Fleet Assignment with Time Windows, Spacing Constraints and Time Dependent Profits

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Given the sets of flights and aircraft of an airline carrier, the fleet assignment problem consists in assigning the most profitable aircraft type to each flight. An extension to this problem is to allow for some flexibility on flight departure times. These time adjustments increase the number of possible connections and therefore improve the solution quality in terms of profits or in the reduction of the number of aircraft used to cover all flights. However some adjustments should be done with care. For instance, if the departure times of two flights serving the same origin-destination pair of cities become too close, passenger demands may be overestimated. Since passenger demands depend on departure times, flight profits may as well be inaccurate in case of a large flexibility. Therefore it is important to control not only the size of the time windows on flight departures but also the spacing out between flights serving the same O-D pair and the demand estimations.

In this paper we propose a model for the fleet assignment problem with time windows in which short spacings between certain flights are penalized while profit estimations integrate both time departure and aircraft type. We consider a periodic scheduling horizon.

1 Problem Description

A flight is characterized by a pair of origin-destination stations, an aircraft type and a time window on the departure time. Given a heterogeneous fleet and a scheduling horizon, we are looking for the best assignment of aircraft types to flights. Restrictions on time windows may apply for some aircraft, for example, to respect curfew imposed at a number of stations.

An aircraft itinerary is a sequence of flights between consecutive O-D stations for which
departure times within the given time windows allow for sufficient connection times. We assume here that the minimum connection time depends on the first flight and the aircraft type. In practice, it also depends on the second flight, specially for international carriers. Indeed a domestic flight followed by an international one (or vice versa) may require the towing of the aircraft, thus seriously increasing the minimum connection time.

To be feasible, the fleet assignment must be done in such a way that it is possible to construct the itineraries of all aircraft, each flight being covered by the selected type, and that these itineraries can be repeated cyclically over several scheduling horizons.

The optimal assignment maximizes the total anticipated profits taking into account the negative impact of short spacing between the time departures of certain flights as well as the fixed costs incurred on the use of the aircraft fleet. In this paper, anticipated profits of a flight depends not only on the aircraft type but also on the selected departure time.

2 Literature Review

Literature on the fleet assignment problem with a fixed schedule spans over several decades, the most recent papers being those of Subramanian et al. (1994) and Hane et al. (1995). Formulations and solution approaches are very similar. They rely on a mixed integer multi-commodity network flow formulation based on a time-space graph representation that is solved by classical branch-and-bound. Barnhart et al. (1998a) also solve the fixed schedule version but the authors introduce maintenance constraints in the model. A branch-and-bound approach is used to solve it. Each node of the search tree corresponds to the linear relaxation of a set partitioning problem solved by column generation (see Barnhart et al., 1998b), where the column generator is a shortest path problem. Columns in the set partitioning problem refers to feasible aircraft itineraries.

Desaulniers et al. (1997) introduce time windows on flight departures for the fleet assignment problem. The multi-commodity model now involves time variables. It is also solved by branch-and-bound and column generation except that the column generator is a time constraint shortest path problem, see Desrosiers et al. (1995). In Rexing et al. (2000), time windows are discretized, hence creating copies of each flight in the underlying graph representation. The column generator turns out to be a a shortest path problem on an acyclic graph.

The model proposed in this paper for the fleet assignment problem as described in the previous section follows the general vehicle routing and crew scheduling framework presented in Desaulniers et al. (1998). As above, multi-commodity flows, branch-and-bound and column generation are used, one difficulty being the generation of the columns, or equivalently, the feasible aircraft itineraries. This is done using a specialized time constrained shortest path problem involving time window restrictions and linear node cost functions to account for flight spacing constraints as well as time dependent profit estimations.

Ioachim et al. (1998) propose an efficient dynamic programming algorithm to solve that type of constrained shortest path problem. It has already been used in several applications, among them are aircraft routing with schedule synchronization (Ioachim et al., 1999) and simultaneous optimization of flight and pilot schedules in a recovery environment (Stojković and Soumis, 2000). In the former application, flights on certain O-D pairs must
be scheduled at the same time but on different days of a weekly horizon. In the later, small schedule perturbations keep aircraft itineraries the same but flight departure times are modified at a certain cost. These two applications and the one proposed in this paper have in common the fact that linear cost functions are associated with the time variables.

References


