Restoration of Images with Atmospheric Perturbation Simulation

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Abstract

We study model-based restoration of ground-to-ground images. Such images may show strong degradations due to non-linear atmospheric turbulence effects and therefore, any efficient restoration method must take into account some modelling of the turbulence. The first part of the paper is devoted to the realistic simulation of single frames and image sequences degraded by atmospheric turbulence at two different light wavelengths. The second part deals with the comparison of several restoration methods (inverse filtering, Tikhonov regularization, Laplacian regularization and Wiener filtering); a new method is also considered.

1. Introduction

Atmospheric turbulence bringing a major trouble to long distance observation, astronomers have developed techniques like deconvolution or adaptive optics for enhancing degraded images. These techniques are either expensive since they require specific technologies or are specific to vertical observation, from the ground to celestial objects. In the case of long distance ground-to-ground observation, the light has to go through thick atmosphere layers whose physics is rather different from vertical cuts. Physical modelling of the atmosphere may be a long and skill-demanding task. Any restoration technique using such a model would also require recording atmosphere parameters, which is unrealistic. Blind deconvolution, which does not require much knowledge upon the physical phenomena, may not be able to cope with highly nonlinear time-varying phenomena. In this paper, we are concerned with model-based restoration, a numerical model of the perturbation providing parameter values to a restoration technique.

The first part of this paper is devoted to the simulation of single frames and image sequences degraded by atmospheric turbulence at two different wavelengths. The second part deals with the comparison of different restoration methods and their suitability to such images. Conclusion and perspectives are given.

2. Perturbation simulation

2.1. The physical model

Roddier [1] computed degraded image by convoluting the initial image with a random turbulent wavefront. Only phase calculation is considered during the simulation of a degraded wavefront. Indeed the effects of amplitude variations correspond to weak scintillation and are negligible compared to those of phase variations. Dephasing creates effects of blurring, local shifting and warping.

In the case of horizontal propagation in the lowest atmosphere layer, the structure constant $C_N^2$ is approximately the same along the propagation path. $C_N^2$ is the amplitude of the refractive index fluctuations spectrum and it gives the influence of turbulence on optical propagation. It depends on local atmospheric conditions (like temperature, pressure, humidity rate, wind speed or sun rate), on the daily cycle and on the ground conditions. In this paper, a
continental ground is considered. $C_n^2$ varies in the range $10^{-17} \text{m}^{-2/3}$ in case of weak turbulence to $10^{-12} \text{m}^{-2/3}$ for strong turbulence. The typical mean value is about $10^{-14} \text{m}^{-2/3}$ [2].

In the literature atmospheric turbulence is often observed for a $D/r_0$ near 3. $D$ is the pupil diameter and $r_0$ is the Fried parameter [3], i.e. the diameter of the wave coherence area or the maximal diameter of the pupil surface for which the degraded wave can be considered as plane.

In this paper, $\lambda = 550 \text{nm}$, $L = 10 \text{km}$ and $C_n^2 = 10^{-15} \text{m}^{-2/3}$ for the visible light. According to Fried’s formula in the case of horizontal propagation [4]:

$$ r_0 = 3.02 \times \left( \frac{2\pi}{\lambda} \right)^{6/5} L^{-3/5} (C_n^2)^{-3/5}, $$

so $r_0 \approx 4 \text{ cm}$. A pupil whose diameter is about 12 cm would be well-adapted to observe atmospheric turbulence at this wavelength.

In the near infrared domain (3-5 $\mu$m) let us have $\lambda = 4 \mu$m, $L = 5 \text{ km}$ and $C_n^2 = 5.10^{-14} \text{m}^{-2/3}$, which gives $r_0 \approx 6.4 \text{ cm}$. A pupil whose diameter is about 20 cm would be well suited to observe atmospheric turbulence with such parameters.

For the sake of comparison, images with various $D/r_0$ have been computed, which is equivalent to vary the pupil size, the other parameters remaining unchanged.

### 2.2. Generation of a turbulent wavefront

Decomposition of the wavefront phase on the Zernike polynomials basis is a classical technique since these polynomials are often used in optics. Polynomials of low orders stand for the classical aberrations of an optical system.

Zernike polynomials are defined on a disk which radius is unity, in each point $r$ with polar coordinates $(r, \theta)$, by the following expressions:

$$ Z_{j,\text{pair}} = \begin{cases} \sqrt{n+1} R_n^m (r) \sqrt{2} \cos(m \theta) & m \text{ even} \\ \sqrt{n+1} R_n^m (r) \sqrt{2} \sin(m \theta) & m \text{ odd} \end{cases} $$

$$ Z_j = \sqrt{n+1} R_n^m (r) \quad m = j $$

with:

$$ R_n^m (r) = \sum_{s = \left\lfloor \frac{-n}{2} \right\rfloor}^{\left\lceil \frac{n}{2} \right\rceil} \frac{(-1)^s (n-s)!}{2^s s! [((n+m)/2-s)!((n-m)/2-s)!]^{1/2}} r^{n+m-2s}. $$

Each Zernike polynomial is the product of an angular function by a radial polynomial $R_n^m (r)$. One can either use the order number $j$ or the index $m$ and $n$ named respectively azimuthal frequency and radial order. These are integers with $m \leq n$ and $n-|m|$ even. The order number $j$ of a given polynomial is linked to $m$ and $n$ by the following relations:

$$ n = \left\lfloor \sqrt{8j^2 - 7j + 1}/2 \right\rfloor $$

$$ m = \begin{cases} 1 + 2 \left\lfloor (j - 1 - \left\lceil (n+1)/2 \right\rceil)/2 \right\rfloor & \text{odd} \\ 2 \left\lfloor (j - \left\lceil (n+1)/2 \right\rceil)/2 \right\rfloor & \text{even} \end{cases} $$

where $\lfloor x \rfloor$ is the integer part of $x$.

![Figure 1: Comparison of the first Zernike polynomials with optical aberrations](image)

In this example the first Zernike polynomials from $Z_0$ ($n=0$, $m=0$) to $Z_5$ ($n=5$, $m=5$) are shown. Some polynomials are compared to classical optical aberrations. The polynomial $Z_4$ is the one which has the highest influence on the image degradation. It takes the aspect of defocusing. The "piston" mode corresponds to the polynomial $Z_1$ which is constant.

The phase of a turbulent wavefront on a pupil with radius $R$ is given by:

$$ \varphi(r, \theta) = \sum_j a_j Z_j \left( \frac{r}{R}, \theta \right). $$

Coefficients are the projections of $\varphi$ on the basis functions:

$$ a_j = \frac{1}{S} \int \int P(r) \varphi(r) Z_j(r/R) \, dr, $$

where $S$ is the pupil area.
where $P(r)$ is the transmission function of the pupil with radius $R$ and surface $S$.

The simulation consists in randomly generating a set of coefficients such that the phase is compatible with Kolmogorov statistics (it is the case with atmospheric turbulence). In order to check that the simulated phase $\varphi(r)$ follows Kolmogorov statistics, one calculates the phase structure function which is the phase fluctuation variance between two points distant of $\rho$:

$$
D_\varphi(\rho) = \langle |\varphi(r+\rho) - \varphi(r)|^2 \rangle,
$$

and it is compared to the structure function phase [3]:

$$
D_\varphi(\rho) = 6.88 \left(\frac{\rho}{r_0}\right)^{5/3},
$$

where $\rho = |\rho|$.

Figure 2: Comparison between a simulated wavefront phase and a Kolmogorov phase

Zernike coefficients of an atmospheric wavefront can be considered as random Gaussian variables with zero mean [5]. Nevertheless, a wavefront cannot be directly simulated by only generating these coefficients: Zernike polynomials are not statistically independent [1]. Noll derived an expression of this covariance [5] which, in the Fourier domain, is a Zernike matrix representation of the Kolmogorov phase spectrum.

To obtain statistically independent random variables, one has to carry out the calculation in an orthonormal basis where vectors are fully decorrelated: we choose the basis formed by the Karhunen-Loeve polynomials. Although they have not an analytical expression, they can be decomposed onto Zernike basis like any wavefront which is simply done by diagonalizing the covariance matrix whose expression was given by Noll [5]. Coefficients can then be generated in this new basis by independent random drawings. Zernike coefficients are obtained by going back to the original basis, and thus respect the Kolmogorov model.

The simulated turbulent phase can be written as a decomposition on the Zernike polynomials basis where the pupil dimension is implicitly normalized compared with its radius. In this decomposition the “piston” mode (which corresponds to the mean phase) is excluded. The sum starts at order $j = 2$ up to a number $J-1$ of Zernike polynomials. In the rest of this paper $J = 135$. To satisfy Shannon’s criterion, a pupil of 64 pixels diameter was used.

Figure 3: Simulated turbulent wavefront

Once a wavefront of this kind is computed, one has just to perform convolution with the scene image to obtain the corresponding atmospheric degraded image.

2.3. Anisoplanatism or isoplanatism?

Atmospheric turbulence perturbations on optical beams vary according to these beams propagation direction. Consequently turbulence effects compensation methods (a posteriori restoration methods and adaptive optics methods) are limited to a reduced field angle named isoplanatic domain.

This limitation is due to anisoplanatism, i.e. the perturbation depends on the position in the observed object field since the beams propagating from different points in the field pass through different layers of the atmosphere [6] (Fig.4). The beams are subject to different perturbations: this is called anisoplanatism. The gray zone corresponds to the atmosphere area where the two beams are degraded in the same manner: it is the isoplanatic domain. It is very small compared to the whole propagation path.
2.4. Case of horizontal propagation

In the particular case of horizontal propagation along a path of length $L$, the structure constant $C_N^2$ is almost constant and according to Fried we have [4]:

$$r_0 = 3.02 \times \left( \frac{2\pi}{\lambda} \right)^{-6/5} L^{-3/5} \left( \frac{C_N^2}{\lambda^2} \right)^{-3/5},$$

$$\theta_0 = 0.95 \times \left( \frac{2\pi}{\lambda} \right)^{-6/5} L^{-3/5} \left( \frac{C_N^2}{\lambda^2} \right)^{-3/5},$$

where $r_0$ is the Fried parameter and $\theta_0$ is the maximal isoplanatic angle ($\theta_0$ can also be seen as the minimal angle from which we are in the case of anisoplanatism). Thus $r_0 = 3.18 L \theta_0$ according to the scheme in figure 5.

We are in the case of distant observation ($L$ is in the range 5-10 km according to the wavelength). For a $1024 \times 1024$ matrix camera, a pixel from a scene of 200 m $\times$ 200 m wide represents about 20 cm of the real scene. If $\alpha$ denotes the pixel resolution, then $\alpha \approx 20$ µrd. Using Fried's formula [4]:

- in the visible domain: $\theta_0 \approx 0.51$ µrd,
- in the infrared domain: $\theta_0 \approx 4.02$ µrd.

In both cases, $\alpha$ is larger than the isoplanatic angle $\theta_0$; we are obviously in the case of anisoplanatism.

2.5. Simulated images examples

In this section, short exposure visible and IR perturbation simulations are compared. Initial images are shown in fig. 6.

In the case of full anisoplanatism, each pixel is degraded by a different random wavefront. From a classical image in the visible domain, spatially and temporally decorrelated degradations have been simulated with different values for $D/r_0$.

In figure 7, one can observe a very important degradation as $D/r_0$ reaches 3. For $D/r_0 = 1$ the image becomes granular while foreseeing details on the background. For a ratio $D/r_0 = 0.1$ the granularity is much lighter and fine details becomes clearer. And for $D/r_0 = 0.01$ the granularity disappears and gives way to a slight blurring.

In the case of full isoplanatism, all pixels are degraded by the same random wavefront. From the same original image, degradations in full isoplanatism have been simulated with different ratios $D/r_0$. We can observe on these images (Fig. 8) a blurring and a global shifting more and more important while the ratio $D/r_0$ increases.

In the case of full isoplanatism, all pixels are degraded by the same random wavefront. From the same original image, degradations in full isoplanatism have been simulated with different ratios $D/r_0$. We can observe on these images (Fig. 8) a blurring and a global shifting more and more important while the ratio $D/r_0$ increases.

As in the visible domain, spatially and temporally decorrelated degradations due to atmospheric turbulence have been simulated on an original infrared image with various $D/r_0$. 
two men are particularly degraded. For $D/r_0 = 1$ we can also observe a strong granularity and details seem to be more difficult to identify in the infrared domain than in the visible domain. For $D/r_0 = 0.1$ there remains a slight granularity and details in the background appear; for $D/r_0 = 0.01$ the granular aspect vanished and gave way to a slight blurring.

From the same original infrared image, degradations in full isoplanatism have been simulated with different ratios $D/r_0$ for comparison.

Once more we can observe (Fig. 10) an increase of blurring and global shifting with the ratio $D/r_0$. For $D/r_0 = 3$ we have an important blurring implying a loss of details, but no granular aspect.

Due to the intrinsic definition of infrared images (less contrast and less details), there will be less degradation in the infrared domain. However we can observe the same kind of perturbation in the visible domain and on the observed objects in the infrared domain: especially a granular aspect for anisoplanatism, and blurring and global shifting for isoplanatism.

Figure 7: increasing random degradations (visible domain)

![Image of increasing random degradations](image)

Figure 8: increasing random degradations (visible domain)

![Image of increasing random degradations](image)

![Image of increasing random degradations](image)

Figure 9: increasing random degradations (infrared domain)

One can observe (Fig. 9) a very strong degradation for $D/r_0 = 3$. This infrared image has a dark and uniform background which permits us to observe the closeness between this image and an astronomical one. The bottom does not seem really degraded while the
Moreover, whatever the considered images set, the better ratio \( D/r_0 \) to represent fully developed atmospheric turbulence seems to be the one equal to 3 even if any degradation simulation with larger ratio \( D/r_0 \) is shown here.

A possible algorithm improvement in the case of local isoplanatism (i.e. between full anisoplanatism and full isoplanatism) would consist of replacing the global shifting by a local one (real situation) which would create local deformations in different directions. This would better fit the case where the pixel resolution is close to the isoplanatic angle.

3. Temporal evolution

In this section we are interested in simulation of temporal turbulence evolution. The exposure time is considered to be smaller than 5 ms, it is chosen small enough to “freeze” the turbulence. To have a long exposure image, it is enough to take the average image of a temporally correlated short exposure images set.

As in [7], short exposure image sequences have been simulated taking into account temporal fluctuations of atmospheric turbulence. The principle is to filter a set of temporally independent random coefficients \( a_j \) and to obtain another set whose power spectral density obeys the following equation [8]:

\[
\omega_j(\nu) = \frac{1}{V} \int_{-\infty}^{\infty} \text{M}_j(V, f_x, f_y) \text{W}_j(V, f_x, f_y) \, df_x,
\]

where \( \nu \) is the temporal frequency, \( f = (f_x, f_y) \) is the spatial frequency, \( V \) is the wind speed oriented along the \( x \) axis, \( w_\omega(\nu) \) is the temporal power spectral density of coefficient \( a_j \) at the order \( j \), \( \text{W}_j \) represents the phase spatial power spectrum, and for a Kolmogorov turbulence it is given by:

\[
\text{W}_j(f) = 0.023 \, r_0^{-5/3} \, f^{-11/3},
\]

and finally \( \text{M}_j(f) \) is the Fourier transform of the Zernike polynomial \( Z_j \) and its modulus is [8]:

\[
\text{M}_j(f) = \sqrt{n+1} \frac{2J_{n+1}(\lambda D)}{\pi df} \begin{cases} 
\sqrt{2} |\cos \theta| & j \text{ even and } m \neq 0, \\
\sqrt{2} |\sin \theta| & j \text{ odd and } m \neq 0,
\end{cases} \quad m = 0,
\]

\((J_l(x)) \) is the \( l^{th} \) kind Bessel function of order \( l \).

Let us give an example of temporal simulation with the following parameters:

- acquisition frequency: \( f = 200 \) fps
- exposure time: 5 ms
- wind speed: \( V = 6 \) m/s (about 22 km/h)
- propagation path length: \( L = 10 \) km
- Fried parameter: \( r_0 = 4 \) cm
- pupil diameter: \( D = 12 \) cm
- wavelength: \( \lambda = 550 \) nm

We can observe on fig. 8 that each signal forms a white noise. Random Zernike coefficients \( a_j \) have been generated independently by Roddier’s method [1], so they are completely temporally decorrelated. We can also note that the coefficients values interval decreases

![Figure 10: increasing random degradations (infrared domain)](image)

![Figure 12: 500 decorrelated random Zernike coefficients related to Z4, Z10 and Z20 (from top to bottom)](image)
while the Zernike polynomial order increases. This is due to the fact that the most influent polynomials are those at first orders. The more the order increases, the less influence the corresponding polynomial has, the nearest of zero the corresponding random coefficient absolute value is.

Figure 13: 500 random correlated Zernike coefficients related to Z4, Z10 and Z20 (from top to bottom)

When comparing this figure with the previous one, we can observe a clear temporal correlation between the random coefficients $a_j$. Their evolution is slower, then visually more representative of atmospheric turbulence. However we can note that the higher the Zernike polynomial order is, the less important the temporal correlation is. This is due to the fact that the correlation time decreases with the rising of radial order of polynomials [8].

Even if each frame of the sequence seems much degraded we can see that the average image is clearly less degraded. This is due to the fact that random coefficients follow a gaussian law in the Karhunen-Loève basis. Some of them are then cancelled during the averaging. Moreover the average image is a long exposure one: the perturbation does not look like granularity but rather like a strong blurring. Some details appear like the tower in the background.

Figure 14: 100 frames sequence in full anisoplanatism with $D/r_0 = 3$

4. Image Restoration

Four classical restoration techniques have been assessed: inverse filtering, Tikhonov regularization, Laplacian regularization and Wiener filtering.

These results clearly show the limited efficiency of these methods. The inverse filter is particularly unsuited due to its great sensibility to noise. The Tikhonov regularization gives the best PSNR but the edges are more visible with the Wiener filter in spite of a badly restored background.

We evaluated a new method [9] whose principle is to detect a local space-varying point spread function related to atmospheric turbulence. The PSF is found by the use of a Wiener filter acting on windowed regions-of-interest of a reference image and each image frame of the sequence. The reference image is initially the average of the sequence but is updated after each deconvolution pass of the complete sequence. The process is repeated until the difference between the two last average images is minimized (see result on fig. 15).

In spite of the presence of a subsisting blurring, this result is visually better than previous ones, which is confirmed by a higher PSNR. The strength of this method results from the fact that the average image is used, giving a good first approximation of the true image. Secondly the local Wiener filter enables to
better recover edges and details. For instance one can distinguish columns of the buildings on the back.

In the second part of this paper, different classical restoration methods have been tested: the inverse filter, the Tikhonov regularization, the Laplacian regularization, and the Wiener filter. These methods give poor results. A more recent method called “Windowed-Wiener” has been tested. One could include in this algorithm a readjusting step for sequences with camera or scene motion, which would give a better reference image.

![Inverse filter PSNR = 17.2 dB](image1)

![Tikhonov regularization PSNR = 18.7 dB](image2)

![Laplacian regularization PSNR = 18.6 dB](image3)

![Wiener filter PSNR = 17.8 dB](image4)

**Figure 14: restoration results**

![Windowed-Wiener result (PSNR = 21.6 dB)](image5)

**Figure 15: Windowed-Wiener result (PSNR = 21.6 dB)**

5. Conclusion

We studied simulation and restoration of atmospheric perturbation in the case of horizontal ground observation at a large distance. Physical characteristics of atmospheric turbulence were recalled. The simulation algorithm was explained and adapted to both the anisoplanatic and isoplanatic cases. Simulations have been made with various ratios \( D/r_0 \) at two wavelengths. Global remarks have been made and differences have been noted for the infrared case where less degradation appears. Moreover, image sequences have been simulated, taking into account temporal evolution of the atmospheric turbulence. The method to obtain such a sequence was explained and an example was given. We noted that the simulation algorithm could be improved by introducing spatial correlation for the case of local isoplanatism.

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