Study on the Spatial Resolution of EEG – Effect of Electrode Density and Measurement Noise

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Abstract—The spatial resolution of electroencephalography (EEG) is studied by means of inverse cortical EEG solution. Special attention is paid to the effect of electrode density and the effect of measurement noise on the spatial resolution. A three-layer spherical head model is used as a volume conductor to obtain the source-field relationship of cortical potentials and scalp potential field. Effect of measurement noise is evaluated with truncated singular value decomposition (TSVD). Also simulations about different electrode systems’ ability to separate cortical sources are performed. The results show that as the measurement noise increases the advantage of dense electrode systems decreases. Our results suggest that in clinical measurement environment it is always beneficial to use at least 64 measurement electrodes. In low-noise realistical measurement environment the use of even 256 measurement electrodes is beneficial.

Keywords—EEG, inverse problem, cortical potentials, spatial resolution

I. INTRODUCTION

The purpose of the present study was to examine the spatial resolution of electroencephalography (EEG). The spatial resolution of traditional 10-20 –electrode system is insufficient for modern brain research. Nowadays measurement systems of up to 256 electrodes exist and there is general interest to evaluate the increase in spatial resolution obtained with them. The objective of the present study was to evaluate the relationship between the number of EEG electrodes, measurement noise and the accuracy of the inverse cortical potential distribution. In the first approach the spatial resolution is studied with a theoretical approach based on applying truncated singular value decomposition (TSVD). In the second approach the results obtained with TSVD are visualized to demonstrate the effects of electrode density and measurement noise on the separability of cortical sources.

The spatial resolution of EEG is affected by blurring caused by volume conductor effects. Especially the contribution of the low-conducting skull is significant. If the spatial resolution is poor, localization of complex electrical activation is difficult. Accurate source localization is important in research on evoked potentials and spontaneous brain activity. Also in planning tumor and epilepsy surgery, precise localization of the areas causing symptoms is of importance. To achieve improvement the number of measurement electrodes needs to be increased and also some spatial enhancement method should be applied to the signal. One of the most commonly applied spatial enhancement methods is to solve the cortical potential distribution, which gives an improved idea of the locations of electrical sources within the brain.

Some research has been conducted where the accuracy of the EEG inverse solution or the spatial resolution of EEG has been studied as a function of the distance between EEG electrodes [1]. For example in [2] and in [3], the benefits of increasing the electrode number in quite sparse electrode systems in the presence of noise are demonstrated. Recently Lantz et al. [4] studied how many electrodes are needed to localize epileptic sources. They studied electrode systems including up to 123 electrodes and found benefits of increasing the electrodes up to this number.

II. METHODOLOGY

A. Volume conductor model

To obtain the source-field relationship of cortical potentials and scalp EEG field, the three-layer spherical Rush and Driscoll head model was applied as a volume conductor. The model included the layers of scalp, skull and grey matter. The radii of the spheres were 92 mm, 85 mm and 80 mm respectively. The resistivity ratio between the tissues was different from the Rush and Driscoll model, being 1:15:1 [5].

Although analytical methods could be applied to spherical model, we chose to construct a finite difference model (FDM) of the head with a software developed at the Ragnar Granit Institute [6]. In [7] it has been proven that this FDM method works correctly compared to analytical model. The FDM is also advantageous because in our future studies the model can be easily constructed from segmented magnetic resonance images.

From the FDM constructed, the cortical surface was so defined that it formed a closed surface. Thus the volume inside this surface could be omitted based on Gauss’s law. To decrease the computational load of the forward transfer matrix, groups of adjacent nodes were combined to form source areas on the cortical surface. In this manner the...
TABLE I
Electrode systems studied. In the first column the interelectrode distance is given. In the third column the corresponding number of electrodes in the realistic electrode systems is given and in the fourth the number of electrodes placed on the entire surface of the spherical model is given.

<table>
<thead>
<tr>
<th>Electrode distance (mm)</th>
<th>Reference</th>
<th>Realistic electrode system</th>
<th>Electrode system on the spherical surface</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>[8]</td>
<td>21 electrodes</td>
<td>34 electrodes</td>
</tr>
<tr>
<td>33</td>
<td>[8]</td>
<td>64 electrodes</td>
<td>104 electrodes</td>
</tr>
<tr>
<td>23</td>
<td>[8]</td>
<td>128 electrodes</td>
<td>232 electrodes</td>
</tr>
<tr>
<td>16</td>
<td>[9]</td>
<td>256 electrodes</td>
<td>462 electrodes</td>
</tr>
<tr>
<td>11</td>
<td>[10]</td>
<td>512 electrodes</td>
<td>938 electrodes</td>
</tr>
</tbody>
</table>

The cortical surface was divided into equal-sized source areas. The number of source areas varied from 32 to 8450 between different simulations. We studied five different electrode systems listed in Table 1. The distances between electrodes were 60 mm, 33 mm, 23 mm and 16 mm and 11 mm. These are the distances in the commonly applied 10-20-system [8] and the systems of 64 [8], 128 [8], 256 [9] and 512 [10] electrodes, respectively. In our study we placed the electrodes evenly over the entire spherical scalp surface according to the electrode distance. The resulting numbers of electrodes in the model were 34, 104, 232, 462 and 938, corresponding to the previously mentioned realistic systems. Thus the number of electrodes on half sphere corresponds to realistic electrode systems. The electrodes were placed as equidistant as possible within the limits posed by the discrete model.

The forward problem of EEG can be described with the equation:

\[ \mathbf{b} = \mathbf{A}\mathbf{x} \]  

(1)

where \( \mathbf{b} \) is a vector containing information on the measured EEG field, \( \mathbf{x} \) is a vector containing information on the source. \( \mathbf{A} \) is the forward transfer matrix containing the information on the volume conductor.

The inverse problem, where the purpose is to solve the \( \mathbf{x} \), does not have a unique solution. Because the system is ill-posed, a regularization method is needed. In the method developed here, TSVD is applied to study the inversion of the matrix \( \mathbf{A} \). We studied the cortical potential distribution as a solution to the inverse problem. It can be considered as an equivalent source, because it is the electric field produced by all the electrical sources within the brain.

We obtained the forward transfer matrices with the FDM solver. The potential on each electrode location was found by computing the electric field generated by each source area one by one. The results were combined into a forward transfer matrix.

### B. Spatial Resolution

In addition to being a convenient method to handle the ill-posedness of the forward transfer matrix, TSVD is a practical means of studying the effect of measurement noise on the spatial resolution. We adopted the method to study the effect of measurement noise from Schneider et al. [11], who applied it to electrocardiography (ECG). In this method the nullspace of the matrix \( \mathbf{A} \) consists of the basis vectors which lead to the signal smaller than the measurement noise:

\[ \frac{\sigma_i}{\sigma_e} < \|\mathbf{e}\|/\|\mathbf{b}\| \]  

(2)

where \( \mathbf{e} \) is the measurement error in \( \mathbf{b} \), including noise. The relation in Equation 2 is known as relative noise level (NL). The nullspace starts with the index \( i \) of the first normalized singular value smaller than the relative noise level. The basis vectors of the source space belonging to the nullspace cannot be reconstructed. As the relative noise level increases the number of reconstructable basis vectors decreases. This results in a more smeared inverse solution of the source potential distribution on the cortical surface, i.e. the spatial resolution decreases.

We applied TSVD to all calculated forward transfer matrices and solved the number of reconstructable basis vectors for each transfer matrix. We estimated the noise levels from EEGs measured under anesthesia [12] and applied the values of 0.01, 0.02, 0.04 and 0.1 for the relative noise levels (NLs).

In addition to comparing the number of reconstructable basis vectors, we also compared the systems of 128, 256 and 512 electrodes by visualizing their ability to separate cortical sources in the presence of noise. Also in this case the relative noise levels 0.01, 0.02, 0.04 and 0.1 were studied. In the case where the cortical surface was divided into 8450 source areas, we placed two unit sources on the cortical surface. Thus the area of each source is 9.5 mm².

We calculated the forward solution and then the inverse solution by applying TSVD. The selection on the number of reconstructable basis vectors was made based on Equation 2. When studying the resulting inverse cortical potential distribution, the bigger is the difference between the voltage of the two peaks and the voltage of the valley between them, the more probable the two sources can be separated and the better is the spatial resolution. In the present study we don’t specify numerical values for spatial resolution of different electrode systems.

### III. Results

The numbers of reconstructable basis vectors at relative noise levels of 0.01, 0.02, 0.04 and 0.1 are sketched in Figure 1. From Figure 1(a) we can see, that if the NL=0.01, we can obtain better spatial resolution by applying 512 electrodes. As the NL increases the benefit of denser electrode system decreases. When NL = 0.1, it is enough to
Figure 1. Number of reconstructable basis vectors is plotted as a function of the number of source areas. The NL is 0.01 (a), 0.02 (b), 0.04 (c) and 0.1 (d).

Figure 2. Separability of two cortical sources. The NL = 0.01. Electrode numbers are 512 (a), 256 (b) and 128 (c). The distance between sources is 26.5 mm. The cortical potential values on the vertical axis are relative. The potentials are sketched on the radius of the spherical cortical surface model, where the sources are located. The original sources are located at points 60 and 67.

Figure 3. Separability of two cortical sources. The NL = 0.02. Electrode numbers are 512 (a), 256 (b) and 128 (c). The distance between sources is 30.5 mm. The cortical potential values on the vertical axis are relative. The potentials are sketched on the radius of the spherical cortical surface model, where the sources are located. The original sources are located at points 61 and 69.
apply 64 electrodes to obtain the best possible spatial resolution.

In Figure 2 the distance between sources is 26.5 mm and NL=0.01. There are differences in spatial resolution of different systems. The more there are electrodes, the bigger is the voltage difference between the peaks and the valley. In Figure 3 the distance between two sources is 30.5 mm and NL=0.02. Now the two sources can be separated with all 128, 256 and 512 electrode systems.

IV. DISCUSSION

The spatial resolution can be studied by calculating the number of reconstructable basis vectors as illustrated in Figure 1 or by studying the solutions of inverse problems on the cortical surface as illustrated in Figure 2 and Figure 3. By comparing Figure 1(a) to Figure 2 and Figure 1 (b) to Figure 3, the similarity is clearly seen. Based on Figure 1 (a) there is a clear difference between the 512, 256 and 128 – electrode systems to separate cortical sources, when the NL=0.01. The same effect is also seen in Figure 2, where the distance between the cortical sources is same and NL=0.01. The same effect can be seen when comparing Figure 1(b) to Figure 3. However, it could be thought that the performance of 128 electrodes should be slightly worse than denser systems. Because the discretization of the cortical surface model is rough the differences of these electrode systems are not as visible as we would have expected. In further studies the cortical surfaces should be more accurately discretized. However, already from this model, the differences in the spatial resolution can be seen. Also in order to approximate real values for spatial resolution, a more realistic model than a three-layer spherical model, should be applied.

We have applied realistical noise estimates in our study. Based on the results it is evident that the advantages of high-resolution EEG devices are highly dependent on the amount of noise in the measurement.

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REFERENCES