_i) = 0, \;\; i = 0, \cdots, n.$, implying $q_i = q^{O_2}, \forall i$, where q^{O_2} is uniquely determined by $p((n+1)q^{O_2}) - C'(q^{O_2}) = 0$.[5]

Also note that if we set $s_0 = s_1 = -q^{O_2} p'((n+1)q^{O_2})$, then $q_i = q^{O_2}, \forall i$ is a solution to the system of equations (26)-(27). This means when the government chooses the uniform subsidy policy $s_0 = s_1 = -q^{O_2} p'((n+1)q^{O_2})$, the firms' equilibrium outputs are socially optimal. Since the government's objective is to maximize the social welfare, the best the government can achieve are the socially optimal outputs, therefore $s_0 = s_1 = -q^{O_2} p'((n+1)q^{O_2})$ is an optimal subsidy policy for the government.

4.3 General Heterogeneous Objectives

In the baseline model, we assume there are only two types of firms: private firm and (partially privatized) public firm. In this subsection, we further extend our model to allow for more than 2 types of firms. To be more specific, we allow for each firm i to have a different degree of privatization, $\alpha_i \in [0, 1]$, where $i = 0, \cdots, n$. The optimal subsidy profile will be $(s_0, \cdots, s_n)$, where s_i is the subsidy for firm i, $i = 0, \cdots, n$.

Assuming convex production function $C(\cdot)$ and general inverse demand function $p(\cdot)$, we can write firms' profit, consumer surplus, and social welfare as follows.

$$\pi_i = q_i p(\sum_{j=0}^{n} q_j) - C(q_i) + s_i q_i, \;\; i = 0, \cdots, n. \tag{28}$$

$$CS = \int_{t=0}^{\sum_{i=0}^{n} q_i} p(t)\, dt - (\sum_{i=0}^{n} q_i) p(\sum_{i=0}^{n} q_i). \tag{29}$$

$$\begin{aligned} W &= CS + \sum_{i=0}^{n} \pi_i - \sum_{i=0}^{n} s_i q_i & (30)\\ &= \int_{t=0}^{\sum_{i=0}^{n} q_i} p(t)\, dt - \sum_{i=0}^{n} C(q_i). & (31) \end{aligned}$$

[5] Note that $p((n+1)q)$ is strictly decreasing in q and $C'(q)$ is weakly increasing in q, thus $g(q) \equiv p((n+1)q) - C'(q)$ is strictly decreasing in q. Since by assumption $g(0) = p(0) - C'(0) > 0$ and $\lim_{q \to +\infty} p((n+1)q) - C'(q) < 0$, we know by continuity of $g(\cdot)$ that $g(q) = 0$ has a unique solution $q^{O_2} \in (0, +\infty)$.

$$u_i = \alpha_i W + (1-\alpha_i)\pi_i \quad (32)$$

$$= \alpha_i \int_{t=0}^{\sum_{j=0}^n q_j} p(t)\,dt - \alpha_i \sum_{j=0}^n C(q_i) + (1-\alpha_i) q_i p(\sum_{j=0}^n q_j) - (1-\alpha_i)C(q_i) + (1-\alpha_i)s_i q_i,\ \ i = 0, \cdots, n. \quad (33)$$

Similarly, the two-stage game is solved by backward induction. In stage 2, given the optimal subsidy levels $(s_0, \cdots, s_n)$, all firms simultaneously choose their output levels q_i to maximize their respective objectives. The first order conditions are given by

$$\frac{du_i}{dq_i} = [p(\sum_{j=0}^n q_j) - C'(q_i)] + (1-\alpha_i)(s_i + q_i p'(\sum_{j=0}^n q_j)) = 0,\ \ i = 0, \cdots, n. \quad (34)$$

Note that the necessary and sufficient conditions for socially optimal output profile $(q_0, \cdots, q_n)$ are $q_i = q^{O_2}, \forall i$, regardless the objective functions of different firms.

Also, it is easy to see that if we set $s_i = -q_i p'(\sum_{i=0}^n q_i) > 0$ $(i = 0, \cdots, n)$, the first order conditions for the Cournot competition will coincide with the first order conditions under social optimum. Therefore, an optimal subsidy should be uniform such that $s_i^* = -q^{O_2} p'((n+1)q^{O_2}), \forall i$, and social optimum is achieved under such a subsidy policy with output $q_i = q^{O_2}, \forall i$. Also note that if $\alpha_i = 1$, s_i^* can be any real number.

Theorem 1 *With homogeneous production technology $C(q_i)$ and inverse demand function $p(\cdot)$, assuming $C'(q_i) > 0$, $C''(q_i) \geq 0$, $p'(\cdot) < 0$, $p(0) > C'(0)$, $\lim_{Q \to +\infty} p(Q) = 0$, regardless firms' publicization levels $(\alpha_0, \cdots, \alpha_n) \in [0,1]^{n+1}$, $i = 0, \cdots, n$, under the optimal subsidy policy every firm has the same output level $q_i^* = q^{O_2}$, and the optimal subsidy is uniform such that*

$$s_i^* \begin{cases} = -q^{O_2} p'((n+1)q^{O_2}) & if\ \alpha_i \in [0,1) \\ \in R & if\ \alpha_i = 1 \end{cases}.$$

5 Optimal Subsidy under Heterogeneous Cost

Based on the analyses from previous sessions, one can see that uniform optimal subsidy result essentially relies on the assumption of homogeneous production technology among all firms. If the firms have different cost functions, the socially optimal output levels for different firms can be different, which will result in different optimal subsidy levels for different firms. We summarize this result in the following theorem and put the proof in the appendix.

Theorem 2 *With heterogeneous publicization level $\alpha_i \in [0,1]$, heterogeneous production technology $C_i(q_i)$ and inverse demand function $p(\cdot)$, assuming $C_i'(q_i) > 0$, $C_i''(q_i) \geq 0$,*

$p'(\cdot) < 0$, $p(0) > C_i'(0)$, $\lim_{Q\to+\infty} p(Q) = 0$, $i = 0, \cdots, n$, *firms' equilibrium outputs* $(q_0^*, \cdots, q_n^*)$ *are uniquely determined by*

$$p(\textstyle\sum_{j=0}^n q_j^*) - C_i'(q_i^*) = 0, \ \ i = 0, \cdots, n \ ;$$

and the government's optimal subsidies $(s_0^*, \cdots, s_n^*)$ *are such that*

$$s_i^* \begin{cases} = -q_i^* p'(\sum_{j=0}^n q_j^*) & \text{if } \alpha_i \in [0,1) \\ \in R & \text{if } \alpha_i = 1 \end{cases} .$$

As the theorem shows, the optimal subsidy is not uniform across firms. In particular, the subsidy for each firm is dependent on the socially optimal level of production for that firm, which in turn depends on that firm's cost function.

6 Concluding Remarks

During the current period of relative economic growth slowdown in comparison to previous decades in China, the government's policy of favorable subsidization of state-owned enterprises has been publicly challenged by both foreign and domestic private firms. Our study solves for the optimal subsidy distribution among private and state-owned firms, in a setting in which government can choose to differentiate the subsidies. Generalizing some aspects of previous studies on optimal subsidy for mixed oligopoly, mainly by endogenizing the choice of relative subsidy, we show that in fact the socially optimal policy is the uniform per-unit production subsidy across all firms.

We show that this result is robust to non-linear demand functions, any increasing and convex cost function, composition of state-owned and private firms in the market, and heterogeneity of state-owned emphasis on welfare maximization. We also analyze the case of heterogeneous production technology and show that under this situation the optimal subsidy is not uniform.

Although the result may be counterintuitive to the conventional wisdom on minimizing government interference in competitive markets, the efficiency result can be understood through the influence that the subsidies have on competitive incentives between state-owned and private firms. In other words, if the government is to provide a subsidy to the state-owned firm to assist it in its social welfare maximizing objective, the optimal policy involves an equal subsidy to the private firms, in order to maintain a competitive market environment. Since the government's objective is aligned with that of the state-owned enterprise, it will find this policy ideal for its own social welfare maximizing goals.

Our analysis raises an important insight in the management of economies with state-owned industrial components, which to our knowledge may not have been raised before. That is, conditional that the government would like to maintain a state-owned sector of the economy, it should also carefully consider the subsidy policy to the private sector in order to maintain proper strategic incentives between the different types of firms.

There are several directions for extension of this work. First, our current analysis has assumed simultaneous Cournot competition among the firms. However, in some realistic settings, either the state-owned firm or one of the private firms may be the Stackelberg leader. Also, we have assumed here that the government is not budget constrained in its allocation of subsidies, but is willing to implement the optimal subsidy scheme for social efficiency. Future work may consider some of the practical constraints that government may face in implementing the optimal policy.

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Appendix

Proof of Proposition 1:

Proposition 1: Firm 0's output $q_0(s_0, s_1)$ is increasing in its subsidy s_0 and decreasing in private firms' subsidy s_1. Private firm i's output $q_i(s_0, s_1)$ is increasing in its subsidy s_1 and decreasing in firm 0' subsidy s_0.

Proof. Given $q_0(s_0, s_1) = \frac{(k+1)a+(1-\alpha)(n+k+1)s_0-ns_1}{(k+1)^2+nk+(1-\alpha)(n+k+1)}$, we have

$$\begin{aligned}
\frac{\partial q_0(s_0, s_1)}{\partial s_0} &= \frac{(1-\alpha)(n+k+1)}{(k+1)^2+nk+(1-\alpha)(n+k+1)} > 0, \\
\frac{\partial q_0(s_0, s_1)}{\partial s_1} &= \frac{-n}{(k+1)^2+nk+(1-\alpha)(n+k+1)} < 0.
\end{aligned}$$

Given $q_i(s_0, s_1) = \frac{(k+1-\alpha)a-(1-\alpha)s_0+(k+2-\alpha)s_1}{(k+1)^2+nk+(1-\alpha)(n+k+1)}$, $i = 1, \cdots, n$, we have

$$\begin{aligned}
\frac{\partial q_i(s_0, s_1)}{\partial s_0} &= \frac{-(1-\alpha)}{(k+1)^2+nk+(1-\alpha)(n+k+1)} < 0, \\
\frac{\partial q_i(s_0, s_1)}{\partial s_1} &= \frac{(k+2-\alpha)}{(k+1)^2+nk+(1-\alpha)(n+k+1)} > 0.
\end{aligned}$$

■

Proof of Proposition 2:

Proposition 2: When firm 0 is at least partially privatized ($\alpha \in [0, 1)$), the optimal subsidy levels for all firms are the same: $s_0^* = s_1^* = s^* \equiv \frac{a}{n+k+1}$; When firm 0 is fully owned by the government ($\alpha = 1$), the optimal subsidy level for private firms is $s_1^* = \frac{a}{n+k+1}$ and the optimal subsidy level for firm 0 can be any number, that is $s_0^* \in R$.

Proof. Comibing equations (7)-(10), we obtain the following two equations:

$$\begin{aligned}
Bs_1 - Cs_0 &= Da, \\
Es_0 - Fs_1 &= Ga,
\end{aligned}$$

where

$$\begin{aligned}
B &= (k+1-\alpha)\left[(k+1)^2+nk+(1-\alpha)(n+k+1)\right]-(1-\alpha)(k+2-\alpha)+nk, \\
C &= (1-\alpha)\left[(n+k+1)k-(1-\alpha)\right], \\
D &= (1-\alpha)(k+1-\alpha)+k(k+1), \\
E &= (1-\alpha)\left[(n+k+1)^2k+(k+1)^2+nk\right], \\
F &= n\left[(n+k+1)k-(1-\alpha)\right], \\
G &= (1-\alpha)\left[(k+1)^2+nk\right]-nk.
\end{aligned}$$

Note that $B > 0, D > 0, F > 0, C \geq 0, E \geq 0$ and G can be positive or negative depending on the value of α.

When $\alpha = 1$, we have

$$\begin{aligned}
B &= k\left[(k+1)^2 + nk\right] + nk, \\
C &= 0, \\
D &= k(k+1), \\
E &= 0, \\
F &= n(n+k+1)k, \\
G &= -nk.
\end{aligned}$$

Thus, the optimal value of s_0 can be any number and we can solve for the optimal value of s_1 as follows:

$$s_1^* = \frac{Da}{B} = -\frac{G}{F} = \frac{a}{n+k+1}.$$

When $\alpha \in [0,1)$, the optimal values of s_0 and s_1 are uniquely determined by the following two equations:

$$\begin{aligned}
s_0^* &= \frac{DF+BG}{BE-CF}a, \\
s_1^* &= \frac{DE+CG}{BE-CF}a.
\end{aligned}$$

It is easy to show that

$$DF + BG = DE + CG = \frac{BE-CF}{n+k+1}.$$

Thus we have

$$s_0^* = s_1^* = \frac{a}{n+k+1}.$$

■

Proof of Theorem 2:

We can write firms' profit, consumer surplus, and social welfare as follows.

$$\pi_i = q_i p(\textstyle\sum_{j=0}^n q_j) - C_i(q_i) + s_i q_i, \;\; i = 0, \cdots, n. \tag{35}$$

$$CS = \int_{t=0}^{\sum_{i=0}^n q_i} p\,(t)\,dt - (\textstyle\sum_{i=0}^n q_i) p(\sum_{i=0}^n q_i). \tag{36}$$

$$\begin{aligned}
W &= CS + \textstyle\sum_{i=0}^n \pi_i - \sum_{i=0}^n s_i q_i &(37)\\
&= \int_{t=0}^{\sum_{i=0}^n q_i} p\,(t)\,dt - \textstyle\sum_{i=0}^n C_i(q_i). &(38)
\end{aligned}$$

$$u_i = \alpha_i W + (1-\alpha_i)\pi_i \tag{39}$$

$$= \alpha_i \int_{t=0}^{\sum_{j=0}^{n} q_j} p(t)\,dt - \alpha_i \sum_{j=0}^{n} C_i(q_i) + (1-\alpha_i) q_i p(\sum_{j=0}^{n} q_j) - (1-\alpha_i) C_i(q_i) + (1-\alpha_i) s_i q_i,\ \ i = 0,\cdots,n. \tag{40}$$

The two-stage game is solved by backward induction. In stage 2, given the optimal subsidy levels $(s_0, \cdots, s_n)$, all firms simultaneously choose their output levels q_i to maximize their respective objectives. The first order conditions are given by

$$\frac{du_i}{dq_i} = [p(\sum_{j=0}^{n} q_j) - C_i'(q_i)] + (1-\alpha_i)(s_i + q_i p'(\sum_{j=0}^{n} q_j)) = 0,\ \ i = 0,\cdots,n. \tag{41}$$

Note that $\frac{\partial W(q_0,\cdots,q_n)}{\partial q_i} = p(\sum_{j=0}^{n} q_j) - C_i'(q_i)$ and $\frac{\partial^2 W(q_0,\cdots,q_n)}{\partial q_i^2} = p'(\sum_{j=0}^{n} q_j) - C_i''(q_i) < 0$, for $i = 0,\cdots,n$. Therefore, the necessary and sufficient conditions for socially optimal output profile are

$$p(\sum_{j=0}^{n} q_j) - C_i'(q_i) = 0,\ \ i = 0,\cdots,n\ ,$$

from which the socially optimal output profile $(q_0^*, \cdots, q_n^*)$ are uniquely determined.

Also, it is easy to see that if we set $s_i = -q_i^* p'(\sum_{j=0}^{n} q_i^*) > 0$ $(i = 0,\cdots,n)$, then $(q_0^*, \cdots, q_n^*)$ is a solution to the system of equations (41). This means when the government chooses the uniform subsidy policy $s_i = -q_i^* p'(\sum_{j=0}^{n} q_i^*)$, the firms' equilibrium outputs are socially optimal. Since the government's objective is to maximize the social welfare, the best the government can achieve are the socially optimal outputs, therefore $s_i = -q_i^* p'(\sum_{j=0}^{n} q_i^*)$ is an optimal subsidy policy for the government.

Obviously if $\alpha_i = 1$, s_i^* can be any real number.