Suppressing Harmonic Powerline Interference Using Multiple-Notch Filtering Methods with Improved Transient Behavior

Jacek Piskorowski

Department of Electrical Engineering
West Pomeranian University of Technology, Szczecin
37 Gen. Sikorski St., 70-313 Szczecin, Poland
e-mail: jacek.piskorowski@zut.edu.pl

Abstract

In signal processing it is often required to have multiple-notch filters which simultaneously possess a very selective magnitude response and a transient response of short duration. Multiple-notch filters are usually used to remove the powerline hum and its harmonics. It is known, that sharper notches are obtained by either using more coefficients in FIR filters, or by placing the pole closer to the unit circle in IIR filters. However, increasing the filter selectivity also increases the duration of the transient process in the filter after the action of the excitation. This paper presents and compares two methods for suppressing the transient response in multiple-notch filters. As an example, these methods have been used to eliminate the powel ine noise from the ECG signal. Both filtering techniques are characterized by improved transient behavior compared to the traditional multiple-notch filters. In the first part of the paper a concept of digital IIR multiple-notch filters whose pole radius changes with time is proposed. Owing to a temporary change in the value of the pole radius, the transient can be considerably
reduced. In the second part of this review the technique which uses the vector projection to find better initial values for time-invariant multiple-notch filter is presented. Simulations verifying the effectiveness of the proposed IIR multiple-notch filter with time-varying pole radius and the filter with non-zero initial conditions are presented and compared to the performance of the traditional time-invariant filter using an ECG signal with unwanted fixed-frequency harmonic interferences as a study case.

**Keywords:** Multiple-notch filters, digital IIR filters, powerline interference, transient behavior, parameter-varying systems, ECG signal.

1. **Introduction**

   Notch filters find many applications in the field of signal processing, where one requires efficient estimation and elimination of particular frequency components or narrowband sinusoidal interferences from the input signal, leaving the nearby frequency components unchanged. Typical applications include rejection of the 50/60 Hz AC interference in low-voltage biomedical measurement signals [1, 2, 3], estimation of speech signals within noise [4], elimination of powerline interference on telephone cables under frequency-varying conditions [5], and canceling acoustic feedback oscillations in audio amplification systems [6]. Multiple-notch filters are used for the removal of multiple narrowband or multiple frequency interference, one popular application being in harmonic cancellation [7, 8].

   Notch filters can be constructed as either FIR or IIR filters. Generally, FIR notch filters cannot have very narrow bandwidth due to the fact that FIR filters are non-recursive filters. However, Deshpande et al. [9] demonstrated
that it is possible to construct highly narrow rejection bandwidth FIR notch filters using an IIR filter as a prototype. IIR filters are recursive filters. The configuration of the pole positions enables IIR filters to perform bandpass or notch filtering with very narrow bandwidths.

Digital notch filters come in a variety of different forms that can be used for various purposes and that achieve different results. Fixed notch filters are set to a given frequency and filter the signals at that frequency. Tunable notch filters [10] are similar to the fixed notch filters as they are set to a given frequency. However, tunable notch filters have a range of frequencies that they can be set to and then fixed at that frequency. On the other hand, adaptive notch filters (ANF) [1, 3, 11, 12, 13, 14, 15] are used in situations when the characteristics of processed signals are variable in frequency and depend on events over time. Such filters have time variant coefficients which are updated continuously by an optimization criterion. A detailed characterization of various adaptive notch filtering methods can be found in [16].

Signal processing by traditional digital filtering techniques brings some problems, such as transient response at the beginning of the signal. Its duration depends mainly on the filter order, the cutoff frequency, and the approximation method of the magnitude response. This causes problems when particularly short signals are filtered or when the initial part of a processed signal is of great importance. In this case, the useful signal can be considerably distorted owing to the transient response or it can be entirely lost in the transient.

Many applications require notch filters which simultaneously possess a very selective magnitude response and a transient response of short dura-
tion. In the design of these systems, however, selective magnitude response and transient response of short duration are design specifications which are contradictory to each other and, therefore, are difficult to tune simultaneously. In the literature one can find only a few works which examine the problem of the reduction of the transient duration in digital IIR notch filters. In [17], Pei and Tseng present a technique for suppressing the transient states of IIR notch filters, which uses the vector projection to find better initial values for time-invariant notch filters. When a notch or multiple-notch filter is used to eliminate the powerline interference in the recording of ECG signals, the performance of the notch filter with transient suppression is better than that of the conventional notch filter with arbitrary initial conditions. The improvements with this technique are at the cost of additional computation load at the beginning of filtering. Another technique to reduce the transient response of notch filters has been proposed in [18, 19]. In these papers single-notch filters with zero initial conditions and time-varying quality factor have been considered.

The simultaneous improvement of the filter properties in the time and frequency domains is not possible, taking into account traditional time-invariant notch filters. However, it is possible to attain a significant reduction of the transient response duration of a notch filter to a given input signal by varying its pole radius with time. One of the aim of this paper is to design a parameter-varying IIR multiple-notch filter which will be able to eliminate given harmonic interfering signals with fixed frequencies as fast as possible without deteriorating in the long term its selectivity characteristics, as demonstrated by Pei and Tseng [17]. The improvement attained by this class
of filters is based on a temporary change in the value of their pole radius \( r \).

The technique proposed to accelerate the response of the filter has little to do with the philosophy behind adaptive filtering (adjustment of the parameters of the filter by means of optimization techniques to obtain a particular behaviour at the filter output). Instead, the technique used for parameter variation considered to optimize the transient behaviour of the parameter-varying filters is already based on a predefined control rule which will be applied whenever the response of the filter is expected to show a distortion due to its own transient behaviour. However, on the other hand, most of the adaptive notch filtering algorithms use a time-varying function of the pole radius in order to increase the robustness of the (adaptive) algorithm and to improve its convergence rate [12, 15].

The strategy proposed for the variation of the filter parameters was used previously in the past with some modifications in a number of applications, mainly in the analog technique. In [20], a concept of \( Q \)-varying analog continuous-time single-notch filter with improved transient response has been presented. Moreover, in [21], a parameter-varying lowpass filter was used to eliminate the oscillatory response exhibited by load cells used in weighing applications. Another parameter-varying filter was used in [22] to reduce the time employed in the acquisition of evoked potentials generated through auditive stimuli. In addition, the parameter-varying technique was also used in [23, 24] to reduce the transient of lowpass filters with compensated group delay response.

The rest of this paper is organized as follows: in Section 2, digital IIR multiple-notch filters are discussed in more detail. The idea of the IIR
multiple-notch filter with time-varying pole radius is described in Section 3. In Section 4, examples of using the proposed multiple-notch digital filter to filtering an ECG signal with unwanted fixed-frequency harmonic interferences are presented. The comparison of the proposed parameter-varying multiple-notch filter with the suppression technique proposed by Pei and Tseng is shown in Section 5. Then, in Section 6, the quantitative error measure to evaluate the performance of the proposed filtering method is presented. Finally, some concluding remarks are given in Section 7.

2. Digital IIR multiple-notch filters

Generally, the input of the multiple-notch filter has the following form

\[ x(n) = s(n) + \sum_{k=1}^{M} A_k \sin(n \Omega_{N_k} + \phi_k) = s(n) + d(n) \]  

(1)

where \( s(n) \) is a desired signal and \( d(n) \) is a sum of sinusoidal interference signals with frequencies \( \Omega_{N_k} \in (0, \pi) \), phase shifts \( \phi_k \), and amplitudes \( A_k \) for \( k = 1, \ldots, M \), where \( M \) is the number of sinusoidal interference signals. In order to extract \( s(n) \) from the corrupted signal \( x(n) \) without distortion, the specification of an ideal multiple-notch filter is given by

\[ |H(e^{j\Omega})| = \begin{cases} 
0 & \text{for } \Omega = \Omega_{N_k}, \ k = 1, \ldots, M \\
1 & \text{for } \Omega \neq \Omega_{N_k} 
\end{cases} \]  

(2)

where \( \Omega_{N_k} = 2\pi f_{N_k}/f_s \) \((k = 1, \ldots, M)\) are the \( M \) specified digital notch frequencies, \( f_{N_k} \) are the analog notch frequencies, and \( f_s \) is the sampling frequency.

There are many methods that can be used to design IIR multiple-notch filters. One of the simplest and most useful techniques for designing notch
filters is the pole/zero placement method. The location of the poles and zeros of the filter determine the behavior of the filter. The pole/zero placement design procedure is based on a radial representation of the filter transfer function. For multiple-notch filters the zeros are placed on the unit circle at the prescribed notch frequencies $\Omega_{N_k}$, i.e. $z_k = e^{\pm j\Omega_{N_k}}$ and the poles, at the same frequencies, very close to the unit circle, i.e $p_k = re^{\pm j\Omega_{N_k}}$, where $r$ is the pole radius. For stability, it is required that $r < 1$. Of course, the closer $r$ is to unity, the sharper are the notches and the more constant is the response at the other frequencies.

The transfer function of a multiple-notch filter with $M$ notches can be written in the following form

$$H(z) = \prod_{k=1}^{M} \frac{1 - 2 \cos \Omega_{N_k} z^{-1} + z^{-2}}{1 - 2r \cos \Omega_{N_k} z^{-1} + r^2 z^{-2}} = \frac{B(z)}{B(r^{-1}z)}$$

(3)

where $B(z) = \sum_{k=0}^{2M} b_k z^{-k}$ is a symmetrical polynomial, that is $b_0 = b_{2M} = 1$ and $b_{2M-k} = b_k$ ($k = 0, \ldots, M - 1$). Given the notch frequencies $\Omega_{N_k}$ ($k = 1, \ldots, M$), the coefficients $b_k$ can be computed by using a recursive relationship [25].

As previously mentioned, the higher the value of the pole radius $r$, the narrower the bandwidth and the greater the selectivity of the filter. However, as the pole radius is increased, the transient duration of the filter is also increased. This phenomenon is particularly noticeable at low frequencies, where the transient process duration can take as long as several seconds. Fig. 1 demonstrates how the value of the pole radius influences the frequency response and the transient response of the single-notch filter.

In some situations, a relatively small pole radius in the multiple-notch
Figure 1: Frequency response (a) and transient response (b) of the single-notch filter for various values of the pole radius $r$. 
filter can be used, which, as a result, gives the transient at an acceptable level. However, for the small pole radius the amplitude of frequency components close to the unwanted notch frequency will be affected by the filter to some extent. Therefore, if the notch frequencies are close or inside the useful signal frequency spectrum, the filter with relatively high pole radius should be applied.

3. IIR multiple-notch filter with time-varying pole radius

In order to improve the time domain response of the multiple-notch filter, it was assumed that its pole radius $r$ is varied in time. The IIR multiple-notch filter with time-varying pole radius may be mathematically represented by the following time-varying difference equation:

$$y(n) = b_0x(n) + b_1x(n-1) + \ldots + b_{2M}x(n-2M) - r(n)b_1y(n-1) - \ldots - r^{2M}(n)b_{2M}y(n-2M).$$  \hspace{1cm} (4)

In equation (4), $x(n)$ and $y(n)$ are the input and output of the filter, respectively, $b_0, \ldots, b_{2M}$ are the filter coefficients and $M$ is the number of notches. Function $r(n)$ defines the time-varying radius of the poles corresponding to the notch frequencies. It should be noticed that the notch frequencies $\Omega_{N_k}$, included in coefficients $b_1, \ldots, b_{2M-1}$, are not time-varying since it is important to preserve the damping of the same notch frequencies throughout the ongoing filtering processes.

It is well known that for smaller values of the pole radius, the duration of the transient behavior of the multiple-notch filter is diminished. If this rule is taken as a departure point, it may be concluded that in order to improve the
dynamic behavior of the multiple-notch filter, a temporary decrease of the pole radius has to take place when the filter is expected to display transient behavior at its output. By analogy to the previous works [23, 24] devoted to the parameter-varying continuous-time filters, the function responsible for the variation of the pole radius has been formulated as follows:

\[
r(n) = \overline{r} \cdot \left[ 1 + (d_r - 1) \cdot \exp \left( -\frac{n}{v_{fs}} \right) \right], \quad n \geq 0
\]

where \( \overline{r} = \lim_{n \to \infty} r(n) \) is the pole radius which comes from the multiple-notch filter approximation. Parameter \( v \) can be denoted as the exponential variation rate of function \( r(n) \). The coefficient \( d_r \) defines the variation range of the function \( r(n) \). This parameter is given by:

\[
d_r = \frac{r(0)}{\overline{r}}
\]

and it is always smaller than unity since \( r(0) \) is smaller than \( \overline{r} \).

In most of the ANF algorithms the pole radius is also a time-varying function [12, 15]. The reason is that the pole radius determines the bandwidth of the filter notches. Practically, if no \textit{a priori} information is available on the input noise, when the notches are too narrow, the adaptive algorithm may not converge. On the other hand, a larger pole radius will lead to less excess mean square error (MSE) after convergence. Therefore an exponential function is often used for \( r(n) \) by letting \( r \) grow from an initial value \( r(0) \) to the desired value \( \overline{r} \).

One of the important problems of the synthesis of the multiple-notch filter with time-varying pole radius is to evaluate the parameters of the function \( r(n) \). As it has been mentioned before, the function of \( r \) is characterized by variation range \( d_r \) and variation rate \( v \).
So as not to alter the transfer characteristics of the multiple-notch filter when its transient behavior has to be reduced in duration, function (5) should settle to the value \( r \) during the time interval \([0, t_s]\). Parameter \( t_s \) stands for the settling time of the original time-invariant filter for an assumed accuracy factor \( \alpha \).

The dynamics of the multiple-notch filter may be improved when the settling time \( t_{sf} \) of the function (5) will approximately equal the settling time of the original time-invariant filter \( t_s \), i.e.

\[
t_{sf} \approx t_s
\]

The settling time is measured with the defined maximum permissible error \( \alpha \). For an error of 2\%, \( \alpha = 0.02 \). From this last relation it is possible to select the exponential variation rate \( v \) of the function \( r(n) \).

Suitable value for \( d_r \) must be determined via computer simulations because a complete analytical solution of the difference equation given in (4) is unfortunately not available. The search strategy for this value is described below. First, an interval \([d_{r_{\text{min}}}, 1]\) must be proposed in which the optimal value of \( d_r \) will be searched. Using \( d_r = d_{r_{\text{min}}} \), the exponential variation rate \( v \) associated with this value should be calculated. In this step it is assumed that the parameter \( \alpha \) is already known and given as a design specification. Once a space search has been defined, the optimal value of \( d_r \) is selected by means of an optimization strategy in which the total efficiency factor \( \eta \) has to be minimized. The efficiency factor \( \eta \) is expressed as the product of the time efficiency factor \( \eta_t \) and the frequency efficiency factor \( \eta_{BW} \)

\[
\eta = \eta_t \cdot \eta_{BW}
\]
The time efficiency factor $\eta_t$ is defined as the ratio of the settling time $\tilde{t}_s$ of the multiple notch filter with varying pole radius and the settling time $\bar{t}_s$ of the original time-invariant filter. On the other hand, the frequency efficiency factor $\eta_{BW}$ stands for the ratio of the 3 dB notch bandwidth $BW$ of the original time-invariant filter and the 3 dB notch bandwidth $BW_0$ of the time-invariant filter which corresponds to the parameter-varying filter for $t = 0$

$$\eta_t = \frac{\tilde{t}_s}{\bar{t}_s}, \quad \eta_{BW} = \frac{BW}{BW_0}. \quad (9)$$

Therefore, the optimal value for the variation range $d_r$ is found for maximum $\eta$. A higher value of the efficiency factor $\eta$ implies that there has been an improvement in the transient behavior of the designed parameter-varying filter.

4. Simulation Results

In this section, as an example the proposed IIR multiple-notch filter with time-varying pole radius will be used to remove the 60 Hz interference and its harmonics from an ECG signal.

4.1. Double-Notch Filter

Firstly, let us consider the ECG signal distorted by the 60 Hz power line interference and its 120 Hz harmonic. Fig. 2 presents the clean ECG signal (a) and its distorted version (b) taken from the MIT-BIH ECG signals database. The distortion is artificially added to the ECG signal which is known in advance. In order to reject the powerline distortions the double-notch filter with time-varying pole radius will be applied. The notch frequencies of the
filter are set to 60 Hz and 120 Hz. The sampling frequency $f_s$ and the filter pole radius $r$ have been assumed to be 1 kHz and 0.98, respectively, whereas the parameters for function (5) have been chosen as follows: $d_r = 0.9$ and $v = 2.0$. The amplitude response for the time-invariant prototype of the considered double-notch filter is presented in Fig. 3.

Fig. 4 presents the comparison of responses to the notch frequencies for the double-notch filter with time-invariant and time-varying pole radius. This figure demonstrates respectively the response for the 60 Hz powerline interference frequency, for 120 Hz harmonic, and for the sum of these distortions. It is evident that the double-notch filter with time-varying pole radius is able to suppress the notch frequencies considerably faster than the traditional filter with time-invariant pole radius.

Fig. 5 presents the effectiveness of the proposed double-notch filter in the removal of the powerline distortions from the ECG signal. It is clearly evident that the transient duration of the proposed filter is considerably reduced and the selectivity of the filter is at an acceptable level. According to [17], functionality analysis of the proposed filter has been also carried out for various beginnings of the ECG signal. It has been assumed that the ECG signal begins at a flat segment, the T wave, and the QRS complex. The results of simulations of the proposed time-varying double-notch filter and the traditional time-invariant filter are presented in Fig. 6. It is evident that the more flat the beginning of the ECG signal is, the greater transient reduction can be obtained.
Figure 2: (a) Clean ECG signal. (b) ECG distorted by the 60 Hz powerline interference and its 120 Hz harmonic

Figure 3: Amplitude response for the time-invariant prototype of the double-notch filter
Figure 4: Transient responses for the traditional time-invariant (dashed line) and proposed (solid line) double-notch filters
Figure 5: Comparison of ECG signal filtering using traditional and proposed double-notch filters
Figure 6: Comparison of ECG signal filtering using traditional time-invariant (dashed line) and proposed (solid line) double-notch filters for various beginnings of the ECG.
4.2. Triple-Notch Filter

In this subsection, let us consider the ECG signal distorted by the 60 Hz powerline interference and its 120 Hz and 180 Hz harmonics, which is presented in Fig. 7. In order to reject the powerline distortions the triple-notch filter with time-varying pole radius will be applied. In analogy to the previous subsection, the sampling frequency $f_s$ and the filter pole radius $r$ have been assumed to be 1 kHz and 0.98, respectively, whereas the parameters for function (5) are as follows: $d_r = 0.91$ and $v = 2.4$. The amplitude response for the time-invariant prototype of the considered triple-notch filter is presented in Fig. 8.

Fig. 9 presents the comparison of responses to the notch frequencies for the triple-notch filter with time-invariant and time-varying pole radius. This figure demonstrates respectively the response for the 60 Hz powerline interference frequency, for 120 Hz harmonic, for 180 Hz harmonic, and for the sum of these distortions. Similarly to the double-notch filter presented in the previous subsection it is clearly evident that the triple-notch filter with time-varying pole radius is able to suppress the notch frequencies considerably faster than the traditional filter with time-invariant pole radius.

Fig. 10 presents the effectiveness of the proposed triple-notch filter in the removal of the powerline distortions from the ECG signal. It can be seen that the use of the proposed filter causes that the transient duration is considerably reduced and the selectivity of the filter is at an acceptable level. In the analogy to the double-notch filter if the ECG signal begins at a more flat segment the transient response of the triple-notch filter can be better reduced. The results of simulations of the proposed time-varying
Figure 7: ECG distorted by the 60 Hz powerline interference and its 120 Hz and 180 Hz harmonics

Figure 8: Amplitude response for the time-invariant prototype of the triple-notch filter

triple-notch filter and the traditional time-invariant filter are presented in Fig. 11.
Figure 9: Transient responses for traditional (dashed line) and proposed (solid line) triple-notch filters.
Figure 10: Comparison of ECG signal filtering using traditional and proposed triple-notch filters
Figure 11: Comparison of ECG signal filtering using traditional time-invariant (dashed line) and proposed (solid line) triple-notch filters for various beginnings of the ECG
5. Suppression technique proposed by Pei and Tseng

As it has been mentioned in the introduction, in the literature one can find other methods for transient suppression in IIR multiple-notch filters. One of them has been proposed by Pei and Tseng in [17]. The suppression technique proposed by Pei and Tseng can be divided into two steps. First, the vector projection is used to decompose the first $L$ samples of an input signal into sinusoidal interference component and the useful signal component (ECG signal in the case of [17]). Then the useful signal component (ECG signal) is used as initial values of the multiple-notch filter to perform the filtering operation. As a result, the transient response can be considerably reduced.

The algorithm for the technique proposed by Pei and Tseng is as follows. Given the measurement signal $x(n)$, the number of its samples $N$, and the traditional multiple-notch filter designed for specified notch frequencies $\Omega_{Nk}$, $k = 1, \ldots, M$ and pole radius $r$, it is necessary to [17]:

1. construct $L \times 1$ input data vector $\mathbf{X}$

$$\mathbf{X} = [x(0)\ x(1)\ \ldots\ x(L-1)]^T \quad (10)$$

2. construct $L \times 2M$ matrix $\mathbf{A}$

$$\mathbf{A} = \begin{bmatrix}
1 & \cos(\Omega_{N1}) & \cos(2\Omega_{N1}) & \ldots & \cos[(L-1)\Omega_{N1}]\\
0 & \sin(\Omega_{N1}) & \sin(2\Omega_{N1}) & \ldots & \sin[(L-1)\Omega_{N1}]\\
1 & \cos(\Omega_{N2}) & \cos(2\Omega_{N2}) & \ldots & \cos[(L-1)\Omega_{N2}]\\
0 & \sin(\Omega_{N2}) & \sin(2\Omega_{N2}) & \ldots & \sin[(L-1)\Omega_{N2}]\\
\vdots & \ldots & \ldots & \ldots & \ldots \\
1 & \cos(\Omega_{NM}) & \cos(2\Omega_{NM}) & \ldots & \cos[(L-1)\Omega_{NM}]\\
0 & \sin(\Omega_{NM}) & \sin(2\Omega_{NM}) & \ldots & \sin[(L-1)\Omega_{NM}]
\end{bmatrix}^T \quad (11)$$
3. calculate projection matrix \( P \)

\[
P = A(A^TA)^{-1}A^T
\]  
(12)

4. calculate first \( L \) output samples by

\[
[y(0) \ y(1) \ldots \ y(L-1)]^T = (I - P)X
\]  
(13)

where \( I \) is the identity matrix

5. calculate \( L + 1 \ldots N \) output samples by

\[
y(n) = b_0 x(n) + b_1 x(n - 1) + \ldots + b_{2M} x(n - 2M) - r b_1 y(n - 1) - \ldots - r^{2M} b_{2M} y(n - 2M).
\]  
(14)

Figs. 12 and 13 present the comparison of the filtering results using the proposed multiple-notch filters with time-varying pole radius and the method proposed by Pei and Tseng. The analysis has been carried out for the double and triple-notch filter for various beginnings of the ECG signal. It has been assumed that \( r = r = 0.98, \ N = 1000, \) and \( L = 17 \) (in order to cover the period of the 60 Hz distortion signal). It can be seen that the proposed time-varying filter is more effective in the time domain than the method proposed by Pei and Tseng. This improvement has been achieved at the cost of insignificant reduction of the filter selectivity in the initial phase of filtering.

6. Performance evaluation

According to [17], a good way to evaluate the performance of the proposed approach for the reduction of the transient duration is to calculate the mean
Figure 12: Comparison of ECG signal filtering using Pei and Tseng method (dashed line) and proposed double-notch filter (solid line) for various beginnings of the ECG
Figure 13: Comparison of ECG signal filtering using Pei and Tseng method (dashed line) and proposed triple-notch filter (solid line) for various beginnings of the ECG
square error ($MSE$) for the different test cases. The mean square error may be defined as [17]

$$MSE = \frac{1}{B} \sum_{n=n_0+1}^{n_0+B} |y(n) - s(n)|^2$$

(15)

where $B$ is the window size and $n_0$ is the starting time. Usually the smaller the mean square error $MSE$ is, the closer the multiple-notch filter output $y(n)$ is to the ECG signal $s(n)$. That is, a small $MSE$ value implies a short transient state. Tables I and II present the mean square error $MSE$ for the filtering using traditional time-invariant multiple-notch filter, Pei and Tseng method, and the multiple-notch filter with time-varying pole radius proposed in this paper. The analysis has been carried out for the double and triple-notch filter for various beginnings of the ECG signal. It is clearly evident that generally the proposed multiple-notch filter with time-varying pole radius is more effective in the sense of the reduction of the transient response than traditional time-invariant filter and the method proposed by Pei and Tseng.

The values of the mean square error listed in Tables I and II show that the multiple-notch filter with time-varying pole radius is sensitive to the beginning of the measured signal, similarly like in the method proposed by Pei and Tseng. The smallest $MSE$ can be observed when the ECG signal begins at the flat segment. In this case the method proposed by Pei and Tseng is a little more effective than the one described in this paper. If the ECG signal begins at a sharper segment the MSE is higher and the greater disproportion between the time-varying filter and the method proposed by Pei and Tseng may be observed.
Table 1: The Mean Square Error $MSE$ for various filtering methods and various beginnings of the ECG signal ($n_0 = 0$, $B = 1000$). Double-notch filter

<table>
<thead>
<tr>
<th>ECG beginning</th>
<th>Traditional Filter</th>
<th>Pei &amp; Tseng method ($L = 17$)</th>
<th>Time-Varying Filter</th>
</tr>
</thead>
<tbody>
<tr>
<td>flat segment</td>
<td>0.0029</td>
<td>0.0007</td>
<td>0.0011</td>
</tr>
<tr>
<td>T wave</td>
<td>0.0026</td>
<td>0.0015</td>
<td>0.0010</td>
</tr>
<tr>
<td>QRS complex</td>
<td>0.0036</td>
<td>0.0077</td>
<td>0.0020</td>
</tr>
</tbody>
</table>

Table 2: The Mean Square Error $MSE$ for various filtering methods and various beginnings of the ECG signal ($n_0 = 0$, $B = 1000$). Triple-notch filter

<table>
<thead>
<tr>
<th>ECG beginning</th>
<th>Traditional Filter</th>
<th>Pei &amp; Tseng method ($L = 17$)</th>
<th>Time-Varying Filter</th>
</tr>
</thead>
<tbody>
<tr>
<td>flat segment</td>
<td>0.0025</td>
<td>0.0016</td>
<td>0.0012</td>
</tr>
<tr>
<td>T wave</td>
<td>0.0023</td>
<td>0.0038</td>
<td>0.0013</td>
</tr>
<tr>
<td>QRS complex</td>
<td>0.0032</td>
<td>0.0112</td>
<td>0.0021</td>
</tr>
</tbody>
</table>
7. Conclusion

In this paper, the parameter-varying technique has been used in order to generate a new class of digital parameter-varying IIR multiple-notch filters with reduced transient response. The proposed multiple-notch filters with time-varying pole radius possess selective magnitude response and transient response of short duration. It was demonstrated that this new class of filters achieved a considerable reduction of the duration of the transient response compared to the traditional time-invariant filter which was used as a prototype. As an example, the proposed multiple-notch filter with time-varying pole radius was used to remove the 60 Hz interference and its harmonics from the ECG signal. The results of simulations confirmed that by using the proposed time-varying multiple-notch IIR filter, both the transient duration and the selectivity of the filter are at an acceptable level. The filter proposed in this paper has been compared with the method proposed by Pei and Tseng. The results of simulations and the quantitative evaluation have proved that the multiple-notch filter with time-varying pole radius is more effective in the transient suppression process. However, if a particular application excludes a temporary reduction in the filter selectivity the method proposed by Pei and Tseng will be a better choice at that time.

Although this work has demonstrated that the proposed concept of the filter with time-varying pole radius may be indeed used to improve the dynamic behavior of a multiple-notch filter, there are still some challenges that require further research work. Particularly, it would be very desirable to obtain fully analytical formulas (if there are any) for optimal parameters of the function $r(n)$.  

29
Acknowledgment

The author would like to thank the anonymous reviewers who contributed to improve the quality and clarity of this paper with their comments during the revision process.

This work has been supported by the Ministry of Science and Higher Education of the Republic of Poland under grant contract N N505 484740.

References


